Prediction of turbulent heat transfer using Physics Informed Machine Learning

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I. Introduction

Accurate prediction of turbulent heat transfer is crucial for numerous engineering applications, including heat exchanger design, cooling systems, and thermal management in nuclear power plants. Computational fluid dynamics (CFD) approaches, while accurate, may become computationally expensive particularly for complex geometries and flow conditions. Conversely, purely data-driven machine learning methods require extensive datasets that may not be available in practical scenarios. Physics-Informed Neural Networks (PINNs), first introduced by Raissi et al. [2], offer a promising alternative by embedding physical laws directly into the neural network architecture, thereby reducing data requirements while maintaining physical consistency.

Aim of this work is to develop a PINN framework for predicting temperature fields in turbulent flows using the RANS formulation. The main objectives are: (1) to demonstrate effective physics integration through PDE residual minimization, (2) to enable adaptive parameter learning from sparse data, and (3) to implement robust data normalization strategies that maintain mathematical consistency during training. A key innovation of this approach is the iterative treatment of dimensionless numbers and turbulent coefficients as a learnable parameters. The approach achieves high accuracy using minimal training data.

II. Large Eddy Simulation (LES)

To generate high-quality training and validation data wall-resolved Large Eddy Simulation (LES) of turbulent forced convection in a channel was performed using OpenFOAM CFD software. The computation is carried out in the channel geometry of $4\delta \times 2\delta \times 2\delta$ with the resolution of $180 \times 80 \times 80$, in x, y, z for a Reynolds number of ~ 4000, which is based on the mean centreline velocity U_c, and the channel half-width δ. The first mesh point away from the wall is at $y^+ \approx 0.15$. One equation dynamic subgrid-scale model is used in the computation for the velocity field. The computational setup is close to the open-channel DNS case by Kim et al. [1]. Cyclic boundary conditions are applied at the inlet to establish fully developed turbulent inlet conditions. This study presents results for the isothermal wall configuration, employing constant molecular and turbulent Prandtl numbers of $Pr=1$ and $Pr_t=0.9$, respectively. The resulting dataset serves for training and validation of the PINN model, offering detailed data across the channel height at multiple time instances.

III. Physics Informed Neural Network (PINN)

PINNs enforce compliance with governing partial differential equations (PDEs) through automatic differentiation, enabling the network to respect fundamental physical principles also in regions with sparse or no training data. The PINN architecture consists of a multilayer perceptron that approximates the solution to a PDE while simultaneously satisfying the governing equations through additional loss terms. The neural network $N(t,x;\theta)$ approximates the temperature field $T(t,x)$, where θ represents the network parameters. The PDE residual is computed using automatic differentiation:

$$R(t,x)=\frac{\partial N}{\partial t}-Pe^{-1}\frac{\partial^2 N}{\partial x^2}+U_{\text{con}}\frac{\partial N}{\partial x} \tag{1}$$

The physics loss enforces $R(t,x)=0$ at randomly sampled collocation points throughout the domain:

$$L_{\text{physics}}=\frac{1}{N_r}\sum_{i=1}^{N_r}\left|R(t_i,x_i)\right|^2 \tag{2}$$

where N_r is the number of residual points.

III. *A.* Adaptive Parameter Learning

A key innovation of this approach is the iterative procedure to evaluate dimensionless numbers and coefficients for the PINN. In the following we discuss the approach applied to the Péclet number as a learnable parameter in the temperature equation. This enables the network to automatically discover effective transport properties from the data:

$$Pe = P(\theta_{Pe}) \quad (3)$$

where θ_{Pe} are dedicated network parameters for the Péclet number. The parameter is constrained to remain positive through a ReLU activation:

$$Pe = max(0.01, \theta_{Pe}) \quad (4)$$

This adaptive learning capability allows the model to account for effective turbulent transport that may not be captured by empirical correlations, making it particularly valuable for complex flow configurations where transport coefficients are not well-established. The optimization simultaneously updates both the neural network weights and the physical parameters:

$$\theta^{(k+1)}, P_e^{(k+1)} = \underset{\theta, P_e}{arg\,min}\left[\alpha L_{\text{data}}(\theta, P_e) + \beta L_{\text{physics}}(\theta, P_e)\right] \quad (5)$$

where L_{data} is the data loss function.

III. B. Network Architecture and Training

The neural network employs a multilayer perceptron

$$N(t,x) = W_L \sigma\left(W_{L-1}\sigma\left(\ldots\sigma\left(W_1[t,x]^T + b_1\right)\ldots\right) + b_{L-1}\right) + b_L \quad (6)$$

where $\sigma(z) = \ln(1+e^z)$ is the softplus activation function. The training employs the Adam optimizer.

IV. Results

The PINN approach demonstrates very good predictive performance across different time instances and spatial locations. Figure 1 shows the mean temperature profile prediction, comparing the PINN solution with OpenFOAM LES data and training points. The PINN solution shows very good agreement with LES reference data. The learned Péclet number demonstrates the model's ability to adaptively discover effective transport coefficients. Performance metrics show relative L2 Error of less than 5% across all test cases. The learned Péclet number converges while capturing flow-specific variations that improve prediction accuracy.

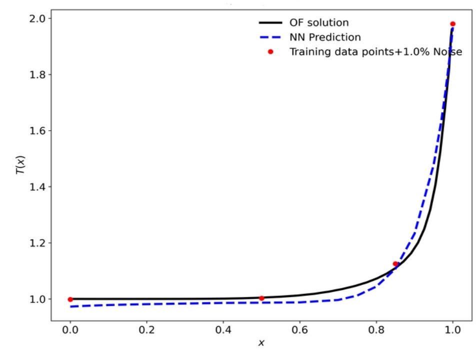

Figure 1: Mean Temperature profiles

V. Conclusions

This work successfully demonstrates the application of Physics-Informed Neural Networks to turbulent heat transfer prediction with several key advantages. The approach achieves high accuracy with minimal training data, requiring only 64 points for the entire 3D domain. The adaptive learning of transport coefficients like the Péclet number and turbulence model coefficients provides valuable approach for engineering heat and flow predictions where transport coefficients are not well-established. Once trained, the model offers fast inference suitable for real-time applications, making it computationally efficient for practical deployment where CFD is computationally expensive and experimental data is limited. Current limitations include RANS PINN formulation which induce the need for careful tuning of loss function coefficients.

References

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