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An undergraduate lab experiment on matched filtering as used in gravitational wave detection

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The detection of gravitational waves using matched filtering algorithms is just one example of the critical role that computational data analysis plays in contemporary physics. We present an undergraduate laboratory experiment where students apply matched filtering methods to detect weak signals in their noisy measurement data. The experiment is based on a Michelson interferometer and requires little additional equipment. To make the experiment accessible to students without programming skills, we developed an open-source graphical user interface for the matched filter search, which is also presented in this article. Finally, we demonstrate the ability to detect weak signals obtained with the experimental setup described here.
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I. INTRODUCTION

Contemporary physics experiments are hard to imagine without the extensive use of computer-based data analysis. The Laser Interferometer Gravitational-Wave Observatory (LIGO) is no exception. Searching the continuous stream of observational data for a faint trace of transient gravitational wave signals was not done by physicists staring at it with the naked eye. The discovery of the first gravitational wave, ultimately rewarded with a Nobel Prize, was initially made by well-designed computer algorithms with important parts based on a matched filter.[1–3] The example of gravitational wave detection shows that data analysis needs to be covered in undergraduate physics courses to help students experience how physics is actually done. Here, we present exercises involving matched filtering in a LIGO analogy experiment that is based on the work of Ugolini *et al.*[4]

A matched filter correlates the measured data with a template of known shape to detect its presence in the data. Using correlation has two significant advantages: first, it enables the detection of templates in noisy data, even when the noise amplitude exceeds the amplitude of the template being sought. Second, the correlation quantifies the similarity between the data and the template, providing a simple form of automatic pattern recognition and enabling the calculation of probabilistic quantities such as a false alarm rate. The method was developed in the 1940s with the advent of radar technology. Today, its application has been extended not only to gravitational wave detection[1] but also to a wide variety of other fields in physics and beyond. Applications range from astrophysical observations and cosmology[5–7] to photonics,[8] and from seismology[9] to medical applications in neuroscience[10,11] and cancer detection,[12] to name just a few.

Despite the importance of the matched filter for the successful detection of gravitational waves and its role in data analysis in contemporary physics, we found no documented implementations for an undergraduate laboratory. There are several suggestions for how students can extract information from a gravitational wave in a classroom setting based on the analysis of properties of these waves, such as frequencies or amplitudes.[13–16] The starting point for these approaches is a clean measurement (no noise) that is known to be a gravitational wave, which is far from where gravitational wave astronomers have to start their journey. The problem of extracting the gravitational waveform from noisy data is addressed in a simplified way by Larson *et al.*,[17] where data with low noise amplitudes are compared to templates by eye, and by Farr *et al.*,[18] where sinusoids are detected using the Fourier transform. The Fourier transform is also mentioned by Ugolini *et al.*[4] in a set of laboratory activities designed to teach experimental techniques for gravitational wave detection, but without further details on data analysis. In addition to the Fourier transform for signals of constant frequency, Gardner *et al.*[19] demonstrate an implementation of the Viterbi algorithm for wandering frequency signals and a variety of filters for music and speech, including the Wiener filter, which is somewhat related to the matched filter. Outside the context of gravitational wave detection, Tang and Wang[20] proposed a student experiment to detect periodic signals in noisy data using autocorrelation detection, also related to the matched filter. In this article, we want to demonstrate an experimental approach to the matched filter that allows students to detect weak signals in their measurement data, mirroring the detection of gravitational waves by the LIGO detectors.

The experiment is based on the LIGO analogy lab of Ugolini *et al.*,[4] which is built around a Michelson interferometer. We use a piezo actuator to move one of the mirrors of the interferometer to generate a signal as if a gravitational wave were being measured. At low amplitudes, the signal may disappear in the noisy detector output but can be recovered using a matched filter analysis. This analysis can be done using the graphical user interface[21] that we have developed to simplify the experiment and thus extend the target audience to students without programming skills.

This manuscript is organized as follows: we begin with a brief description of the matched filter in Sec. II, followed by a description of typical gravitational waves that will serve as models for the signals we use in the experiment in Sec. III. We provide a detailed description of the setup and the software required for the experiment in Sec. IV. Finally, in Sec. V, we demonstrate typical results obtained with the setup and report on our experience with the experiment.

II. MATCHED FILTER

The matched filter is an important tool for detecting weak signals in a noisy dataset. The matched filter returns the correlation of the measured data with an expected template to which the filter is matched. If this particular template has been measured, but with excessive noise, the correlation between the measurement data and the template will still be high. On the other hand, if the measurement contains only noise, the correlation with the template should be significantly lower. The correlation indicates the likelihood that a particular template has been measured. It also provides a simple tool for automatically screening large amounts of data: only sections where the correlation exceeds a predefined threshold are candidate detections. Along with these advantages, the matched filter has an obvious disadvantage: to compute the correlation, you need templates of the expected signals. Thus, the matched filter cannot be used to answer general questions like "Is there something interesting in my data?," but only specific questions like "Is there a signal of this particular shape in the data?" This specificity might seem like a deal-breaker for applications in fields like gravitational wave astronomy, where it is impossible to know in advance what the next source will be and, therefore, what the next signal will look like. However, there is a workaround: create large databases (called "template banks") that contain all the possible forms of gravitational waves and search for each of them at all times.

Two aspects must be considered to keep the computational effort within reasonable limits. First, a sensible choice of an optimal template bank must be made[22,23] to sufficiently represent each of the infinitely many gravitational waves that are possible in a finite template bank. For example, the first gravitational wave detection was made with a template bank of about 250 000 templates.[1] Second, even after specifying a finite template bank, there are many correlations to compute. Consequently, the Fourier transform is used to further reduce computation requirements because the correlation between the data and the template is equivalent to the convolution of the data with the complex conjugate of the time-reversed template. The convolution theorem allows us to compute the pointwise product in the Fourier domain instead of the convolution integral in the time domain. As the length N of the arrays of interest grows, switching to the Fourier domain and back speeds up the computation. This speed-up is due to the fast Fourier transform algorithms, which scale as $\mathcal{O}(N \log N)$. In contrast, the straightforward evaluation of the correlation integral would scale as $\mathcal{O}(N^2)$. Thus, when implemented on a computer, the matched filter is a multiplication in the frequency domain and the filter is represented by the Fourier transform of the time-reversed template. (Complex conjugation is not necessary for real-valued templates.)

It does not seem appropriate for this article to go into the details of the implementation of the matched filter, but a basic mathematical description may be helpful. Nowadays, matched filter analysis is typically performed on a computer, where the measurement data and templates are processed as discrete time series, and our experiment is no exception. Thus, we will call the measurement data $s[n]$ and the template $h[n]$, where n denotes the sampling index. The data analysis is based on the normalized cross-correlation, which we call the "match" $m[n]$:

$$m[n] = \frac{\sum_k s[n+k]\ \ h[k]}{\sqrt{\sum_k s[n+k]^2 \cdot \sum_k h[k]^2}}. \tag{1}$$

In Eq. (1), we use k for sampling indices that are summed over. Starting at any point n in the data $s[n]$, the normalized quantity $m[n]$ is a measure of how similar subsequent samples are to the shape of the template, regardless of amplitude. Looking at this equation in the abstract mathematical terms of a vector space of possible datasets and templates, with the dot product as the inner product, $\langle s[k], h[k] \rangle = \sqrt{\sum_k s[k] \cdot h[k]}$, one could identify the match m as the cosine of the angle between the measured data $s[n+k]$ and the template $h[k]$. It makes intuitive sense to use the angle between the two "vectors" (data and template) as a measure of the similarity of their shape, although a thorough mathematical foundation is needed to justify its use for signal detection. This is beyond the scope of this article, but has been given in a very readable way by Turin.[24] A derivation of why correlation is in a sense ideal for detection[25] can be found therein, as well as a discussion of probabilistic interpretations of the results, such as probability of detection and false alarm. In addition to Eq. (1) and the measured data, we need to know what the templates should look like before we can make a detection with a matched filter. This will be covered in Sec. III.

III. GRAVITATIONAL WAVES

General relativity predicts that accelerated masses emit gravitational waves, similar to the electromagnetic waves emitted by accelerated charges. These gravitational waves travel at the speed of light and manifest themselves as a stretching and contracting of space. The relative deformation $h = \Delta L/L$ ("strain") is used to quantify this effect, where the length L is stretched or squeezed by the amount ΔL. This strain is extremely small, practically insignificant for any process on Earth or even in the solar system. Even the signals produced when two black holes collide, an event in which enormous masses are violently accelerated, cause a strain on the order of only 10^{-21} and smaller here on Earth, far too weak to be measured by ordinary high-precision instruments. Extremely specialized detectors like LIGO, built and refined over decades, are needed to measure these signals. Matched filtering is one of the many techniques used to improve their detection sensitivity. As discussed in Sec. II, a template bank of possible signals is required in order to apply the matched filter. The template bank is made based on approximate solutions to Einstein's equations, and the software we have written will generate the templates, as will be described in Sec. IV. However, in this section, we give a general description of what these templates look like. For a quantitative description suitable for undergraduates, we refer to Rubbo *et al.*[13] and to the LIGO Scientific Collaboration and the Virgo Collaboration.[14]

In the experiment presented here, we focus on gravitational waves from compact binary coalescence events because these are the ones that have been measured by the LIGO detectors so far. The term compact binary refers to systems of two so-called compact objects, either two black holes, two neutron stars, or one of each kind. When two of these objects approach each other, they are bound by gravity and they begin to orbit around their common center of mass, just as planets and stars do. As they orbit, they will emit gravitational waves. Figure 1 shows what these gravitational waves typically look like near coalescence. These gravitational waves can be divided into three phases: inspiral, merger, and ringdown. Although the two bodies are gravitationally bound and orbiting each other, they are in the first phase of inspiral for a very long time. Initially, they are far away from each other on large orbits with long orbital periods and relatively small accelerations. Therefore, the gravitational wave is of low frequency and amplitude. The energy dissipated is also relatively small, so the two objects initially approach each other very slowly. However, over a long period of time, they will come closer and, as they do, the orbital frequency and acceleration will increase, leading to a steady increase in the amplitude of the gravitational wave. This is a self-reinforcing cycle, as more and more energy is radiated away, bringing the bodies closer together and increasing the amplitude of the gravitational wave. Only in the last few seconds, after millions or even billions of years of orbit, are the amplitudes and frequencies high enough to be detected by the LIGO detectors. At this point, the bodies are orbiting each other up to more than 100 times per second while being only a few hundred kilometers apart. Now they are finally close enough to merge, and the amplitude of the gravitational wave peaks before it quickly rings down and fades out. The inspiral phase is by far the longest one, with amplitude and frequency increasing steadily. If you were to play these waveforms through your speakers, you would hear a sound that might remind you of a chirping bird, which is why they are called "chirps." It is a pleasant coincidence that the frequencies of these chirps are within the range of human hearing, and we encourage our students to listen to some of them[26] while preparing for the experiment. In this section, we have explained the origins of the waveforms of the gravitational waves. We will now describe the experimental setup and the software we created to detect them.

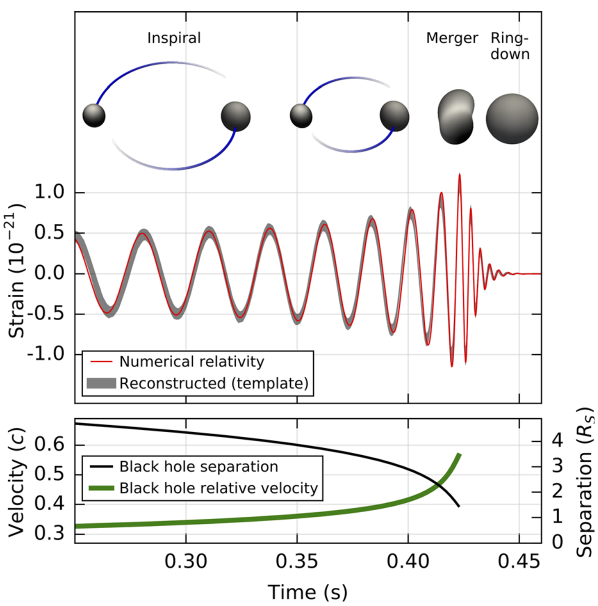

Fig. 1. The first gravitational wave ever detected was emitted by a system of two merging black holes, as illustrated in this figure. The upper panel shows the estimated horizons of the black holes through the three phases of the process (inspiral, merger, ringdown), along with the estimated waveform of the gravitational wave. The lower plot shows the effective separation in units of Schwarzschild radii and the effective relative velocity v/c. This figure is reprinted from Abbott *et al.*, Phys. Rev. Lett. **116**(6), 061102 (2016). Copyright 2016 Authors, Creative Commons Attribution 3.0 License (Ref. 1).

IV. IMPLEMENTATION OF THE MATCHED FILTER IN AN UNDERGRADUATE LAB

The matched filter experiment is part of a LIGO analogy lab based on a Michelson interferometer. To detect real gravitational waves, we would need to measure relative length changes of about 10^{-21} and smaller, which is not feasible in an undergraduate lab experiment. Instead, we use our software to create waveforms similar to gravitational wave measurements and inject them into the interferometer through a mirror mounted on a piezo actuator. In this way, we can choose the signal amplitudes to be around and below our noise level to get into the regime where the advantages of the matched filter become apparent. Throughout the experiment, the output intensity of the interferometer is measured by a photodiode and recorded on a computer. After the measurement, our software is used again to perform the matched filter analysis. In Subsections IV A and IV B, we describe the experimental setup and then the software used to generate the waveforms and perform the matched filter analysis.

A. Experimental setup

A standard Michelson interferometer is the basis for the matched filter experiment as shown in Fig. 2. In a Michelson interferometer, a laser beam is split by a beamsplitter into two beams that propagate in orthogonal directions. Mirrors are placed in each arm of the interferometer to reflect these beams back to the beamsplitter. They interfere and leave the

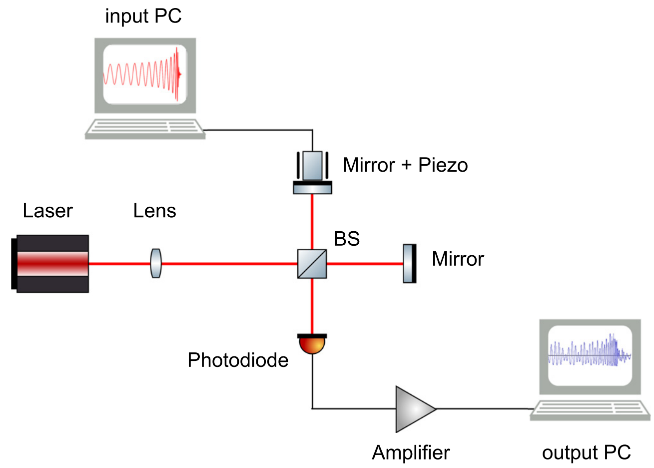

Fig. 2. Schematic overview of the experimental setup. The setup is a Michelson interferometer into which signals can be fed by moving one of its mirrors with a piezo actuator. The output is measured by a photodiode and recorded on a PC where it is later analyzed using matched filtering methods.

beamsplitter at its two possible outputs. A lens (focal length 5 mm) is placed between the laser and the beamsplitter to expand the beam and thereby broaden the interference pattern. The interference patterns at the outputs change as the relative length of the two arms changes. While the other arm remains unchanged, the length of one arm can be changed by moving the mirror in that arm with a piezo actuator. The mirror is attached directly to the piezo with superglue as shown in Fig. 3(b).[27] Thus, it moves when a voltage is applied to the piezo. The source of this voltage is the audio output of a computer that plays audio files shaped like gravitational waves. The computer audio output can be amplified to provide a wider dynamic range for the signals, but this is not necessary since the goal of the experiment is to measure weak signals anyway. The intensity of the interferometer output is measured by a photodiode, amplified by an ordinary audio amplifier, and recorded by the sound card of another computer. Special laboratory equipment is not needed to amplify and record these signals because their frequencies are within the range of human hearing and can be handled well by standard audio equipment. The recorded audio signals are then analyzed using matched filtering methods. We have written software for this purpose, as described in Subsec. IV B.

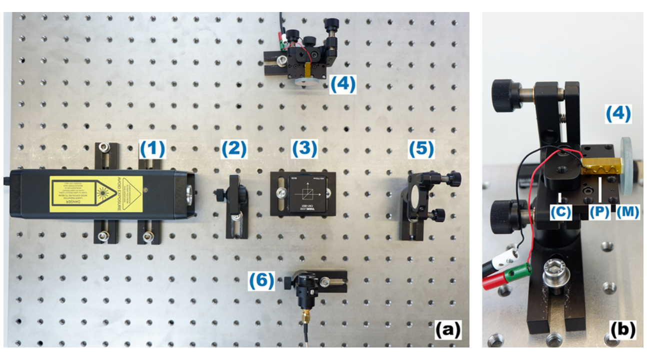

Fig. 3. Photographs of our experimental setup. On the left (a) is the interferometer, consisting of a laser (1), a lens (2, $f = 5$ mm), a beamsplitter (3), a mirror glued on a piezo (4), another mirror (5), and a photodiode (6). The detail on the right (b) is the mirror (M) glued to the piezo (P). The second end plate of the piezo is glued to a small metal block as a counterpart (C), which is screwed to the mount.

The Michelson interferometer used is based on the same components as the one by Ugolini *et al.*,[4] but additional parts can be found in Table I. The interferometer consists mostly of professional physics laboratory equipment, but we have also successfully performed the experiment with an interferometer from a high school supplier and a ~$1 photodiode from an electronics retailer. There are no special requirements for any part of the interferometer as long as it can produce a stable interference pattern and the photodiode can clearly distinguish between light and dark fringes. The only critical part is the movable mirror because the recorded intensity variations in this experiment must closely match the gravitational wave templates in the matched filter, otherwise the detection will fail. Gluing a mirror directly to a piezo proved to be an adequate choice to limit signal deformations. Compared to placing the mirror mount on a piezo-controlled translation stage, this reduces the inertia of the moving parts, allowing the system to respond much more quickly and accurately to the applied voltage. Ugolini *et al.*[4] describe two techniques to improve the performance of the interferometer, namely, locking and folding the interferometer arms, but neither is necessary for the experiments presented here. It is not necessary to increase the sensitivity of the interferometer by folding its arms, since the goal of the experiment is to recover weak signals. However, locking the arms could be helpful by preventing interferometer drift and providing a linear error signal. Our measurements take only a few seconds, so interferometer drift is not a major concern, but a linear error signal is appealing. For our experiments, it is essential that the measured data match the input templates, so a linear dependence of the interferometer output on its input would be desirable. Without further measures, the relationship between the input and the output intensity of a Michelson interferometer is not linear, but cosine-squared-shaped. Again, it is due to the weakness of the signals that this nonlinearity does not play a dominant role and does not prevent detection. In practice, interferometer locking and a linear signal are not required, as long as the signal amplitude is small enough to keep the output well within a fringe of the interference pattern. Overall, the inclusion of these techniques is not necessary for the success of the experiments described here, but at the same time, it would not be in conflict. In fact, since these are standard techniques for improving interferometric measurements, and they are used for the LIGO detectors, they could add educational value to the experiment, given a suitable audience. However, as shown in Sec. V, in our laboratory course, we perform these experiments without locking and folding, leading to satisfactory

Table I. Special components used in the described experiment and approximate prices (as of April 2024). In addition, a Michelson interferometer with a photodiode is required. Similar parts from other manufacturers can replace all components without loss of functionality.

Piece	Manufacturer/ vendor	Part specification	Approximate price (in US $)
Piezo	Thorlabs	PK2FQP2	100
Piezo counterpart	Thorlabs	KCP05	20
Piezo mount	Thorlabs	KM100PM	88
Audio amplifier	Thomann	the t.amp E-400	125

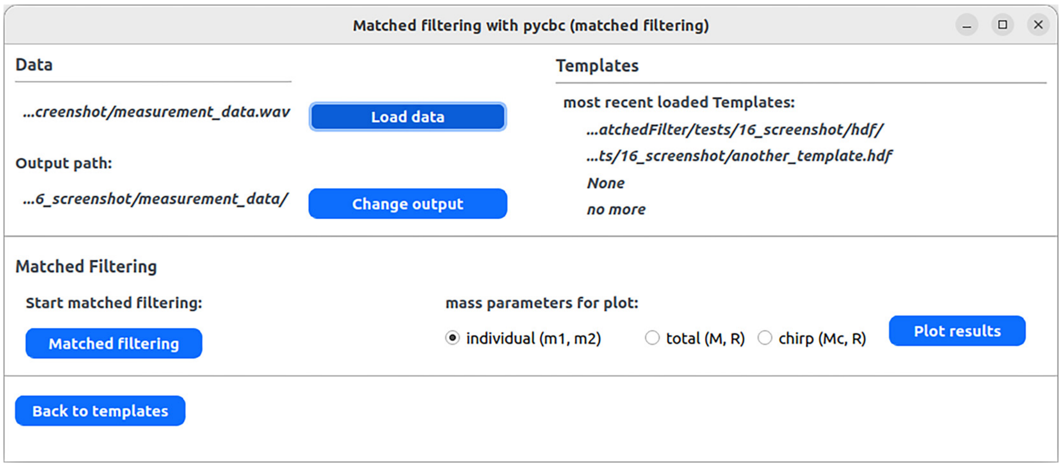

Fig. 4. A screenshot of the matched filtering screen of the MatchedFilter software. In the upper left segment, the user can select a data file to analyze and an output path to save the results. On the top right, previously loaded templates are listed. The matched filtering is started in the middle segment on the left, and on the right, 3D plots of the results over the parameter space can be generated. The button on the bottom left allows the user to go back and load more templates into the template bank.

results. Before looking at these results, we introduce the software used to analyze them.

B. Software

To make matched filtering feasible for students without programming skills, we created an open-source Python program called MatchedFilter, which is available as a free download on github.[21] Its current version is also included as the supplementary material[28] to this article. The software performs three critical tasks in our experiment: creating the signals to be injected into the interferometer, creating the templates needed for the matched filtering, and performing the matched filtering itself. It provides a graphical user interface (GUI) for some of the functions of the PyCBC library.[29–33] The PyCBC library is a free Python package used by the LIGO-Virgo collaboration to search for gravitational waves. Figure 4 shows what the GUI looks like while the user enters the information for the matched filtering.

The PyCBC library is only available for Linux and macOS. Therefore, our software relies on the Docker engine to create a cross-platform application that can also run on Windows. The Docker engine creates and manages containers, which are basically lightweight virtual machines. In our case, they run Linux to access the PyCBC library. The applications inside these containers communicate with the main application on the host by reading and writing files to the host's local drive. A simplified flow chart of the complete matched filtering process with the MatchedFilter software, including template creation, is shown in Fig. 5. The Docker engine must always be running when trying to use the MatchedFilter software on Windows. We currently recommend using the Docker engine on macOS as well, as there is an unresolved error that may cause it to break otherwise. The Docker engine is not required on Linux.

The MatchedFilter software can create two types of templates, but in both cases, it calls the waveform.get_td_waveform function of the PyCBC library. The first type of templates are time domain templates, which are used as input signals for the interferometer. They are saved as `wav` files. The second type of templates are frequency domain templates, which are used as templates in the matched filtering process. These are saved as `hdf` files. The shape of the templates is defined by the masses of the compact objects emitting the gravitational wave, which the user selects when creating the templates. Although the shape of real gravitational waves from compact binaries depends on more than just the masses of the objects involved (e.g., their spins, orbital eccentricity, and the inclination of their common plane with respect to the observer), our GUI only allows for the masses to be changed. This avoids overtaxing students who are not experts in this fie ld and allows the results to be visualized in a 3D plot, since the parameter space has only two parameters. Masses are measured in units of solar masses in the MatchedFilter software, and they can be specified by one of three different parameter pairs: The simplest way is to specify the masses of the two compact objects m_1 and m_2 individually. Alternatively, the total mass $M = m_1 + m_2$ or the chirp mass $M_c = \sqrt[5]{(m_1 m_2)^3/(m_1 + m_2)}$

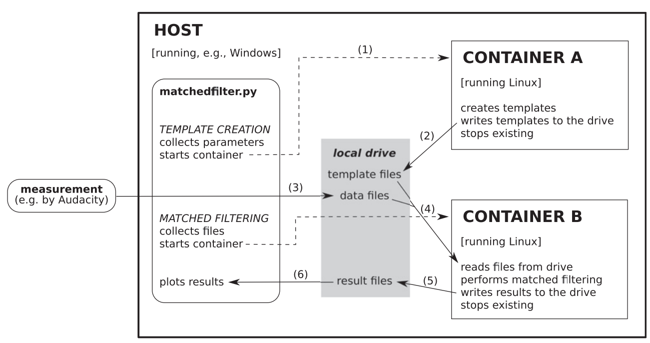

Fig. 5. A flowchart illustrating the basic structure of the MatchedFilter software. The main application is matchedfilter.py, which can be run on any platform. (1) Calling its template creation function will make a Docker container A (2) to create the templates for the template bank and write them to the local drive. (3) External software is used to write measurement data to the drive during the experiment. (4) The matched filtering function creates another container B in which the matched filtering is performed. (5) The container writes the results to the drive, where they can be read with any text editor. (6) They can also be plotted using the MatchedFilter software. The Docker containers are necessary to ensure operability on Windows systems, since crucial tasks are done by functions of the PyCBC library, which is only available for Linux and MacOS.

together with the mass ratio $r = m_2/m_1$ can be used. The chirp mass is a mass parameter commonly used in gravitational wave detection because it is closely related to the actual waveform of the gravitational wave. Note that the MatchedFilter software will only create a template if the total mass M is between 0.5 and 100 solar masses, and it will always rearrange the masses so that $m_1 > m_2$ and $r = m_2/m_1 < 1$.

To perform a matched filter search with the MatchedFilter software, the user must first load template files stored on the computer into the software's internal template bank. If there are no suitable template files on the computer, they can first be created with the software as described in the previous paragraph. The template bank is reset each time the MatchedFilter GUI is closed, so templates must be loaded each time the software is restarted. The next step is to load a `wav` file containing the measurement data to be analyzed. Then, the "Matched filtering" button starts the matched filtering process. The MatchedFilter software calls the filter.matched_filter_core function of PyCBC's Filter subpackage for each template in the template bank to determine the maximum match to the data. The MatchedFilter creates a table containing the maximum match and its time of occurrence for each template. It also creates a plot of the template and the data around the merger time for at least the five best matching templates. Additionally, the user can create a 3D plot of the match against the two-dimensional parameter space for a quick overview of the results. The 3D plot cannot be permanently saved, but can be re-created at any time from the tabular results file using the MatchedFilter software without having to repeat the matched filtering process.

V. EXPERIMENTS AND RESULTS

Using the setup and software shown, the students perform three similar experiments, each with a slightly different focus. As a first experiment, the students play a template of their choice, measure it with the interferometer, and analyze it with the MatchedFilter software. It should come as no surprise if the matched filter output peaks for the template chosen by the students. We think it is worth starting here anyway in order to familiarize the students with the workflow and to instill confidence in the ability of the matched filter to detect and discriminate templates that would be difficult to see with the naked eye. An example measurement is shown in Fig. 6, and the corresponding results are presented in Fig. 7.

In the second experiment, an unlabeled template selected by the instructor is measured and analyzed by the students. Thus, the students do not know in advance what mass parameters were used to create the waveform. They can be asked to guess these parameters by visually comparing their measurement to some templates, but they will immediately realize that they can only reject some very different ones. As can be seen in Fig. 6, there is no way to tell by eye which of the many similar-looking templates was measured, resulting in a wide range of candidate parameters. The matched filter search, on the other hand, will always reveal the exact template that was measured by assigning it the highest match, and a plot of the results against the template parameters will show a narrow region of good matches around that template, as shown in Fig. 7. This plot would reveal the most likely mass parameters of the emitting system, even if the exact template was not in the template bank. This exercise simulates the search for real gravitational waves, where it can never be known in advance which signal will be measured next. It demonstrates the power of the matched filter method to detect signals anyway and to estimate key parameters of the emitting systems.

For this second experiment, we provide students with a template bank containing the exact template provided by the instructor and about 50 other templates coarsely distributed in parameter space. We use this approach because our students are not familiar with the chirp mass and its strong influence on the shape of the gravitational wave. Templates with the same chirp mass will typically match a given set of measured data similarly, even if their mass parameters are otherwise quite different. An example is shown in Table II. Students might be confused if they were to analyze their data with a template bank that contained not only the measured template but also several templates with similar chirp

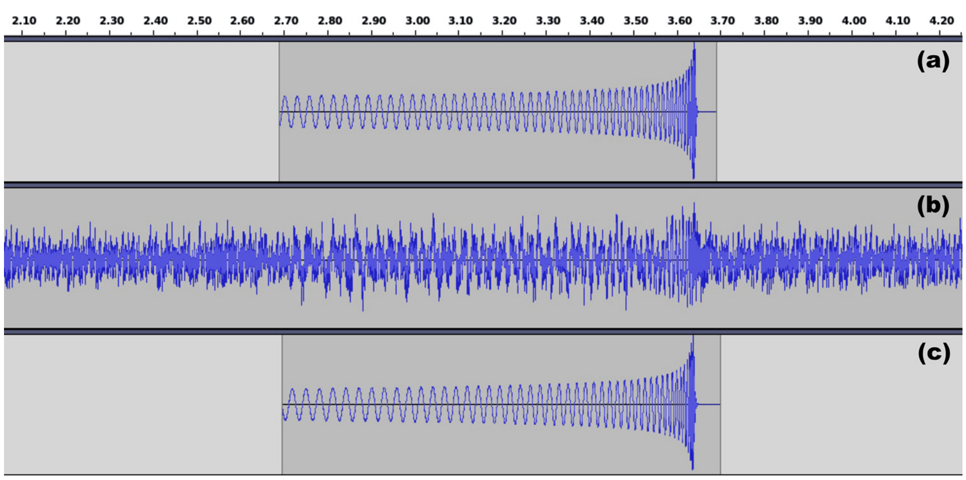

Fig. 6. Comparison of the signal injected into the interferometer (a) (mass parameters $m_1 = m_2 = 12.5$ solar masses), the interferometer output measured at the same time (b), and another signal that was not involved (c) ($m_1 = m_2 = 15.0$ solar masses). Looking at the measured data by eye, one could guess that some kind of signal was measured, but it would be impossible to say, which. A matched filter analysis favored the injected signal (a) above 952 other templates with a match of 0.55. The similar-looking template (c) had a match of only 0.29. The plotted results of this analysis are shown in Fig. 7. This figure was generated using the software Audacity (Ref. 34). The amplitudes of the datasets are arbitrary and unrelated. The time in seconds since the start of the measurement is shown at the top.

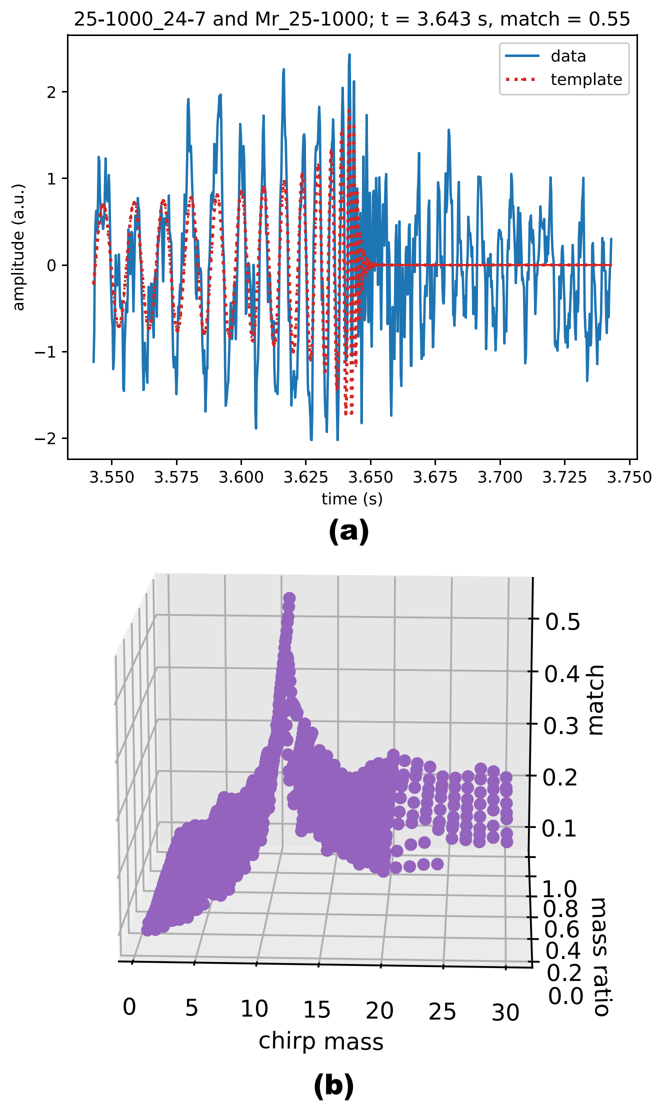

Fig. 7. (Color online) Two different plots for the results obtained by the MatchedFilter software for the measurement shown in Fig. 6. The mass parameters for the measured signal are $m_1 = m_2 = 12.5$ solar masses, corresponding to a chirp mass of 10.9 solar masses and a mass ratio of 1. In a matched filter search with a template bank of 953 templates, the template with these mass parameters best matches the measurement. (a) Measured data (blue, solid line) and the best matching waveform (red, dotted) scaled by its match. Both are plotted together at the time of the merger. (b) The match of the templates in the template bank plotted against the template parameters: the chirp mass (in units of solar mass) and the mass ratio of the merging objects. In the MatchedFilter software, the 3D plot can be studied from any angle.

masses. If they do not realize that these templates are indeed close neighbors in parameter space, it will appear that the matched filter has failed to favor only templates close to the measured template. An appropriate template bank helps to avoid this fallacy and to demonstrate the power of the matched filter method to an audience that is not too well versed in gravitational wave generation. On the other hand, if you are teaching a more detailed series that focuses on gravitational waves rather than experimental techniques, you can find several resources on how to address the influence of the chirp mass on the waveform,[13–16] and it may then be instructive for your students to design and optimize the template bank themselves. The 3D plot of the match against the template chirp mass and mass ratio should provide an easy

Table II. Matched filter results (match) for the data shown in Fig. 6 for selected templates, showing the dependence of the match on the chirp mass. The template in the first row was actually measured. The templates in the second and third rows have the same chirp mass and also match the data reasonably well despite their different mass ratio. Although the templates in the last two rows may seem closer to those in the first row, the different chirp masses result in a much worse match. All masses in units of solar masses.

Mass 1	Mass 2	Chirp mass	Mass ratio	Match
12.5	12.5	10.9	1.0	0.55
15.0	10.5	10.9	0.7	0.49
18.0	9.0	11.0	0.5	0.45
15.0	12.5	11.9	0.83	0.29
11.0	11.0	8.7	1.0	0.28

way to evaluate and improve the quality of the selected set of templates.

However, the matched filter can do more than just tell you which template is most likely. It can also identify glitches that do not resemble any of the templates. For the third experiment, we ask students to record data while not playing a template, but disturbing the measurement, for example by gently touching the interferometer while collecting data. Figure 8 shows an example measurement and the 3D plot of the match against the template parameters. For such a measurement, this plot may not always be perfectly flat, since the glitch may contain some frequency components that are also present in some of the templates. However, the match should not significantly exceed, say, 0.2. For signals unrelated to any of the templates, the match is typically between 0.1 and 0.2. A recording of a gravitational wave template with parameters similar to those in the template bank, on the other hand, should at least achieve a match of about 0.4, and could even climb up to a match of more than 0.95 if it is a clear recording.

The experiment has been developed, tested, and refined over the past 3 years. Our students were generally very curious to learn how to detect the extraordinarily weak gravitational wave signals. The context of gravitational wave detection seems to be very appealing, and the analysis of their measurements seems to encourage engagement with the topic of data analysis, which students are not always enthusiastic about when treated only theoretically. The icing on the cake is the use of software developed and used in the search for real gravitational waves.

With this software, it typically takes only a few seconds to perform the matched filter search for short measurements (<10 s) and small template banks (about 50 templates). That's what we usually do. However, even analyzing a 1-min measurement with a template bank of more than 1200 templates took less than 2 min on a standard laptop.

VI. CONCLUSION

The matched filter plays a crucial role in the detection of gravitational waves and in many other fields of study. We have shown how to use a Michelson interferometer for an instructional laboratory experiment that introduces students to matched filtering. The experiment fits naturally into an advanced course on LIGO detections or interferometry but can also be done independently. Students will learn how to use the matched filter to recover signals from extensive noise, estimate parameters, automate detection, and

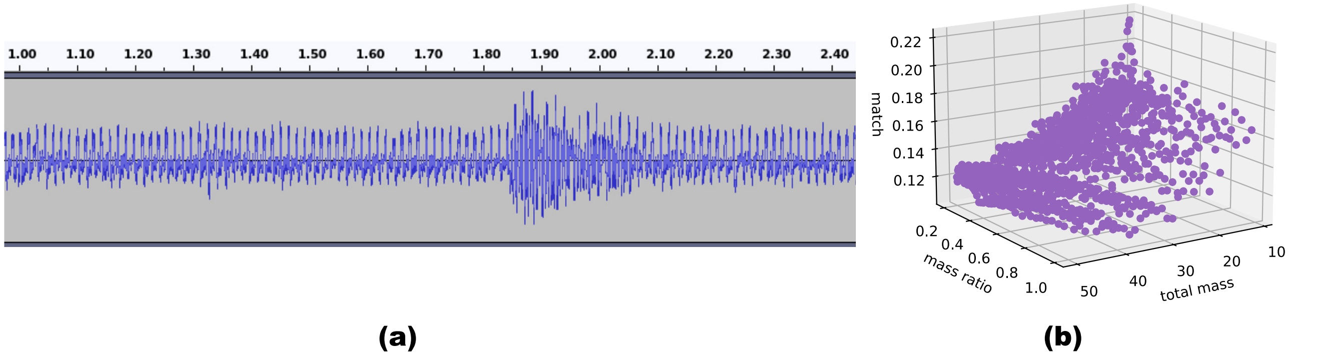

Fig. 8. (a) Data measured when no gravitational wave-like signal was injected into the interferometer, but the optical table was touched. (b) 3D plot of the match of these data with the templates against their mass parameters, using a template bank of 1231 templates. The match increases for smaller masses and more extreme mass ratios, but with a maximum below 0.22, it never comes close to the maximum match for real signals such as 0.55 in Fig. 7. Note the different scaling of the vertical axis ("match") in the two figures.

distinguish between glitches and physically meaningful events. To allow students without programming skills to participate in the experiment, we have developed free and open-source software based on professional gravitational wave detection software. This expands the possible participants to include first-year students, student teachers, and even students in high school labs or extracurricular learning spaces. This experiment uses equipment often found in undergraduate optics labs, making it affordable for various educational institutions. We believe that this experiment has the potential to demonstrate the importance of computational data analysis in contemporary physics experiments, and to facilitate engagement with this exciting topic through the fascinating context of gravitational wave astronomy.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

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[25]The matched filter is the linear filter that maximizes the signal-to-noise ratio in the presence of additive stationary white noise. Even if the noise is not white, but has a known power spectral density, a slight modification called "whitening" still makes it the ideal filter.

[26]See <https://youtu.be/QyDcTbR-kEA> LIGO Lab Caltech: MIT, “The sound of two black holes colliding,” youtube.com (2016).
[27]We had good results with a very small drop of the superglue-type industrial adhesive “Schwanheimer Industriekleber” (<https://schwanheimer-industrie-kleber.de/industrial-adhesives/industrial-adhesive-no.100/63/schwanheimer-industrial-adhesive-no.100/20g>) based on ethyl-2-cyanoacrylate (ECA). However, we would like to thank an anonymous reviewer and pass on the suggestion to try a very thin layer of vacuum-safe epoxy if superglue ever fails.
[28]See the supplementary material online for the Python code of the MatchedFilter software.
[29]See https://pycbc.org/ and <https://zenodo.org/doi/10.5281/zenodo.596388> for “Project Web Site.”
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