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Why the Statutory Retirement Age Is Too Low in a Democracy

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ABSTRACT

This paper examines the normative criteria for setting the statutory retirement age (SRA) under majority voting. It finds that when the working population forms the democratic majority, the SRA tends to be set inefficiently low. This inefficiency stems from a positive fiscal externality in pay-as-you-go pension systems: workers determining the SRA do not consider the benefits that a higher retirement age provides to current pensioners. Using a continuous-time overlapping generations model, the paper demonstrates that a Pareto improvement can be achieved by raising the SRA–provided it is accompanied by compensatory transfers from pensioners to workers at the time of the increase.

JEL Classification: D72, H55, J26

1 | Introduction

The statutory retirement age (SRA) is a critical component of public pay-as-you-go (PAYG) pension systems. It determines retirement timing for the vast majority of workers, even when early retirement options exist (Seibold 2021). Thus, it is essential that the SRA closely reflects the interests of the population. In democracies, the SRA is determined by majority rule. This paper examines the normative properties of democratically determined retirement ages. The paper argues that when the working population holds a democratic majority, the SRA is inefficiently low.

This finding is driven by a positive fiscal externality associated with raising the SRA. The externality arises because pensioners and workers are affected quite differently by an SRA increase in a PAYG pension system. Specifically, raising the SRA benefits current pensioners by increasing their pensions without requiring additional work, because pensioners are usually protected by a grandfather rule, which implies that they do not have to return to work if the SRA increases. For workers, in contrast, a higher SRA entails a trade-off: while it increases future pension benefits, it also requires extended labor force participation and reduces the duration of benefit receipt. When deciding on the SRA, workers optimally balance these advantages and disadvantages concerning their lifetime utility. However, they fail to account for the fact that those already retired at the time of the SRA increase also benefit from it. This is because in a PAYG system, the additional funds generated by a higher retirement age are directly translated into higher pension payments.

The positive fiscal externality associated with a higher retirement age shows similarities with the gain retired individuals enjoy when a PAYG pension scheme is introduced or expanded. In such cases, those already retired or near retirement receive a pension without having contributed much, if anything, to the scheme. Older individuals, therefore, support such policies and favor intensive intergenerational redistribution. Consequently, democratically determined PAYG pension systems tend to over-redistribute between young and old individuals. This was first pointed out by Browning (1975) and later elaborated upon by, for example, Hu (1982), Sjoblom (1985), Boadway and Wildasin (1989), and Cooley and Soares (1999).

What is unique about the SRA is that the political conflict line is clearly between workers and pensioners. This can be illustrated by comparing a contribution rate increase with an SRA increase for a 20-year-old and a 40-year-old worker. The 20-year-old is more affected by a contribution rate increase than the 40-year-old because they have to pay the higher rate for 20 years longer. Therefore, the 20-year-old is more likely to vote against a contribution rate increase than the 40-year-old, whose interests are more closely aligned with those of pensioners. Raising the SRA by half a year, on the other hand, affects both workers similarly, as it extends their working lives by the same duration. Although both the advantages and disadvantages of an SRA increase materialize earlier for an older worker than for a younger worker, the trade-off is essentially the same for both. This line of conflict between workers and pensioners concerning an increase in the SRA may help to explain why workers and unions are so clearly opposed to such an increase.

The intergenerational conflict associated with an SRA increase is fundamentally different from the intergenerational conflict associated with an expansion of the PAYG pension system, which is otherwise emphasized in the literature. In this literature, the interests of workers continuously converge with the interests of pensioners as they age because past pension contributions represent sunk costs, so that old-age pension benefits gain importance relative to pension contributions. Higher old-age pensions are, therefore, welcomed not only by pensioners but also by older workers. This literature often stresses that the socially optimal retirement system is the one most preferred by the youngest individuals as they consider the benefits and costs of the system over their entire life. Central to the conflict between workers and pensioners over the SRA, in contrast, is that an individual's perspective on the SRA changes drastically when they retire. While workers weigh the benefits of a higher retirement age against the costs of a longer working life, pensioners only see advantages in a higher retirement age. The important aspect here is the grandfathering of retirement rights, which means that pensioners do not have to return to work when the SRA is raised. Rationally, pensioners should, therefore, have an interest in subsequent working generations working as long as possible. However, workers (young and old) are also failing to appreciate the full costs and benefits of increasing the SRA because they ignore the positive effect on current pensioners.

In principle, pensioners have an interest in both higher contribution rates leading to higher pension benefits and a higher SRA. However, while higher contribution rates, ceteris paribus, only lead to higher intergenerational redistribution, a higher SRA creates additional income. The present paper shows that this additional income can be used to ensure that not only pensioners but also workers benefit from a higher SRA.

The paper examines an SRA determined by majority vote. First, it identifies age-specific preferences regarding the SRA. To do this, it considers a model of overlapping generations with deterministic life expectancies and no intragenerational differences in wage income. The SRA is also the effective retirement age, so voluntary early or delayed retirement is not considered. The paper shows that all individuals have single-peaked preferences regarding the SRA and considers scenarios in which the median voter is a worker. It shows that, depending on the historical SRA, there is a voting equilibrium in which all workers prefer the same level of SRA. The paper demonstrates that this SRA is inefficiently low. A Pareto improvement is possible if a higher retirement age is linked to compensation for workers for their longer working lives. Thus, policies designed to raise the SRA should be coupled with compensation for workers.

To make the conflict between workers and pensioners regarding the level of the SRA as transparent as possible, the present analysis is based on a series of simplifying assumptions. The paper assumes a logarithmic utility function. As a result, different wage levels for different age cohorts do not lead to a change in the division of work and leisure time that is considered individually optimal. With a logarithmic utility function, the income and substitution effects of a change in the wage rate neutralize each other. This ensures that there are no cohort effects but only aging effects with regard to the level of the SRA that is individually perceived as optimal. This assumption is restrictive, but it allows us to analyze age-specific preferences for the SRA that depend solely on the age of the individuals and not on the general wage rate changing over time. Alternatively, it could be assumed that the wage rate remains constant over time.

Furthermore, the paper confines attention to intergenerational redistribution by assuming away intragenerational wage differences. The pension system, therefore, only redistributes between young and old individuals. In the pension literature, such pension systems are referred to as Bismarckian systems as opposed to Beveridge systems. In a Bismarckian system, workers acquire entitlements to a future old-age pension, the amount of which depends on the contributions previously made. Accordingly, workers with higher wages acquire larger entitlements than workers with lower wages. Ceteris paribus, the rate of return of a Bismarckian system for each worker is determined by the growth rate of the wage bill, that is, by the sum of the growth rates of labor productivity and the labor force. In a Beveridge system, on the other hand, all pensioners receive a pension of the same amount. Therefore, a Beveridge system implies intragenerational redistribution between workers with higher and lower wages. Accordingly, workers with lower wages earn a higher return on their contributions, and workers with higher wages earn a lower return than in a Bismarckian system. In principle, an increase in the SRA also leads to the positive fiscal externalities described above in a pension system that also has an intragenerational redistribution component. The logic that current pensioners benefit from an increase in the SRA, which is not taken into account by workers, remains. However, the interests of lower-wage workers may differ from those of higher-wage workers, so that the intergenerational fiscal externality discussed here is overshadowed by diverging intragenerational interests.

If the SRA preferences of workers differ, either because cohort effects are allowed or because there are intragenerational distribution issues, the clear Pareto improvement from a higher SRA established here cannot be derived. Pensioners remain the winners of a higher SRA, but losers (and, possibly, winners) among workers are much more difficult to identify and to compensate. However, the positive fiscal externality of an increase in the SRA remains. Also, in the presence of cohort effects or intragenerational distribution conflicts, workers do not internalize the positive effect of an increase in the SRA on current pensioners and, therefore, have an incomplete perspective on the advantages of a higher SRA. From a social perspective, the fiscal externality associated with an increase in the SRA therefore renders the SRA too low even in a more general context.

The positive fiscal externality highlighted in this paper has not yet been considered in the theoretical literature on the political economy of the SRA. Related papers address intergenerational distributional effects, population aging, and tax distortions on individual retirement decisions. Lacomba and Lagos (2007), as well as Lacomba and Lagos (2010), shed light on preferences regarding the SRA in a population of individuals with different wages. They show that a less intragenerationally redistributive pension system increases political support for a higher SRA. Like this paper, they consider a continuous-time model with overlapping generations in the tradition of Sheshinski (1978). Lacomba and Lagos (2006) and Casamatta and Gondim (2011) study the effects of population aging in such a model, assuming homogeneous wages within an age cohort. Casamatta and DePaoli (2012) consider the interplay of unemployment and pension policies. Like in the present paper, they find that the median voter chooses a retirement age lower than the optimal one. In their paper, this is due to the fact that the median voter prefers a high contribution rate, which makes earlier retirement more desirable. A somewhat different strand of literature looks at the political economy of retirement funding, examining how the tax rate used to fund the pension system affects individual retirement decisions. This literature includes Casamatta et al. (2005) and Leroux (2010).

The remainder of the paper is organized as follows: Section 2 establishes the model. Section 3 derives the political equilibrium and develops the central inefficiency result. Section 4 contains some concluding remarks.

2 | The Model

The population consists of overlapping generations. Time is continuous. At each time t, the oldest generation dies and a new generation is born. The population grows at the constant rate n, the interest rate on savings, r, is time-invariant, and labor productivity evolves at the constant rate g.

Individuals live for T periods, inelastically supply one unit of labor in the first R periods, and are retired in the remaining $T - R$ periods.

Consider an individual of age $A < T$ at time t, that is, an individual born at time $t - A$. The remaining lifetime utility of an individual of age A at time t is

$$U_{A,t} = \int_0^{T-A} u[c_{A,t}(\theta)]e^{-\rho\theta}d\theta - \int_0^{\max\{0,R-A\}} z(A+\theta)e^{-\rho\theta}d\theta, \tag{1}$$

where u denotes instantaneous utility from consumption and $\rho \geq 0$ is the individual discount rate. Instantaneous utility from consumption is assumed to be logarithmic, that is, $u(c) = \ln c$. $c_{A,t}(\theta)$ denotes consumption at time $t + \theta$ of an individual born at time $t - A$. The function $z = z(A)$ denotes the disutility the individual derives from working at the age of A. It is assumed that z is strictly positive and differentiable.

At time t the present value of remaining lifetime income and wealth of an individual born at time $t - A$ is given by

$$\begin{aligned} B_{A,t} = S_{A,t} + \int_0^{\max\{0,R-A\}} (1-\tau)w(t+\theta)e^{-r\theta}d\theta \\ + \int_{\max\{0,R-A\}}^{T-A} \pi(t+\theta)e^{-r\theta}d\theta, \end{aligned} \tag{2}$$

where $w(t+\theta)$ is the wage rate at time $t+\theta$, so that $w(t+\theta) = w(t)e^{g\theta}$. Further, τ is the contribution rate of the public pension scheme, which is assumed to be time-invariant, $\pi(t+\theta)$ is the public pension benefit at time $t+\theta$, and $S_{A,t}$ is the amount of savings or debt that an individual born at time $t - A$ has accumulated until time t. It is assumed that individuals are not endowed with any inherited wealth or debt at the beginning of their (economic) life, so that $S_{0,t} = 0$ for all t.

An individual of age A at time t chooses a flow of periodic consumption $c_{A,t}$ that maximizes remaining lifetime utility $U_{A,t}$, taking the constraint into account that the present value of remaining lifetime consumption does not exceed $B_{A,t}$. This leads to the following consumption function

$$c_{A,t}(\theta) = \frac{\rho B_{A,t}}{1 - e^{-\rho(T-A)}} e^{(r-\rho)\theta}, \text{ for } \theta \in [0, T-A]. \tag{3}$$

The public pension system is based on a PAYG scheme and balances at each point in time. The budget constraint of the system at time t reads

$$\int_0^R \tau w(t)e^{-n\theta}d\theta = \int_R^T \pi(t)e^{-n\theta}d\theta,$$

which is equivalent to

$$\pi(t) = \tau \frac{1 - e^{-nR}}{e^{-nR} - e^{-nT}} w(t). \tag{4}$$

For further reference, define the contribution rate that leads to an equalization of net labor income and public pension benefits at each time t. This contribution rate, denoted as $\bar{\tau}$ in what follows, is implicitly defined by $(1-\bar{\tau})w(t) = \pi(t)$. Substituting for $\pi(t)$ by means of (4), it follows that

$$\bar{\tau} = \frac{e^{-nR} - e^{-nT}}{1 - e^{-nT}}. \tag{5}$$

For $\tau < \bar{\tau}$, net labor income exceeds the pension benefit at each time t, implying a replacement rate of less than 100%, whereas for $\tau > \bar{\tau}$ the opposite holds true.

Combining Equations (1–4), indirect remaining lifetime utility at time t of an individual born at time $t - A$ can be written as

$$V_{A,t} = \int_0^{T-A} \left[\ln\left(\frac{\rho B_{A,t}}{1 - e^{-\rho(T-A)}} \right) + (r - \rho)\theta \right] e^{-\rho\theta} d\theta - \int_0^{\max\{0,R-A\}} z(A + \theta) e^{-\rho\theta} d\theta. \tag{6}$$

Herein, the present value of remaining lifetime income $B_{A,t}$ is determined by

$$B_{A,t} = S_{A,t} + w(t)\left[(1 - \tau)\frac{1 - e^{-(r-g)\max\{0,R-A\}}}{r - g} + \tau \frac{1 - e^{-nR}}{e^{-nR} - e^{-nT}} \frac{e^{-(r-g)\max\{0,R-A\}} - e^{-(r-g)(T-A)}}{r - g} \right]. \tag{7}$$

Savings that an individual aged A has accumulated until time t, $S_{A,t}$, are predetermined by decisions made before time t. In particular, they depend on the SRA in effect until time t. This SRA is referred to as the historical SRA and denoted by $\bar{R}$. For further reference, cumulative savings of a worker, i.e., an individual aged $A < \bar{R}$ at time t are specified in detail. These savings result from the difference between cumulated labor income and cumulated consumption as follows

$$S_{A,t} = \int_0^A (1 - \tau) w(t) e^{(r-g)\theta} d\theta - \int_0^A c_{0,t-A}(\theta) e^{r\theta} d\theta.$$

Considering Equations (3) and (7) and $S_{0,t-A} = 0$, $S_{A,t}$ can be written as

$$S_{A,t} = w(t) e^{(r-g)A} \left\{ (1 - \tau)\frac{1 - e^{-(r-g)A}}{r - g} - \frac{1 - e^{-\rho A}}{1 - e^{-\rho T}} \times \left[(1 - \tau)\frac{1 - e^{-(r-g)\bar{R}}}{r - g} + \tau \frac{1 - e^{-n\bar{R}}}{e^{-n\bar{R}} - e^{-nT}} \frac{e^{-(r-g)\bar{R}} - e^{-(r-g)T}}{r - g} \right] \right\}. \tag{8}$$

In what follows, the subscript t will be omitted for notational simplicity as long as confusion can be ruled out. Thus, V_A denotes the remaining indirect utility of an individual of age A at time t, B_A the remaining lifetime budget, and S_A the individual's wealth or debt.

3 | Majority Voting

This section characterizes the SRA as the outcome of a majority vote at time t. In doing so, the following two assumptions are made. First, it is assumed that all individuals vote on the SRA under the assumption that the SRA does not change again during their lifetime. Second, a grandfathering clause applies. This states that for individuals who are already retired at time t, i.e, individuals of age $A \geq \bar{R}$, the historical SRA, $\bar{R}$, continues to apply. This means that people who have already retired do not have to return to work to qualify for pension benefits if the SRA is raised.

To identify a voting equilibrium, in a first step, the age-specific individual preferences regarding the SRA are worked out. The following lemma describes how the indirect utility function V_A changes depending on R.

Lemma 1.

i. *V_A is strictly concave in R for all $A \in [0, \bar{R})$ if $\tau \leq \bar{\tau}$, $r \geq n + g$, $z' \geq 0$, and ρ is sufficiently small.*

ii. *V_A is strictly increasing in R for all $A \in [\bar{R}, T]$.*

Proof. See the Appendix. □

Lemma 1 distinguishes between workers and pensioners. Lifetime utility of pensioners clearly increases as a result of a raise in the SRA. This is because pension benefits increase when the retirement age is raised. Because current pensioners do not need to work longer for this, they benefit from a higher retirement age. The effect of a higher SRA on the lifetime utility of workers is more involved. If the net replacement rate of the public pension scheme does not exceed 100 percent ($\tau \leq \bar{\tau}$), if the interest does not fall short of the growth rate of the aggregate wage bill ($r \geq n + g$), if the disutility derived from working does not decrease with age ($z' \geq 0$), and if the time preference rate ρ is sufficiently small, lifetime utility of all individuals younger than $\bar{R}$ is strictly concave in R. All these conditions are sufficient but not necessary. However, most of these conditions are either intuitive or reasonable. The net replacement rate condition guarantees that workers' remaining lifetime resources increase with a longer working time. If the net replacement rate were larger than 100 percent, the possibility arises that workers could increase their remaining lifetime resources by retiring earlier rather than later. If the interest rate r was smaller than the growth rate of the aggregate wage bill, $n + g$, the returns to private savings would be smaller than the implicit returns from the public pension system, the macroeconomy would suffer from inefficiently high savings, and individuals would be better off by extending the public pension system. The conditions related to the disutility of work, z, and the time preference rate, ρ, are of a more technical nature. Specifically, $z' < 0$ would imply that the marginal disutility of work decreases as the SRA increases. In such a case, the trade-off between higher lifetime income and prolonged disutility from work would weaken as the SRA rises, potentially leading to a scenario where workers may no longer have a clear preference to retire. Similarly, the time preference rate ρ should not be excessively high, as a higher ρ reduces the present value of the disutility associated with an extended working life. Consequently, a high ρ could also result in a diminished marginal disutility from a higher SRA.[1] The following example demonstrates that the preferences of workers may lose the concavity property when ρ becomes large.

Example 1. Let $A = 0$, $T = 50$, $w(t) = 1$, $r = .03$, $n = .01$, $g = 0$, $\tau = .2$, and $z(A + \theta) = 3$. Figure 1 plots V_A as a function of R for $\rho = .01$ and for $\rho = .1$, respectively.

Lemma 1 implies that preferences of workers and pensioners regarding the SRA differ considerably. Pensioners invariably benefit from an increase in the SRA. This is because raising the SRA results in higher periodic pension benefits without requiring current pensioners to work longer. Workers, in contrast, face a trade-off. A higher retirement age extends the period of wage earning and increases the periodic pension benefits they will receive in the future. At the same time, however, it lengthens the time spell during which disutility accrues from work and shortens the duration of pension benefit receipt.

To specify the SRA most preferred by workers, consider an individual of age $A < \bar{R}$ and let R_A denote the most preferred SRA of this individual. Then, R_A maximizes this individual's indirect lifetime utility V_A as defined in Equation (6). To characterize R_A, differentiate V_A with respect to R. If the individual prefers neither immediate retirement nor lifelong work, R_A is determined by the following first-order condition

$$\frac{1}{B_A}\frac{\partial B_A}{\partial R}\frac{1}{\rho}(1 - e^{-\rho(T-A)}) - z(R_A)e^{-\rho(R_A-A)} = 0, \qquad (9)$$

where R_A is uniquely determined in light of Lemma 1. This condition provides, for a given historical SRA, $\bar{R}$, the most preferred SRA of an individual of age A, R_A. Technically, condition (9) implies a function f_A such that $R_A = f_A(\bar{R})$. The next lemma establishes a fixed point property of the function f_A, which proves useful for further analysis.

Lemma 2. *Let the conditions stated in Lemma 1 hold. Then, the function f_A contains a single and stable fixed point $R^* < T$, which is independent of A.*

Proof. See the Appendix. □

Lemma 2 states that all workers prefer an SRA equal to R^* if the historical SRA satisfies the condition $\bar{R} = R^*$. To provide intuition for this result, note that R^* represents the SRA most preferred by a worker aged $A = 0$, regardless of the historical retirement age $\bar{R}$ (see the remark at the end of the proof of Lemma 2 in the Appendix). If the historical SRA is indeed R^*, the worker's most preferred SRA remains R^* as they age. This is because individual preferences, given the assumption of exponential discounting, are time-consistent. Therefore, older workers also prefer an SRA of R^* when the historical SRA is already R^*.

However, while a worker aged $A = 0$ prefers an SRA of R^* regardless of the historical retirement age, older workers will only prefer an SRA of R^* if the historical retirement age is already set at R^*. This is because older workers have already aligned their savings with the historical retirement age. Lemma A2 in the Appendix shows that savings S_A negatively depend on the historical SRA. Therefore, older workers will have saved more than they would have under R^* if $\bar{R} < R^*$, and less if $\bar{R} > R^*$. If they have saved more than they would have under R^*, they prefer a lower SRA than R^*, as the additional savings enable them to retire earlier. Conversely, if they have saved less than they would have under R^*, they prefer a higher SRA than R^*. In summary, older workers prefer an SRA closer to their existing savings plan. Therefore, the older a worker is, the closer the worker's preferred SRA is to the historical SRA.[2] However, older workers also prefer lowering the SRA if the historical SRA is higher than R^*, and augmenting the SRA if the historical SRA is lower than R^*. Technically, this is reflected in the fact that R^* is a stable fixed point.

It should be noted that the result that all workers, regardless of age, prefer the same SRA for $\bar{R} = R^*$ does not depend solely on their time-consistent preferences. This result also depends on the assumption of a logarithmic utility function. The logarithmic utility function implies that the income and substitution effects of a higher wage rate on the demand for leisure offset each other, so that the wage level has no influence on the preference with regard to the SRA. In the present model,

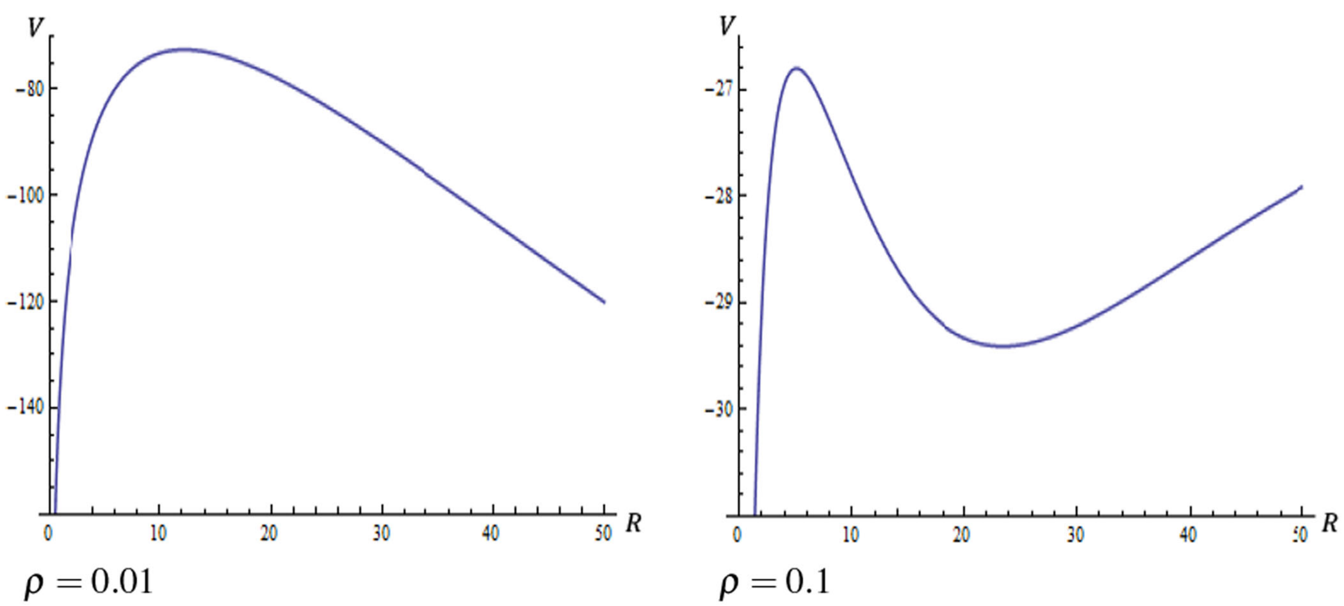

FIGURE 1 | SRA preferences of workers.

workers may differ in the wage level they receive over the course of their working lives. This is not directly due to their respective age, but rather to the point in time at which they enter working life. If the substitution effect of a higher wage rate is larger than the income effect, later-born cohorts prefer a higher SRA in the case of positive wage growth than earlier-born cohorts, and vice versa in the case of negative wage growth.[3] Accordingly, the above-defined function f would not have an age-independent fixed point. The crucial point here is that the logarithmic utility function excludes cohort effects on the leisure-work time decision because of changing wage rates. Alternatively, one could assume that the wage rate does not change over time.

Finally, it should be noted that the first-order condition (9) determines R_A in the case when the individual neither wants to retire immediately nor to work their whole life. If the disutility from working at the end of life, $z(T)$, is sufficiently large, all individuals want to retire during their lifetime. This condition is taken as satisfied here. Otherwise, the retirement age issue would be difficult to motivate. Furthermore, if the historical SRA, $\bar{R}$, is smaller than R^*, then all workers want the SRA to be raised, i.e., no worker wants to retire immediately. In this case, condition (9) determines the most-preferred R_A for all $A \in [0, \bar{R})$. However, if $\bar{R}$ is higher than R^*, then workers at an age close to $\bar{R}$ will want to retire immediately. For these workers, $R_A = A$ then holds. Though for these workers, the most preferred SRA is then no longer determined by (9), it still holds that all workers want a lowering of the SRA for $\bar{R} > R^*$.

Lemma 1 implies that all individuals have single-peaked preferences regarding the SRA if the conditions stated there are met. In a majority vote, therefore, the SRA preferred by the median voter prevails. In the present model, the median voter is an individual whose age corresponds to the median age of the population. Given the lifespan T and the population growth rate n, the median age is determined by $M = -\frac{1}{n}\ln[(1 + e^{-nT})/2]$. In what follows, it is assumed that $M < \bar{R}$, so that individuals of median age are workers. Given the historical retirement age $\bar{R}$, the SRA determined in a majority vote therefore assumes the value R_M. This is either determined by condition (9), or it is $R_M = M$ if a worker of age M wants to retire directly.

The level of R_M is contingent upon the historical SRA, $\bar{R}$. According to Lemma 2, R_M assumes the value R^* if $\bar{R} = R^*$. If, on the other hand, $\bar{R}$ differs from R^*, Lemma 2 implies that R_M is closer to R^* than $\bar{R}$ is to R^*. Repeated majority voting would therefore eventually lead to an SRA at the level of R^* prevailing. Therefore, in what follows the present analysis considers a scenario where $\bar{R} = R^*$. The result of a majority vote can then be summarized as follows.

Proposition 1. *Let the conditions stated in Lemma 1 hold and $\bar{R} = R^*$. Then, R^* is the outcome of a majority vote if $M < R^*$.*

In the next step, the welfare implications of the voting equilibrium described in Proposition 1 are explored. As the SRA R^* maximizes the utility of current workers in the sense that condition (9) is satisfied, a small increase in the retirement age at R^* would produce only a second-order negative effect on their utility. Current pensioners, on the other hand, would benefit from a first-order positive effect by increasing the retirement age, since their utility strictly increases in the SRA. Therefore, as the next proposition states, a Pareto improvement is possible if an increase in the SRA starting from R^* is combined with a compensation scheme for workers.

Proposition 2. *The SRA R^* is inefficiently low. A Pareto improvement can be achieved by raising the SRA if it is accompanied by compensatory transfers from pensioners to workers at the time of the increase.*

Proof. See the Appendix. □

The inefficiency highlighted in Proposition 2 stems from a positive fiscal externality inherent in a PAYG pension system. When current workers decide on the retirement age, they weigh the pros and cons for their own lifetime utility. However, they neglect the fact that current pensioners already gain from a higher retirement age through increased pension benefits without additional labor.[4] With an increase in the SRA, pensioners enjoy gains that are substantial enough to potentially compensate workers for their losses.

It should be noted that a straightforward Pareto improvement is not guaranteed when a more general utility function is considered, which results in workers of varying ages having heterogeneous preferences over SRA levels. In such cases, workers whose preferred SRA deviates from the voting equilibrium's SRA experience a first-order effect, either positive or negative, from an SRA increase. Nevertheless, the fiscal externality discussed here persists even in a more generalized model, as workers lack incentives to internalize the welfare effects of a higher SRA on current pensioners.

4 | Concluding Remarks

This paper has shown that the SRA determined by majority voting is inefficiently low when workers have a democratic majority. The background is a positive fiscal externality. When workers decide on the SRA, they do not take into account that current pensioners already benefit from it. If neither cohort effects on the work-leisure decision nor intragenerational distribution conflicts are present, a Pareto improvement is possible if the SRA is increased and, at the same time, current workers are compensated by a tax on pensioners.

The result derived in this paper has policy significance against the background of population aging in many countries. It suggests that an increase in the SRA will be easier to implement if current workers receive compensation at the time of the increase. The approval of a higher SRA by the already retired population is given anyway. Indeed, Bittschi and Wigger (2023) provide empirical evidence that once individuals reach retirement age, they are significantly more in favor of a higher SRA.

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Data Availability Statement

The author has nothing to report.

Endnotes

[1] As shown in Equations (A4) and (A5), the present value of the marginal disutility of work associated with an increase in R is determined by $-z(R)e^{-\rho(R-A)}$ and the derivative with respect to R by $[\rho z(R) - z'(R)]e^{-\rho(R-A)}$. Therefore, both $z' < 0$ and a high ρ would open up the possibility that the present value of the marginal disutility of work decreases as the SRA increases.

[2] See Lacomba and Lagos (2007) for a similar result.

[3] For example, if instantaneous utility from consumption u takes the form $u = (c^{1-\eta} - 1)/(1-\eta)$, where η denotes the inverse of the intertemporal elasticity of substitution (for $\eta = 1$, this function becomes the logarithmic function assumed here), then for $\eta < 1$ the demand for leisure decreases with a higher wage rate, while it increases for $\eta > 1$. For $\eta < 1$, younger cohorts therefore prefer a higher SRA with positive wage growth and older cohorts a lower SRA.

[4] It might be worth considering whether the result changes with intergenerational altruism. If young individuals are altruistic toward the elderly, they might also benefit from a higher SRA. However, they would then internalize this effect when determining the SRA. Further analysis is required to assess the extent to which such altruism mitigates the fiscal externality and weakens the case for a Pareto improving SRA increase.

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Appendix

The following two lemmata will be used in the subsequent proofs of the lemmata and propositions in the text.

Lemma A1.

i. *B_A is strictly increasing in R for all $A \in [0, \bar{R})$ if $\tau \leq \bar{\tau}$.*

ii. *B_A is strictly concave in R for all $A \in [0, \bar{R})$ if $\tau \leq \bar{\tau}$ and $r \geq n + g$.*

iii. *B_A is strictly increasing in R for all $A \in [\bar{R}, T]$.*

Lemma A2. *$S_{A,t}$ is strictly decreasing in $\bar{R}$ for all $A \in (0, \bar{R})$ if $\tau \leq \bar{\tau}$.*

Proof of Lemma A1. *Proof of* i. Differentiate (7) with respect to R for $A \in [0, \bar{R})$, taking into account that S_A is predetermined at time t. This leads to

$$\frac{\partial B_A}{\partial R} = w(t)e^{(r-g)A}\Omega(\tau, R), \tag{A1}$$

with

$$\begin{aligned}\Omega(\tau, R) = {} & (1-\tau)e^{-(r-g)R} \\ & + \tau e^{-nR}\frac{n}{e^{-nR} - e^{-nT}}\frac{e^{-(r-g)R} - e^{-(r-g)T}}{r-g} \\ & + \tau e^{-nR}\frac{n(1-e^{-nR})}{(e^{-nR} - e^{-nT})^2}\frac{e^{-(r-g)R} - e^{-(r-g)T}}{r-g} \\ & - \tau e^{-(r-g)R}\frac{1-e^{-nR}}{e^{-nR} - e^{-nT}}.\end{aligned} \tag{A2}$$

Observe that $x(1 - e^{-xR}) > 0$ for all x as well as $(1 - e^{-xR})/(e^{-xR} - e^{-xT}) > 0$ and $(e^{-xR} - e^{-xT})/x > 0$ for all x and $R < T$. For $\tau \leq \bar{\tau}$ it follows that

$$\begin{aligned}\Omega(\tau, R) \geq {} & (1-\bar{\tau})e^{-(r-g)R} \\ & + \tau e^{-nR}\frac{n}{e^{-nR} - e^{-nT}}\frac{e^{-(r-g)R} - e^{-(r-g)T}}{r-g} \\ & + \tau e^{-nR}\frac{n(1-e^{-nR})}{(e^{-nR} - e^{-nT})^2}\frac{e^{-(r-g)R} - e^{-(r-g)T}}{r-g} \\ & - \bar{\tau}\, e^{-(r-g)R}\frac{1-e^{-nR}}{e^{-nR} - e^{-nT}}.\end{aligned} \tag{A3}$$

Substituting $\bar{\tau}$ by Equation (5), it follows that

$$\begin{aligned}\Omega(\tau, R) \geq \tau e^{-nR}\Bigg[& \frac{n}{e^{-nR} - e^{-nT}}\frac{e^{-(r-g)R} - e^{-(r-g)T}}{r-g} \\ & + \frac{n(1-e^{-nR})}{(e^{-nR} - e^{-nT})^2}\frac{e^{-(r-g)R} - e^{-(r-g)T}}{r-g}\Bigg] > 0.\end{aligned}$$

This proves i. □

Proof of ii. Since B_A is C^2 in R for all $A \in [0, \bar{R})$, it will be shown that $\partial^2 B_A/\partial R^2 < 0$ for all $A \in [0, \bar{R})$ if $\tau \leq \bar{\tau}$ and $r \geq n + g$. Differentiation of Equation (A1) with respect to R yields

$$\frac{\partial^2 B_A}{\partial R^2} = w(t)e^{(r-g)A}[\alpha(\tau) + \beta + \gamma + \delta + \phi + \psi + \sigma + \zeta(\tau) + \mu],$$

where

$$\begin{aligned}
\alpha(\tau) &= -(r-g)(1-\tau)e^{-(r-g)R},\\
\beta &= -\tau n e^{-nR}\frac{n}{e^{-nR} - e^{-nT}}\frac{e^{-(r-g)R} - e^{-(r-g)T}}{r-g},\\
\gamma &= \tau n e^{-nR}\\
&\quad \frac{ne^{-nR}(e^{-(r-g)R} - e^{-(r-g)T}) - (r-g)^2 e^{-(r-g)R}(e^{-nR} - e^{-nT})}{(r-g)(e^{-nR} - e^{-nT})^2},\\
\delta &= -\tau n^2 e^{-2nR}\frac{e^{-(r-g)R} - e^{-(r-g)T}}{(r-g)(e^{-nR} - e^{-nT})^2},\\
\phi &= -\tau(1 - e^{-nR})n^2 e^{-nR}\frac{e^{-(r-g)R} - e^{-(r-g)T}}{(r-g)(e^{-nR} - e^{-nT})^2},\\
\psi &= \tau(1 - e^{-nR})ne^{-nR}\\
&\quad \times\frac{2ne^{-nR}(e^{-(r-g)R} - e^{-(r-g)T}) - (r-g)^2 e^{-(r-g)R}(e^{-nR} - e^{-nT})}{(r-g)(e^{-nR} - e^{-nT})^3}\\
&\quad ,\\
\sigma &= \tau n e^{-nR}e^{-(r-g)R}\frac{1}{e^{-nR} - e^{-nT}},\\
\zeta(\tau) &= \tau(1 - e^{-nR})(r-g)e^{-(r-g)R}\frac{1}{e^{-nR} - e^{-nT}},\\
\mu &= \tau(1 - e^{-nR})ne^{-nR}e^{-(r-g)R}\frac{1}{(e^{-nR} - e^{-nT})^2}.
\end{aligned}$$

Now observe that $\alpha(\tau) + \zeta(\tau) \leq \alpha(\bar{\tau}) + \zeta(\bar{\tau})$ for $\tau \leq \bar{\tau}$. Substituting $\bar{\tau}$ by (5), it follows that $\alpha(\bar{\tau}) + \zeta(\bar{\tau}) = 0$, so that

$$\frac{\partial^2 B_A}{\partial R^2} \leq w(t)e^{(r-g)A}[\beta + \gamma + \delta + \phi + \psi + \sigma + \mu].$$

Manipulation of γ leads to

$$\gamma = \frac{\tau n^2 e^{[-2n-(r-g)]R}}{(e^{-nR} - e^{-nT})^2}\left[\frac{1 - e^{-(r-g)(T-R)}}{r-g} - \frac{1 - e^{-n(T-R)}}{n}\right].$$

Since

$$\frac{1 - e^{-(r-g)(T-R)}}{r-g} \leq \frac{1 - e^{-n(T-R)}}{n}$$

for $r - g \geq n$ with $=$ if $r - g = n$, it follows that $\gamma \leq 0$ for $r - g \geq n$. Adding ψ and μ leads to

$$\phi + \mu = \frac{2\tau(1-\tau)n^2 e^{[-2n-(r-g)]R}}{(e^{-nR} - e^{-nT})^3}\left[\frac{1 - e^{-(r-g)(T-R)}}{r-g} - \frac{1 - e^{-n(T-R)}}{n}\right],$$

so that, by the same argument, $\phi + \mu \leq 0$ for $r - g \geq n$. Adding δ and σ leads to

$$\delta + \sigma = \frac{\tau n^2 e^{[-2n-(r-g)]R}}{(e^{-nR} - e^{-nT})^2}\left[\frac{1 - e^{-(r-g)(T-R)}}{r-g} - \frac{1 - e^{-n(T-R)}}{n}\right],$$

so that $\delta + \sigma \leq 0$ for $r - g \geq n$. It follows that

$$\frac{\partial^2 B_A}{\partial R^2} \leq w(t)e^{(r-g)A}(\beta + \phi),$$

if $\tau \leq \bar{\tau}$ and $r \geq n + g$. Since

$$\begin{aligned}
\beta + \phi = &-\tau n^2 e^{-nR}\frac{e^{-(r-g)R} - e^{-(r-g)T}}{(r-g)(e^{-nR} - e^{-nT})}\\
&-\tau(1 - e^{-nR})n^2 e^{-nR}\frac{e^{-(r-g)R} - e^{-(r-g)T}}{(r-g)(e^{-nR} - e^{-nT})^2} < 0,
\end{aligned}$$

it follows that $\partial^2 B_A/\partial R^2 < 0$ if $\tau \leq \bar{\tau}$ and $r \geq n + g$. This proves ii. □

Proof of iii. Differentiate Equation (7) with respect to R for $A \in [\bar{R}, T]$ again taking into account that S_A is predetermined at time t. This leads to

$$\frac{\partial B_A}{\partial R} = w(t)\tau e^{-nR}\frac{n(1 - e^{-nT})}{(e^{-nR} - e^{-nT})^2}\frac{1 - e^{-(r-g)(T-A)}}{r-g} > 0.$$

This proves iii. □

Proof of Lemma A2. Differentiate Equation (8) with respect to $\bar{R}$. This leads to

$$\frac{\partial S_A}{\partial \bar{R}} = -w(t)\frac{1 - e^{-\rho A}}{1 - e^{-\rho T}}e^{(r-g)A}\Omega(\tau, \bar{R}),$$

where $\Omega(\cdot,\cdot)$ is defined in Equation (A2). Sinde Ω is strictly positive, $S_{A,t}$ is strictly decreasing in $\bar{R}$ for all $A \in (0, \bar{R})$ if $\tau \leq \bar{\tau}$. □

Proof of Lemma 1. *Proof of* i. Since V_A is C^2 in R for $R \in [0, \bar{R})$, it suffices to show that $\partial^2 V_A/\partial R^2 < 0$ if $\tau \leq \bar{\tau}$, $r \geq n + g$, and ρ sufficiently small. Differentiation of Equation (6) with respect to R for $A \in [0, \bar{R}]$ leads to

$$\frac{\partial V_A}{\partial R} = \frac{1}{B_A}\frac{\partial B_A}{\partial R}\frac{1}{\rho}(1 - e^{-\rho(T-A)}) - z(R)e^{-\rho(R-A)}, \tag{A4}$$

and

$$\begin{aligned}
\frac{\partial^2 V_A}{\partial R^2} = &\left[-\frac{1}{B_A^2}\left(\frac{\partial B_A}{\partial R}\right)^2 + \frac{1}{B_A}\frac{\partial^2 B_A}{\partial R^2}\right]\frac{1}{\rho}(1 - e^{-\rho(T-A)})\\
&+ [\rho z(R) - z'(R)]e^{-\rho(R-A)}.
\end{aligned} \tag{A5}$$

Since $B_A > 0$ and finite, $\lim_{\rho\to 0}\frac{1}{\rho}(1 - e^{-\rho(T-A)}) = T - A$ by L'Hospital's Rule, and $\partial B_A/\partial R$ and $\partial^2 B_A/\partial R^2$ independent of ρ, it follows that

$$\lim_{\rho\to 0}\frac{\partial^2 V_A}{\partial R^2} = -(T-A)\left[\frac{1}{B_A^2}\left(\frac{\partial B_A}{\partial R}\right)^2 - \frac{1}{B_A}\frac{\partial^2 B_A}{\partial R^2}\right] - z'(R).$$

Since $\partial^2 B_A/\partial R^2 < 0$ if $\tau \leq \bar{\tau}$ and $r \leq n + g$ by Lemma A1. ii, it follows that

$$\lim_{\rho\to 0}\frac{\partial^2 V_A}{\partial R^2} < 0,$$

and i. follows by a standard continuity argument. □

Proof of ii. Differentiation of Equation (6) with respect to R for $A \in [\bar{R}, T]$ leads to

$$\frac{\partial V_A}{\partial R} = \frac{1}{B_A}\frac{\partial B_A}{\partial R}\frac{1}{\rho}(1 - e^{-\rho(T-A)}).$$

Since $\partial B_A/\partial R > 0$ for all $A \in [\bar{R}, T]$ by Lemma A1. iii, it follows that $\partial V_A/\partial R > 0$ for all $A \in [\bar{R}, T]$. This proves ii. □

Proof of Lemma 2. In light of Lemma 1, condition (9) implies a continuous and differentiable function $f_A : [A, T] \to [A, T]$ such that $R_A = f_A(\bar{R})$. Since f_A maps the closed interval $[A, T]$ into itself (note that condition (9) can also hold at $R_A = A$ or at $R_A = T$), it has a fixed point given by $R_A^* = f_A\left(R_A^*\right)$. To show that R_A^* is uniquely determined, implicitly differentiate Equation (9) to get

$$\frac{dR_A}{d\bar{R}} = f_A'(\bar{R}) = \frac{\frac{1}{B_A^2}\frac{\partial S_A}{\partial \bar{R}}\frac{\partial B_A}{\partial R}\frac{1}{\rho}(1 - e^{-\rho(T-A)})}{\left[-\frac{1}{B_A^2}\left(\frac{\partial B_A}{\partial R}\right)^2 + \frac{1}{B_A}\frac{\partial^2 B_A}{\partial R^2}\right]\frac{1}{\rho}(1 - e^{-\rho(T-A)}) + \rho z'(R_A)e^{-\rho(R_A - A)}},$$

which is positive if ρ is sufficiently small, since $\partial B_A/\partial R > 0$ and $\partial^2 B_A/\partial R^2 < 0$ by Lemma A1 and $\partial S_A/\partial \bar{R} < 0$ by Lemma A2.

To show that R_A^* is a stable fixed point of f_A, it will be shown that $f_A'\left(R_A^*\right) < 1$. Observe that

$$\frac{\partial S_A}{\partial \bar{R}} = -\frac{\partial B_A}{\partial R}\frac{A}{T}$$

for $\bar{R} = R_A$, which can be inferred from Equations (7) and (8). Thus,

$$f_A'(R_A) = \frac{\frac{1}{B_A^2}\left(\frac{\partial B_A}{\partial R}\right)^2\frac{A}{T}\frac{1}{\rho}(1 - e^{-\rho(T-A)})}{\left[-\frac{1}{B_A^2}\left(\frac{\partial B_A}{\partial R}\right)^2 + \frac{1}{B_A}\frac{\partial^2 B_A}{\partial R^2}\right]\frac{1}{\rho}(1 - e^{-\rho(T-A)}) + \rho z'(R_A)e^{-\rho(R_A - A)}},$$

which is smaller than 1 if ρ is sufficiently small.

To show that R_A^* does not depend on A, so that $R_A^* = R^*$, combine Equations (7) and (8) for $\bar{R} = R_A$. Then, B_A can be written as

$$B_A = w(t)e^{(r-g)A}\frac{e^{-\rho A} - e^{-\rho T}}{1 - e^{-\rho T}}\Delta(\tau, R_A),$$

with

$$\Delta(\tau, R_A) = (1-\tau)\frac{1 - e^{-(r-g)R_A}}{r - g} + \tau\frac{1 - e^{-nR_A}}{e^{-nR_A} - e^{-nT}}\frac{e^{-(r-g)R_A} - e^{-(r-g)T}}{r - g}. \tag{A6}$$

Then, considering Equation (A1), condition (9) becomes

$$\frac{\Omega(\tau, R_A)}{\Delta(\tau, R_A)}\frac{1 - e^{-\rho T}}{\rho} - z(R_A)e^{-\rho R_A} = 0.$$

Since Ω and Δ do not depend on A, R_A is also independent of A. Note that for $A = 0$, condition (9) reads

$$\frac{1}{B_0}\frac{\partial B_0}{\partial R}\frac{1 - e^{-\rho T}}{\rho} - z(R_0)e^{-\rho R_0} = 0.$$

Since B_0 does not depend on $\bar{R}$, it follows that $R_0 = R^*$ irrespective of $\bar{R}$, i.e., irrespective of the historical SRA. □

Proof of Propostion 2. Let $\bar{R} = R^*$ and consider a permanent increase in R at R^* at time t. When R is increased, individuals of age $A \geq R^*$, i.e., pensioners, pay a one-time age-dependent lump-sum tax $\mu(\theta)$, $\theta \in [R^*, T]$. Remaining lifetime income of an individual aged $A \geq R^*$ then becomes

$$B_A^\mu = B_A - \mu(A), \text{ with } \frac{\partial B_A^\mu}{\partial R} = \frac{\partial B_A}{\partial R} \quad \text{and} \quad \frac{\partial B_A^\mu}{\partial \mu(A)} = -1. \tag{A7}$$

Note that individuals of age $A \geq R^*$ will be taxed according to age, as younger pensioners benefit from an increase in R for a longer time spell than older pensioners. Replacing B_A by B_A^μ in Equation (6) yields indirect lifetime utility of an individual aged $A \geq R^*$, denoted by V_A^μ. Totally differentiating V_A^μ with respect to R and $\mu(A)$ leads to

$$dV_A^\mu = \frac{\partial V_A^\mu}{\partial R}dR + \frac{\partial V_A^\mu}{\partial \mu(A)}d\mu(A).$$

The lump-sum tax is chosen so that the remaining lifetime utility of each individual aged $A \geq R^*$ remains unchanged after the increase in R, that is, $dV_A^\mu = 0$ for all $A \geq R^*$. Considering Equations (6) and (A7), it follows that

$$\frac{d\mu(A)}{dR} = \frac{\partial B_A}{\partial R}, \tag{A8}$$

which is strictly positive by Lemma A1. iii.

Individuals of $A < R^*$, that is, workers, receive a one-time lump-sum transfer ω as compensation for an increase in R. The lump-sum transfers do not depend on the specific age of the worker, but are the same for all workers. It is financed by the lump-sum tax levied on pensioners so that

$$\int_0^{R^*}\omega e^{-n\theta}d\theta = \int_{R^*}^T \mu(\theta)e^{-n\theta}d\theta.$$

Differentiation with respect to R leads to

$$\frac{d\omega}{dR} = \frac{n}{1 - e^{-nR^*}}\int_{R^*}^T \frac{d\mu(\theta)}{dR}e^{-n\theta}d\theta,$$

which is strictly positive in light of Equation (A8).

With the lump-sum transfer ω remaining lifetime income of an individual aged $A < R^*$ becomes

$$B_A^\omega = B_A + \omega, \text{ with } \frac{\partial B_A^\omega}{\partial R} = \frac{\partial B_A}{\partial R} \quad \text{and} \quad \frac{\partial B_A^\omega}{\partial \omega} = 1. \tag{A9}$$

Replacing B_A by B_A^ω in Equation (6) yields the indirect lifetime utility of an individual aged $A < R^*$, denoted by V_A^ω. Totally differentiating V_A^ω with respect to R and ω leads to

$$dV_A^\omega = \frac{\partial V_A^\omega}{\partial R}dR + \frac{\partial V_A^\omega}{\partial \omega}d\omega.$$

Considering Equations (A4) and (A9) this becomes

$$dV_A^\omega = \left[\frac{1}{B_A^\omega}\frac{\partial B_A}{\partial R}\frac{1}{\rho}(1 - e^{-\rho(T-A)}) - z(R)e^{-\rho(R-A)}\right]$$
$$dR + \frac{1}{B_A^\omega}\frac{1}{\rho}(1 - e^{-\rho(T-A)})d\omega,$$

which can be written as

$$dV_A^\omega = \frac{B_A}{B_A^\omega}\left[\frac{1}{B_A}\frac{\partial B_A}{\partial R}\frac{1}{\rho}(1 - e^{-\rho(T-A)}) - z(R)e^{-\rho(R-A)}\right]$$
$$dR + \frac{1}{B_A^\omega}\frac{1}{\rho}(1 - e^{-\rho(T-A)})d\omega.$$

Considering Equation (9), the term in square brackets vanishes for $R = R^*$. Therefore,

$$dV_A^\omega = \frac{1}{B_A^\omega}\frac{1}{\rho}(1 - e^{-\rho(T-A)})d\omega > 0.$$

Thus, a small increase in R at R^* accompanied by a transfer from pensioners to workers makes workers better off without making pensioners worse off. □