

In Situ Tunable Interaction with an Invertible Sign between a Fluxonium and a Post Cavity

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Quantum computation with bosonic modes presents a powerful paradigm for harnessing the principles of quantum mechanics to perform complex information processing tasks. In constructing a bosonic qubit with superconducting circuits, nonlinearity is typically introduced to a cavity mode through an ancillary two-level qubit. However, the spurious heating of the ancilla has impeded progress toward fully fault-tolerant bosonic qubits. The ability to *in situ* decouple the ancilla when not in use would be beneficial but has so far only been realized with tunable couplers or additional parametric drives. This work presents a novel architecture for quantum information processing, comprising a three-dimensional post cavity coupled to a fluxonium ancilla via a readout resonator. The intricate energy-level structure of this system results in a complex landscape of interactions, the sign of which can be tuned *in situ* by the magnetic field threading the fluxonium loop without the need for additional elements. Our results could significantly advance the lifetime and controllability of bosonic qubits.

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I. INTRODUCTION

Among the various platforms explored for quantum computing, bosonic qubits represent a compelling approach. Unlike traditional qubit systems, which use two-level systems, bosonic qubits leverage the infinite-dimensional Hilbert space of harmonic oscillators,

allowing for the encoding of quantum information in more complex and potentially more robust ways. Encoding techniques such as Gottesman-Kitaev-Preskill (GKP) [1–3], cat states [4,5], and kitten [6] and binomial states [7] have already demonstrated how bosonic qubits can store and process information with high fidelity in a single harmonic oscillator. Similarly, there are schemes for distributing information across two bosonic modes, i.e., dual-rail erasure qubits [8]. All these methods can significantly reduce the detrimental effect of the inherent loss mechanisms in the system, resulting in qubits with enhanced storage capabilities and reduced hardware control stack.

Superconducting microwave cavities are a great candidate to host bosonic qubits as their lifetimes are enhanced

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by at least an order of magnitude compared to conventional junction-based superconducting qubits, such as transmons and fluxonia. To control such a bosonic mode, a nonlinearity is typically introduced to the cavity through a transmon qubit [9], referred to as an ancilla. However, it has been shown that this ancilla is the main limitation of the bosonic qubit due to residual excitations out of the computational space and its chaotic behavior under strong drives [10]. This has motivated a search for alternatives: removing the transmon overall [11] or using an ancilla bosonic qubit [12]. While both approaches succeed in their objective, they come with disadvantages. The latter uses an increased control complexity, i.e., a bigger instrument control stack is needed, while the former suffers from weaker interaction strengths leading to longer experiment times. A perfect replacement would be an ancilla that can be decoupled selectively from the bosonic mode after the state preparation and tomography. So far, this has been realized by introducing either additional parametric drives [13,14] or elements such as tunable couplers [15].

In this work, we couple a $\lambda/4$ high- Q post cavity to a fluxonium ancilla qubit [16] via its readout resonator. The complex energy landscape reveals novel transversal interactions between the bosonic mode and its ancilla, which can be tuned *in situ* between negative and positive values smoothly through zero. This system allows for both sufficient interaction strength required for the control of the storage mode and for decoupling the high- Q cavity from the fluxonium, removing the main source of dephasing of the bosonic qubit.

II. EXPERIMENTAL SETUP

In our experiment, schematically shown in Fig. 1(a), an aluminum post cavity serves as the memory mode. The control is provided by a fluxonium [17,18] [Fig. 1(b)], consisting of an aluminum–aluminum oxide–aluminum (Al–AlO_x–Al) Josephson junction (Fig. 1(b) right inset), shunted by a granular aluminum (grAl) superinductance (see the fabrication details in Appendix B). The fluxonium is inductively coupled to a readout resonator via a shared grAl section (Fig. 1(b), left inset). The readout resonator shares a capacitance with the cavity. The additional control knob for the magnetic bias of the fluxonium is provided by a small superconducting coil sitting on top of a home-built magnetic flux hose [19], which guides the field into the superconducting enclosure.

The fundamental mode of the coaxial cavity sits at frequency $\omega_C/2\pi \approx 4.5$ GHz, much lower than the cutoff frequency of the waveguide section above it, resulting in its high quality factor [20,21]. The measured lifetime of this mode with the qubit coupled to it is $T_1^C = 210 \pm 40$ μ s (see Appendixes F and G). According to finite-element simulations, the higher cavity modes start at about 13.37 GHz. These modes spread more into the waveguide section,

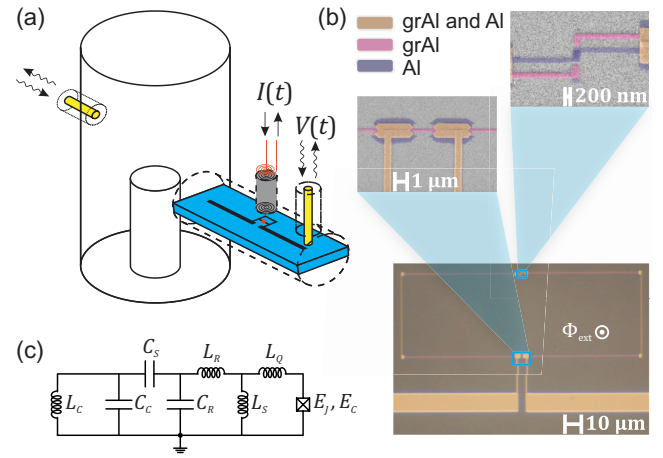


FIG. 1. The experimental setup. (a) The device consists of a high- Q cavity, fluxonium (the junction is marked with a red cross), a readout resonator on a sapphire chip (blue), a flux hose (gray) and superconducting coil (red), and coupling pins (yellow). (b) A false-colored scanning-electron-microscope (SEM) image of the sample around the fluxonium. The colors correspond to the thin-film material; violet for aluminum (Al), pink for granular aluminum (grAl), and orange for overlapped grAl and Al material. (c) A lumped-element model of the setup. The cavity (L_C , C_C) is capacitively coupled (C_S) to the resonator (L_R , C_R), which in turn is inductively coupled (L_S) to the fluxonium qubit consisting of a Josephson junction (E_J , E_C) shunted by a superinductance (L_Q).

giving them lower quality factors as they have a higher participation ratio with the lossy seam at the top of the cavity. The qubit is tunable in the range $\omega_Q/2\pi \in [1.02, 13.45]$ GHz, meaning that it can also couple to the higher cavity modes (see Appendix C).

The short section of grAl, a high-kinetic-inductance material [22], serves as a mutual inductance between the qubit and the $\lambda/2$ readout resonator. The first resonator mode has a current antinode at the center, leading to higher participation of the kinetic inductance, while the second and all other even resonator modes have a current node at the same spot and the participation of the grAl piece is negligible. Consequently, the qubit couples only to the fundamental readout-resonator mode at $\omega_R/2\pi \approx 6.8$ GHz and all other odd modes if we assume pure inductive coupling and no spurious capacitive coupling. Both the fundamental cavity and resonator modes are coupled to the control instruments, with rates $\kappa_C^{\text{ext}}/2\pi = 233 \pm 7$ Hz and $\kappa_R^{\text{ext}}/2\pi = 549 \pm 2$ kHz, respectively.

III. SYSTEM MODEL

The system Hamiltonian is comprised of three subsystems, namely, the fluxonium $\hat{H}_Q$, the readout resonator $\hat{H}_R$, and the high- Q cavity $\hat{H}_C$, connected

by their respective interactions $\hat{H} = \hat{H}_Q + \hat{H}_R + \hat{H}_C + \hat{H}_{QR} + \hat{H}_{RC}$. The fluxonium has the Hamiltonian

$$\hat{H}_Q = 4E_C \hat{n}^2 + \frac{E_L}{2} (\hat{\phi} + \varphi_{\text{ext}})^2 - E_J \cos(\hat{\phi}), \quad (1)$$

where $\hat{n}$ is the charge-number operator, $\hat{\phi}$ is its conjugate phase operator, and $\varphi_{\text{ext}} = 2\pi \Phi_{\text{ext}}/\Phi_0$ is the reduced external flux, with $\Phi_0 = h/2e$ being the flux quantum. The capacitive energy can be written in terms of the total qubit capacitance, which is dominated by the junction, $E_C = e^2/2C_Q$, while the inductive energy contains the contribution of the grAl superinductance and the shared grAl inductance $E_L = \phi_0^2/(L_Q + L_S)$, where $\phi_0 = \Phi_0/2\pi$ is the reduced flux quantum. E_J is the Josephson energy of the junction $E_J = \phi_0 I_c$, with I_c being the critical current of the junction.

Determining the full system Hamiltonian requires different approaches depending on the physical dimensions of its constituents compared to the wavelengths of interest. If the whole system were to satisfy the lumped-element approximation [23], an equivalent circuit model could be drawn [as shown in Fig. 1(c)] and a full Hamiltonian diagonalization would be possible [24]. However, this method is unsuitable in our case, as the post cavity is a distributed element. An ideal way to quantize distributed-element circuits is black-box quantization (BBQ) [25] but it necessitates only weakly anharmonic modes, which are incompatible with the fluxonium. Instead, we solve the Hamiltonian in the normal-mode basis for the resonator and the cavity,

i.e., $\hat{H}_R = \hbar\omega_R \hat{a}^\dagger \hat{a}$ and $\hat{H}_C = \hbar\omega_C \hat{b}^\dagger \hat{b}$, with ω_R and ω_C being the fundamental-mode frequencies of the resonator and the cavity, respectively, and $\hat{a}$ and $\hat{b}$ being their annihilation operators, respectively. The inductive coupling between the qubit and the resonator is modeled as $\hat{H}_{QR} = -\hbar g_{QR} \hat{\phi}(\hat{a}^\dagger + \hat{a})$, where g_{QR} is the interaction strength. Additionally, the capacitive coupling between the resonator and the post cavity is $\hat{H}_{RC} = \hbar g_{RC}(\hat{a}^\dagger - \hat{a})(\hat{b}^\dagger - \hat{b})$ with the interaction strength g_{RC} .

We calculate the energy eigenspectrum and the dispersive shifts of the system Hamiltonian using a numerical toolbox (scQubits [26]). The system model is fitted to experimental data obtained through qubit spectroscopy (Fig. 2). From the fit, we first acquire the relevant fluxonium energies, E_C , E_L , and E_J in Eq. (1). Subsequently, we construct the Hilbert space by adding the relevant harmonic oscillators and their respective interactions. This step does not alter the fluxonium spectrum significantly except in the regions in which the levels of the qubit and the harmonic oscillators cross. This fitting work flow gives us the full interaction Hamiltonian. With this information, we can predict the dispersive shifts between the subsystems as a function of the flux threading the fluxonium qubit. The complete analysis and details of the simulation methods can be found in Appendix C.

IV. EXPERIMENTAL RESULTS

The qubit frequency is measured with two-tone spectroscopy. We monitor the resonator resonance while

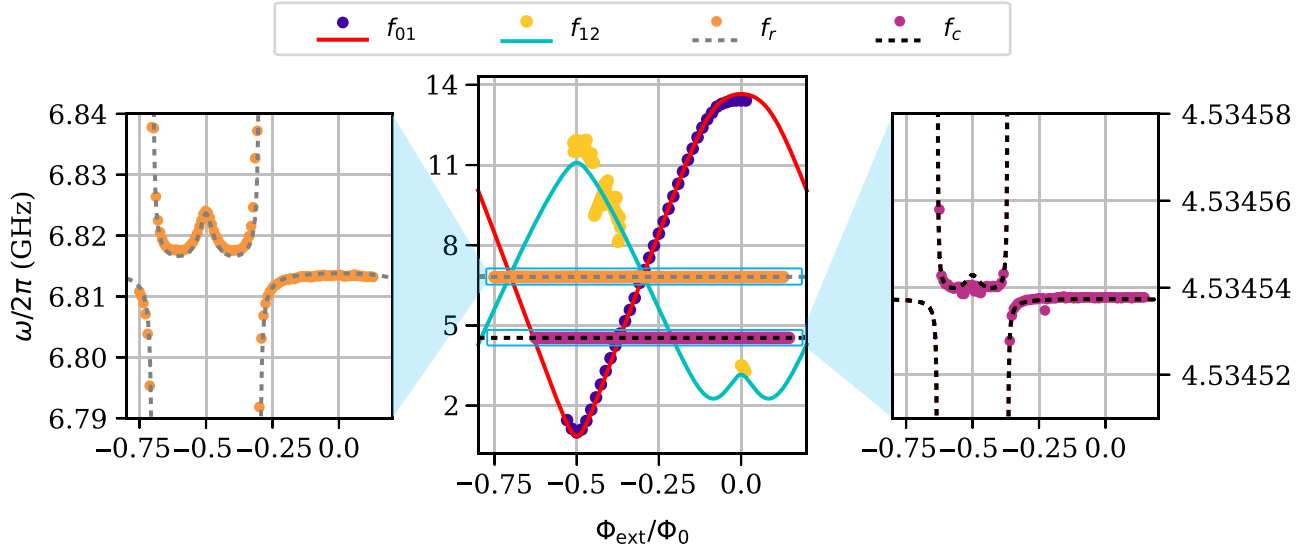


FIG. 2. The system spectrum. The plot in the center shows the qubit frequency measured as a function of the flux bias. By fitting this spectrum, we obtain the qubit energies $E_C/h = 3.5$ GHz, $E_L/h = 1.014$ GHz, and $E_J/h = 10.8$ GHz. The measured data for the fundamental transition are plotted in purple, together with their fit in red, while the higher transition f_{12} is in yellow with its simulation in cyan. The plots to the left and right show the readout-resonator and high- Q -cavity spectra, respectively, measured as a function of the flux. The qubit is tuned through both the readout resonator and the high- Q cavity, resulting in avoided crossings with coupling strengths $g_{QR}/2\pi \approx 25.2$ MHz and $g_{QC}/2\pi \approx 1.5$ MHz. The dashed lines correspond to their fits.

sweeping the frequency of a second tone with a signal generator. At the qubit frequency, the resonator shifts and our readout signal changes. We repeat the procedure for different applied magnetic fields and fit each data set to a Lorentzian curve to extract the qubit resonance frequency. The higher transition f_{12} is extracted in the same way around $\Phi_{\text{ext}}/\Phi_0 = -0.5$, where the nonzero excited-state population of the qubit allows direct excitation to the next higher level. We fit these data using scQubits (Fig. 2 center) and obtain the qubit energies $E_C/h = 3.5$ GHz, $E_L/h = 1.014$ GHz, and $E_J/h = 10.8$ GHz. The readout-resonator and cavity frequencies have been extracted from direct spectroscopy measurements. The Hamiltonian is then constructed by including the harmonic oscillators and the interactions. The numerical results of the diagonalization are manually fitted to the cavity and resonator spectroscopy (Fig. 2 right and left, respectively) as a function of the external magnetic field. The avoided crossings between the resonator and the qubit, and between the cavity and the qubit, give us the interaction strengths $g_{QR}/2\pi \approx 25.2$ MHz with an accuracy of about 10% and $g_{RC}/2\pi \approx 8$ MHz with an accuracy of about 15%. The splitting of $2g_{QC}/2\pi \approx 3$ MHz that we expect from the avoided crossing in Fig. 2 (right) is a measure of the effective coupling rate between the qubit and the cavity, which is mediated by the resonator and determined by both g_{QR} and g_{RC} .

Near the high sweet spot $\Phi_{\text{ext}}/\Phi_0 = 0$, there is a discrepancy between the model and the measured data. A possible explanation is the higher modes of the cavity, which we expect around 13.37 GHz, 13.73 GHz, and 13.78 GHz. Each of those modes will couple to the qubit via the readout resonator. In addition, the symmetry of the second additional mode will allow it to leak into the tunnel of the qubit and couple directly with the fluxonium. Owing to all the additional interactions at those frequencies, the model fails to fully capture the qubit behavior at high frequencies. For further discussion on the limitations of this model, see Appendix C.

With the fits, the dispersive shift between the qubit and the resonator, χ_{QR} , and between the qubit and the cavity, χ_{QC} , can be calculated as depicted in Fig. 3. We extract the dispersive shift χ_{QR} from two-tone spectroscopy on the resonator, while the χ_{QC} measurements have been performed in the time domain. The different methods are required because χ_{QR} is higher than the qubit linewidth at any external flux and χ_{QC} is lower.

Large displacements on the cavity enable the extraction of the Hamiltonian parameters [27] even when the dispersive shift is smaller than the qubit decoherence rate. Due to the cavity coupling to the qubit, the displaced coherent state will start rotating in phase space around the origin. After a fixed time, we displace the state back to the vacuum to obtain the acquired phase. The rotation frequency of the cavity state about the origin will be different for ground and

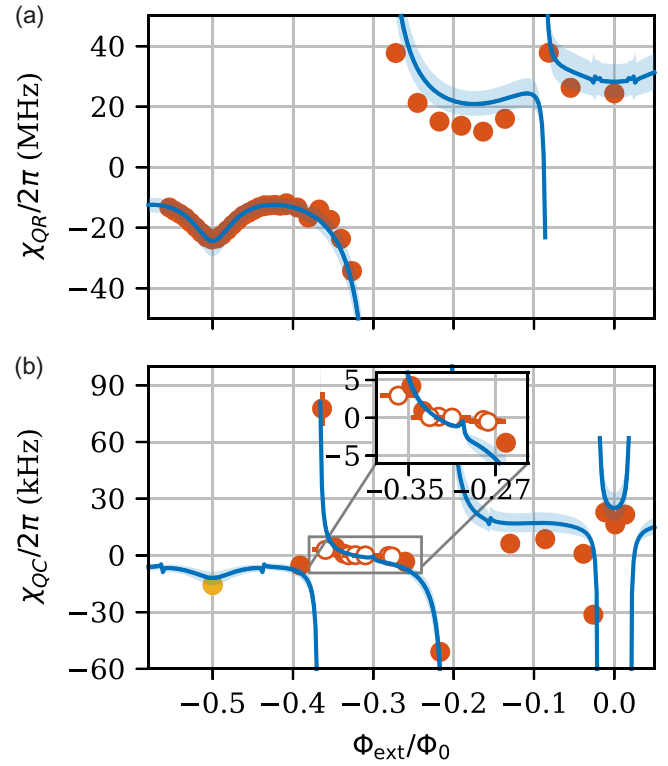


FIG. 3. The system dispersive shifts. (a) The dispersive shift between the qubit and the readout resonator as a function of the flux bias. The shaded region corresponds to a 10% variation in the coupling strength g_{QR} . (b) The interaction between the qubit and the high- Q cavity as a function of the flux bias. The yellow point is measured with an echoed conditional displacement sequence, while the rest (red) are extracted from qubit spectroscopy as a function of photons in the cavity measured in the same (full) or consecutive (empty) cooldown. The shaded region is a confidence interval, which corresponds to 15% uncertainty of the coupling strength g_{RC} . Note the smooth transition between positive and negative values of the interaction around $\Phi_{\text{ext}}/\Phi_0 = -0.33$.

excited qubit states and will depend on the dispersive shift (for more details, see Appendix F). With this procedure, we obtain the dispersive shift at the low sweet spot [yellow data point in Fig. 3(b)]. The other points are extracted from qubit spectroscopy as a function of photons in the cavity.

The extracted data are plotted together with the numerical-model predictions in Fig. 3. We note that in Fig. 3(b), the dispersive shift χ_{QC} is smoothly tunable between positive and negative values through zero, meaning that we can switch off the interaction between the fluxonium and the cavity. The feature at $\Phi_{\text{ext}} \approx -0.3\Phi_0$ stems from the crossing between the fluxonium and the readout resonator (see Fig. 2 left). It is not detrimental, as the required qubit-frequency change to go from $\chi_{QC}/2\pi = 0$ Hz to this crossing is about 100 times larger than the qubit linewidth, which is limited by flux noise at this flux-bias point. The largest negative dispersive shift

that we have measured is $\min(\chi_{QC})/2\pi = -51 \pm 2$ kHz, which is comparable to other recent works [3,6,27]. Also, the closest to zero that we have measured in a consecutive cooldown is $\min(|\chi_{QC}|)/2\pi = 30 \pm 94$ Hz. While such measured results can also be achieved with a tunable transmon as an ancilla [28], in this setup it is possible to tune the interaction strictly to zero. Furthermore, we can tune to the opposite sign of χ_{QC} , with our largest measured value at 78 ± 9 kHz, similar to tunable transmon implementations using additional elements such as an inductive coupler [29].

When the absolute value of the dispersive shift χ_{QC} is high compared to the relaxation rate of the fluxonium, i.e., $|\chi_{QC}| \gg \Gamma_1^Q$, the storage-mode coherence time will be determined by the lifetime and the thermal population of the ancilla. In the limit $|\chi_{QC}| \ll \Gamma_1^Q$, the cavity dephasing time becomes proportional to χ_{QC}^{-2} , profiting from smaller dispersive shifts [13] (see Appendix H).

For this system to be suitable as a bosonic qubit, further understanding of the fluxonium losses is required. We measure the lifetime of the fluxonium qubit $T_1^Q = 1/\Gamma_1^Q$ as a function of the flux bias and plot the T_1^Q limit from the dominant decay channels [30–32]: dielectric loss, inductive loss, the Purcell limit [33], and quasiparticle loss [34] (see Fig. 4). From the fitted bounds for the quality factors of each dissipation channel, we deduce that the lifetime of the fluxonium at the zero-field spot is limited by dielectric losses. Besides the range in which the qubit is close to resonant with the readout resonator, it is evident that

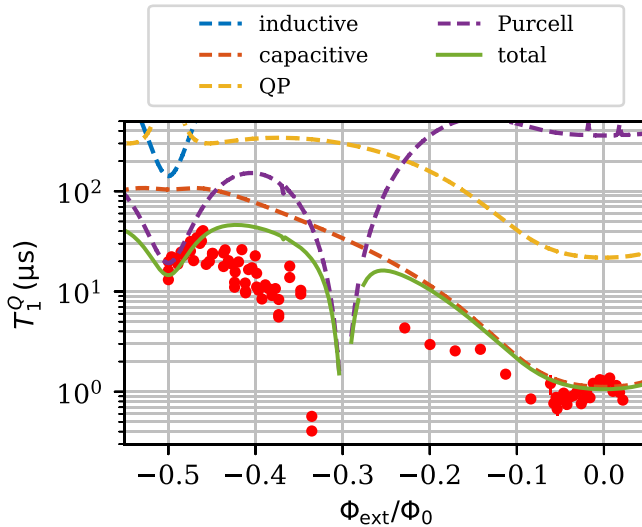


FIG. 4. The fluxonium lifetime and loss mechanisms. The measured fluxonium T_1^Q is plotted over the lifetime limits set by different loss channels: dielectric, inductive, nonequilibrium quasiparticles (QP), and Purcell. The losses are calculated for a qubit-mode temperature of $T_Q = 45$ mK and a resonator-mode temperature of $T_R = 50$ mK. For the exact dissipation model and its temperature dependence, see Appendix D.

the qubit is also Purcell limited [35,36] at $\Phi_{\text{ext}}/\Phi_0 = 0.5$. These limits are further reduced at this flux bias when including the high thermal population of both the qubit and the resonator. The system would greatly benefit from a Purcell filter and improved fabrication techniques. A possible improvement in the fabrication would be to treat the sample with hydrofluoric acid (HF) after fabrication, which has been shown to reduce dielectric losses due to two-level system defects in resonators on silicon [37]. Another possibility is to adapt the fabrication process so that it is compatible with more aggressive surface cleaning by swapping the Al with either tantalum [38,39] or niobium capped by another metal [40]. Nevertheless, in the regions in which the qubit lifetime is compromised by Purcell decay $\Phi_{\text{ext}} \approx -0.3\Phi_0$, the cavity coherence is preserved as the dispersive shift is much lower than the qubit relaxation rate (see Fig. 3). To measure this effect on the cavity dephasing times, fast flux control is required, as the qubit coherence is only sufficient at $\Phi_{\text{ext}} = 0.5\Phi_0$ (see Appendix E).

V. CONCLUSIONS AND OUTLOOK

To summarize, we have demonstrated that a fluxonium is a viable ancilla to a high- Q coaxial cavity. We explore for the first time that this platform realizes a tunable interaction with an invertible sign without using supplementary circuit elements such as tunable couplers or applying parametric drives. This appends another element to the toolbox to control higher-coherence microwave cavities, which could enable novel control strategies for building bosonic codes. The fact that the ancilla can be decoupled from the bosonic mode alleviates the stringent requirement for a high-coherence ancilla without requiring a trade-off with the bosonic-qubit-control capabilities.

Further improvements of the systems involve including a Purcell filter, which would greatly increase the lifetime of the fluxonium. In particular, the intrinsic Purcell protection [41] is compatible with this setup. Adding fast flux control to the flux hose would allow fast *in situ* switching between a flux bias where the dispersive shift is high, allowing for state creation and tomography, and the zero crossing of the dispersive shift, where the cavity is protected from ancilla-induced dephasing. Such a system would be ideally suited for pursuing new quantum gates or creating and evacuating quantum states from the long-lived memory mode.

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DATA AVAILABILITY

The data that support the findings of this paper are openly available [42].

APPENDIX A: EXPERIMENTAL SETUP

A schematic of the cryogenic and microwave setup is shown in Fig. 5. The measurements were conducted in an Oxford Triton DU7-200 cryofree dilution refrigerator system.

The radio-frequency (rf) input coaxial lines were attenuated by 20 dB at the 4-K plate and 10 dB at the still plate. On the base plate, the input signal for the qubit line was attenuated further by 30 dB (a 20-dB attenuator and a Quantum Microwave 10-dB thermalized attenuator) and then filtered with a Mini-Circuits DC-14 GHz low-pass filter. The experiment was in a reflection configuration and a Low Noise Factory single-junction (4–12)-GHz circulator was used. The signal was finally routed through a home-built infrared filter. The input signal for the cavity had the same configuration up to the 100-mK stage. A low-pass K&L filter was used to filter the cavity signal, which was then attenuated by a Quantum Microwave 20-dB thermalized directional coupler and 10-dB attenuator from the same brand. A narrow-band Mini-Circuits (4–4.8)-GHz bandpass filter and an infrared filter were placed just outside the shield. A Quinstar (4–8)-GHz circulator allowed for direct reflection measurements of the cavity.

The sample was thermalized to the base plate via copper clamps. The experiment was shielded by a μ -metal shield and enclosed by a copper can, covered with Eccosorb paint on the inside. In the inner can, the sample was surrounded by Eccosorb foam.

The output signal from the resonator was filtered by a (6–10)-GHz Microtronics bandpass filter and then routed through an isolator, a Krytar 10-dB directional coupler, and a Low Noise Factory circulator before reaching a parametric dimer Josephson-junction array amplifier (DJJAA) [43]). The 10-dB directional coupler was used to couple the amplifier pump line. After the parametric amplifier, the output signal went through two Quinstar isolators before being amplified at the 4-K stage by a high-electron-mobility amplifier and again at room temperature. To avoid amplifier saturation, an additional 10-dB directional coupler before the room-temperature amplifier was added to cancel the DJJAA pump tone from the fridge.

A Quantum Machine Operator X+ (OPX+) was used together with a Quantum Machine Octave to generate and

digitize the microwave signals. The OPX+ had arbitrary-waveform generators that produced the pulses used in the experiment, which were then mixed up to the relevant frequencies and amplified in the Octave. For readout, the Octave down-mixed the resonator and cavity pulses and was digitized by the OPX+. In the low-frequency range $f \in [1, 6.8]$ GHz, the fluxonium pulses were up-mixed via a double-superheterodyne [44] scheme that employed two local oscillators and two single-sideband mixers to produce a wide range of frequencies without the need for calibration. For higher qubit frequencies, the pulses were directly mixed by the Octave. The pulse-generation setup included other amplifiers, filters, fast microwave switches, and dc blocks to produce a high amplitude of the wanted microwave signal, reduce leakage of unwanted signals, and prevent ground loops. The flux biasing was done by leading in dc signals via twisted pairs through commercial low-pass filters. Two home-built high-attenuation filters further refined the qubit bias. One flux quantum corresponded to a current $I_{\text{bias}} = 7.35$ mA, allowing us to measure in a span of approximately three flux quanta before the onset of heating, which occurred around ± 10 mA.

APPENDIX B: FABRICATION

The fabrication recipe was adapted from Ref. [18] for the University of Innsbruck Quantum Nano-Zentrum Tirol (QNZT) cleanroom. The fluxonium and resonator were patterned by an electron-beam writer (Raith eLINE Plus 30 kV) on a bilayer resist stack (0.96 μm MMA (8.5) EL13 and 0.24 μm of 950 PMMA A4). The substrate wafer used was a 2-in. sapphire wafer that was piranha-cleaned before processing. To prevent the charging of the substrate, an 8-nm-thick aluminum layer was evaporated onto the surface of the resist stack. After lithography, the aluminum layer was etched in a tetramethyl-ammonium-hydroxide (TMAH) based developer and the resist was developed in a 3:1 solution of isopropyl alcohol (IPA) and deionized (DI) water. Next, 20-nm Al, 30-nm Al, and 40-nm grAl layers were evaporated onto the sample using a Plassys MEB550S electron-beam evaporator. The Al layers were evaporated under an angle with a controlled-oxidation step (29.6 mbar for 5 min) in between the deposition to create the Dolan-bridge junction. The grAl was deposited by evaporating Al under an oxygen flow. Before venting the chamber, the wafer was exposed to a pure-oxygen environment for 10 min at 10 mbar to avoid impurities in the aluminum oxide capping layer on top of the sample. The resist layer was lifted off in *N*-methyl-2-pyrrolidone (NMP) at 80 °C and cleaned in acetone and IPA. Finally, the samples were laser diced with a Coherent EXACTCUT 430 laser dicer.

Immediately after machining, the post cavity had a post length of 14.8 mm, an inner radius of 2.2 mm, and an outer radius of 5.8 mm. The tunnel for the sapphire chip

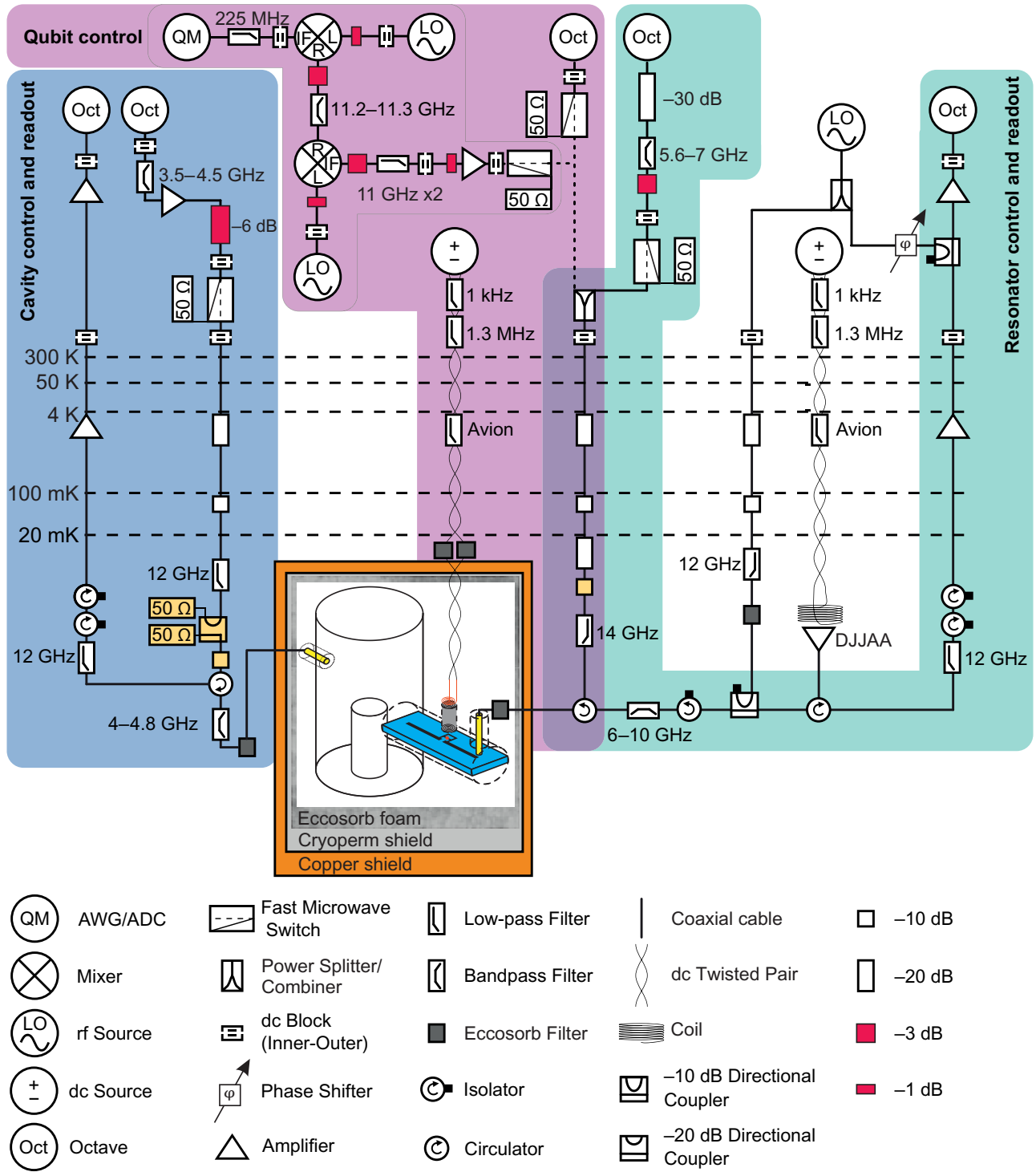


FIG. 5. The wiring diagram of the experiment. The lined part of the qubit control corresponds to the low-frequency microwave setup. Depending on the measurement, either the low- or high-frequency branch is combined with the readout tone or both branches are combined with the readout tone. The dashed lines refer to the different plates of the cryostat. The elements highlighted in yellow are specially thermalized to the base plate. The components related only to the parametric amplifier are not highlighted.

had a diameter of 3.8 mm and was aligned with the post height to maximize the coupling between the high- Q cavity mode and the qubit. The high-purity aluminum cavity was

machined at the Institute for Quantum Optics and Quantum Information Innsbruck mechanical workshop via an electrodischarge machining method. After manufacturing,

the cavity was etched in two rounds with Transene aluminum etchant type A, which removed approximately 150 μm of material. Finally, the cavity was thoroughly washed with DI water and dried.

APPENDIX C: SIMULATIONS

The high nonlinearity of the fluxonium in combination with distributed elements coupled to it makes the system hard to simulate. Finite-element simulators, such as Ansys HFSS, in conjunction with either BBQ or the QISKIT METAL extension, pyEPR [45], have until now failed to simulate highly anharmonic qubits. As we have shown, scQubits provides the possible solution but a few subtleties need to be taken care of.

First, all interaction terms are required, as the rotating-wave approximation cannot be applied in order to obtain the correct interaction strengths. Second, the higher cavity modes cannot be neglected and will influence the bare frequencies of the harmonic oscillators, which are needed for the fit. They will also push the qubit resonance to lower frequencies than anticipated. In Fig. 6, we show the influence of higher cavity modes on the qubit fundamental frequency. Three higher modes of the cavity are included with direct coupling to the resonator and the highest-frequency mode also is coupled directly to the qubit. The coupling strengths have been based on expectation values from the scaling with $\sqrt{\omega}$ and then further increased to amplify their effect. As the frequencies of those modes have never been measured but extracted from simulations, the actual cavity modes could coincide with the qubit levels, which would cause a further shift in the transition f_{01} . Such a coincidence could explain why around $\Phi_{\text{ext}}/\Phi_0 = 0$, the measured qubit frequencies fit better to the cavity mode at f_c^1 .

The final remark concerns the fitting routine. Initially, the nonlinear optimization and curve fitting of the qubit resonance as a function of flux were executed using the PYTHON solver LMFIT. However, the energies from the initial fit were adapted from $E_J/h = 10.34 \pm 0.04$ GHz, $E_C/h = 3.36 \pm 0.04$ GHz, and $E_L = 1.038 \pm 0.003$ GHz to the values from the main text to accommodate for the avoided crossing in Fig. 3(b), observed around the high sweet spot. Upon further investigation, the cause of this crossing was found to be the two-photon qubit transition f_{13} crossing the cavity resonance. The exact location of the higher energy levels plays an important role in how χ behaves. Even small changes in the qubit energies E_J , E_C , and E_L result in new features, such as the aforementioned avoided crossing. For the most accurate predictions of the dispersive shift, the fit must include the higher fluxonium transitions.

APPENDIX D: QUBIT DECAY CHANNELS

In this appendix, we discuss in detail the limitations on the qubit lifetime via the various qubit decay channels

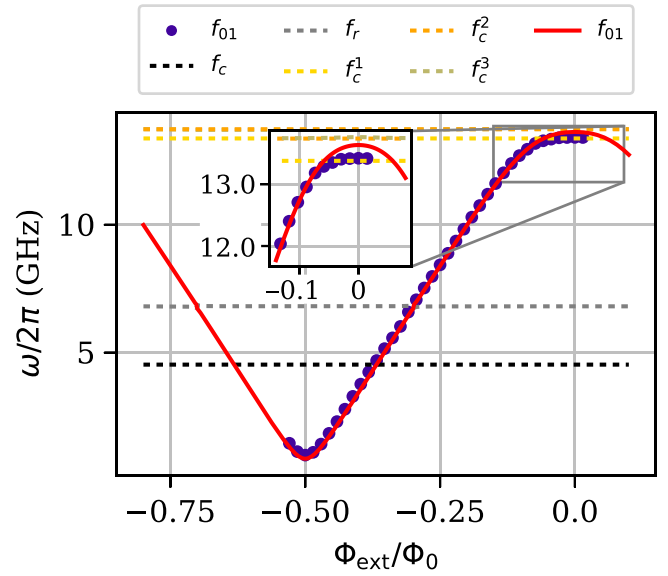


FIG. 6. Fluxonium spectroscopy with simulation curves including higher cavity modes. In the model, apart from the fundamental modes of the cavity f_c and the resonator f_r , three higher cavity modes are included: $f_c^1 = 13.375$ GHz, $f_c^2 = 13.735$ GHz, and $f_c^3 = 13.734$ GHz. They are capacitively coupled to the resonator with rates $g_{RC^{1,2,3}}/2\pi = 16$ MHz. Moreover, the highest-frequency mode at f_c^3 is directly coupled to the qubit. This mode does not affect the frequencies significantly even up to a direct coupling strength of $g_{QC^3}/2\pi = 16$ MHz. The inset shows an enlargement of the region around $\Phi_{\text{ext}}/\Phi_0 = 0$.

[30,31,36]. The decay rate of the qubit is given by Fermi's golden rule [32], of the type

$$\Gamma_{01} = \frac{1}{\hbar^2} |\langle 0|\hat{O}|1\rangle|^2 [S(\omega_{01}) + S(-\omega_{01})]. \quad (\text{D1})$$

The operator $\hat{O}$ is the qubit operator that couples to the lossy bath and $S(\omega_{01})$ is the noise spectral density at the qubit frequency. The first term corresponds to emission into the bath and the second to absorption.

We consider the most common loss mechanisms. For dielectric loss, the flux operator $\hat{O} = \phi_0 \hat{\phi}$ couples to the bath voltage with density

$$S_{VV}(\omega_{01}) + S_{VV}(-\omega_{01}) = \frac{2\hbar\omega_{01}^2 C_J}{Q_{\text{diel}}(\omega)} \coth\left(\frac{\hbar|\omega|}{2k_B T}\right), \quad (\text{D2})$$

where $C_J = e^2/2E_C$ is the junction capacitance, T is the qubit-mode temperature, and k_B is Boltzmann's constant. The quality factor of the dielectric losses is weakly frequency dependent:

$$Q_{\text{diel}}(\omega) = Q_{\text{diel}} \left(\frac{2\pi \cdot 6 \text{ GHz}}{|\omega|} \right)^{0.7}. \quad (\text{D3})$$

Here, the Q_{diel} is the nominal quality factor quoted for 6 GHz.

In the case of inductive losses, the flux operator $\hat{\mathcal{O}} = \phi_0 \hat{\varphi}$ couples to the bath current with density

$$S_{II}(\omega_{01}) + S_{II}(-\omega_{01}) = \frac{\hbar}{LQ_{\text{ind}}} \coth\left(\frac{\hbar|\omega|}{2k_B T}\right), \quad (\text{D4})$$

where $L = \phi_0^2/E_L$ is the fluxonium superinductance and Q_{ind} is the inductive quality factor.

The quasiparticle loss would result from the qubit operator $\hat{\mathcal{O}} = 2\phi_0 \sin\frac{\hat{\varphi}}{2}$ coupling to a noise variable with density

$$S_{\text{qp}}(\omega) + S_{\text{qp}}(-\omega) = 2\hbar\omega \Re\{Y_{\text{qp}}(\omega)\} \coth\left(\frac{\hbar|\omega|}{2k_B T}\right), \quad (\text{D5})$$

with the dissipative part of the Josephson-junction admittance having the form

$$\begin{aligned} \Re\{Y_{\text{qp}}(\omega)\} &= \sqrt{\frac{2}{\pi}} \frac{8E_J}{R_k \Delta_{\text{Al}}} \left(\frac{2\Delta_{\text{Al}}}{\hbar\omega}\right)^{1.5} x_{\text{qp}} \sqrt{\frac{\hbar\omega}{2k_B T}} \\ &\times K_0\left(\frac{\hbar|\omega|}{2k_B T}\right) \sinh\left(\frac{\hbar|\omega|}{2k_B T}\right). \end{aligned} \quad (\text{D6})$$

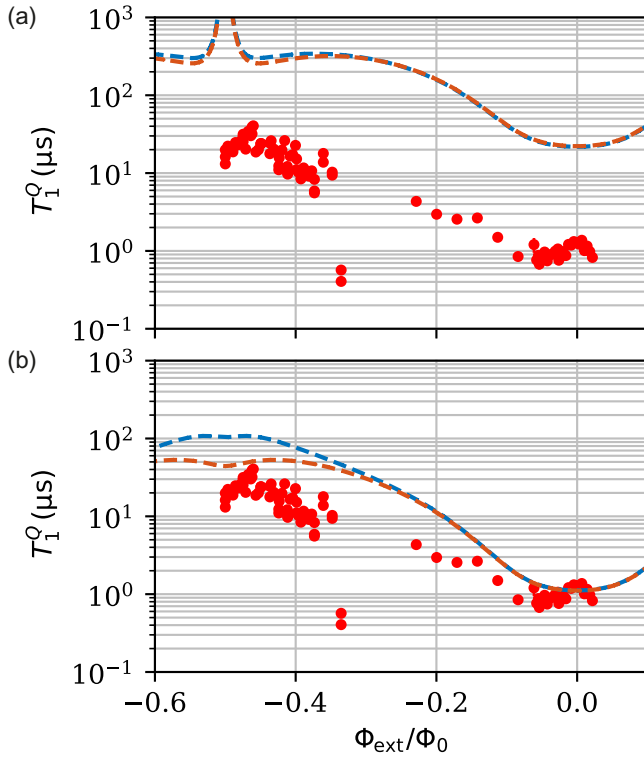


FIG. 7. The temperature dependence of the quasiparticle and dielectric losses for 45-mK (blue) and 112-mK (red) qubit-mode temperatures. (a) The quasiparticle losses for quasiparticle density $x_{\text{qp}} = 1.25 \times 10^{-6}$. (b) The dielectric losses for quality factor $Q_{\text{diel}} = 1.5 \times 10^5$. The biggest influence that the temperature has is around $\Phi_{\text{ext}}/\Phi_0 = -0.5$. Only the respective loss channel for each plot is considered and all other loss contributions are neglected.

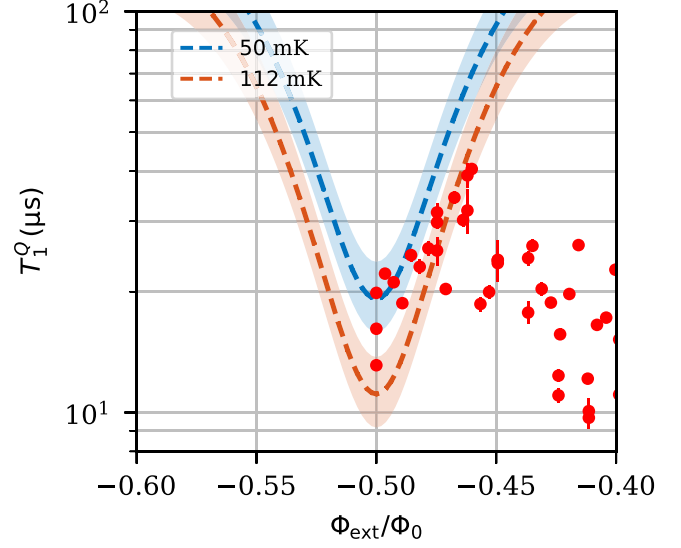


FIG. 8. The adjusted Purcell limit. The temperature of the resonator mode is adjusted from 112 to 50 mK to show the temperature change during the measurements. The shaded regions correspond to a change in the coupling rate g_{OR} of $\pm 10\%$ for each resonator temperature. All other loss mechanisms are neglected.

Here, $R_k = h/e^2$ is the resistance quantum and $\Delta_{\text{Al}} = 1.76T_c^{\text{Al}}k_B$ is the Al superconducting gap with critical temperature T_c^{Al} . The fitted value here is the quasiparticle density x_{qp} relative to the number of Cooper pairs. For Fig. 4, the qubit operator has been transformed to $\hat{\mathcal{O}} = \phi_0 \sin(\frac{\hat{\varphi}}{2} + \frac{\hat{I}}{2})$, where $\hat{I}$ is the identity operator.

The Purcell loss has been calculated according to the supplementary material of Ref. [33]. The absorption rate from the bath of oscillators through the readout resonator is

$$\Gamma_{|0\rangle_Q \rightarrow |1\rangle_Q}^{\text{Purcell}\uparrow} = P_{\text{res}}(0) \kappa n_{\text{th}}(\omega_{01}) |\langle 01|\hat{a}^\dagger|00\rangle|^2, \quad (\text{D7})$$

while the emission rate into the bath is

$$\Gamma_{|1\rangle_Q \rightarrow |0\rangle_Q}^{\text{Purcell}\downarrow} = P_{\text{res}}(0) \kappa [n_{\text{th}}(-\omega_{01}) + 1] |\langle 00|\hat{a}|01\rangle|^2. \quad (\text{D8})$$

Here, $P(n) = [1 - e^{-\hbar\omega_R/k_B T_R}] e^{-n\hbar\omega_R/k_B T_R}$ is the probability of having n photons in a resonator with mode temperature T_R , κ is the total loss rate of the resonator, and $n_{\text{th}}(\omega) = 1/(e^{\hbar\omega/k_B T} - 1)$. The states $|00\rangle$ correspond to the dressed state of the coupled qubit-resonator system with the biggest overlap to $|0\rangle_R|0\rangle_Q$ in the bare basis.

For the fit in Fig. 4, a qubit-mode temperature of $T_Q = 45$ mK, a nominal resonator-mode temperature $T_R = 50$ mK, and the total loss rate at the low sweet spot $\kappa_R/2\pi = 922$ kHz have been used. However, the qubit-mode temperature obtained before the sample was fully thermalized to the base plate of the cryostat was

$T_Q = 112 \pm 2$ mK. The minimum measured qubit temperature was 45 mK, 5 months after the cooldown. In Fig. 7, the effect of the qubit-mode temperature on the quasiparticle and dielectric losses is shown. Depending on the qubit-mode temperature, the quasiparticle density [Fig. 7(a)] and the dielectric quality factors [Fig. 7(b)] need adjustment to fit the same qubit lifetimes. From a qubit-spectroscopy measurement, we set the upper limit on the resonator-mode temperature to $T_R = 112 \pm 29$ mK. For this temperature, the Purcell limit set on the qubit lifetime lies below the measured data. As the biggest contributors to the Purcell decay around $\Phi_{\text{ext}}/\Phi_0 = 0.5$ are the coupling strength g_{QR} and the resonator temperature, we vary both, as shown in Fig. 8. The lower temperature of 50 mK is chosen to be slightly above the minimum measured qubit-mode temperature, while the variation of the coupling strength corresponds to 10% of its nominal value (refer to Table I). Even with a reduced interaction strength, the Purcell limit at $T_R = 112$ mK lies below the measured fluxonium lifetimes, which dictates our choice to use $T_R = 50$ mK instead. For the lower resonator-mode temperature and reduced coupling strength, the qubit lifetime is fully Purcell limited around the low sweet spot.

To justify the limits that we have set on the inductive losses, we plot the bound by both Purcell decay and inductive loss with varying quality factors (Fig. 9). The biggest impact of these dissipation channels is around the low sweet spot. When dielectric loss is included, the total bound is lowered further, imposing a high inductive quality factor to fit the qubit lifetimes around $\Phi_{\text{ext}}/\Phi_0 = -0.5$.

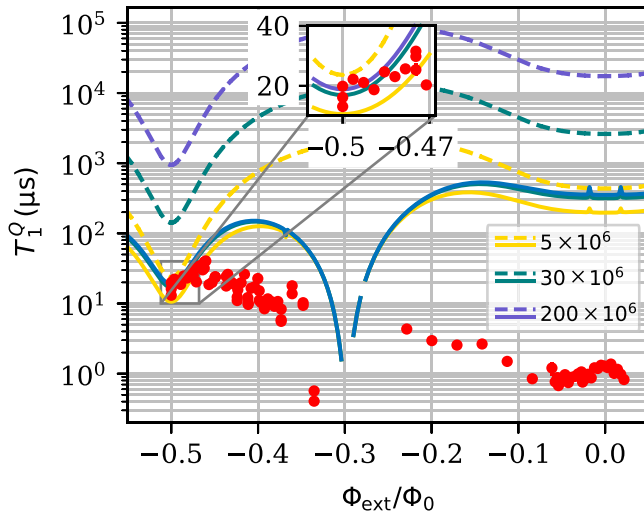


FIG. 9. The limit for Purcell decay and inductive losses with a varying inductive quality factor. The solid line represents the sum of inductive and Purcell loss, while the dashed line is only the limit set by inductive loss. The inset focuses on the illustrated losses around the low sweet spot.

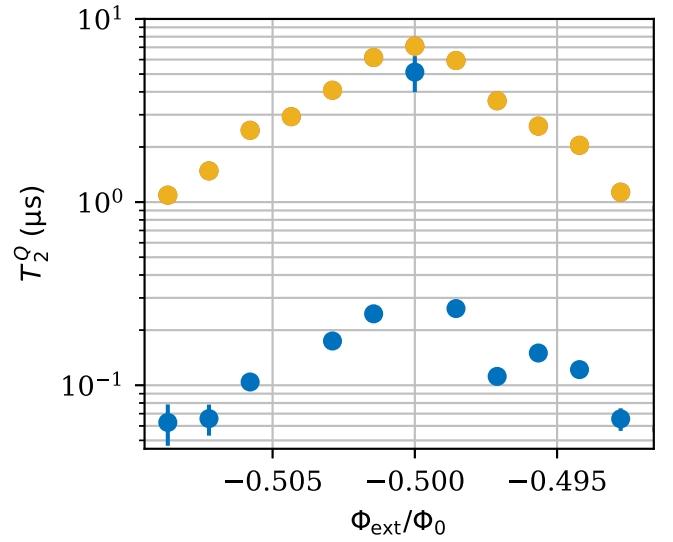


FIG. 10. Fluxonium coherence times from Ramsey T_2^R (blue) and echo T_2^E (yellow) measurements as a function of the flux bias. The sharp drop of T_2^R away from the sweet spot, compared to T_2^E , indicates that the main coherence limitation is low-frequency flux noise.

APPENDIX E: FLUXONIUM DECOHERENCE

Ramsey, T_2^R , and echo, T_2^E , times provide valuable insight into the frequency components of the environment noise to which the qubit couples. Ramsey measurements capture the inhomogeneous dephasing time caused by both environmental flux noise and other sources of low-frequency noise. Echo T_2^E measurements provide complementary information about higher-frequency components of the noise spectrum. We address the flux stability in our system by measuring both the T_2^R and T_2^E times of the fluxonium as a function of the flux bias (see Fig. 10). At the flux sweet spot $\Phi_{\text{ext}} = -0.5\Phi_0$, the qubit frequency is first-order insensitive to flux variations. At this flux bias, the Ramsey time approaches the echo time but decreases rapidly away from the sweet spot. This behavior points toward the main source of dephasing coming from low-frequency $1/f$ flux noise. Further filtering and shielding of the setup as well as the flux-bias line is required to reduce the susceptibility of the fluxonium to flux noise.

APPENDIX F: MEASUREMENT METHODS

The dispersive shift between the resonator and the fluxonium is much higher than both the linewidth of the resonator and the qubit, allowing for easy extraction of the quantity as a function of the flux bias. For Fig. 3(a), several approaches have been used. First, because of the high mode temperature of the qubit, the excited-state population is non-negligible around $\Phi_{\text{ext}}/\Phi_0 = 0.5$, causing the resonator response to split by an amount equal to the dispersive shift. The qubit-mode temperature has been calculated from the depth ratio of the two resonances and

TABLE I. The system parameters. The cavity frequency $\omega_C/2\pi$ has additional uncertainty of 400 kHz coming from the 15% variation in g_{RC} . Due to the dephasing at $\Phi_{\text{ext}}/\Phi_0 = 0.5$ caused by the excited qubit population, the cavity coupling rate and the linewidth have not been measured at this flux bias.

Parameter	Symbol	Value
Josephson energy	E_J/h	10.8 GHz
Inductive energy	E_L/h	1.014 GHz
Capacitive energy	E_C/h	3.5 GHz
Bare high- Q cavity frequency	$\omega_C/2\pi$	4.535854 GHz
Bare readout-resonator frequency	$\omega_R/2\pi$	6.8176 GHz
Qubit-resonator coupling	$g_{QR}/2\pi$	25.2 MHz $\pm$ 10%
Resonator-high- Q -cavity coupling	$g_{RC}/2\pi$	8 MHz $\pm$ 15%
High- Q -cavity lifetime at $\Phi_{\text{ext}}/\Phi_0 = 0.5$	T_1^C	210 $\pm$ 40 μ s
Qubit residual-mode temperature at $\Phi_{\text{ext}}/\Phi_0 = 0.5$	T_Q	112 $\pm$ 2 mK
Qubit excited-state population at $\Phi_{\text{ext}}/\Phi_0 = 0.5$	P_e	39.1 $\pm$ 0.7 %
Resonator residual-mode temperature at $\Phi_{\text{ext}}/\Phi_0 = 0.5$	T_R	110 $\pm$ 30 mK
Readout-resonator linewidth at $\Phi_{\text{ext}}/\Phi_0 = 0.5$	$\kappa_R/2\pi$	922 $\pm$ 3 kHz
Readout-resonator external-coupling rate at $\Phi_{\text{ext}}/\Phi_0 = 0.5$	$\kappa_R^{\text{ext}}/2\pi$	549 $\pm$ 2 kHz
Readout-resonator linewidth at $\Phi_{\text{ext}}/\Phi_0 = 0$	$\kappa_R/2\pi$	956 $\pm$ 4 kHz
Readout-resonator external-coupling rate at $\Phi_{\text{ext}}/\Phi_0 = 0$	$\kappa_R^{\text{ext}}/2\pi$	801 $\pm$ 2 kHz
Cavity linewidth at $\Phi_{\text{ext}}/\Phi_0 = 0$	$\kappa_C/2\pi$	550 $\pm$ 20 Hz
Cavity external-coupling rate at $\Phi_{\text{ext}}/\Phi_0 = 0$	$\kappa_C^{\text{ext}}/2\pi$	233 $\pm$ 7 Hz
Dielectric quality factor at 6 GHz	Q_{diel}	$\gtrsim 0.15 \times 10^6$
Inductive quality factor	Q_{ind}	$\gtrsim 30 \times 10^6$
Nonequilibrium quasiparticle density	x_{qp}	$\lesssim 1.25 \times 10^{-6}$

the measured qubit frequency. At higher qubit frequencies, the qubit cannot be thermally excited by the bath and the resonator has a single resonance. At these flux-bias points, a two-tone measurement has been needed, in which an extra tone with varying frequency around the qubit frequency has been applied while the resonator spectroscopy has been monitored. Once the frequency has been correct, the resonance splitting has been noted. At the high sweet spot $\Phi_{\text{ext}}/\Phi_0 = 0$, χ_{QR} has been extracted from qubit spectroscopy after pulsing the resonator.

Despite the dispersive shift χ_{QC} being smaller than the fluxonium decoherence rate, using large displacement pulses on the cavity enhances the system response, making it possible to extract the Hamiltonian parameters. This technique has been implemented in Ref. [2,27]. When the cavity is displaced to α_0 , due to its coupling to the qubit, the coherent state will start rotating in phase space around the origin. The rotation frequency will depend on the qubit state: $\Delta_g = \Delta - 2K_C\alpha_0^2$ and $\Delta_e = \Delta - \chi_{QC} - (2K_C + 2\chi_{nl})\alpha_0^2$, where Δ is the detuning between the cavity drive and the mode frequency, K_C is the Kerr shift of the cavity, and χ_{nl} is a higher-order dispersive shift.

By applying a selective 1.2- μ s Gaussian pulse, we prepare the qubit in the ground or excited state with the help of the parametric amplifier. The excited-state population in the steady state is around 36.5%; after preparation in the ground state, it is reduced to 27.7%. The preparation of the qubit in an excited state with active reset gives us 73.5% excited-state population, compared

to 51.5% without feedback from the parametric amplifier. The preparation fidelity is limited by the qubit lifetime and the external coupling rate of the readout resonator. Following the state preparation, we displace the cavity with a 40-ns square pulse. After a fixed time of 1 μ s, we try to displace the cavity back to the origin by applying a pulse with varying phases. We repeat this out-and-back procedure 5 times to enhance the measurement sensitivity. At the end of the protocol, we flip the qubit with a narrow-bandwidth π pulse and then read it out with a 2 μ s square pulse. The π pulse is only successful if the phase for the displacement back to the cavity ground state is correct. We repeat the procedure for different displacements α_0 . The correct phase of the cavity pulse is then converted to frequency. We fit the data for $\Delta_e - \Delta_g$ to a linear regression as shown in Fig. 11, where the intercept gives us the dispersive shift at the low sweet spot $\chi_{QC}/2\pi = -15.6 \pm 2$ kHz. Similarly, the detuning can be determined from fitting Δ_g , from where we extract $\Delta/2\pi = -24.21 \pm 0.03$ kHz.

To extract the slope parameters (K_C and χ_{nl}), the cavity drive strength is calibrated. We prepare the qubit in a superposition state by applying a $\pi/2$ pulse. We then apply a sequence similar to the echoed conditional displacement [27], without the π pulse in between, i.e., $\hat{D}(\alpha)\hat{T}(t_w)\hat{D}(-r\alpha)\hat{D}(-r\alpha)\hat{T}(t_w)\hat{D}(\alpha)$, where $\hat{D}(\alpha) = e^{\alpha\hat{a} - \alpha^*\hat{a}^\dagger}$ is the displacement by α , $\hat{T}(t_w)$ allows the system to evolve for time $t_w = 200$ ns, and r is the correction ratio to make the pulses more accurate. During the sequence, the cavity and qubit are entangled. After disentangling

the two subsystems, the trajectories for the ground and excited qubit states will have acquired different phases. We measure this phase by either applying $\hat{X}_{\pi/2}$ or $\hat{Y}_{\pi/2}$ and reading out the qubit population $\langle\sigma_x\rangle$ and $\langle\sigma_y\rangle$. We repeat the sequence for different cavity-pulse amplitudes. To fit the curves, the semiclassical trajectories are numerically simulated, allowing us to extract the enclosed area for each drive power and convert it to the accumulated phase difference, which we later fit to the measured $\langle\sigma_x\rangle$ and $\langle\sigma_y\rangle$. This gives us a conversion factor $\sqrt{\text{const}}$ from Fig. 11 between the pulse voltage and the displacement that we can achieve. For more details on the exact sequence and the semiclassical trajectories, see the supplementary material of Ref. [27].

As this method is time-consuming, only the dispersive shift at $\Phi_{\text{ext}}/\Phi_0 = 0.5$ has been measured this way. As the cavity frequency does not change much as a function of flux threading the fluxonium loop, we use the same photon-number calibration from the measured dispersive shift for the rest of the points in Fig. 3(b). These points have been extracted by monitoring the qubit frequency after the cavity had been displaced to a different coherent state. We fit the qubit resonance shift observed as a function of the photon number and extract the dispersive shift $\chi_{QC}/2\pi$. These subsequent measurements have allowed for fast characterization of the dispersive shift across the entire flux map.

With the gained knowledge of the system, the lifetime of the storage mode has been measured. We displace the cavity and let it decay while recording the characteristic function as we wait (details can be found in the supplementary material of Ref. [2]). The coherent state will decay

from the cavity as $\alpha(t) = \alpha_0 e^{-i\Delta t - \frac{t}{2T_1^C}}$. The characteristic function of such a coherent state in the cavity is described as $\mathcal{C}(\beta, t) = e^{\frac{|\beta|^2}{4}} e^{-i\Im\{\alpha(t)\beta^*\}}$, where β determines the point of phase space where the function is measured. From $\mathcal{C}(\beta, t)$ for $\beta \in \mathbb{R}$, we extract $\alpha(t)$ at each recorded time and fit it to extract the cavity lifetime. This procedure has only been feasible at $0.5\Phi_0$, as the fluxonium coherence time has been insufficient at other flux-bias points.

APPENDIX G: PURCELL LIMIT ON THE CAVITY

The cavity mode is constantly coupled to the read-out resonator, creating the possibility for excitations to leak out via Purcell decay. However, the measured cavity lifetime is below the Purcell limit $T_{|1\rangle_C \rightarrow |0\rangle_C}^{\text{Purcell}} \approx 2.8$ ms, which has been extracted from simulations, as described in Appendix D. This limit is constant throughout the whole flux-bias range, except for the region in which the fluxonium first excited state crosses the fundamental cavity mode. The model assumes no internal losses of the fluxonium.

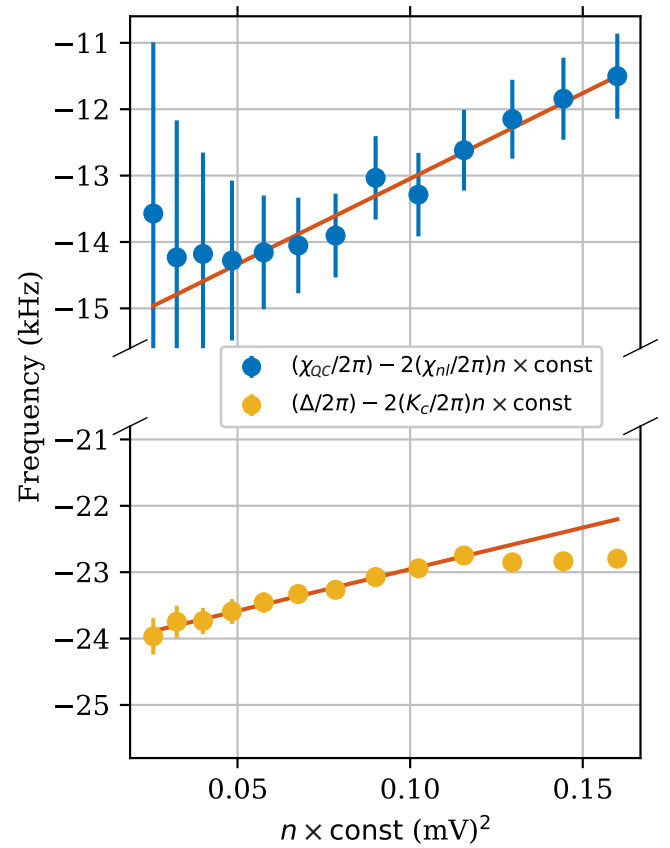


FIG. 11. The cavity-frequency dispersion at $\Phi_{\text{ext}} = 0.5\Phi_0$ as a function of the pulse voltage squared. The rotation frequency $\Delta_g/2\pi$ is plotted in yellow, while the difference $(\Delta_e - \Delta_g)/2\pi$ is plotted in blue. The data sets are fitted by linear regression (red). The voltage squared is proportional to the number of photons in the cavity up to a constant. From the fit, we extract $\Delta/2\pi = -24.21 \pm 0.03$ kHz and $\chi_{QC}/2\pi = -15.6 \pm 2$ kHz, the latter of which we use to find the unknown constant and thus calibrate the photon number n .

APPENDIX H: QUBIT-INDUCED DEPHASING OF THE CAVITY

When the qubit relaxation rate Γ_1^Q is much smaller than the dispersive shift $|\chi_{QC}|$, the cavity dephasing time depends solely on the thermal population of the qubit and its lifetime [13,46]. If this relationship swaps, i.e., $|\chi_{QC}| \ll \Gamma_1^Q$ (around the zero-crossing flux bias), the cavity dephasing becomes proportional to χ_{QC}^{-2} ,

$$T_\phi^C = \frac{\Gamma_1^Q}{\chi_{QC}^2 n_{th}^Q}, \quad (\text{H1})$$

where n_{th}^Q is the thermal population of the fluxonium. If we take our best qubit lifetime $T_1^Q \approx 40$ μ s, this equation becomes valid for a dispersive shift $|\chi_{QC}|/2\pi \ll 4$ kHz. Compared to the limit set by the cavity lifetime of 210 μ s and assuming the same thermal excitation of

the fluxonium as measured at $\Phi_{\text{ext}} = -0.5\Phi_0$, the qubit-induced decoherence becomes negligible for $|\chi_{QC}|/2\pi \ll 1.8$ kHz. In fact, this limit is further raised by the smaller thermal occupation of the qubit at higher frequencies.

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