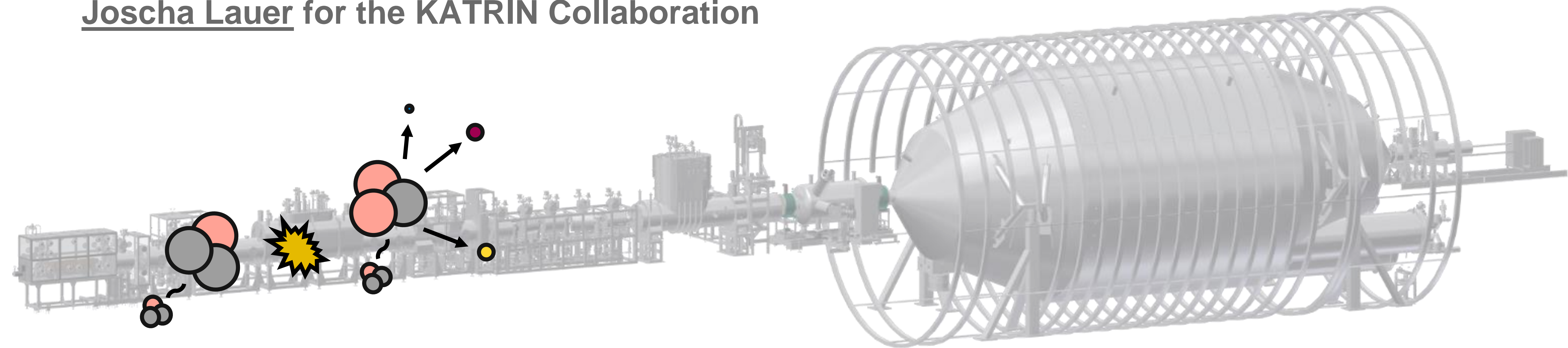




Exploring β -decay with light boson emission in the KATRIN experiment

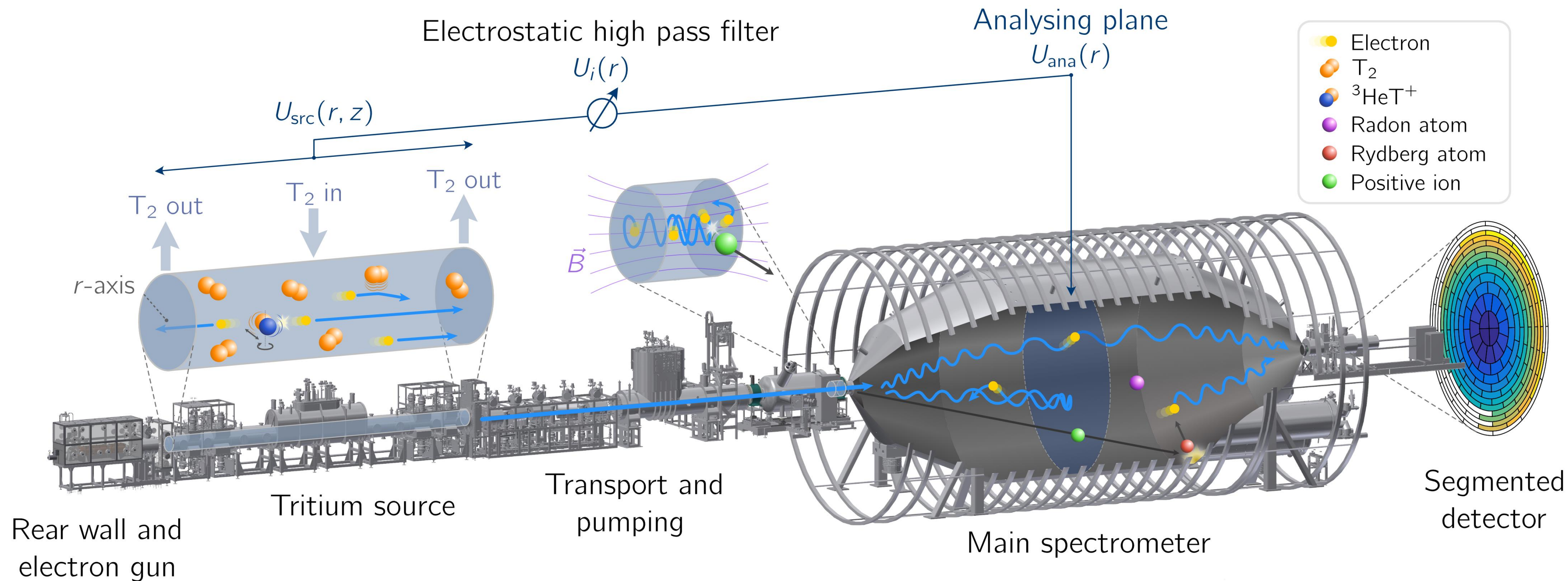
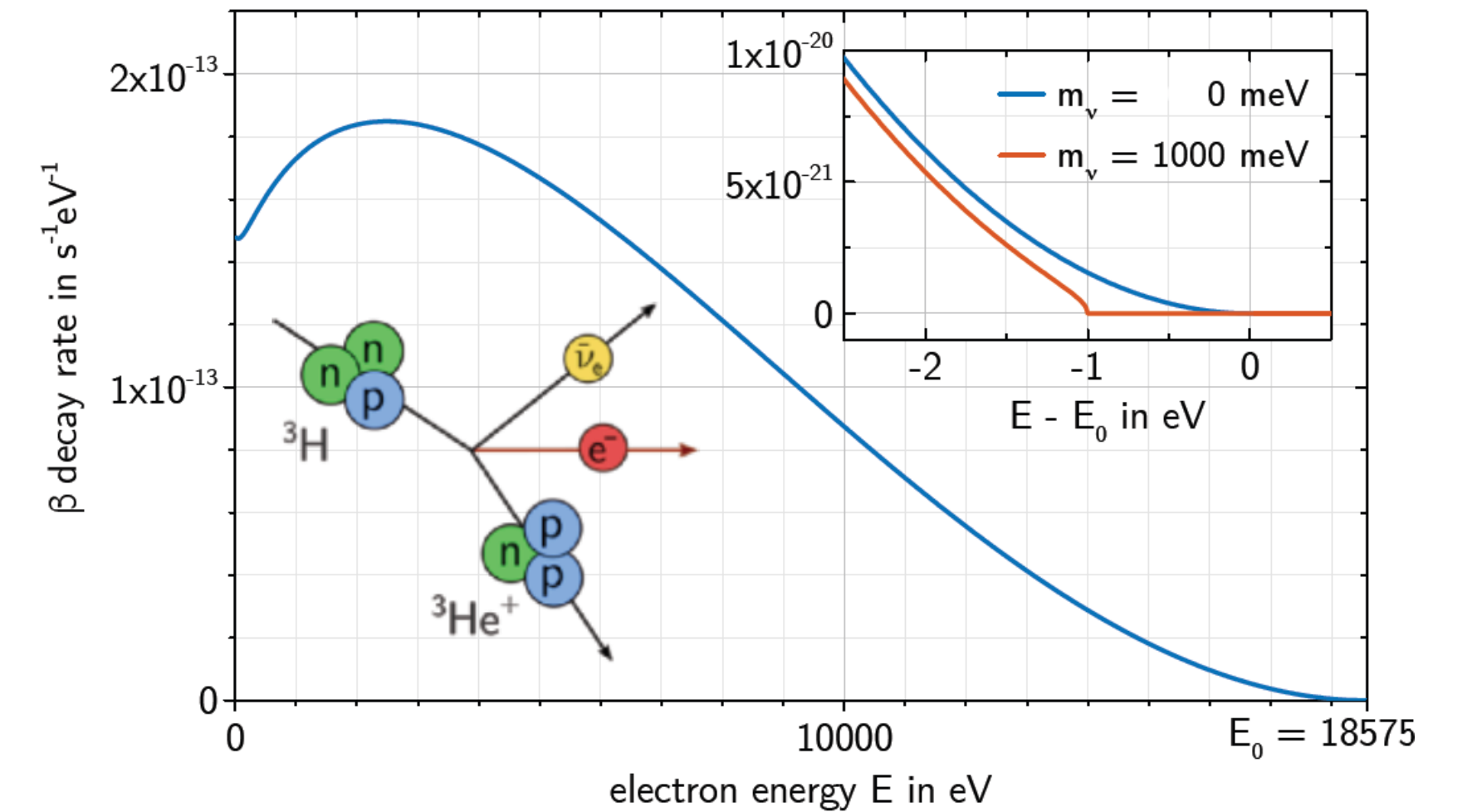
Joscha Lauer for the KATRIN Collaboration



The KATRIN experiment



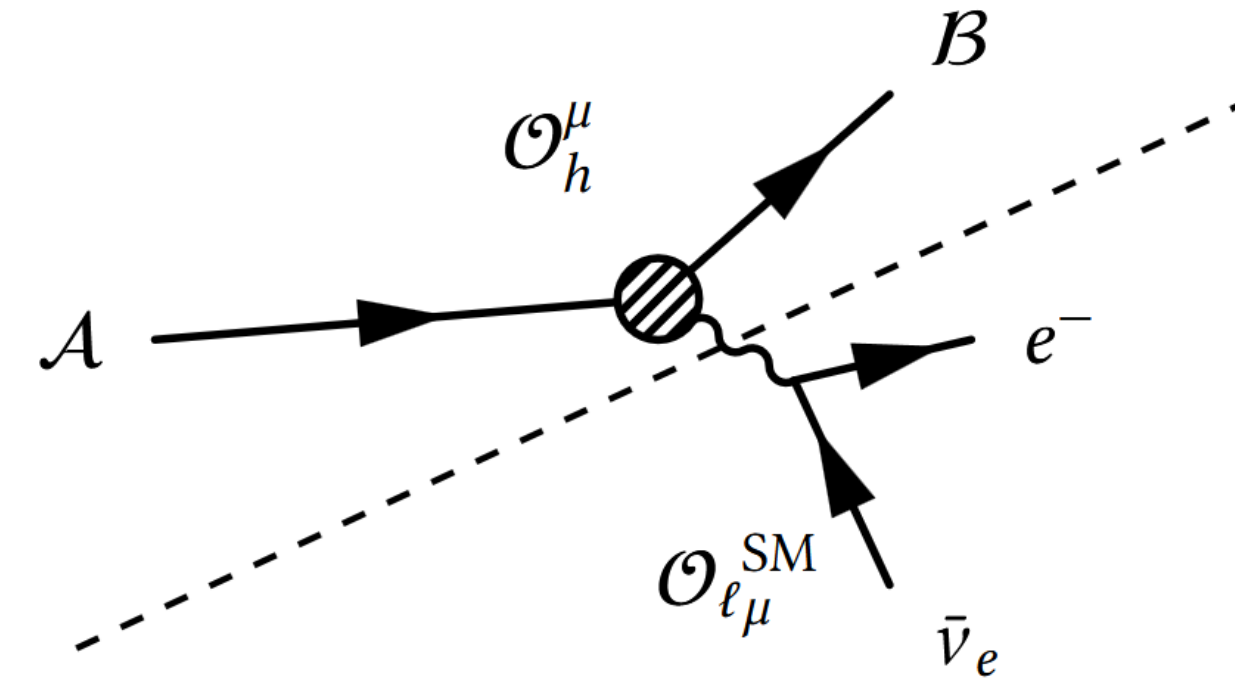
- Observable: effective electron antineutrino mass $m_\nu^2 = \sum_i |U_{ei}|^2 m_i^2$
- Kinematic approach: electron energy spectrum of **tritium β -decay**



Standard Model β -decay of tritium

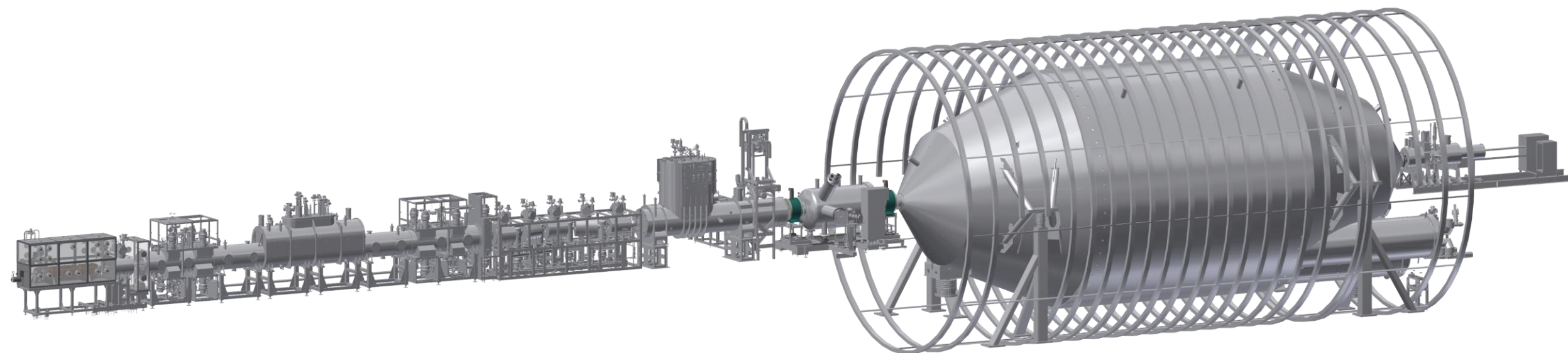
$$\mathcal{A} \rightarrow \mathcal{B} + e^- + \bar{\nu}_e$$

- Fermi's golden rule (decay rate): $d\Gamma = \frac{(2\pi)^4}{2m_{\mathcal{A}}} |\overline{\mathcal{M}}|^2 d\Phi$



$$\mathcal{M} = -\frac{G_F}{\sqrt{2}} (\bar{B} \mathcal{O}_h^\mu \mathcal{A}) (\bar{e} \mathcal{O}_{\ell\mu} \nu)$$

- Differential spectrum $\frac{d\Gamma_\beta}{dE}(E, \mathbf{m}_\nu^2) = C \cdot (E + m_e) \cdot p_e \cdot E_\nu \cdot \sqrt{(E_0 - E)^2 - \mathbf{m}_\nu^2} \cdot \text{Corr}(E)$
- Energy scale: tritium Q-value $\sim \mathbf{E_0 \approx 18.6 keV}$ (kinematic limit)



→ Measurement of **integrated spectrum**
beyond set retarding potential U_{ret}

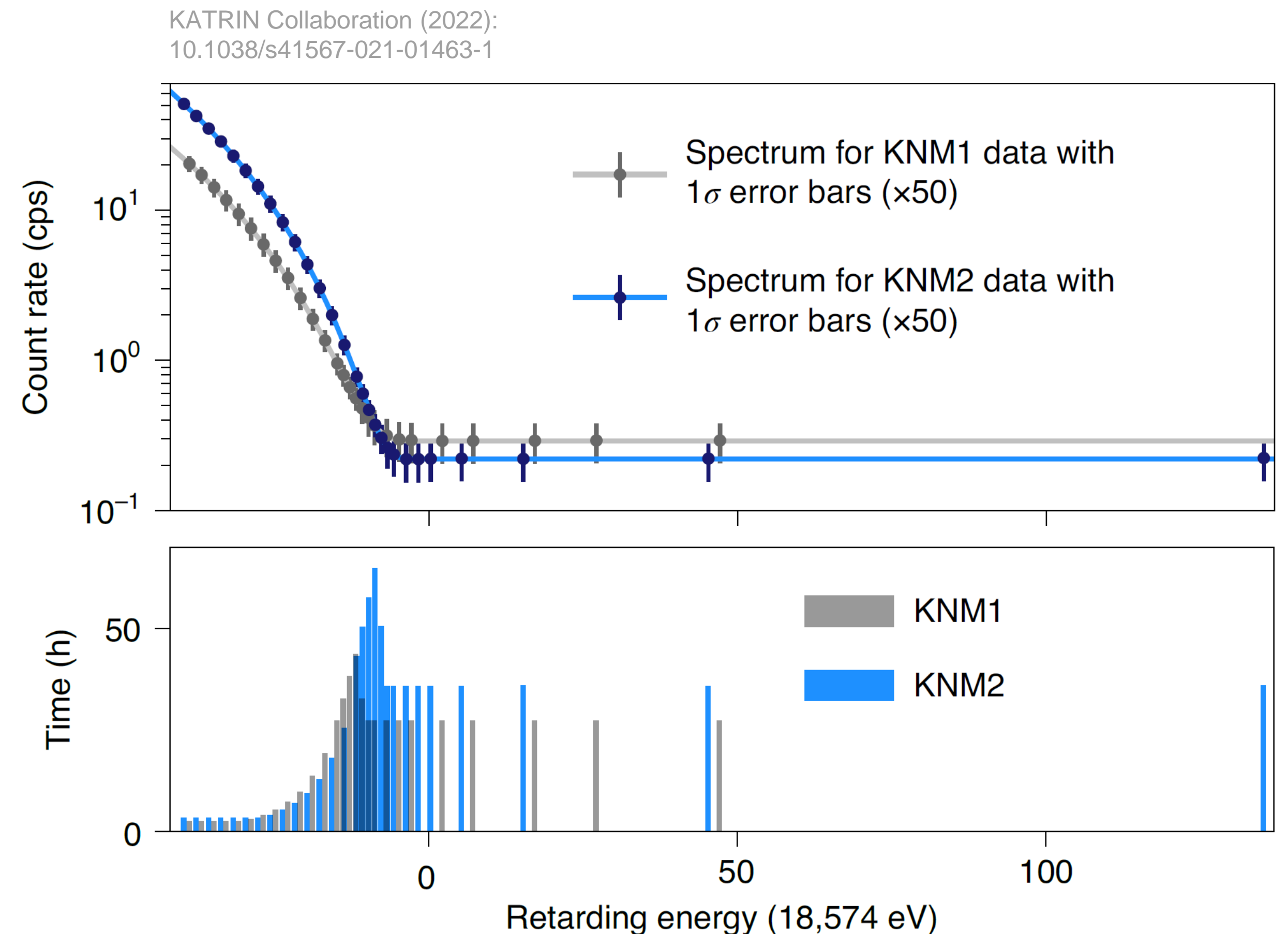
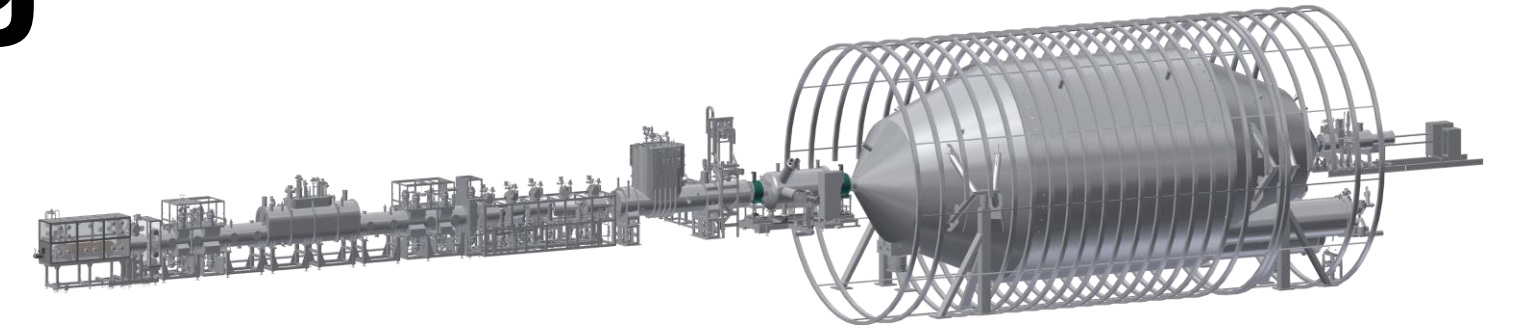
Inferring the neutrino mass – spectrum fitting

- **C++**-based analysis framework *KaFit*
- Precise model evaluation, Gauss-Legendre integration
- $R(qU_{ret}) = A_{\text{Sig}} \int_{qU}^{E_0} f(E - qU_{ret}) \frac{d\Gamma_\beta}{dE}(E, m_\nu^2) dE + R_{\text{Bg}}$
- Four free fit parameters:
 - Neutrino mass m_ν^2
 - Endpoint E_0
 - Amplitude A_{Sig}
 - Background R_{Bg}

$\rightarrow m_\nu < 0.45 \text{ eV (90\% CL)}$

KATRIN Collaboration (2024):
2406.13516

$$\mathcal{A} \rightarrow \mathcal{B} + e^- + \bar{\nu}_e$$



Beyond the SM: emission of new light boson

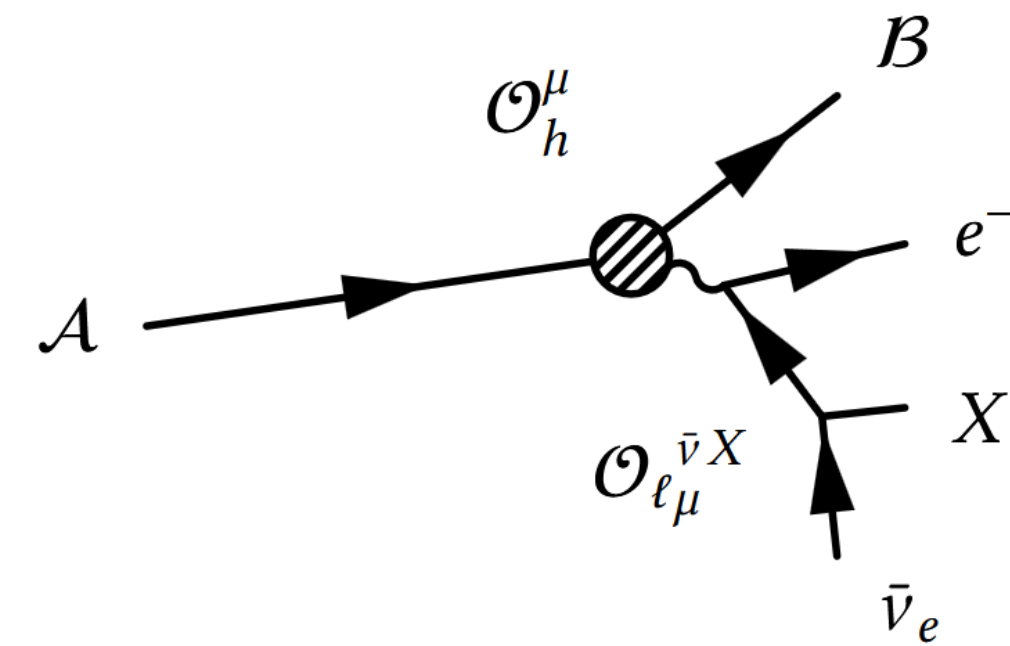
$$\mathcal{A} \rightarrow \mathcal{B} + e^- + \bar{\nu}_e + X$$

- Beyond SM (BSM) theories: new particles (light boson X), interacting with the final state leptons

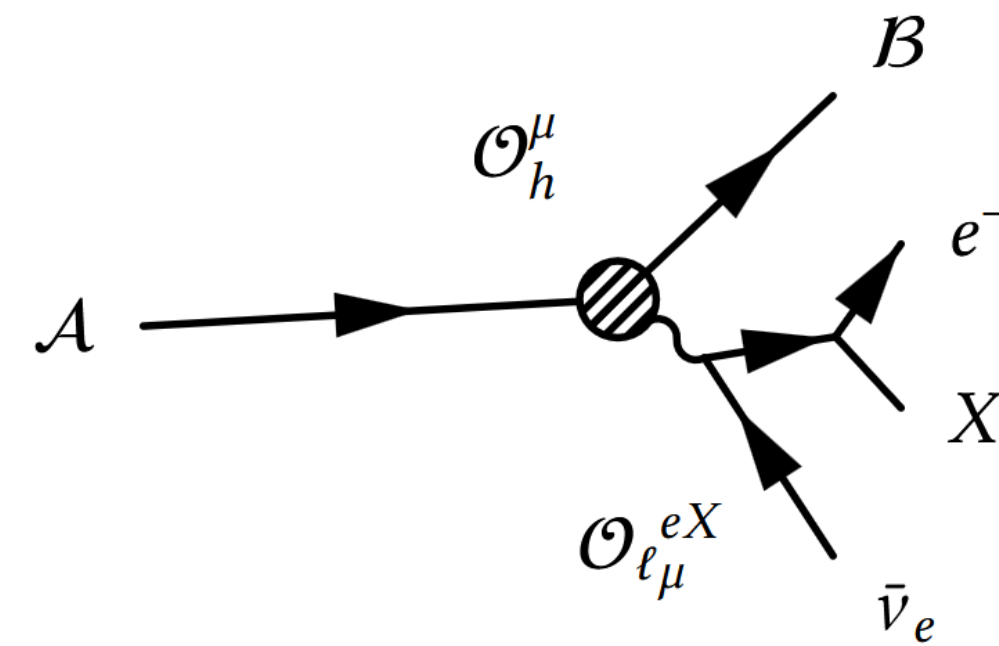
$$ig\bar{\nu}_e\gamma_5\nu_e X, \quad ig\bar{e}\gamma_5 e X, \quad \text{or} \quad g\bar{\nu}_e\gamma^\mu P_L\nu_e X_\mu, \quad g\bar{e}\gamma^\mu e X_\mu, \quad g j_{L_e}^\mu X_\mu$$

ref. Arcadi et al.:

JHEP01(2019)206



(a) boson X coupling to the neutrino $\bar{\nu}$



(b) boson X coupling to the electron e^-

→ Consequence: spectral modification due to emission of the additional real particle X

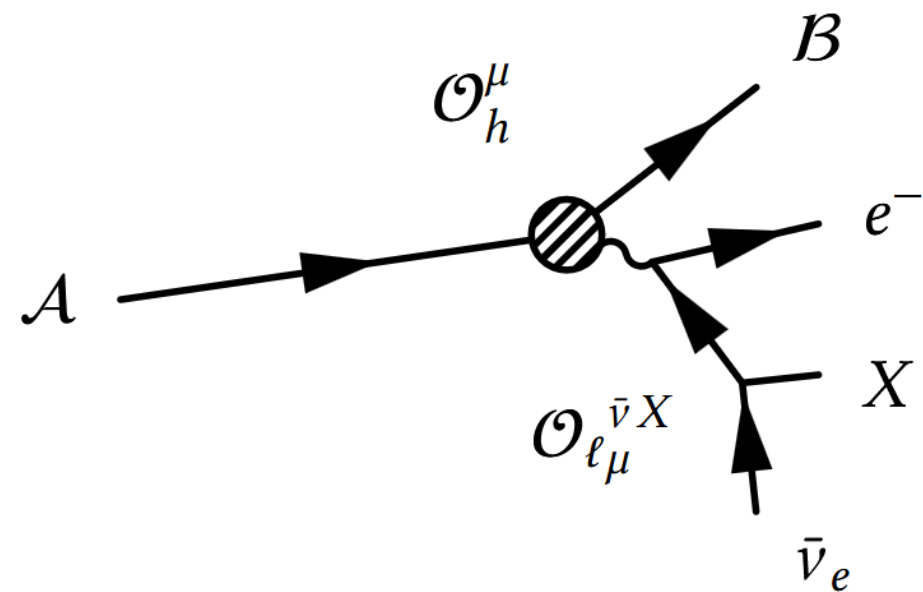
1. *dynamics*: special coupling structures, virtual intermediate leptons

2. *kinematics*: shifted endpoint, four-body final state

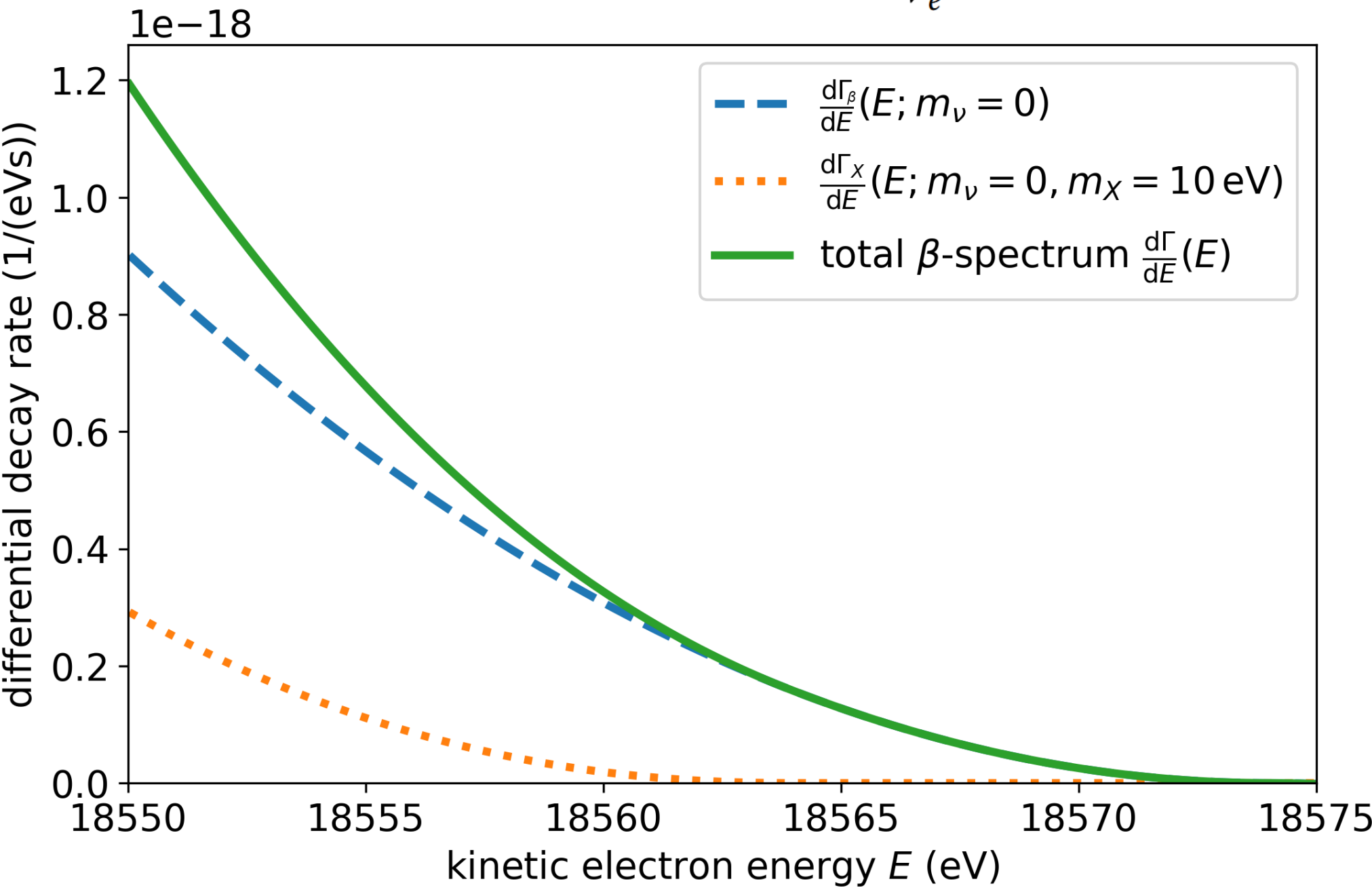
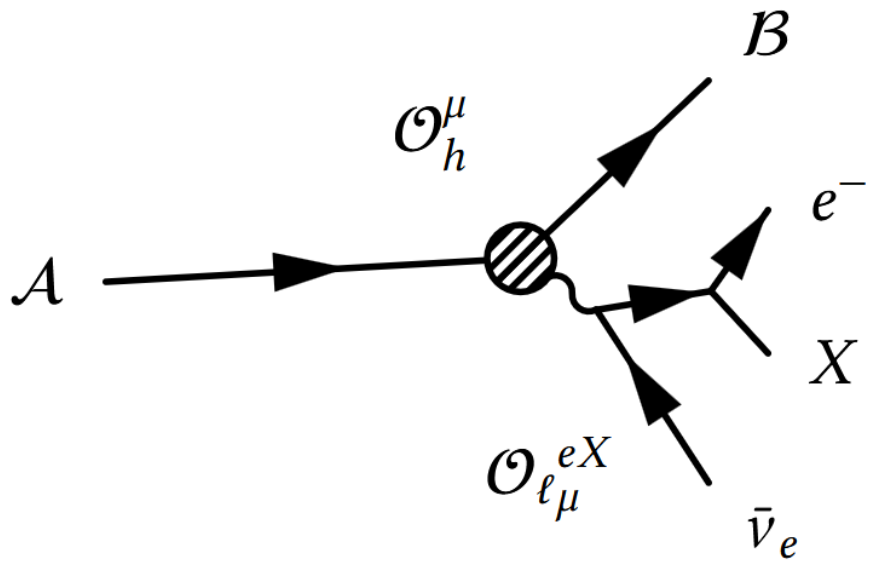
$$d\Gamma = \frac{(2\pi)^4}{2m_{\mathcal{A}}} \overline{|\mathcal{M}|^2} d\Phi$$

Spectral modifications with light bosons

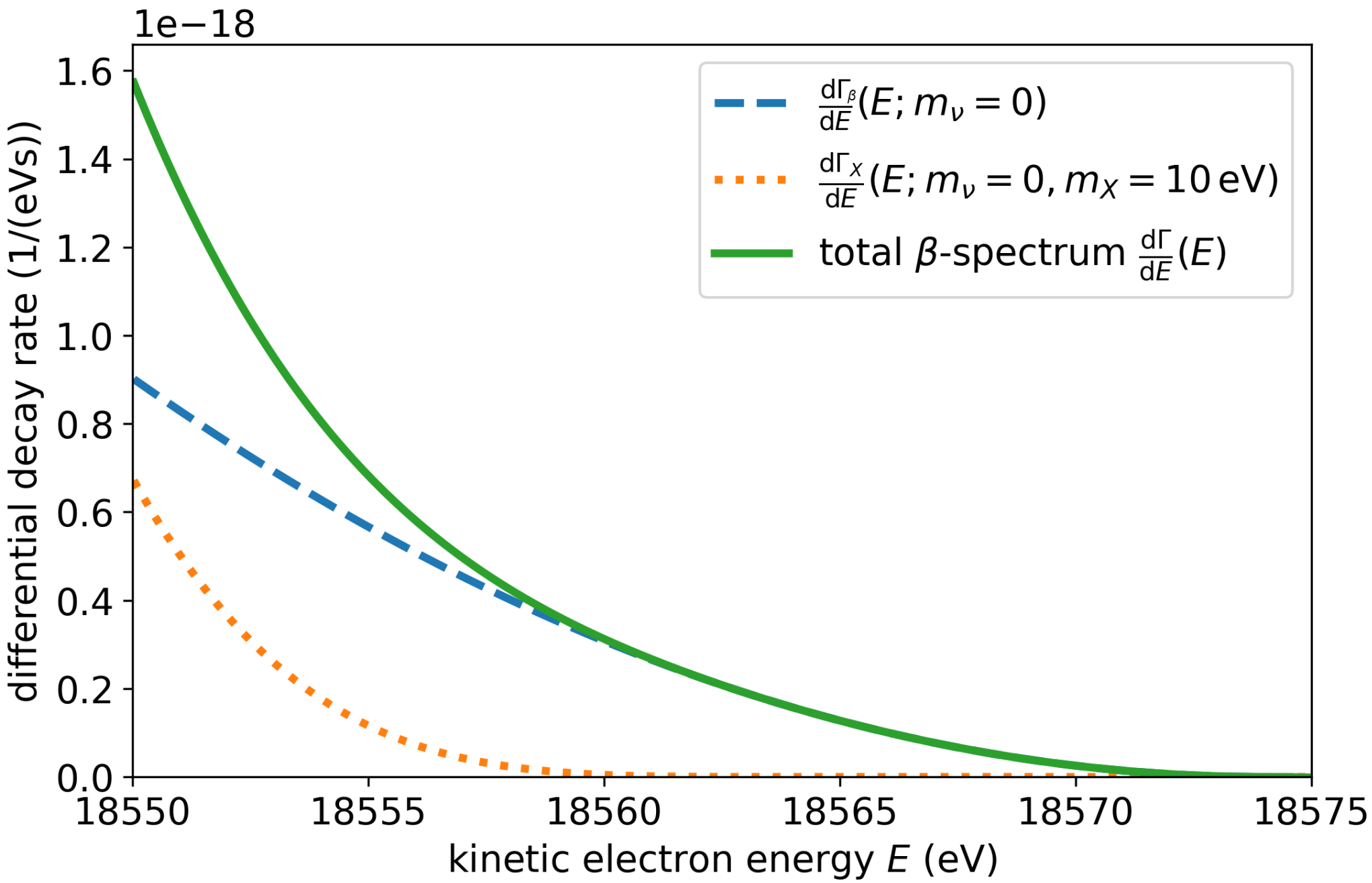
→ Additional decay channel $d\Gamma_{fX}$: $\frac{d\Gamma}{dE_e} = \frac{d\Gamma_\beta}{dE_e} + \frac{d\Gamma_{fX}}{dE_e} \geq \frac{d\Gamma_\beta}{dE_e}$



Example scenarios:
pseudoscalar couplings
 $\mathcal{L} \supset ig_X \bar{\nu} \gamma_5 \nu X, ig_X \bar{e} \gamma_5 e X$



tree-level process:
 $\Gamma \propto g_X^2$ 💡
→ two parameters:
 m_X and g_X



Search for light bosons with KATRIN

JHEP01(2019)206

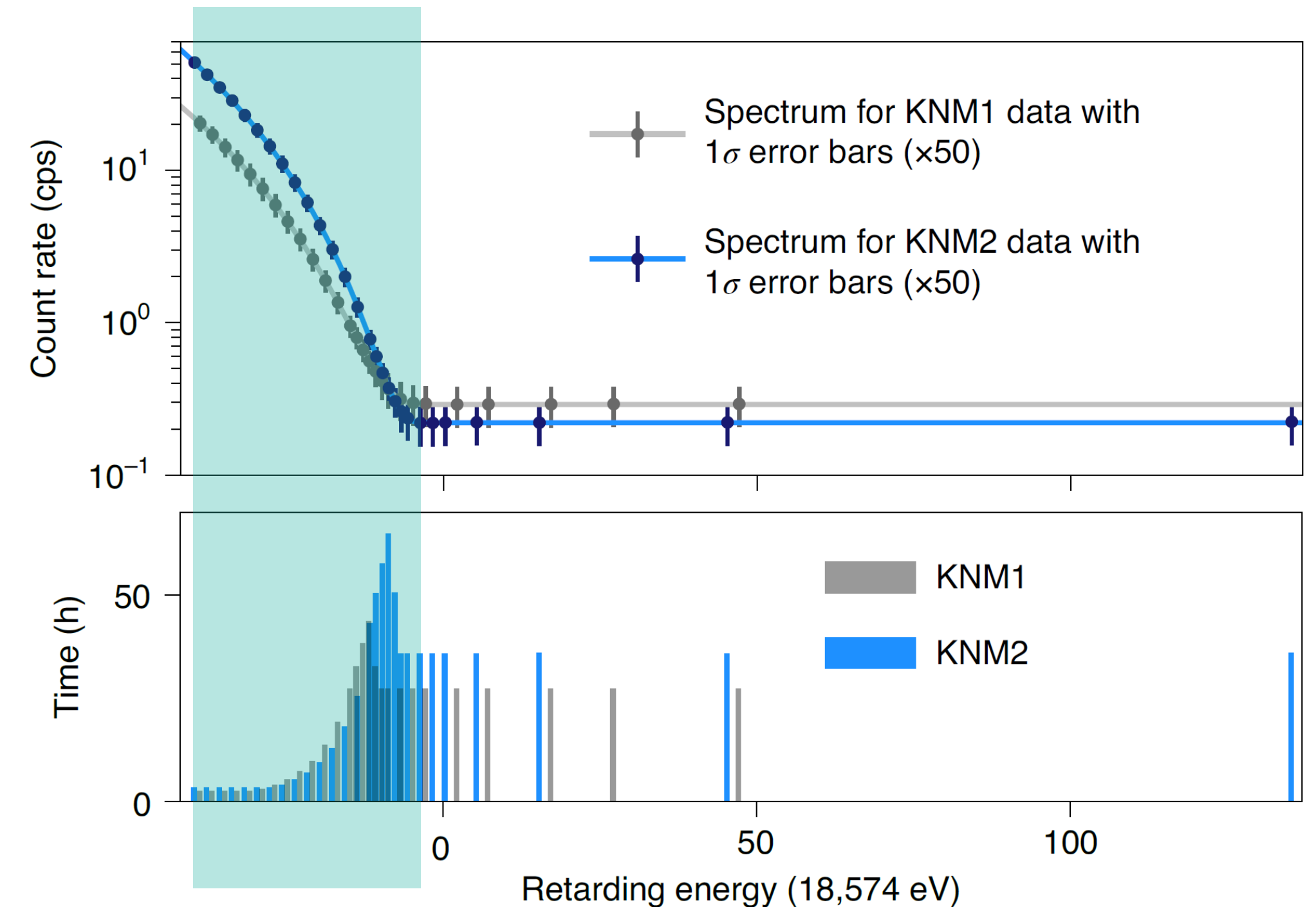
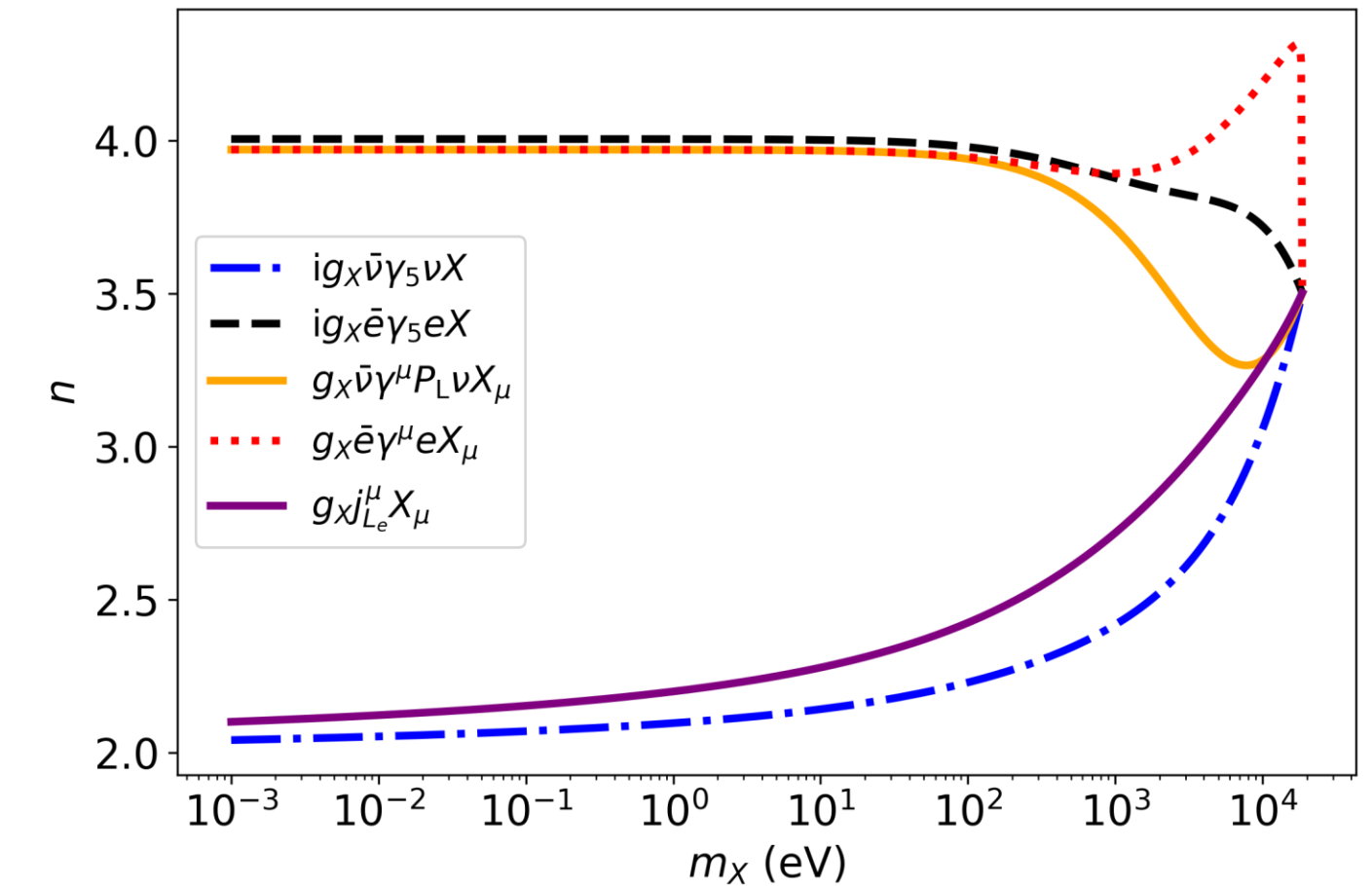
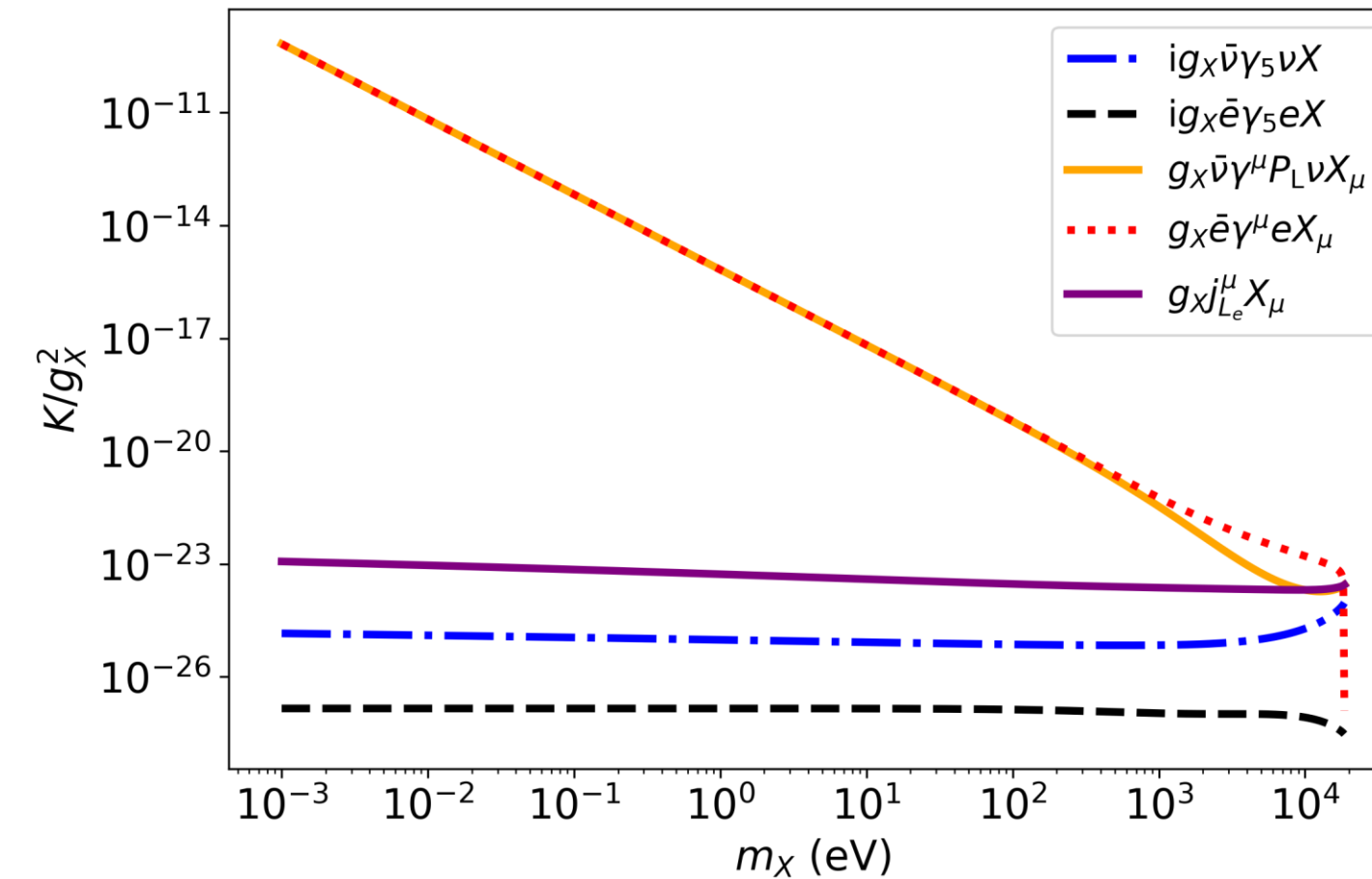
- No general analytic expression for the (differential) rate exists

→ Empirical parametrization by Julian Heeck:

$$\frac{d\Gamma}{dE} = K \sqrt{\frac{E}{m_e}} \left(\frac{E_{\max} - E}{E_{\max} + m_e} \right)^n$$

- Semi-analytic approach for evaluation at certain energies E (with $m_\nu = 0$) → fit of K, n
- Analysis procedure for KATRIN: **likelihood scan** over parameter grid in (m_X, g_X)

→ current KATRIN:
masses $m_X \leq 40$ eV



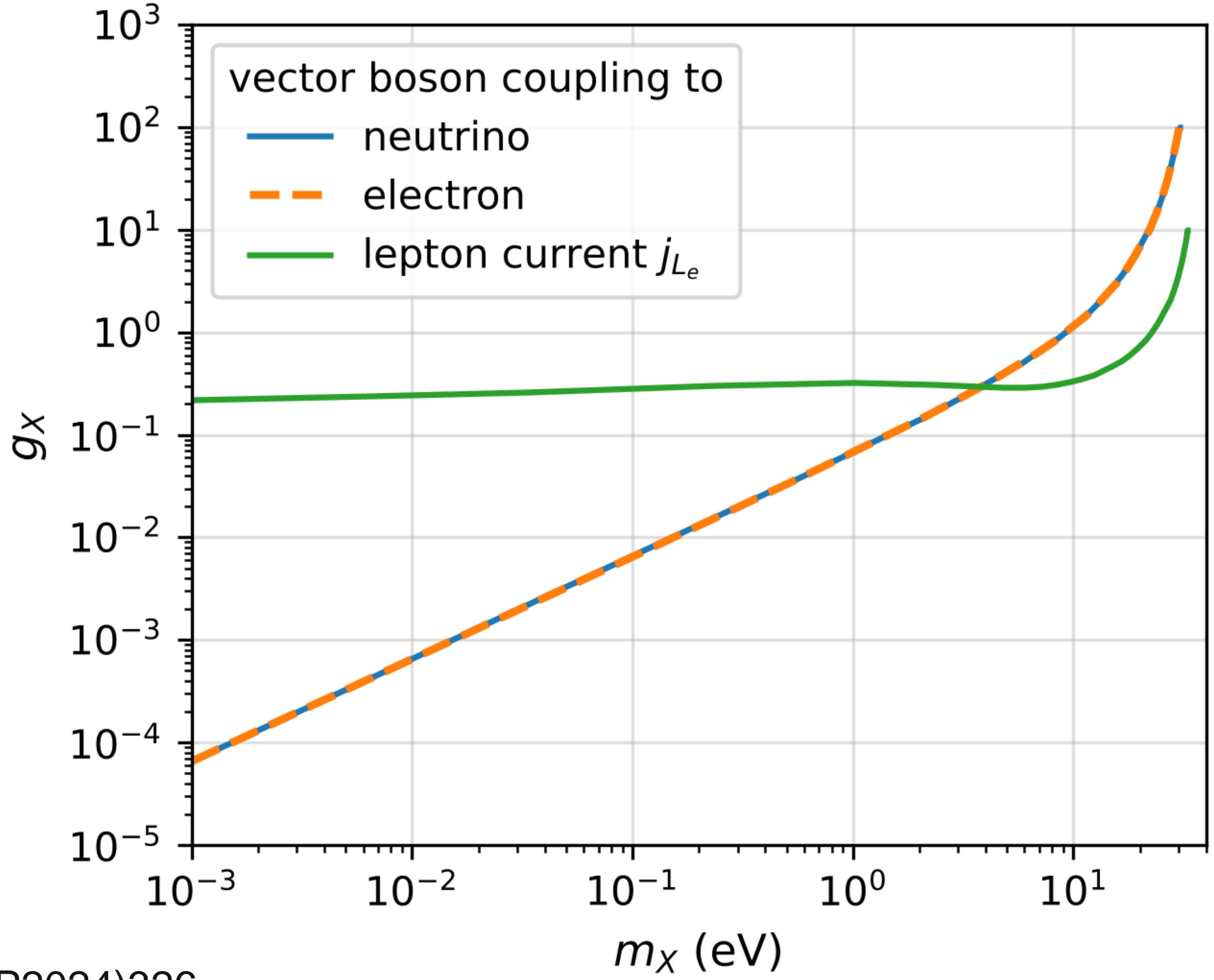
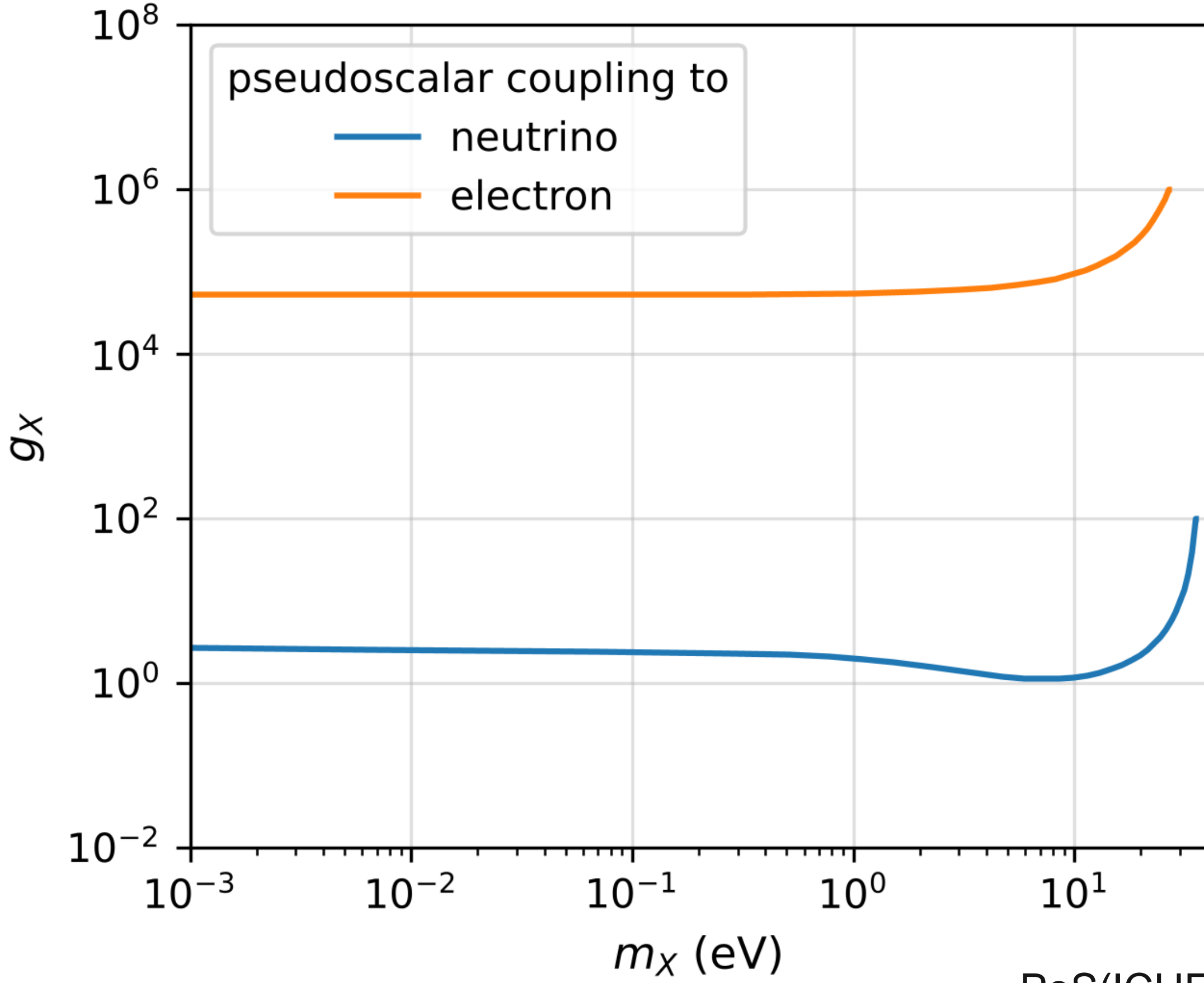
Reference-based light boson *sensitivity* of KATRIN ($m_\nu = 0$)



2nd science run:
 4×10^6 electrons

statistics 
dominated

light boson X coupling: 95% CL sensitivity



PoS(ICHEP2024)326

Exact derivation of the spectra

- **Goal:** extension to explicitly **massive neutrino** ($m_\nu \neq 0$, as opposed to reference)

→ calculation of the spectra from first principle (tree-level): $d\Gamma = \frac{(2\pi)^4}{2m_{\mathcal{A}}} \overline{|\mathcal{M}|^2} d\Phi$

Dynamics:

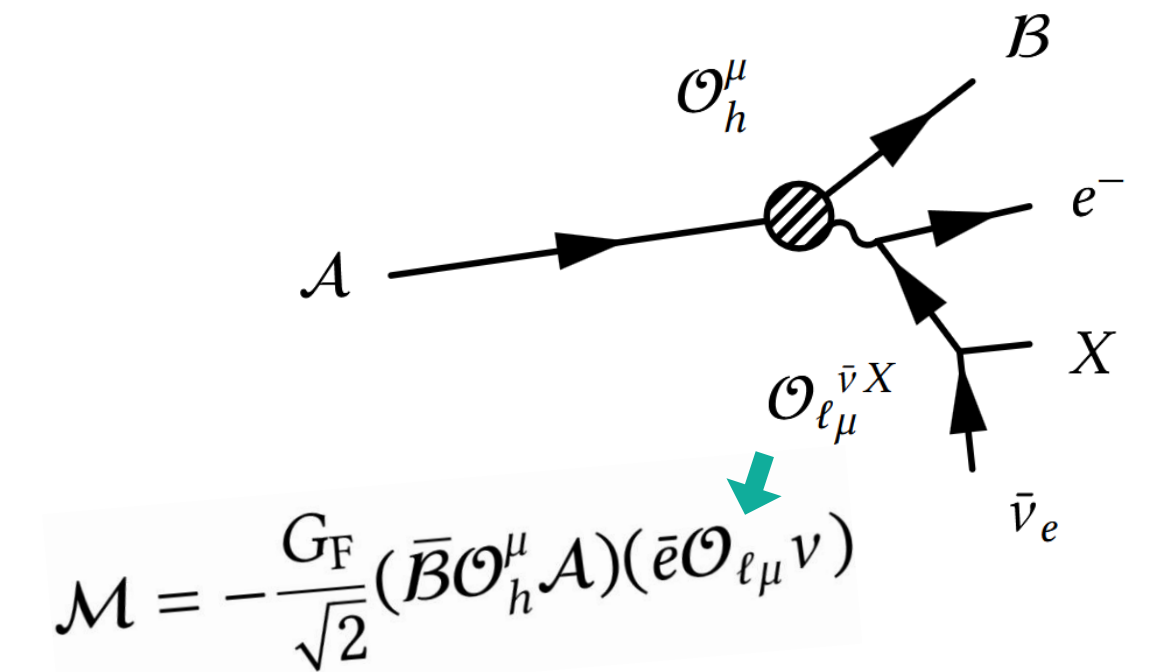
- *Mathematica* with package *FeynCalc* is used to perform the operations for $\overline{|\mathcal{M}|^2} = \overline{\sum |\mathcal{M}|^2} = \frac{1}{2} \sum_{\text{spins}} |\mathcal{M}|^2$

Kinematics:

- Parameterization: 5 independent many-particle mass-squares $M_{i\dots j}^2 = \left(\sum_{k=i\dots j} p_k \right)^2 = (E_i + \dots + E_j)^2 - (\vec{p}_i + \dots + \vec{p}_j)^2$

$$\frac{d\Gamma}{dE_e} = \frac{1}{2^5 (2\pi)^6 m_{\mathcal{A}}^2} \int_{M_{12-}^2}^{M_{12+}^2(E_e)} \int_{M_{34-}^2(E_e, M_{12}^2)}^{M_{34+}^2(E_e, M_{12}^2)} \int_{M_{134-}^2(M_{12}^2, M_{34}^2)}^{M_{134+}^2(M_{12}^2, M_{34}^2)} \int_{M_{14-}^2(E_e, M_{12}^2, M_{34}^2, M_{134}^2)}^{M_{14+}^2(E_e, M_{12}^2, M_{34}^2, M_{134}^2)} \frac{\overline{|\mathcal{M}|^2}}{\sqrt{-B}} dM_{12}^2 dM_{34}^2 dM_{134}^2 dM_{14}^2$$

$$\bar{\nu} \rightarrow 1, \quad X \rightarrow 2, \quad e \rightarrow 3, \quad B \rightarrow 4$$



Spectrum calculation – *numerical integration*

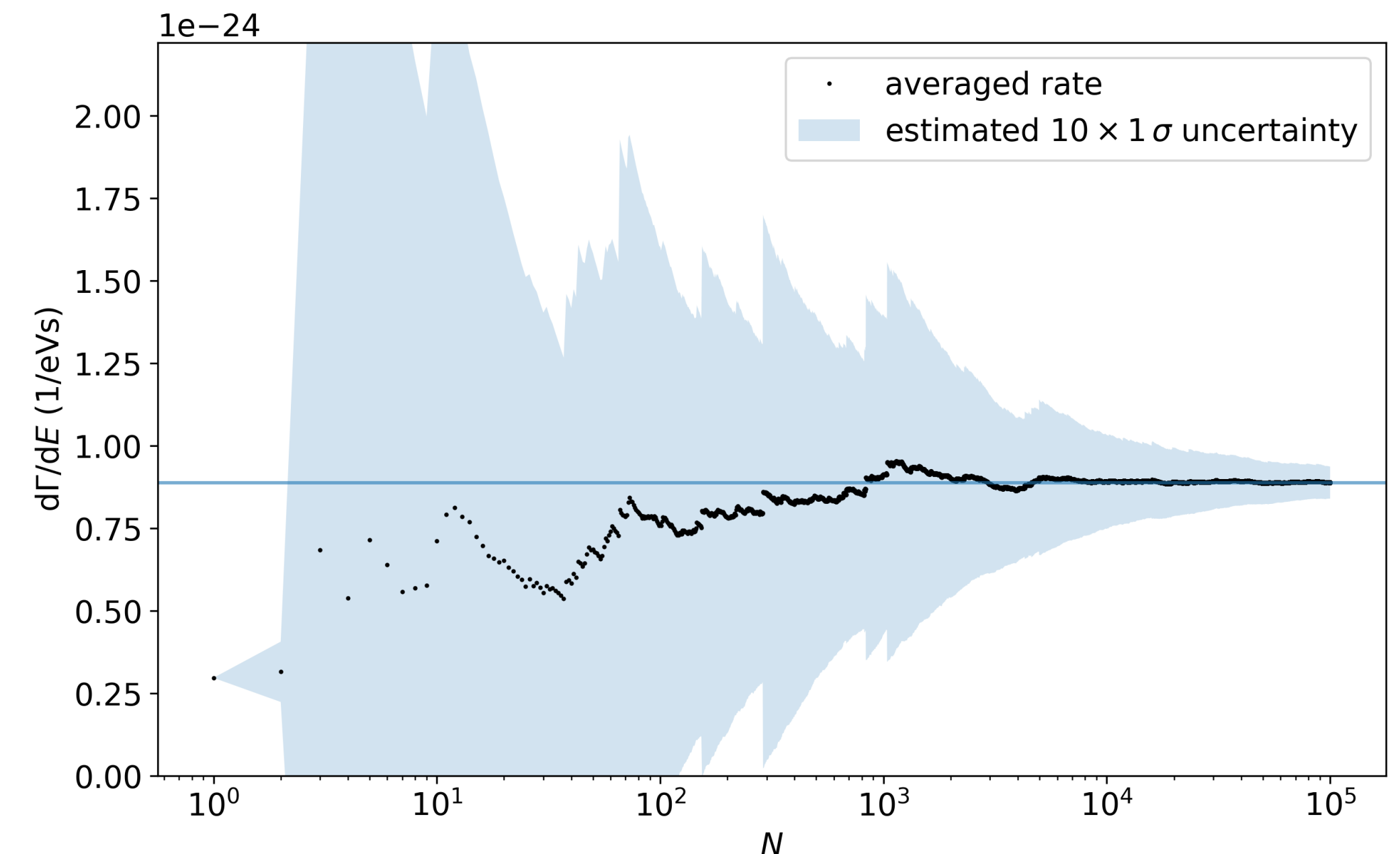
$$\frac{d\Gamma}{dE_e} = \frac{1}{2^5(2\pi)^6 m_{\mathcal{A}}^2} \int_{M_{12-}^2}^{M_{12+}^2(E_e)} \int_{M_{34-}^2(E_e, M_{12}^2)}^{M_{34+}^2(E_e, M_{12}^2)} \int_{M_{134-}^2(M_{12}^2, M_{34}^2)}^{M_{134+}^2(M_{12}^2, M_{34}^2)} \int_{M_{14-}^2(E_e, M_{12}^2, M_{34}^2, M_{134}^2)}^{M_{14+}^2(E_e, M_{12}^2, M_{34}^2, M_{134}^2)} \frac{|\overline{\mathcal{M}}|^2}{\sqrt{-B}} dM_{12}^2 dM_{34}^2 dM_{134}^2 dM_{14}^2$$

- No general exact analytic solution for the integral was found
- Highest level of flexibility and modularity: **MC sampling** of entire phase space

→ statistically converging approximation of the integral, uncertainty $\propto 1/\sqrt{N}$

*Performance
example*

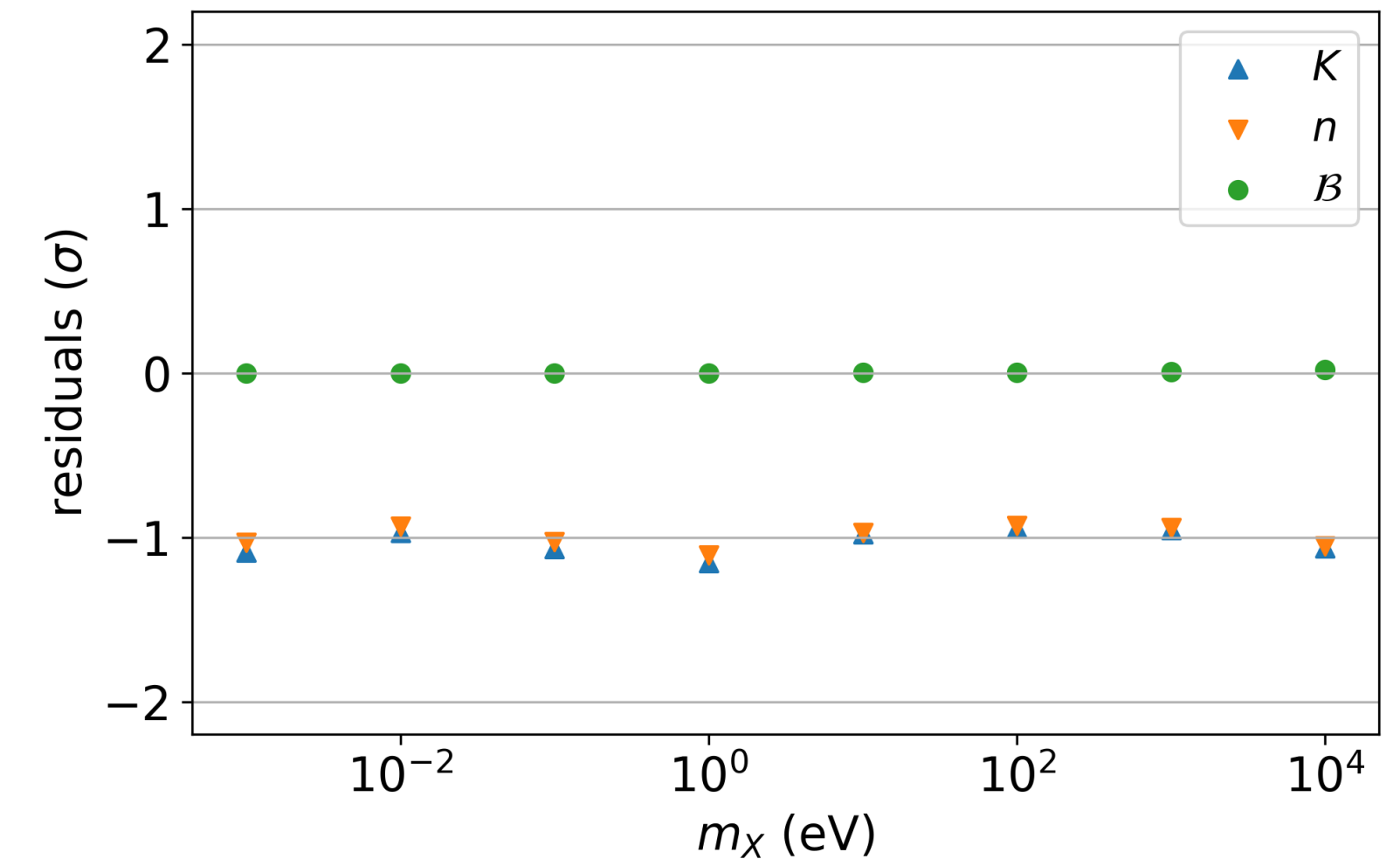
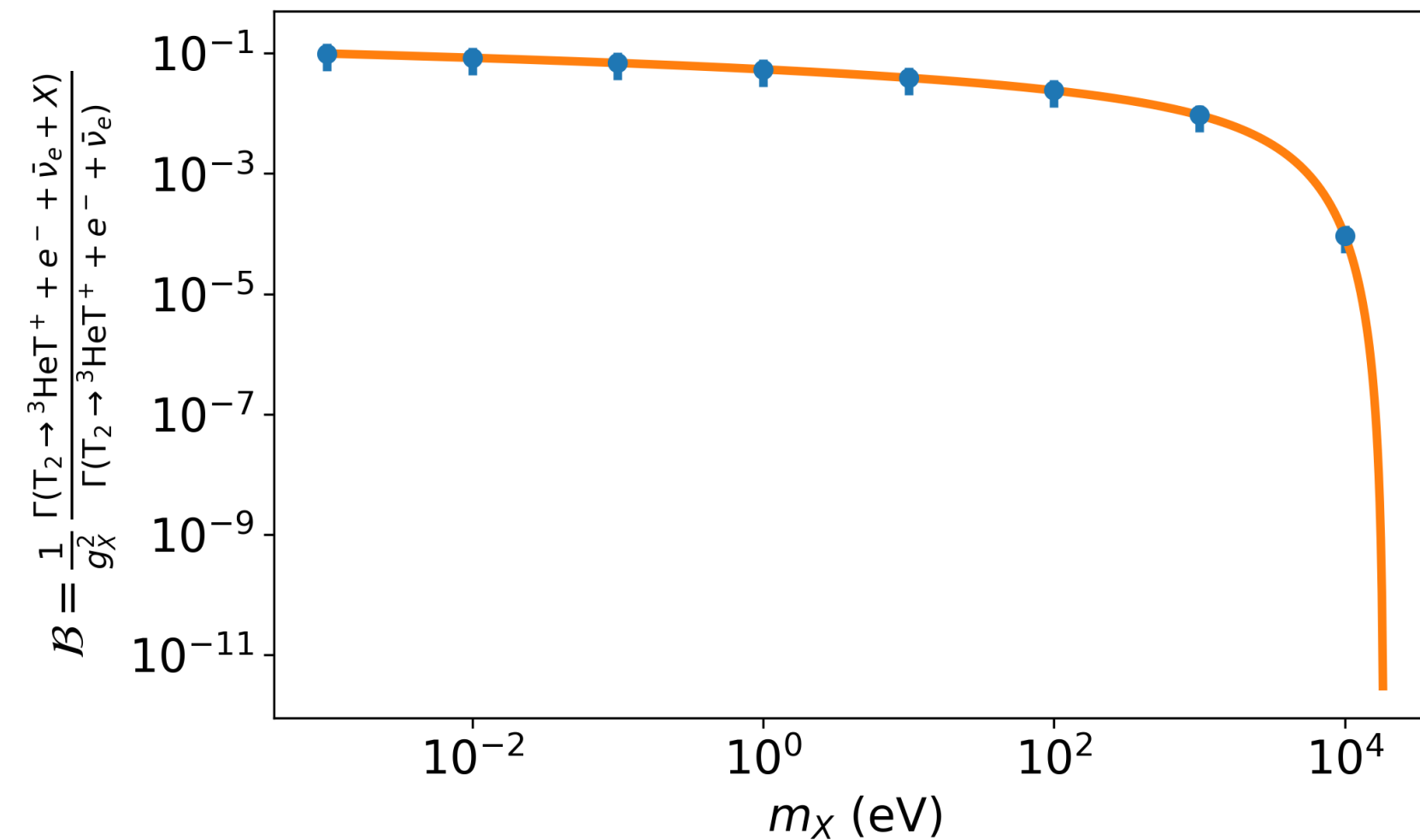
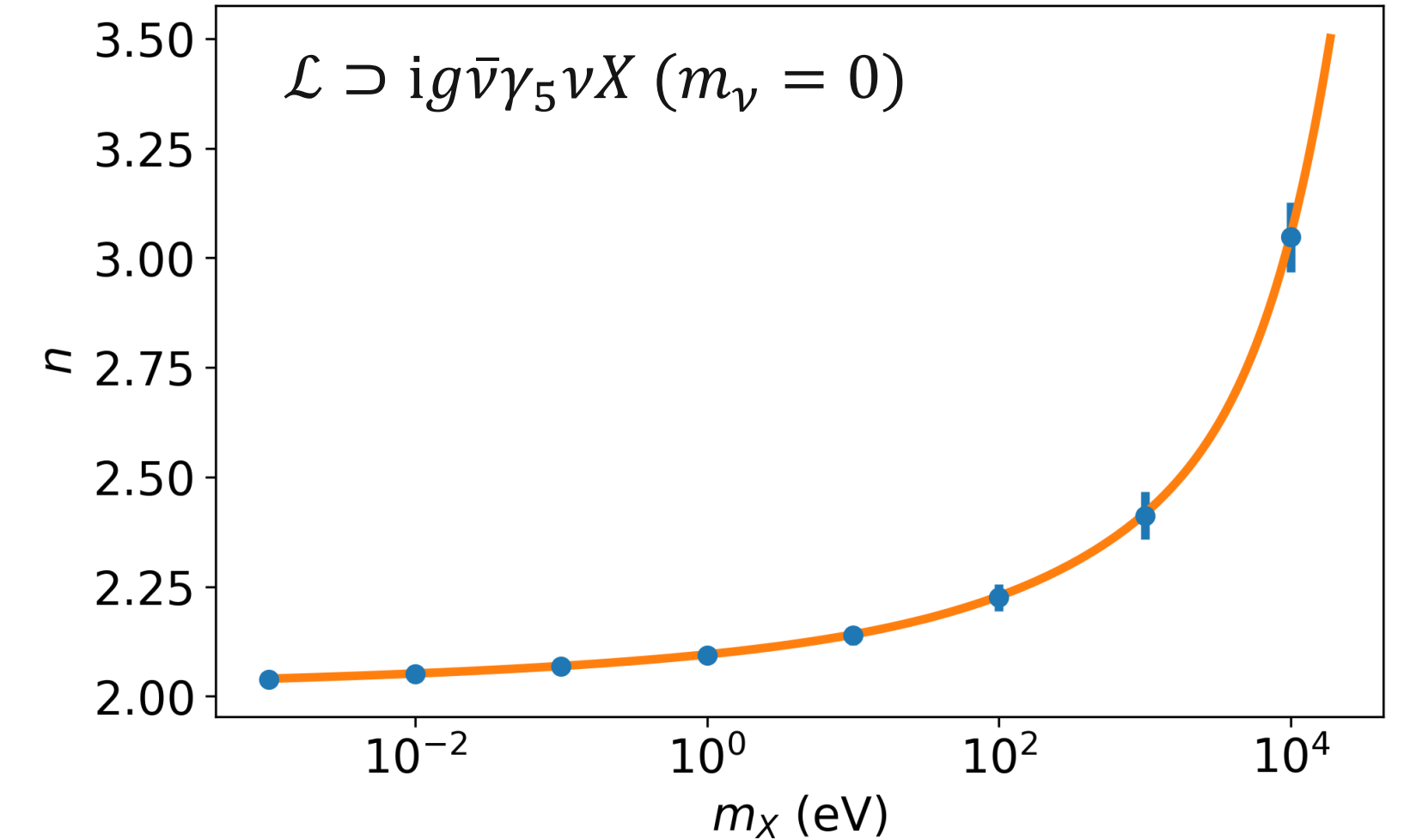
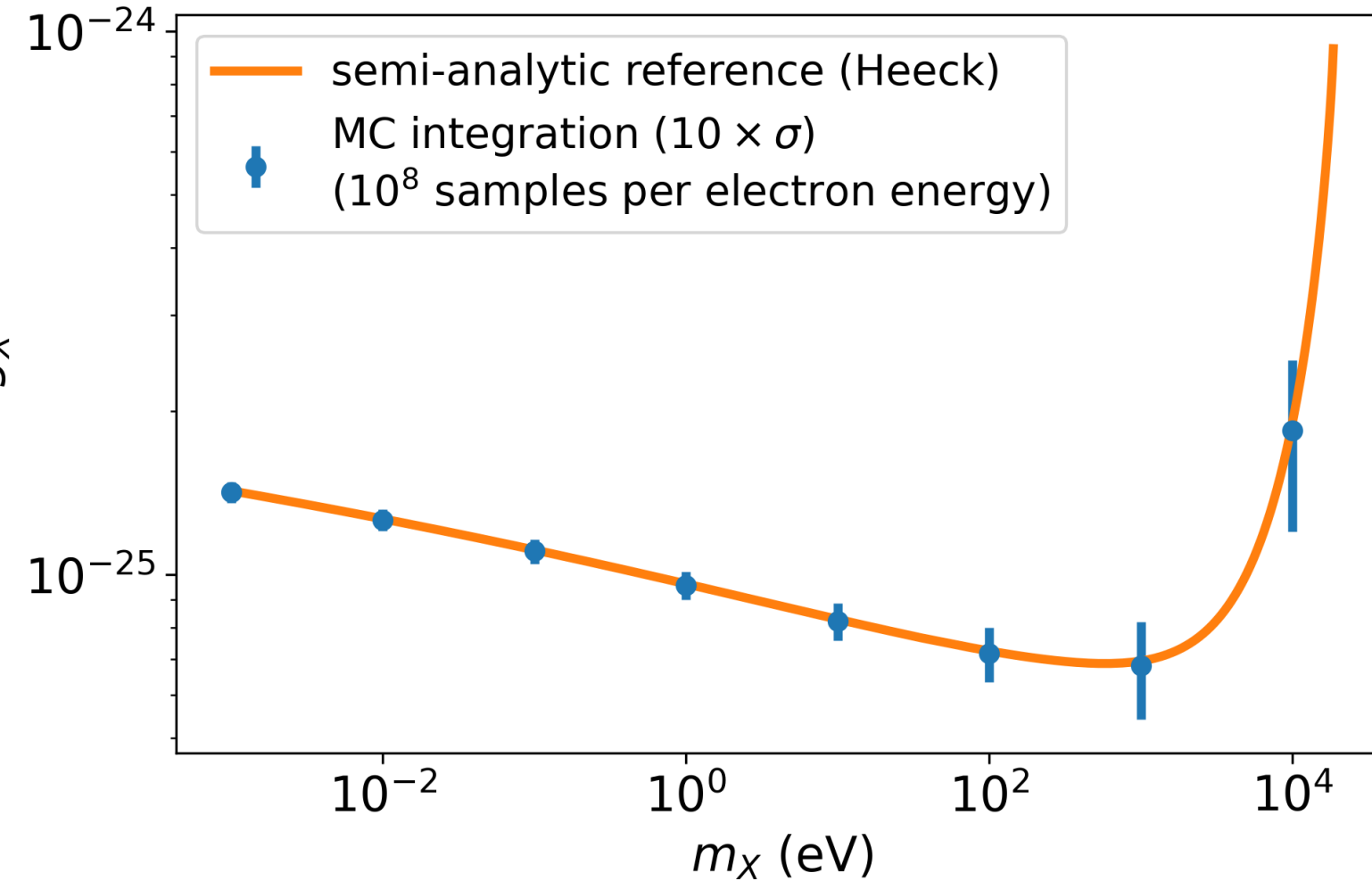
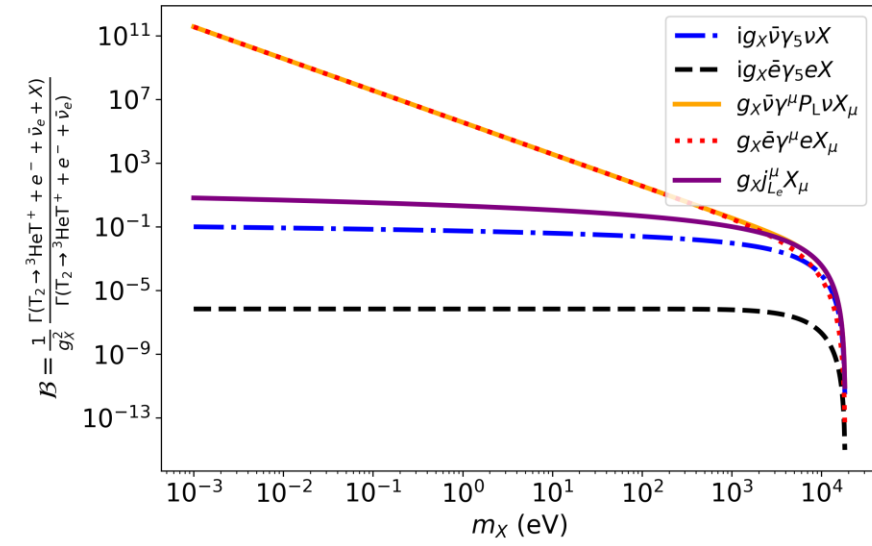
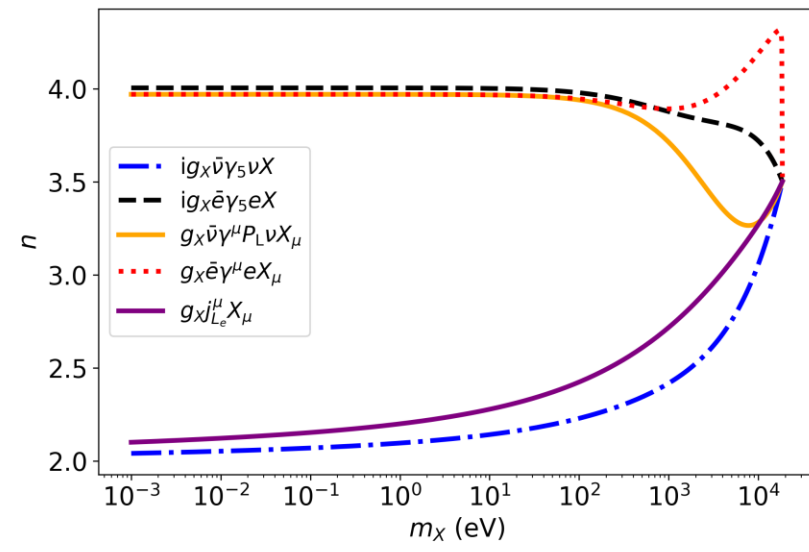
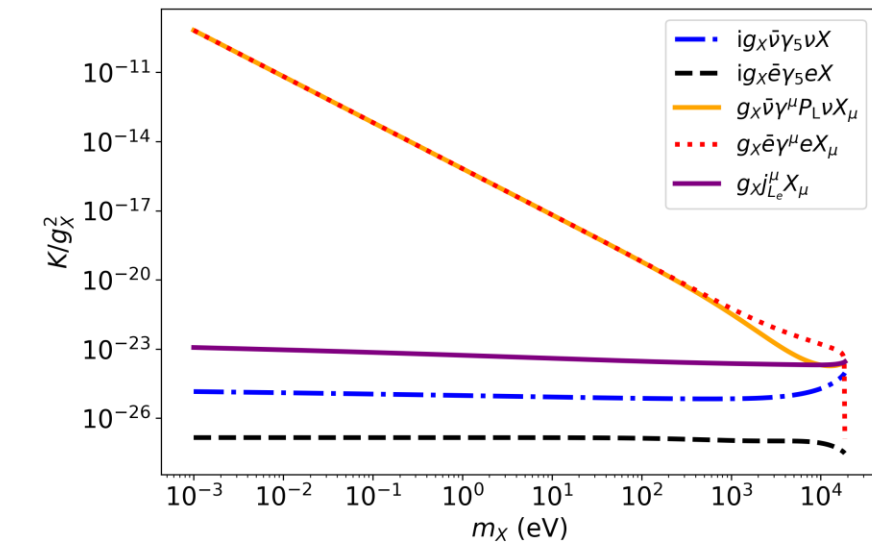
- Computation: **C++** framework with *GNU Multiple Precision Arithmetic Library (GMP)* → numerically extremely stable
- **Downside:** expensive computation for precise calculation at *single electron energy* \neq spectrum



Results – *comparison*

reference JHEP01(2019)206

$$\frac{d\Gamma}{dE} = K \sqrt{\frac{E}{m_e}} \left(\frac{E_{\max} - E}{E_{\max} + m_e} \right)^n$$



- **Good agreement** with reference
- **Problem:** assuming uniform weights instead of actual uncertainties
→ deviations at the endpoint (small contributions)

Endpoint studies

- **Goal:** precise description of the last 60 eV below the maximum energy E_{\max}
→ parametrization not accurate in this region

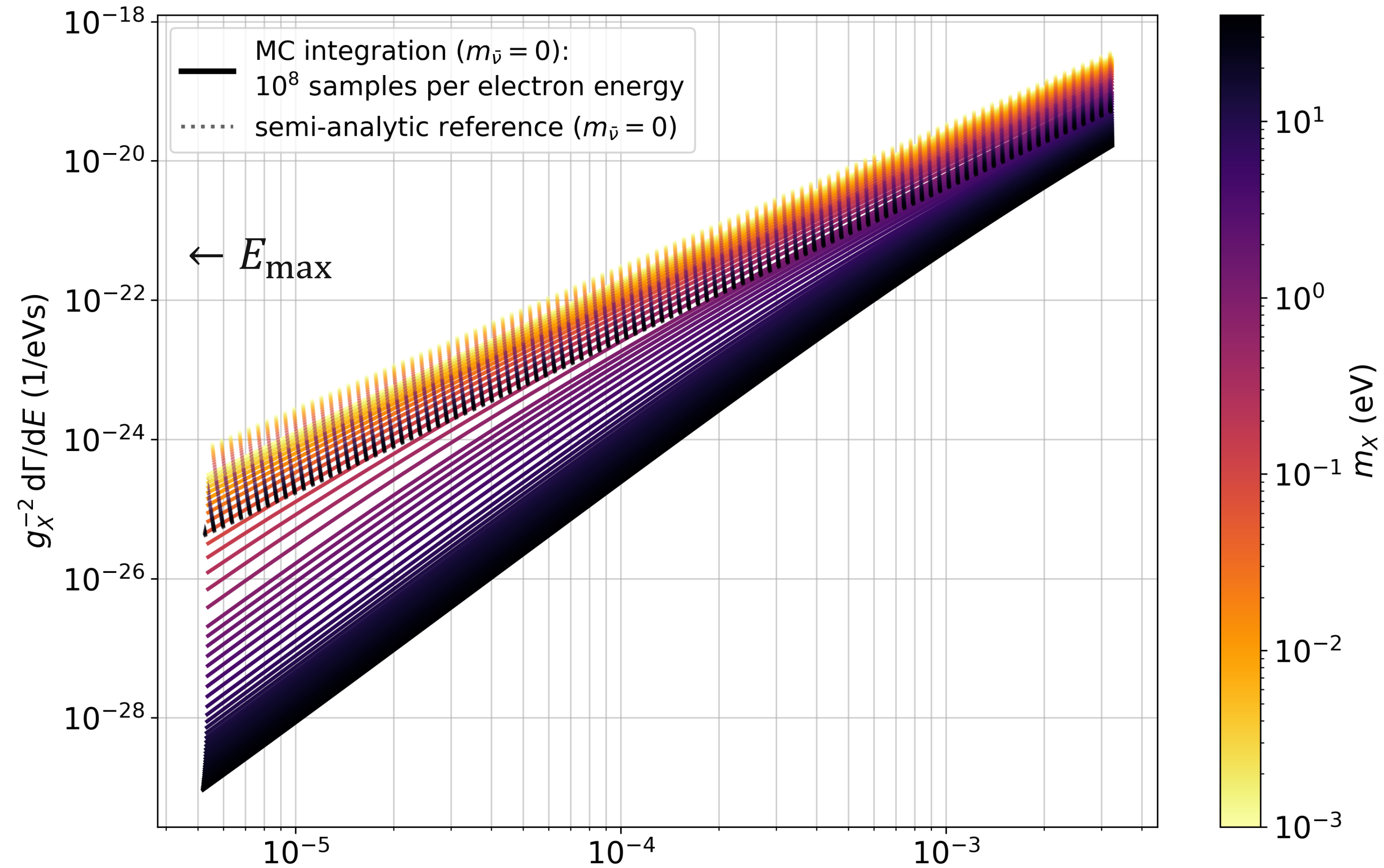
- **Adapted ansatz:** 💡

$$\ln \left(\frac{d\Gamma}{dE} \right) \propto \sum_{j=0}^k \theta_j \ln^j(x)$$

$$\Rightarrow \frac{d\Gamma}{dE} \approx \exp \left(\sum_{j=0}^k \theta_j \ln^j(x) \right) = K \exp \left(\ln(x) \sum_{j=1}^k \theta_j \ln^{j-1}(x) \right)$$

$$= K x^{\sum_{j=1}^k \theta_j \ln^{j-1}(x)} = K \left(\frac{E_{\max} - E}{E_{\max}} \right)^{\underbrace{\sum_{j=0}^{k-1} \theta_{j+1} \ln^j(x)}_{n(E)}}$$

$$\mathcal{L} \supset ig\bar{\nu}\gamma_5\nu X \ (m_\nu = 0)$$



Refined parametrization

- Numerical results are fitted with ansatz of order k :

$$\frac{d\Gamma}{dE} \approx \exp\left(\sum_{j=0}^k \theta_j \ln^j(x)\right)$$

- Parameters K and $n(x)$ are extracted:

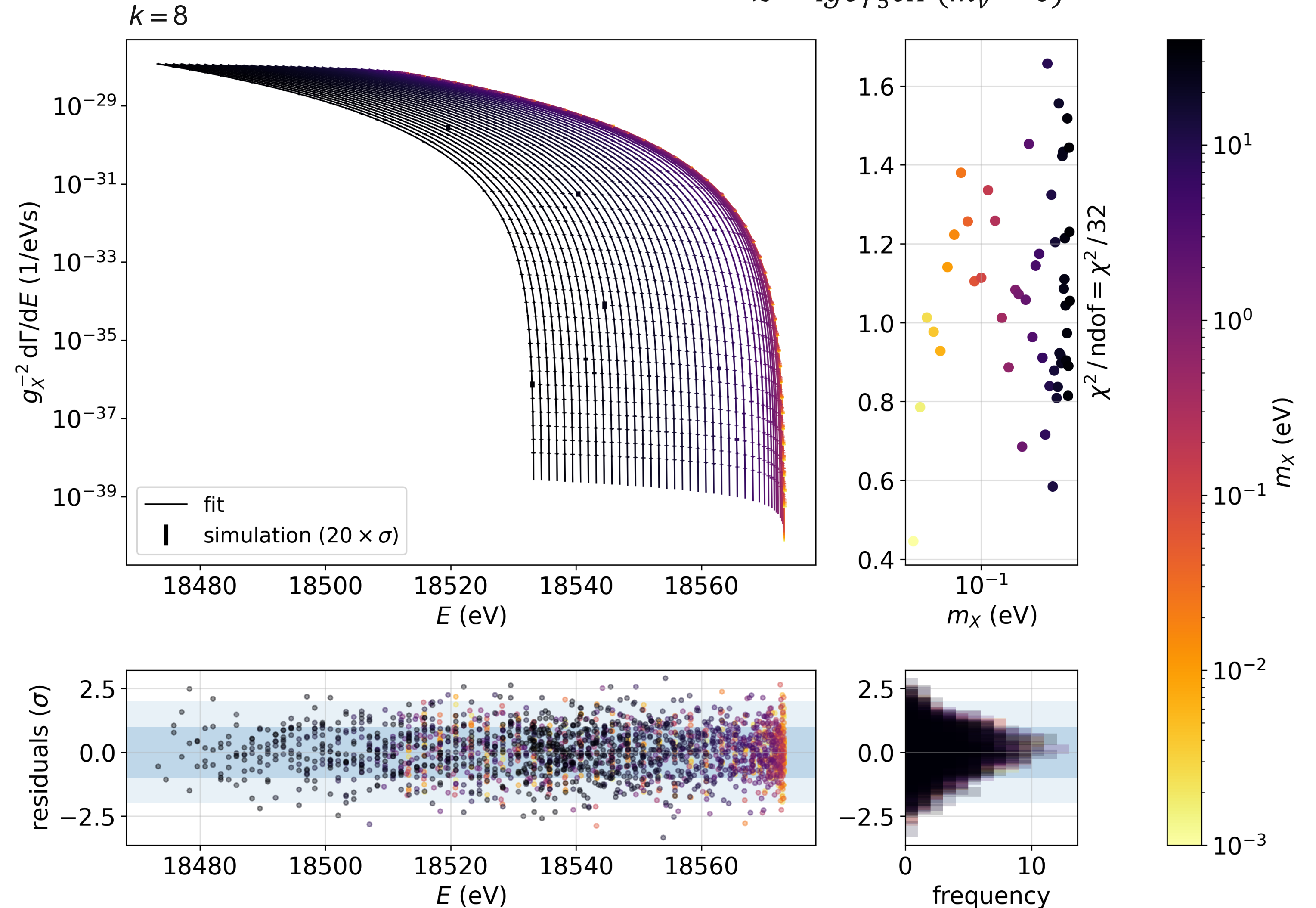
$$K \left(\frac{E_{\max} - E}{E_{\max}} \right)^{\underbrace{\sum_{j=0}^{k-1} \theta_{j+1} \ln^j(x)}_{n(E)}}$$

→ Calculated for **any considered boson scenario** and neutrino masses m_ν from 0 to 1 eV

→ Computations are running on the *bwForCluster NEMO* (Freiburg)

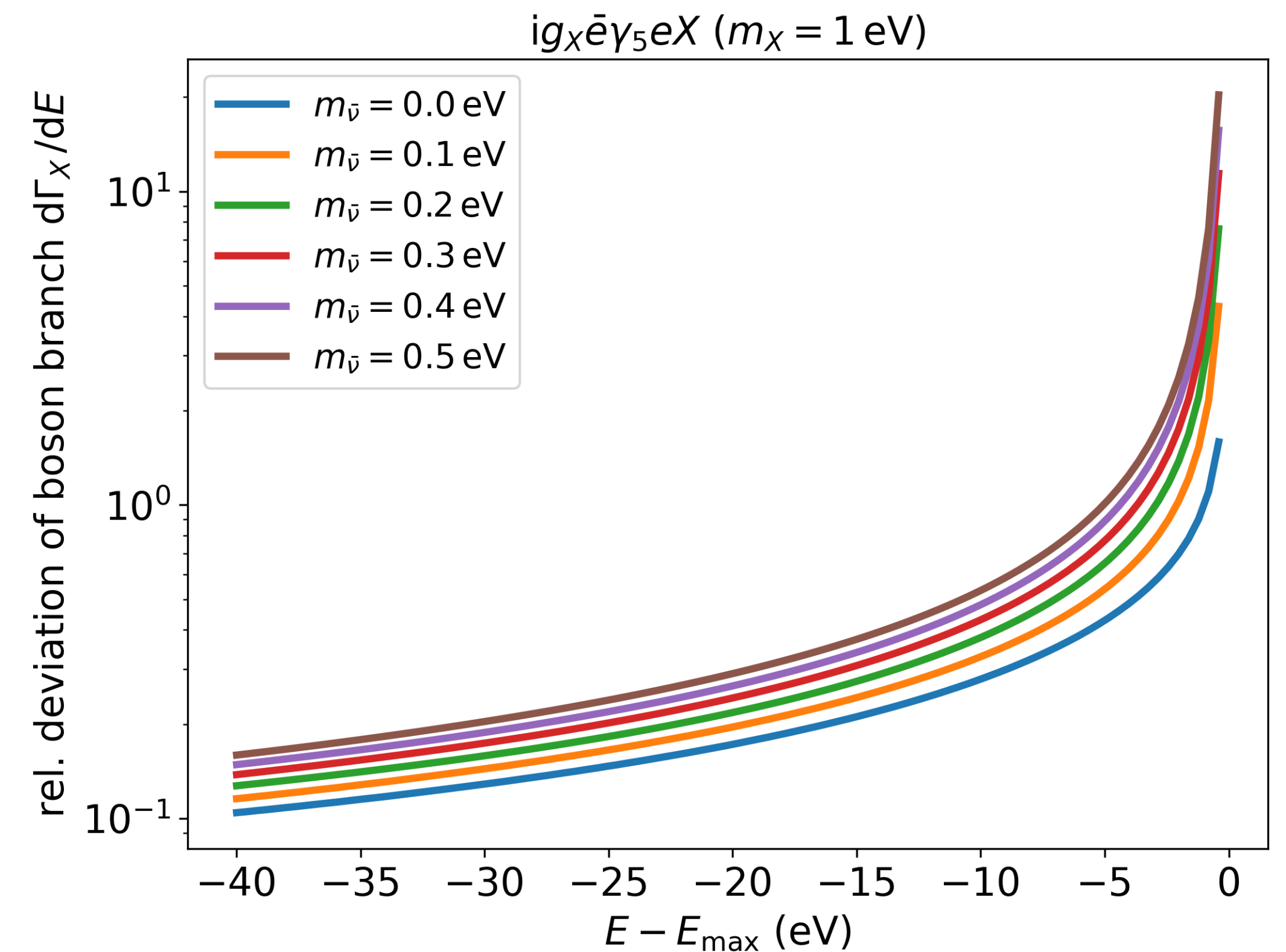
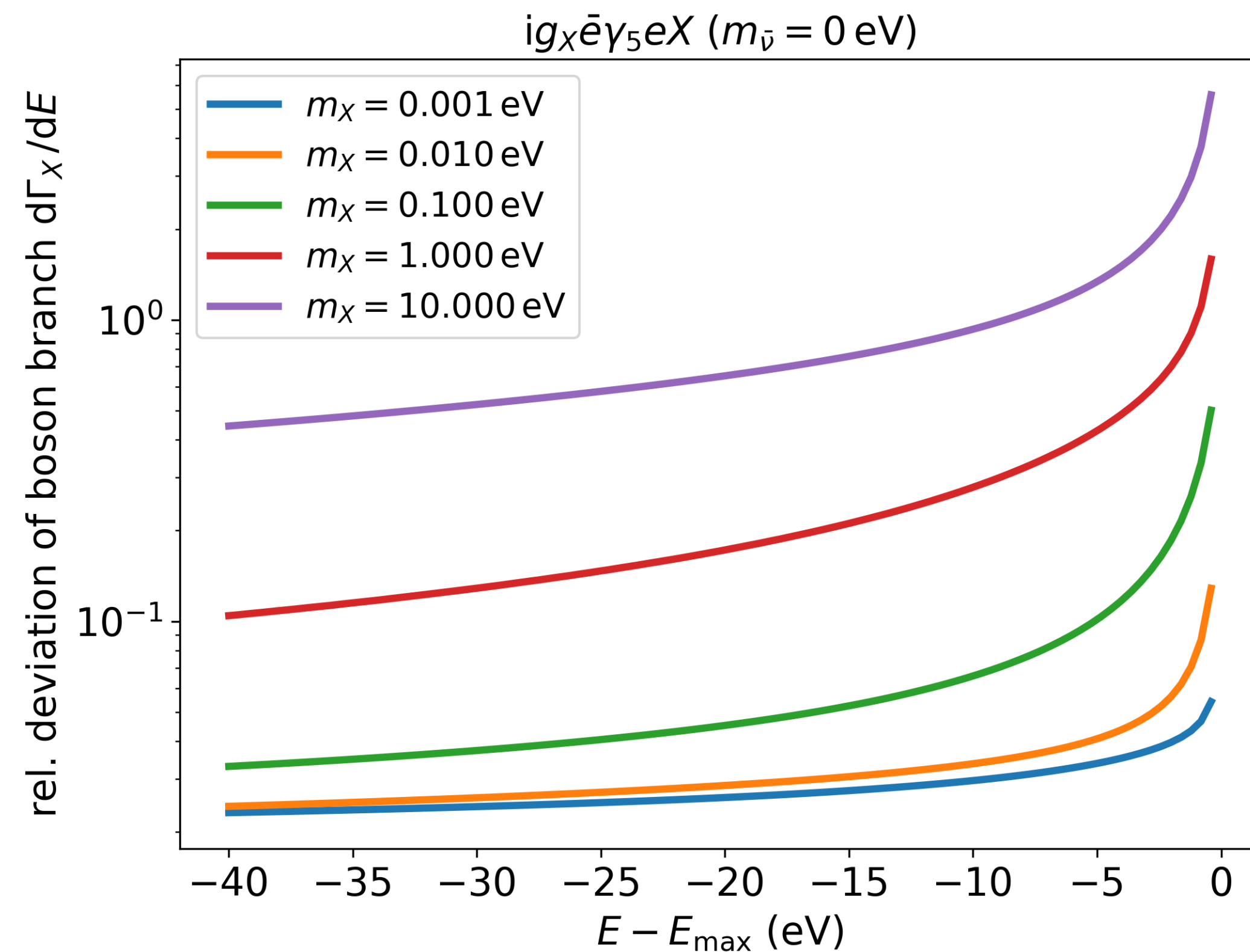
→ Precise analytic model in the sensitive region

Example scenario:
electron-pseudoscalar coupling
 $\mathcal{L} \supset ig\bar{e}\gamma_5 e X (m_\nu = 0)$



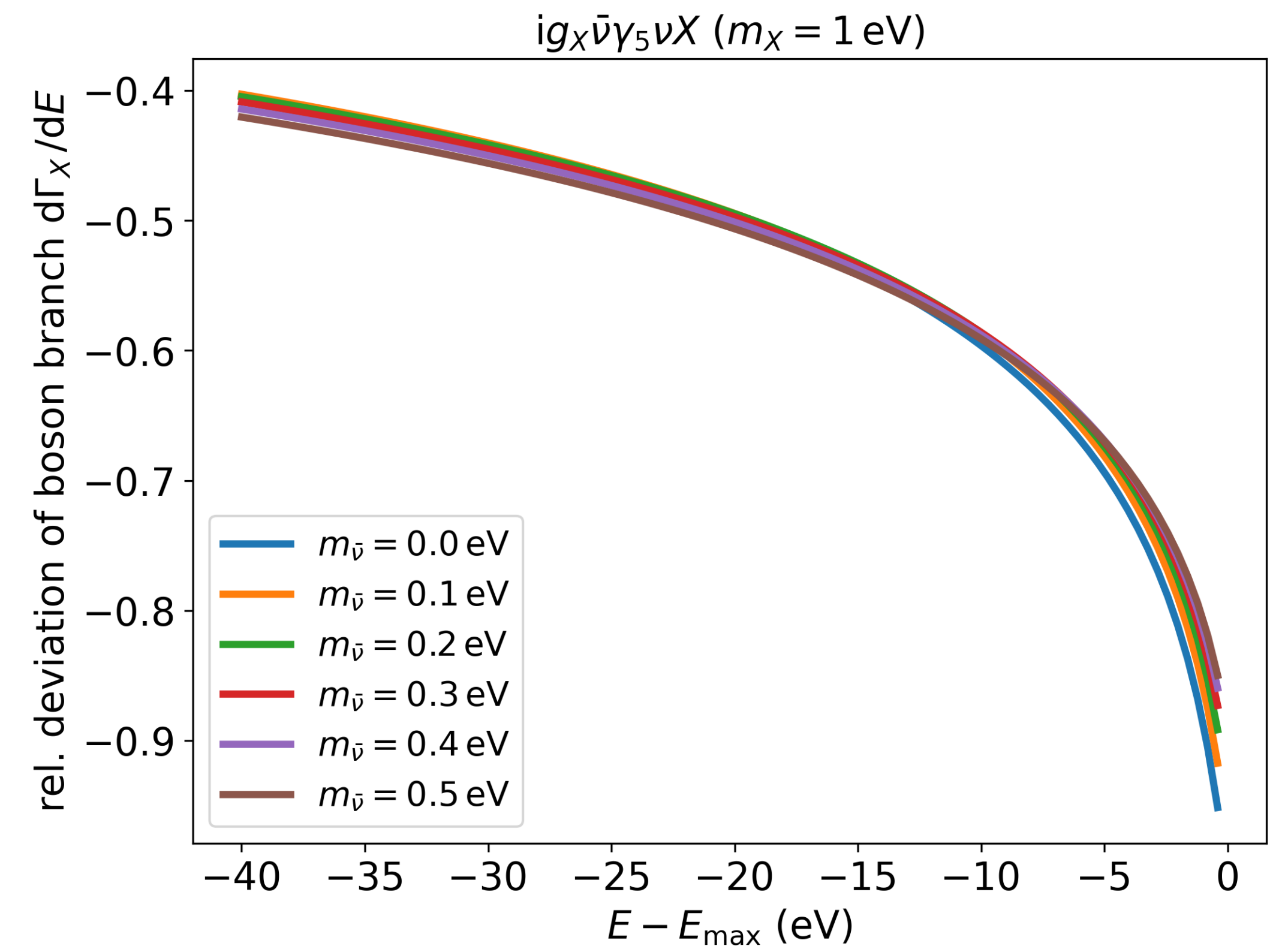
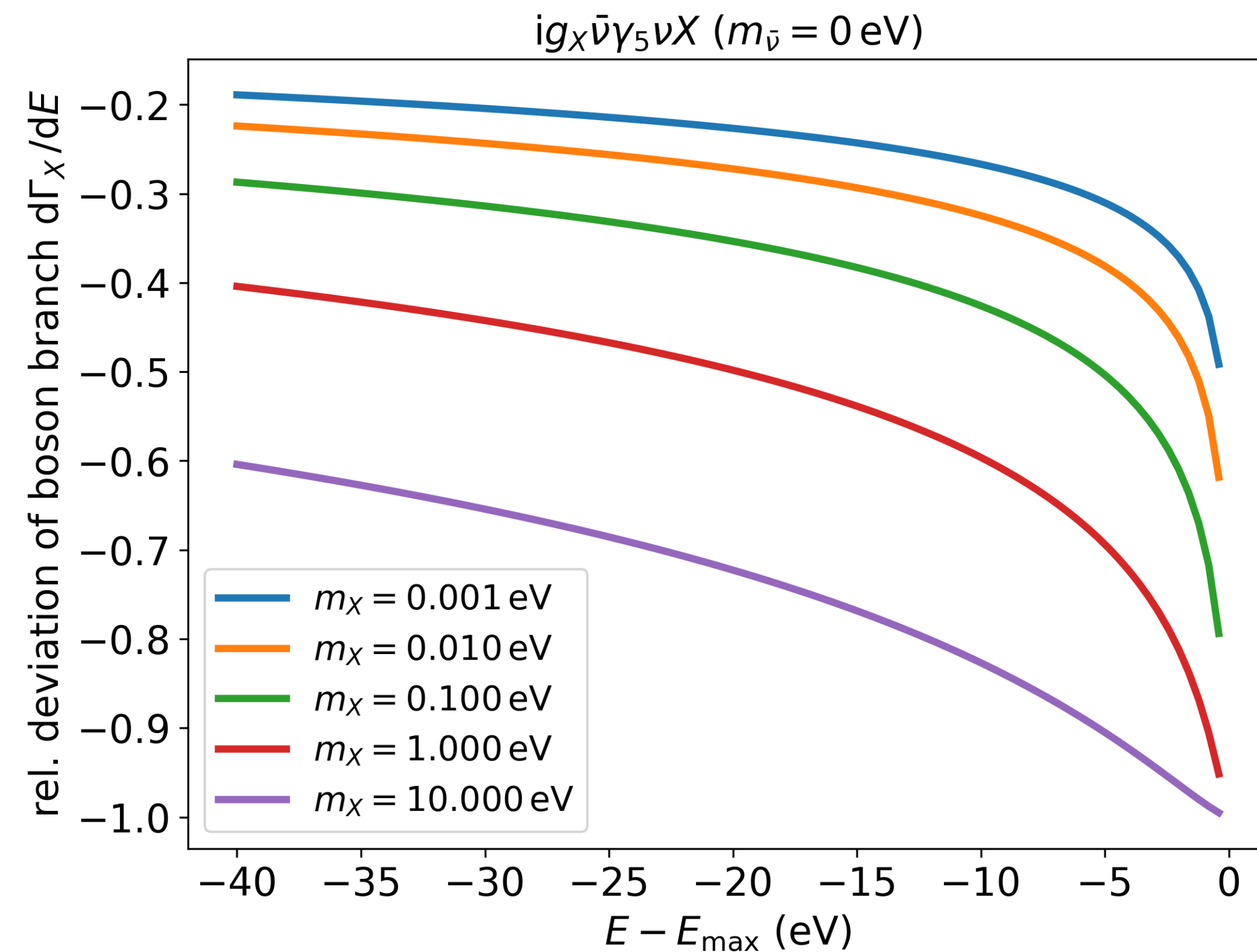
New model: compensated deviations

- Comparison: relative deviation of this work from the simplified model (reference)
- Scenario: **electron-pseudoscalar** coupling



New model: compensated deviations

- Comparison: relative deviation of this work from the simplified model (reference)
- Scenario: **neutrino-pseudoscalar** coupling

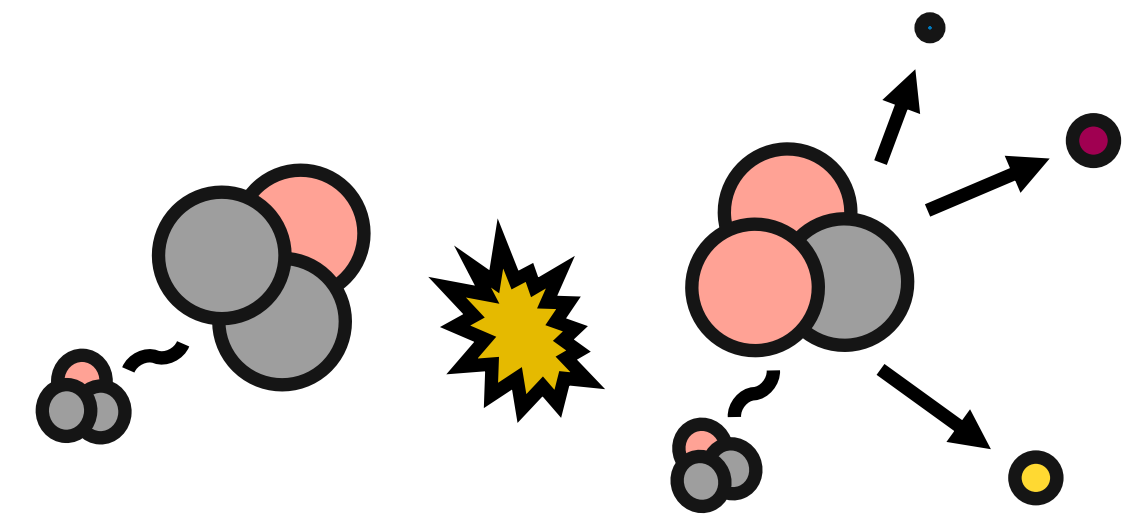


Summary & outlook

- *This work:* a highly **adaptable and modular framework** for boson emission spectrum computation
- Reproduction of the reference values from JHEP01(2019)206, but **deviations in the endpoint region**
- **Refined parameterization** of the tree-level light boson branches compensates deviations close to E_{max}
- Extension to **explicitly massive neutrino** was performed

Outlook:

- Update **analysis framework** according to refined model
- Analysis of a **subset of our data** with respect to imprints of light bosons
- Comparison of **Majorana** neutrino case to current **Dirac** assumption



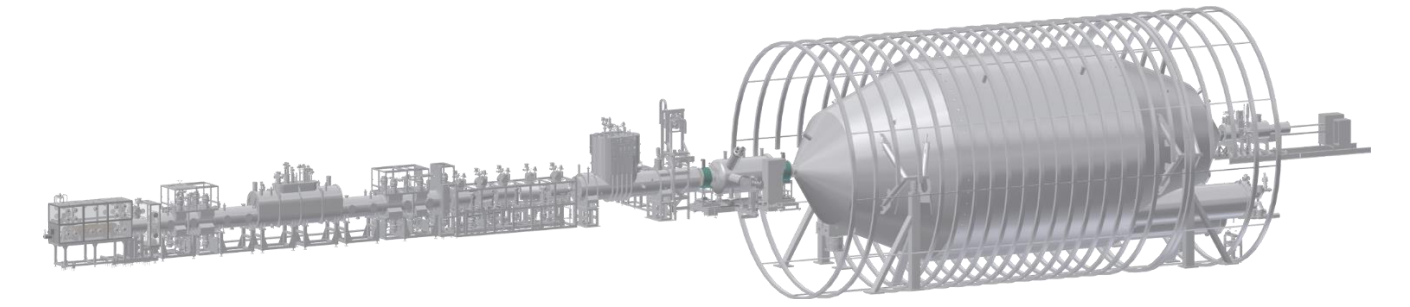
Acknowledgement

This work is supported by the Helmholtz Association, by the Ministry for Education and Research BMBF (grant numbers 05A23PMA, 05A23PX2, 05A23VK2 and 05A23WO6), and by the bwForCluster NEMO.

J. L. is grateful for valuable support by Ferenc Glück and Julian Heeck.

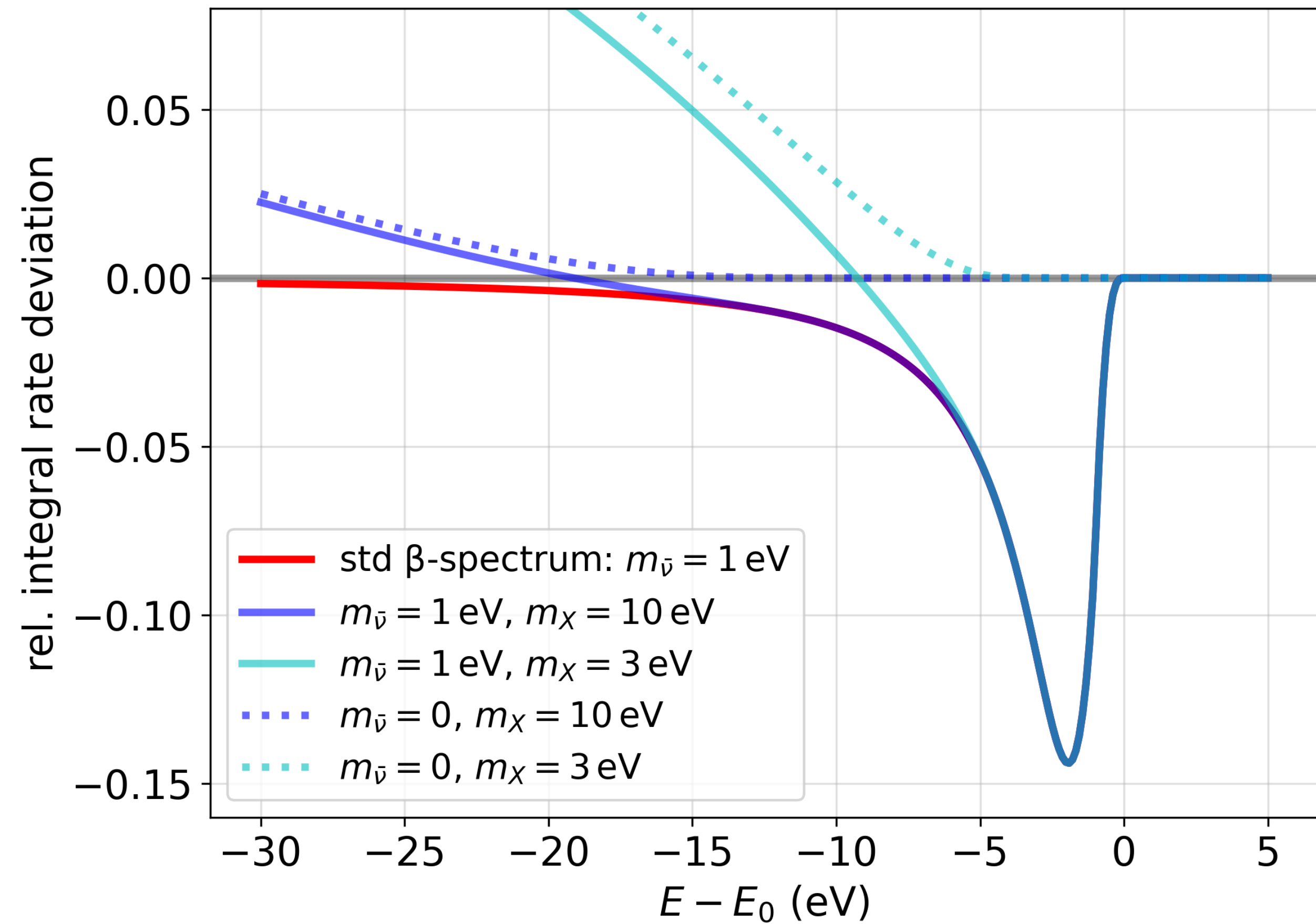
Backup

Refined boson signature



Example scenario:
neutrino-pseudoscalar coupling
 $\mathcal{L} \supset i g \bar{\nu} \gamma_5 \nu X$ ($g_X = 5$)

- The neutrino mass is now respected explicitly in the underlying theory
- Assumption (so far): Dirac neutrino



Fit results

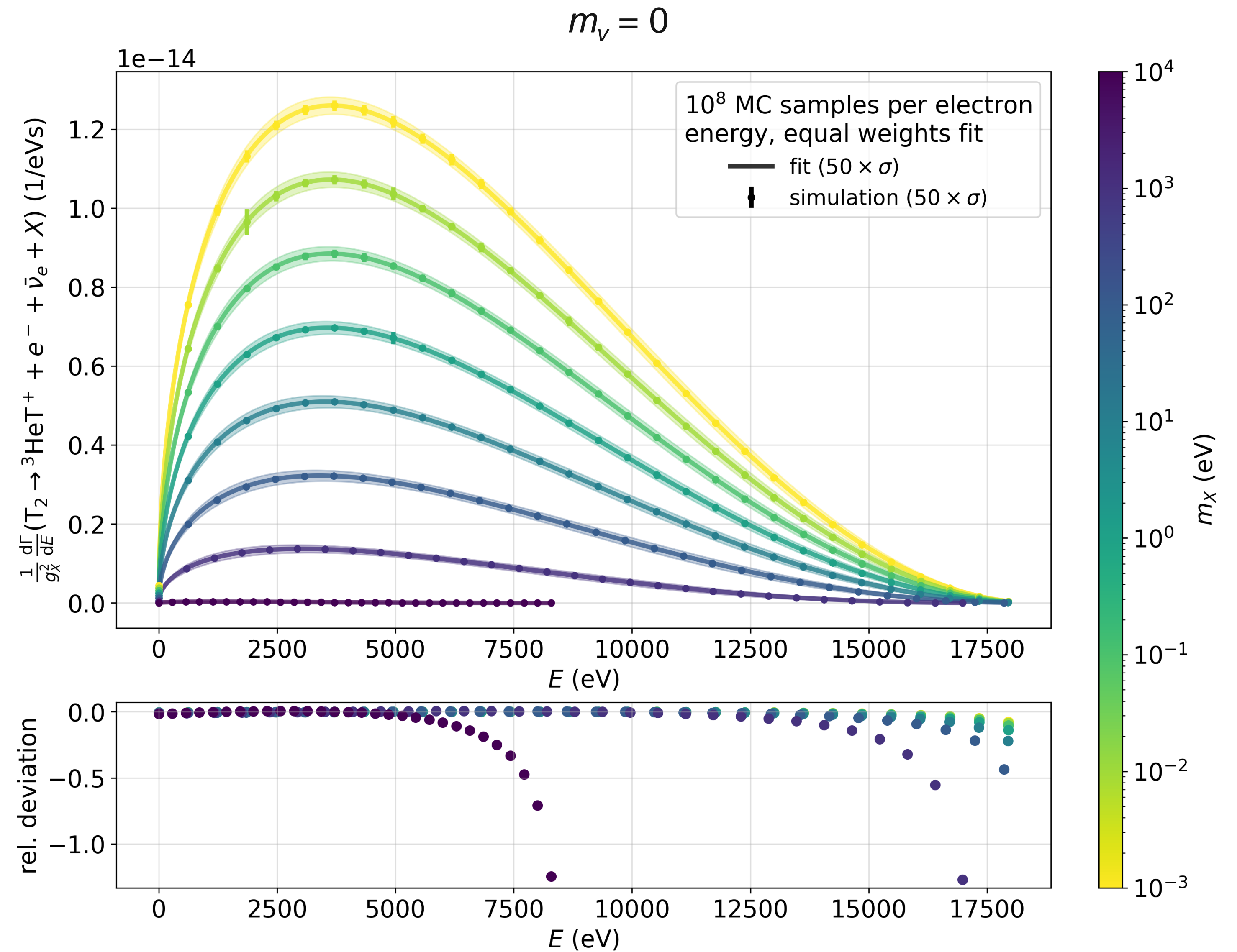
- Fit of proposed relation to full spectra from MC integration:

$$\frac{d\Gamma}{dE} = K \sqrt{\frac{E}{m_e}} \left(\frac{E_{\max} - E}{E_{\max} + m_e} \right)^n$$

- Assumption: **equal weight** for each point

→ extraction of parameters K, n

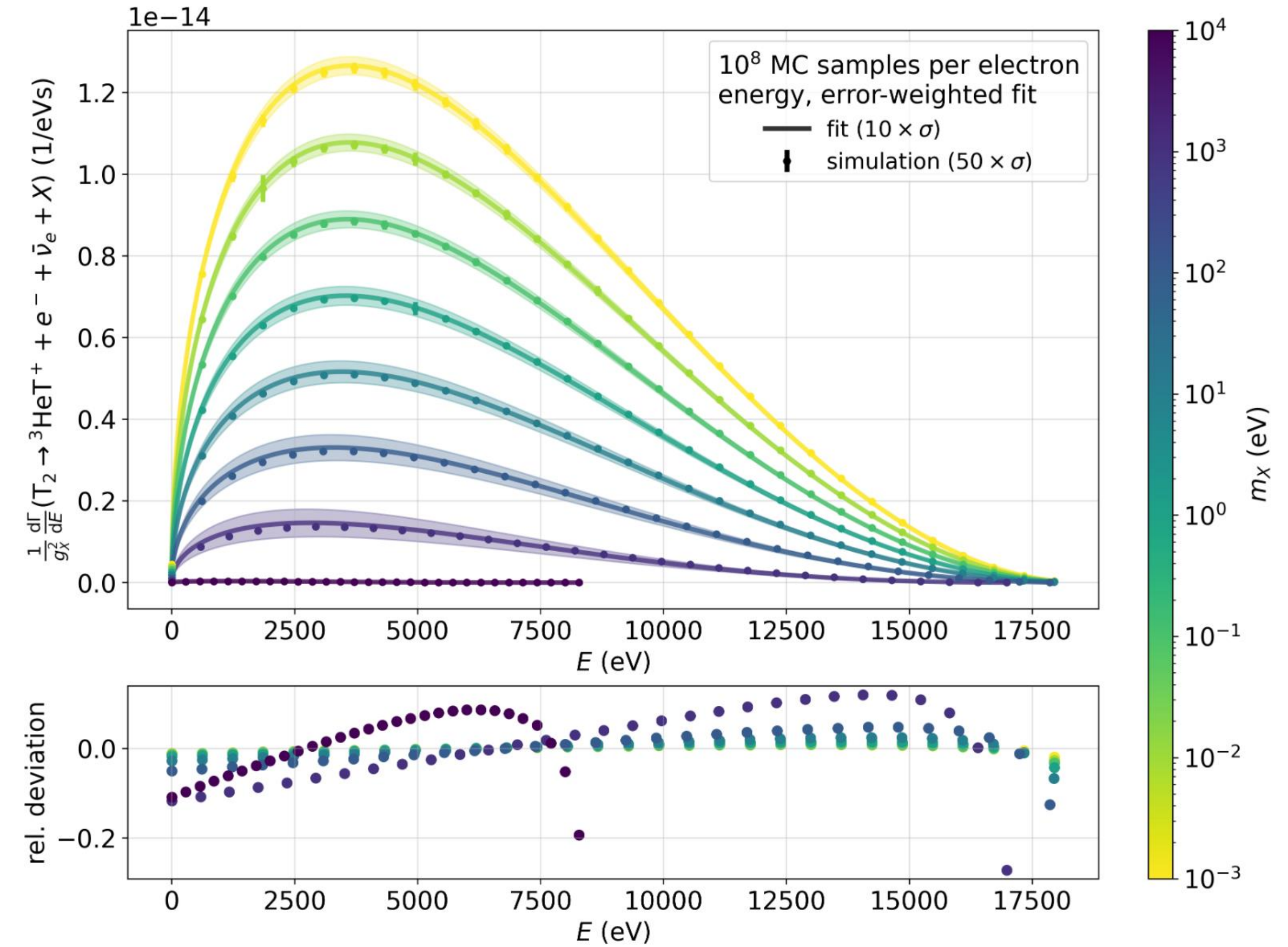
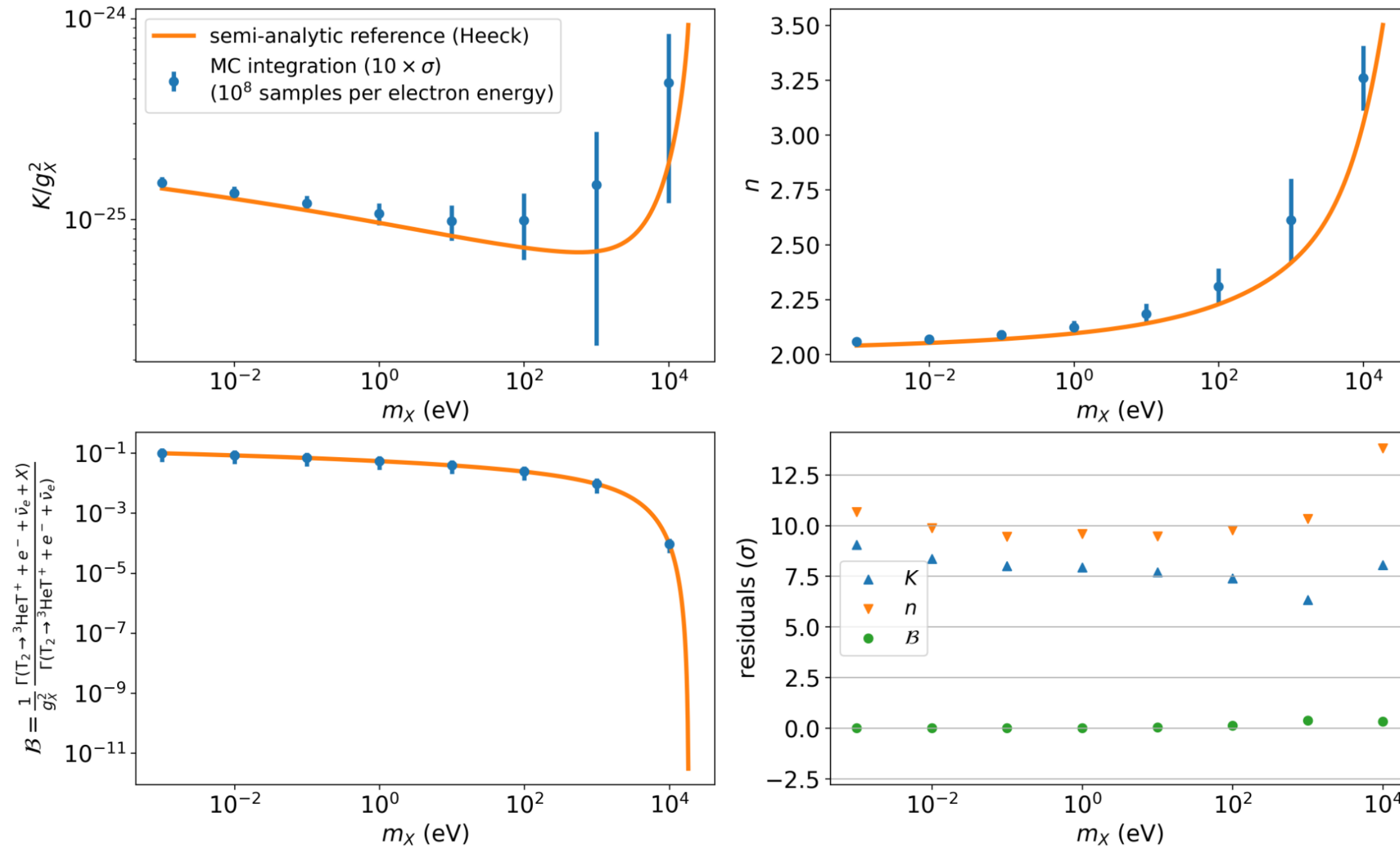
Example scenario:
neutrino-pseudoscalar coupling
 $\mathcal{L} \supset ig\bar{\nu}\gamma_5\nu X$



$$m_\nu = 0$$

$$\frac{d\Gamma}{dE} = K \sqrt{\frac{E}{m_e}} \left(\frac{E_{\max} - E}{E_{\max} + m_e} \right)^n$$

- Fit: each point **weighted with uncertainty**



Example scenario:
neutrino-pseudoscalar coupling
 $\mathcal{L} \supset ig\bar{\nu}\gamma_5\nu X$

$$\begin{aligned}
B = & [M^2Q^2 + N^2P^2 + M^2R^2 + N^2R^2 + P^2Q^2] - 2[M^2QR + N^2PR + MNR^2 + MPQ^2 + NP^2Q] \\
& + 2[MNPQ + MNPR + MNQR + MPQR + NPQR] - 2[M^2Qm + N^2Pn + M^2Rm + N^2Rn + MQ^2q \\
& + NP^2p + MR^2r + NR^2r + P^2Qp + PQ^2q] - 2[MNPm + MNQn + MPRp + NQRq + PQRr] \\
& + 2[MNR(m+n-2r) + MPQ(m+p-2q) + NQP(n+q-2p) + QRM(q+r-2m) + PRN(p+r-2n)] \\
& + [M^2m^2 + N^2n^2 + P^2p^2 + Q^2q^2 + R^2r^2] + 2[MNmn + MPmp + NQnq + PRpr + QRqr] \\
& + 2[MQ(mq+mn+qn+mp+qr-pr) + NP(np+nm+pm+pr+nq-qr) + MR(mr+mp+rp+mn+rq-nq) \\
& + NR(nr+nq+rq+nm+rp-mp) + PQ(pq+pr+qr+pm+qn-mn)] - 2[Mm(mp+mn+qr-pr-nq+2np) \\
& + Nn(nm+nq+pr-pm-qr+2mq) + Pp(pm+pr+nq-mn-qr+2mr) + Qq(qn+qr+mp-mn-pr+2nr) \\
& + Rr(rp+rq+mn-mp-nq+2pq)] + [m^2n^2 + m^2p^2 + n^2q^2 + p^2r^2 + q^2r^2] - 2[m^2np + mn^2q + mp^2r + nq^2r + pqr^2] \\
& + 2[mn pq + mn pr + mn qr + mp qr + np qr], \quad (7)
\end{aligned}$$

where the symbols are defined by

$$M = M_{12}^2, \quad N = M_{34}^2, \quad P = M_{124}^2, \quad Q = M_{134}^2, \quad R = M_{14}^2;$$

$$m = m_3^2, \quad n = m_2^2, \quad p = m_1^2, \quad q = m_4^2, \quad r = E^2.$$

$$\bar{\nu} \rightarrow 1, \quad X \rightarrow 2, \quad e \rightarrow 3, \quad B \rightarrow 4$$

scalar product	IKV representation
$p_1 \cdot p_2$	$\frac{1}{2} (M_{12}^2 - m_1^2 - m_2^2)$
$p_1 \cdot p_4$	$\frac{1}{2} (M_{14}^2 - m_1^2 - m_4^2)$
$p_3 \cdot p_4$	$\frac{1}{2} (M_{34}^2 - m_3^2 - m_4^2)$
$p_2 \cdot p_3$	$\frac{1}{2} (M_{14}^2 - M_{124}^2 - M_{134}^2 + m_A^2)$
$p_1 \cdot p_3$	$\frac{1}{2} (M_{134}^2 - M_{14}^2 - M_{34}^2 + m_4^2)$
$p_2 \cdot p_4$	$\frac{1}{2} (M_{124}^2 - M_{12}^2 - M_{14}^2 + m_1^2)$
$p_A \cdot p_2$	$\frac{1}{2} (m_A^2 + m_2^2 - M_{134}^2)$
$p_A \cdot p_3$	$\frac{1}{2} (m_A^2 + m_3^2 - M_{124}^2)$
$p_A \cdot p_1$	$\frac{1}{2} (M_{134}^2 + M_{12}^2 - M_{34}^2 - m_2^2)$
$p_A \cdot p_4$	$\frac{1}{2} (M_{124}^2 + M_{34}^2 - M_{12}^2 - m_3^2)$

Light bosons: sensitivity comparison

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