ARTICLE IN PRESS

European Journal of Operational Research xxx (xxxx) xxx

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/eor

Invited Review in Celebration of the 50th Anniversary of EURO

Fifty years of power systems optimization

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ARTICLE INFO

Keywords:
Power systems optimization
Power system planning
OR in energy
Monopoly
Market

ABSTRACT

This review paper examines the evolution of power systems optimization over the past fifty years by considering two distinct periods: from 1970 to 1990 and from 1990 to the present. The initial period is typically defined by a centralized power system framework that prevailed around the world. The latter observes a transition to a decentralized structure in a market environment, marked by an increasing integration of renewables and advanced technologies, in addition to maintaining a centralized power system structure for some countries. Since we review the broad topic of power systems optimization, this analysis focuses on the main power system problems — investment, operation planning, operations, control, and forecasting — to define the main research streams. We provide a thorough exploration of the operations research methods applied to specific problem types within each period. Thus, this review not only underscores the pivotal role of operations research in addressing the challenges posed by changing landscapes and advanced technologies but also unveils the transformative journey of power systems optimization along with future research directions.

1. Introduction

Linear programming (LP) has evolved into a practical approach with Dantzig's development of the Simplex Method in 1947 (Dantzig, 1949). Since the late 1980s, the rise of algebraic modeling languages, the widespread accessibility of robust and user-friendly computers, and ongoing improvements in algorithms have all driven the development of operations research, and LP in particular (Bixby, 2002). The application of operations research techniques in power systems has given rise to innovative methodologies, addressing complex challenges inherent to the control, operations, operation planning, and investment of power systems. That is, operations research has played a crucial role to enhance efficiency, ensure reliability, and promote sustainability of power systems around the world. Initially, LP, as an operations research method, was employed to address specific problems within the power system area, such as economic dispatch and expansion planning. Over the last five decades, not only has the power system undergone transformative changes, but operations research has also seen remarkable advancements. The evolution of power systems optimization can be followed through a timeline marked by dynamic shifts in technological perspective, regulatory frameworks, and environmental considerations.

A centralized power system structure was common around the world until the 1990s. In a centralized power system, the system operator holds complete authority over all production, transmission and distribution decisions. The computation of dispatch decisions involves minimizing the overall cost of meeting demand at each node in the network while complying with network and production constraints (Kagiannas et al., 2004). Starting in the 1990s, many power systems around the world underwent a shift towards a liberalized, market-oriented environment, adopting a predominantly decentralized structure. Under this framework, producers have the flexibility to sell their energy in a market (Möst & Keles, 2010). In recent decades, along with the transition to a liberalized market environment, the integration of renewable energy sources, advancements in energy storage systems, the integration of advanced technologies and the growing emphasis on environmental sustainability have introduced new aspects to power systems optimization.

The use of optimization models, algorithms, and decision-support tools has become instrumental in addressing the complexities of power systems over the past fifty years. From 1970 to 1990, power systems optimization mainly relied on LP, even though the first studies incorporating discrete decisions, nonlinearities and uncertain parameters had appeared. During this period, decomposition techniques — as a solution methodology — were employed primarily for investment, unit commitment and reservoir optimization problems. Then, optimization models became more complex due to the increasing need for discrete decisions, nonlinearities, uncertain parameters and decentralization. This evolution led to the growing utilization of mixed-integer linear programming (MILP) and mixed-integer nonlinear programming (MINLP) next to LP. In this context, second-order cone (SOC) and mixed-integer

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https://doi.org/10.1016/j.ejor.2025.05.022
Received 4 March 2024; Accepted 14 May 2025
Available online 27 May 2025

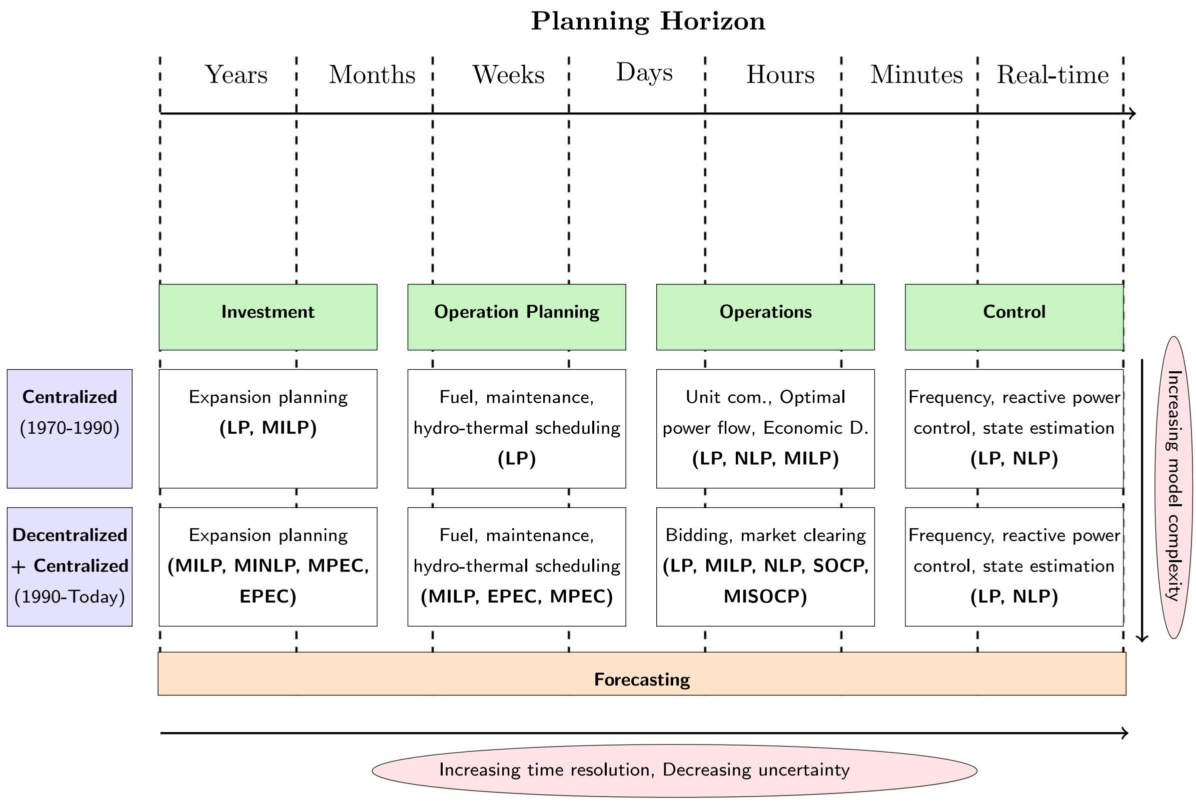

Fig. 1. Components of power system optimization throughout the entire planning horizon.

second-order cone (MISOC) models have also been developed. Alongside an increased nonlinearity representation, equilibrium problems with equilibrium constraints (EPEC) and mathematical programs with equilibrium constraints (MPEC) were introduced in response to the evolving decentralized structure (Fig. 1). In recent times, increasing levels of uncertainty plagued the power systems, leading to the development of various models based on stochastic programming (SP), robust optimization, and fuzzy logic. Considering several factors such as uncertainty representation, probability, and risk degree, the appropriate method must be determined to deal with uncertainty. Given the emerging challenges, new solution methods and algorithms for power systems have been developed to enhance the reliability and robustness of completed solutions.

We would like to point out that this paper focuses on topics related to power systems optimization. Thus, this paper examines the main power system problems, including investment, operation planning, operations, control and forecasting (Fig. 1). While these main problems do not by themselves represent a full power system analysis, they reflect the main streams of research on power systems optimization. Investment problems consider a planning horizon spanning from years to several decades, with decisions related to generating units, transmission lines and distribution systems being determined. Operation planning problems can span from days to years and encompass maintenance planning for lines, transformers and generators, fuel procurement planning and hydro-thermal scheduling. Operations problems, which often extend over minutes to days, determine scheduling and dispatch decisions. Control problems, aimed at ensuring the stability of the entire power system, cover horizons ranging from real-time to minutes. To this end, we review the key features of optimization models and solution methods that have shaped the power systems optimization field in the last fifty years. As pointed out, since this review covers the broad topic of power systems optimization, we have to focus on selected topics, defining the main research streams. This is especially true, as the field of power systems is interdisciplinary at the intersection of economics, electrical engineering and operations research, where we focus on the operations research perspective.

We analyze power systems optimization in two separate sections. The first covers the early period from 1970 to 1990s, characterized by a centralized structure and monopoly nature. The second period, from 1990s to the present, considers decentralized as well as centralized structures, still applicable for some countries. In particular, the second period not only features optimization models and solution methodologies for a decentralized structure, but also encompasses recent methodological developments for a centralized structure. As a result, we structure our analysis into two historical periods to examine the evolution of power systems optimization. Although some solution methodologies remain similar across both operational paradigms, they exhibit significant variations. Along with the transition of power system operational paradigms, a number of aspects require this distinction. These aspects include computational capabilities, operations research approaches, the incorporation of renewable energy sources, the use of advanced technologies, and regulatory and environmental requirements, all of which have seen substantial development since the 1990s.

The remainder of the paper is organized as follows. Section 2 reviews early times (1970–1990) with a focus on the centralized framework. Section 3 discusses recent times (1990–present) to review the decentralized framework and updated methodologies on the centralized structure. Section 4 emphasizes key directions for future research and Section 5 concludes the paper. We summarize the abbreviations in Table 6 at the end of the paper.

2. Early times—Centralized operations and planning (1970–1990)

2.1. Overview

In this section, we focus on the centralized operations and planning from 1970 to 1990. In a centralized framework, the power system operator possesses comprehensive technical and cost-related data, along with complete control over the power system. Despite some advancements in computing machinery and software between 1947 (when LP was developed) and 1990, they remained insufficient to overcome computational limitations for solving large-scale problems (Bixby, 2002). Given the computational capabilities of the period, LP was the predominant optimization approach. The power systems operated within a comparatively stable condition characterized by regulated prices and minimal uncertainty (Botterud et al., 2005). In terms of modeling uncertainty, the forecasting of load and fuel prices presented the primary challenge within this period, while renewable technologies such as wind, solar or biogas were utilized minimally for electricity generation. Therefore, the primary focus in this period was on deterministic models. However, the studies on stochastic models started in this period; they have broadened the scope and depth of operations research applications in power systems since 1990s. This section covers investment, operation planning, operations, control, and forecasting as main power system problems. We note that some power systems continue to operate centrally and recent methodologies have been adapted to their current needs, as explored in Section 3.

2.2. Investment

Power system expansion planning seeks to optimize the investment and retirement strategies for generating units, transmission lines and distribution systems, specifying sizing, location, and timing. The aim under a centralized framework is to ensure the supply of the power demands at lowest cost while considering technical, economic, and environmental constraints (Cho et al., 2022). During this period, notable progress was made in computational methods for expansion planning. It is important to note that incorporating uncertainties into the expansion planning problem, such as future loads, investment costs of production technologies and operation costs (particularly fuel costs) is essential for obtaining reliable solutions (Conejo et al., 2016). The time and spatial resolutions play an integral role in expansion planning models. That is, a high temporal and spatial representation can result in computational complexity, while low resolution might lead to inaccurate representation in the expansion planning problem (Gacitua et al., 2018; Kotzur et al., 2021).

2.2.1. Generation expansion planning—MILP, decomposition

Generation expansion planning (GEP) aims at identifying an optimal power generators portfolio (type, location, and installation timing) to meet future power demand over a long time horizon. Expansion planning models were among the initial applications of LP techniques in the 1950s (Massé & Gibrat, 1957). Given the discrete nature of operational and technical constraints, optimization models have evolved from LP to MILP to effectively address GEP needs (Cho et al., 2022).

There are two main approaches in the literature to formulate GEP problems: static and dynamic. A static approach represents a single decision point at the beginning of the planning horizon and does not decide when to build each generator, but rather whether the generators should be built at all. A dynamic approach determines decisions about generation expansion at different stages in the planning horizon. Thus, it determines if and when each plant will be built. Dynamic approaches lead to more accurate models than static ones; the expected increase in solution quality comes at the cost of increased computational complexity. Dynamic approaches are generally treated as multi-stage problems and solved by decomposition methods (Cote & Laughton, 1979).

The formulation of a dynamic GEP problem (as a MILP model) is as follows (Conejo et al., 2016),

$$(G) \quad \min \sum_{t\in T}\left[\sum_{h\in H}\left[\sum_{i\in G} C^E_{it} p^E_{iht} + \sum_{n\in N} C^N_{nt} p^N_{nht}\right] + \sum_{n\in N} I^N_{nt} p^{N^{max}}_{nt}\right] \tag{1a}$$

$$\text{s.t. } p^{N^{max}}_{nt} = \sum_{q\in Q} u_{nqt} \bar{P}^{N^{max}}_{nqt} \qquad \forall n\in N, t\in T, \tag{1b}$$

$$\sum_{q\in Q} u_{nqt} \le 1 \qquad \forall n\in N, t\in T, \tag{1c}$$

$$\sum_{i\in G} p^E_{iht} + \sum_{n\in N} p^N_{nht} = \sum_{d\in J} P^D_{dht} \qquad \forall h\in H, t\in T, \tag{1d}$$

$$0 \le p^E_{iht} \le P^{E^{max}}_{it} \qquad \forall h\in H, i\in G, t\in T, \tag{1e}$$

$$0 \le p^N_{nht} \le \sum_{\tau\le t} p^{N^{max}}_{n\tau} \qquad \forall h\in H, n\in N, t\in T, \tag{1f}$$

$$p^{N^{max}}_{nt} \ge 0 \qquad \forall n\in N, t\in T, \tag{1g}$$

$$u_{nqt} \in \{0,1\} \qquad \forall n\in N, q\in Q, t\in T, \tag{1h}$$

where $p^{N^{max}}_{nt}$ is a non-negative decision variable to determine the capacity to be built for the new generation unit n in year t. The parameter $\bar{P}^{N^{max}}_{nqt}$ denotes the potential investment block capacity q for the new generation unit n in year t. The binary variable u_{nqt} determines which potential investment capacity q will be considered for the new generation unit n in year t. The non-negative decision variables p^E_{iht} and p^N_{nht} are the power output of existing generating unit i and new generating unit n for each hour h in year t, respectively. The parameter P^D_{dht} is the load of demand d for each hour h in year t; C^E_{it} is the production cost of existing generating unit i, while C^N_{nt} is the production cost of new generating unit n per year t, and I^N_{nt} represents the investment cost. The objective function (1a) refers to the total cost that encompasses generation and investment costs. Constraints (1b) define the capacity of new generation plants. Constraints (1c) ensure that at most one option is selected per year. Constraints (1d) establish the balance between generation and demand. Additionally, constraints (1e) and (1f) set limits on the power quantities supplied by existing and candidate generating units, respectively.

Since the 1950s, LP has been employed as a basic approach for GEP. By utilizing continuous variables, investment decisions were determined to fulfill power demand while considering budget constraints (Massé & Gibrat, 1957). However, operational limitations were not taken into account in these early versions. Due to increased operational and technical constraints, the necessity of incorporating discrete decisions becomes apparent, resulting in MILP models (Gacitua et al., 2018). The incorporation of integer variables in GEP led to a significant computational burden. Extensive research in the literature has studied the utilization of Benders decomposition for solving the GEP problem. Benders decomposition is a natural choice in the context of expansion planning. It decomposes the original problem into a master problem — associated with the investment decisions — and a subproblem— associated with the operational problem, to evaluate the quality of investment trial decisions from the master problem. Originally proposed for LP problems by Benders (Sinske & Rebennack, 2022), it has also been extended to mixed-integer master problems, non-convex master problems, convex subproblems — the so-called generalized Benders decomposition (Bloom, 1983; Geoffrion, 1972) — and general non-convex problems (Füllner & Rebennack, 2022). As such, it is no surprise that extensive research has been conducted to utilize Benders decomposition for solving the GEP problem (Cote & Laughton, 1979). Dynamic Programming has also been used for the capacity expansion problem (Caramanis et al., 1982). In order to address the uncertainty pertaining to GEP, stochastic dynamic programming (Mo et al., 1991) and the Dantzig–Wolfe decomposition (Sanghvi & Shavel, 1986) have been utilized. Gorenstin et al. (1993) employed Benders decomposition to address a two-stage stochastic planning problem, where sub-problems were solved by an earlier methodology of stochastic dual dynamic programming (SDDP). A selection of solution methodologies for solving expansion problems up to the 1990s are presented in Table 1.

2.2.2. Transmission expansion planning—MILP, decomposition

Transmission expansion planning (TEP) seeks to determine the transmission network's expansion decisions, including the timing, locations, and types of transmission lines in order to fulfill future energy demand (Niharika et al., 2016). Its primary objective is to minimize the total costs, including investment, and operational cost, while ensuring system stability, reliability, and compliance with regulatory and environmental requirements.

The formulation of a basic transmission expansion planning problem (as a MILP model) is provided below (Conejo et al., 2016),

$$(T)\quad \min \sum_{t\in T}\left[\sum_{\ell\in L} I^L_{\ell t}x^L_{\ell t} + \sum_{h\in H}\left[\sum_{i\in G} C^E_{it}p^E_{iht} + \sum_{d\in J} C^{LS}_{dt}p^{LS}_{dht}\right]\right] \tag{2a}$$

$$\text{s.t.} \quad \sum_{i\in L^P} I^L_{\ell t}x^L_{\ell t} \le I^{L,max}_t \qquad \forall t\in T, \tag{2b}$$

$$\sum_{t\in T} x^L_{\ell t} \le 1 \qquad \forall \ell \in L^P, \tag{2c}$$

$$\sum_{i\in G^N} p^E_{iht} - \sum_{\ell|s(\ell)=n} p^L_{\ell ht} + \sum_{\ell|r(\ell)=n} p^L_{\ell ht} = \sum_{d\in J^N}\left(P^{D^{max}}_{dht} - p^{LS}_{dht}\right) \qquad \forall n\in N, h\in H, t\in T, \tag{2d}$$

$$p^L_{\ell ht} = B_\ell\left(\theta_{s(\ell)ht} - \theta_{r(\ell)ht}\right) \qquad \forall \ell\in L^E, h\in H, t\in T, \tag{2e}$$

$$p^L_{\ell ht} = \left(\sum_{\tau\le t} x^L_{\ell\tau}\right)B_\ell\left(\theta_{s(\ell)ht} - \theta_{r(\ell)ht}\right) \qquad \forall \ell\in L^P, h\in H, t\in T, \tag{2f}$$

$$-F^{max}_{\ell t} \le p^L_{\ell ht} \le F^{max}_{\ell t} \qquad \forall \ell\in L, h\in H, t\in T, \tag{2g}$$

$$0 \le p^E_{iht} \le P^{E,max}_i \qquad \forall i\in G, h\in H, t\in T, \tag{2h}$$

$$0 \le p^{LS}_{dht} \le P^{D,max}_d \qquad \forall d\in J, h\in H, t\in T, \tag{2i}$$

$$-\pi \le \theta_{nht} \le \pi \qquad \forall n\in N, h\in H, t\in T, \tag{2j}$$

$$x^L_{\ell t} = \{0,1\} \qquad \forall \ell\in L^P, t\in T, \tag{2k}$$

where $x^L_{\ell t}$ is a binary variable to determine whether or not transmission line ℓ is built in year t. The non-negative decision variable p^E_{iht} refers to the generated power from unit i for each hour h in year t, and the continuous decision variable $p^L_{\ell ht}$ is the power flow through transmission line ℓ for each hour h in year t. The non-negative decision variable p^{LS}_d represents a load shedding and the parameter $P^{D^{max}}_{dht}$ is the load of demand d. The decision variable θ_{nht} is the voltage angle at node n; $\theta_{r\ell ht}$ is the voltage angle of the destination-end node of line ℓ and $\theta_{s\ell ht}$ is the voltage angle of the sending-end node of line ℓ. The parameters $I^L_{\ell t}$, C^E_{it}, and C^{LS}_{dt} represent the investment cost of transmission lines, production cost of generating units, and load shedding costs, respectively. The parameter B_ℓ represents the susceptibility of line ℓ, whereas $F^{max}_{\ell t}$ represents its capacity. The objective function (2a) seeks to minimize the total cost, including investment, operational and load-shedding costs. Constraints (2b) enforce that the investment cost in new transmission lines remains within the budget ($I^{L,max}_t$). Constraints (2c) ensure that transmission line ℓ can only be built at most once throughout the planning horizon. Eqs. (2d) force that the demand volume from all demand points at node n, including transfer volumes to and from node n and load shedding volumes, must balance with the generation from all units at node n. Equality constraints (2e) specify the power flows from the existing lines ($\ell \in L^E$). Equality constraints (2f) define the power flows from the prospective lines ($\ell \in L^P$) if a prospective line is invested in at time t or during a previous time period t. Note that constraints (2f) are non-linear, but these are typically reformulated as linear inequalities by using a large enough constant M. Constraints (2g) impose the bounds on power flow. Constraints (2h), (2i) and (2j) set limits on produced power, lost demand and voltage angles respectively.

For transmission expansion planning, LP (Garver, 1970; Pereira et al., 1985) and dynamic programming (Dusonchet & El-Abiad, 1973) have been used since the 1970s. MILP was employed to address transmission expansion planning considering transmission losses, congestion, and operational constraints (Latorre-Bayona & Perez-Arriaga, 1994; Romero & Monticelli, 1994). Benders decomposition (Pereira et al., 1985; Romero & Monticelli, 1994) and heuristics (Latorre-Bayona & Perez-Arriaga, 1994) were used as well. Given that generation and transmission expansion planning are inherently connected, these problems can be jointly solved to achieve a reliable and robust solution for both transmission and generation facilities (Conejo et al., 2016); we discuss in Section 3.2.2.

2.2.3. Distribution expansion planning—MILP

The distribution system is an integral part of the power system, connecting transmission facilities and end-consumers. Distribution expansion planning (DEP) pertains to determining the optimal expansion strategies for the components of the distribution system, *i.e.*, substations and feeders. A collection of substations in the distribution system are coupled with each other by feeders. That is, ensuring sufficient substations' and feeders' capacity is a requirement for distribution expansion planning (Khator & Leung, 1997). The aim of distribution expansion planning is to minimize total cost, encompassing investment, fixed, operation and energy loss costs, subject to operational and network constraints. In the literature, MILP, NLP, and LP are most commonly used to tackle the distribution expansion planning problems.

The general formulation of a distribution expansion planning problem as a MILP model can be represented as follows (Sun et al., 1982),

$$(D)\quad \min \sum_{i\in G}\sum_{j\in B}\left(C^V_{ij}p_{ij} + C^F_{ij}y_{ij}\right) \tag{3a}$$

$$\text{s.t.} \quad \sum_{j\in B} p_{ij} = S_i \qquad \forall i\in G, \tag{3b}$$

$$\sum_{i\in G} p_{ij} = D_j \qquad \forall j\in B, \tag{3c}$$

$$0 \le p_{ij} \le U_{ij}y_{ij} \qquad \forall i\in G, j\in B, \tag{3d}$$

$$y_{ij} \in \{0,1\} \qquad \forall i\in G, j\in B, \tag{3e}$$

where y_{ij} is a binary variable to determine whether the feeder ij is built or not. The non-negative decision variable p_{ij} is the power flow from supply node $i\in G$ to demand node $j\in B$. The objective function (3a) minimizes the total cost that incorporates operational (C^V_{ij}) and investment costs (C^F_{ij}) of feeders. Constraints (3b) ensure that the total power flow from each supply node i to all demand nodes equals the available capacity S_i at that supply node i, while constraints (3c) enforce that the total power flow to each demand node j equals the demand D_j at that demand node j. Constraints (3d) are bound on the power flow through feeder ij, depending on whether the feeder ij is built or not.

Starting in the 1970s, LP has been used for distribution expansion planning. Crawford and Holt (1975) addressed the substation location program using LP. Wall et al. (1979) introduced a linear transshipment model to find the optimal solution of the distribution expansion problem. In Thompson and Wall (1981), a MILP model for distribution planning was solved by a branch-and-bound algorithm. Aoki et al. (1990) presented a "branch-exchange" algorithm to handle a single period distribution planning problem. Quadratic programming and heuristic methods were also employed for the distribution expansion planning (Ponnavaikko et al., 1987).

2.3. Operation planning

Operational planning is a mid-term optimization framework within power systems. During this phase, the planning aspects of maintenance for generators and transmission, fuel procurement, and hydro-thermal scheduling are considered. It is important to note that inaccurate decisions made in the mid-term time frame can have an impact on the operations, potentially leading to unfavorable outcomes.

Table 1
A selection of solution methodologies for solving expansion problems up to the 1990s.

Method	Reference	GEP	TEP	DEP	Comments
Linear Programming	Garver (1970) Pereira et al. (1985)	No Yes	Yes Yes	No No	Very restrictive, as no operational constraints are considered.
Mixed-integer Linear Programming	Aoki et al. (1990)	No	No	Yes	Weak computational performance due to the long planning horizon and physical and operational constraints.
Benders Decomposition	Cote and Laughton (1979) Pereira et al. (1985)	Yes Yes	No Yes	No No	It guarantees an exact solution, however it lacks performance for multi-stage stochastic problem.
Dynamic Programming	Dusonchet and El-Abiad (1973) Caramanis et al. (1982)	No Yes	Yes No	No No	It has restrictions due to computational and memory needs. It depends on priority criteria in case of simplification.
Stochastic Dynamic Programming	Mo et al. (1991)	Yes	No	No	Dynamic equation is used but there is a curse-of-dimensionality for large-scale problems.

2.3.1. Fuel procurement—LP, MILP

Fuel procurement is a pivotal optimization problem for power producers that depend on a primary fuel source for production. Given the significant cost escalations observed in fuel markets since the 1970s (*e.g.*, the oil price shock), fuel prices have become a crucial factor influencing the overall generation cost (Fancher et al., 1986). The objective of the fuel procurement problem is to minimize the overall cost, including transportation, procurement, storage, subject to technical, and operational constraints. Both the quality and quantity of fuel are essential requirements that must be satisfied. In this regard, a blending of same fuels type from the different sources is employed to ensure the quality level of the resulting fuel.

Fuel procurement is typically provided through two sources. The first involves take-or-pay agreements (fuel contracts), which compel power producers to secure only the necessary quantity. The second source is the spot market for short-term transactions, often associated with significantly higher costs. With respect to this, the fuel procurement problem can generally be formulated (as an LP model) as follows (Sun et al., 2021):

$$(F) \qquad \min \sum_{t\in T}\left(\sum_{i\in N} C_i^C g_{it}^C + C_t^P g_t^P\right) \tag{4a}$$

$$\text{s.t.}\ \beta\left(\sum_{i\in N} g_{it}^C + g_t^P\right) \geq D_t \qquad \forall t\in T, \tag{4b}$$

$$N_i^C \leq g_{it}^C \leq M_i^C \qquad \forall i\in N, t\in T, \tag{4c}$$

$$0 \leq g_t^P \leq M^P \qquad \forall t\in T, \tag{4d}$$

where C_i^C is the fuel cost from contract i while C_t^P is the fuel cost from the spot market for each time period t. For each time period t, the continuous variable g_{it}^C is the purchased quantity from fuel contract i, and the continuous variable g_t^P is the purchased quantity from spot markets. The parameter β is the fixed ratio from fuel to power. M_i^C and M^P are upper bounds of purchased quantity from fuel contract i and the spot market, respectively. N_i^C is the minimum quantity to be purchased under the fuel contract. The aim of the model (4a) is to minimize the total cost by getting the fuel via take-or-pay agreement and spot markets. Constraints (4b) enforce that the output of power producers meets the total demand for each time t. Constraints (4c) and (4d) set limits for fuel purchased from fuel contract i and the spot market, respectively.

The integration of fuel procurement and unit commitment leads to MILP models. Dynamic and linear programming (Van Meeteren, 1984), and Lagrangian relaxation (Cohen & Wan, 1987) were employed to solve operation problems that address fuel constraints. The short-term fuel scheduling can be modeled as a network flow optimization problem and solved through heuristic procedures (Kumar et al., 1984). Commonly, demand and fuel prices are treated as uncertain factors for this problem (Fancher et al., 1986).

2.3.2. Maintenance of generation and transmission assets—MILP

The regular maintenance of transmission facilities and generators play a fundamental role in mitigating the risk of unforeseen faults. These faults have the potential to cause unexpected short-term operational interruptions, impacting system reliability. Despite proactive measures, transmission lines and generators may have breakdowns, leading to high operational costs and unmet power demand. In such cases, it becomes imperative to provide corrective maintenance promptly. In the context of a centrally operated system, the maintenance is scheduled centrally by taking into account comprehensive data, including costs, reliability and security details.

The generator maintenance scheduling (GMS) problem involves deciding when to halt operations for maintenance to ensure reliable operations, while minimizing associated costs. Binary variables are employed to determine the status of each generator within each time window. This discrete component transforms the GMS problem into a MILP problem. When its objective function primarily focuses on maintaining reliability and minimizing cost, considerations such as maintenance time window, demand, and network are treated as constraints (Froger et al., 2016). The transmission maintenance schedule (TMS) aims to establish the maintenance schedule for the lines of the system. It can be addressed either in conjunction with the GMS or independently.

The TMS problem can generally be formulated (as a MILP model) as follows (Sun et al., 2021):

$$(M) \quad \min \sum_{t\in T}\sum_{i\in G} C_i p_{it} \tag{5a}$$

$$\text{s.t.} \sum_{i\in G^N} p_{it} - \sum_{j\in J^N} D_{jt} - \sum_{\forall \ell|s(\ell)=n} B_\ell(\theta_{s(\ell)t} - \theta_{r(\ell)t}) + \sum_{\forall \ell|r(\ell)=n} B_\ell(\theta_{s(\ell)t} - \theta_{r(\ell)t}) = 0 \qquad \forall n\in N, t\in T, \tag{5b}$$

$$|B_\ell(\theta_{s(\ell)t} - \theta_{r(\ell)t})| \leq F_{\ell t}^{max} + K_\ell^M x_{\ell t} - K_\ell^F u_{\ell t} \qquad \forall \ell\in L, t\in T, \tag{5c}$$

$$0 \leq P_i^{min} \leq p_{it} \leq P_i^{max} \qquad \forall i\in G, t\in T, \tag{5d}$$

$$x_{\ell t} \leq x_{\ell(t+v)} \qquad \forall \ell\in L, t\in\{1,\dots,T-V+1\}, v\in\{1,\dots,V-1\}, \tag{5e}$$

$$x_{\ell t} = 0 \qquad \forall \ell\in L, t\in\{T-V+2,\dots,T\}, \tag{5f}$$

$$x_{\ell t} \in \{0,1\} \qquad \forall \ell\in L, t\in T, \tag{5g}$$

where the continuous decision variable p_{it} is the power output of generator unit i for each time period t. The binary variable $x_{\ell t}$ represents the planned maintenance for line ℓ for each time period t. The binary indicator $u_{\ell t}$ refers to the existence of an interruption for line ℓ for each time period t, is typically defined as an uncertain parameter. The parameter C_i is the generation cost per unit i; K_ℓ^M reflects the increase in transmission capacity caused by preventive maintenance on line ℓ, while K_ℓ^F shows the reduced transmission capacity as a result of line ℓ's interruption. $\theta_{r\ell t}$ is the voltage angle of the destination-end node of line ℓ for each time t and $\theta_{s\ell t}$ is the voltage angle of the sending-end node of line ℓ for each time t. The parameters B_ℓ and $F_{\ell t}^{max}$ are susceptance and capacity of line ℓ, respectively; D_{jt} is the load of demand point j. V is the minimum duration of maintenance. The objective (5a) minimizes the total cost while satisfying the capacity constraints of generators (5d) and lines (5c), as well as demand requirements (5b). Constraints (5e) and (5f) set a minimum maintenance duration. Constraints (5b) require that the generation-demand balance be maintained at each system node. Constraints (5c) updates the capacity of the transmission line ℓ based on whether preventive maintenance was completed or if a failure occurred. The absolute value relations in constraints (5c) can be adjusted by splitting it into two constraints:

$$-B_\ell(\theta_{s(\ell)t} - \theta_{r(\ell)t}) \leq F_{\ell t}^{max} + K_\ell^M x_{\ell t} - K_\ell^F u_{\ell t} \qquad \forall \ell \in L, t \in T, \quad (6a)$$

$$B_\ell(\theta_{s(\ell)t} - \theta_{r(\ell)t}) \leq F_{\ell t}^{max} + K_\ell^M x_{\ell t} - K_\ell^F u_{\ell t} \qquad \forall \ell \in L, t \in T. \quad (6b)$$

During this period (early years to the 1990s), dynamic programming was typically employed for solving the GMS problem. Since the 1970s, MILP has been used to model GMS and TMS problems (Dopazo & Merrill, 1975). In order to address the computational burden associated with MILP, decomposition methods such as Lagrangian relaxation (Charest & Ferland, 1993), and Benders decomposition (Al-Khamis et al., 1992) were mainly used in the literature. Heuristic methods (Charest & Ferland, 1993) and multi-objective optimization (Kralj & Rajaković, 1994) were also proposed.

2.3.3. Hydro-thermal scheduling—LP, MILP, NLP

Hydro power scheduling is essential to effectively manage power generation from hydro plants. It aims to minimize the total cost by determining decisions including reservoir level, water spillage and water discharge, based on operational and environmental constraints. The Hydro-Thermal Scheduling problem (HTSP) manages power production from a combination of run-off-the-river and hydro reservoir plants as well as thermal plants. Thermal plants are strategically employed to compensate for fluctuations in hydro plants production due to uncertain water inflow, while ensuring the fulfillment of power demand. Note that hydro production has basically zero operational cost, if water is available. As such, the water has a future value, if left in the reservoir. There is an associated opportunity cost when using water for electricity production. This so-called water value depends on various factors, such as the reservoir size, reservoir levels, inflows, inflow uncertainty or thermal generation cost.

The HTSP typically requires an optimization horizon of at least one year to capture the seasonal variation of hydro inflows into the reservoir system. Due to the uncertain inflow, the HTSP is mainly modeled as a multi-stage stochastic problem. However, for the ease of presentation, we consider a deterministic formulation (as an NLP) of a basic HTSP as follows (de Queiroz, 2016):

$$(H) \qquad \min \sum_{t\in T}\left(\sum_{j\in G} C_{jt}^T p_{jt} + C_t^{LS} u_t\right) \quad (7a)$$

$$\text{s.t.} \quad v_{i,t+1} = v_{it} - x_{it} - y_{it} + \sum_{h\in H_i}(x_{ht} + y_{ht}) + A_{it} \quad : \mu_{it} \qquad \forall i \in H, t \in T, \quad (7b)$$

$$\sum_{j\in G} p_{jt} + \sum_{i\in H} f_{it}(x,h) = D_t - u_t \qquad \forall t \in T, \quad (7c)$$

$$V_{min} \leq v_{i,t+1} \leq V_{max} \qquad \forall i \in H, t \in T, \quad (7d)$$

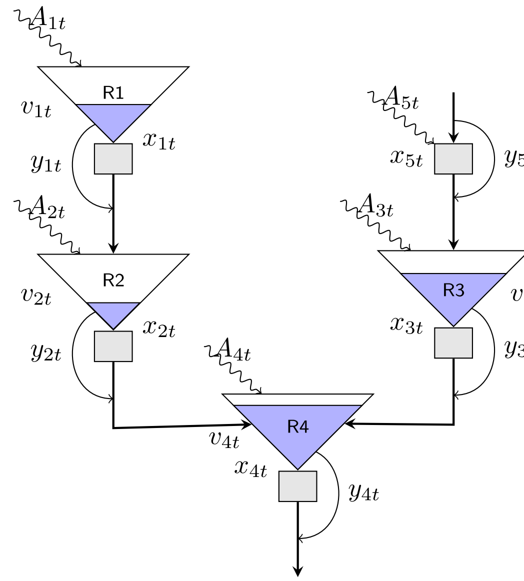

Fig. 2. Hydro system with four reservoirs (R1–R4) and run-off-the-river hydro plant.

$$X_{min} \leq x_{it} \leq X_{max} \qquad \forall i \in H, t \in T, \quad (7e)$$

$$P_{min} \leq p_{jt} \leq P_{max} \qquad \forall i \in H, t \in T, \quad (7f)$$

with the decision variables p_{jt}, y_{it}, x_{it}, v_{it} and u_t. Here, these refer to generated power from thermal plant j, water spillage, water discharge, reservoir level for reservoir i, and unmet demand respectively. Set H_i comprises the hydro plants situated directly upstream of reservoir i. The decision variable y_{ht} is the upstream water spillage, while x_{ht} represents upstream water discharge for all immediate upstream hydro plants. The parameter C_{jt}^T represents the generation cost, while C_t^{LS} is the load shedding cost; A_{it} is the (uncertain) water inflow. The hydro system with four reservoirs is presented in Fig. 2. The power output is determined by the hydro production function, which is actually a non-linear function of the water discharge (x_{it}) and net water-head (h_{it}^{net}). The hydro production function can be formulated as follow (Diniz & Maceira, 2008):

$$f_{it}(x,h) = \eta\rho_i h_{it}^{net} x_{it} \quad (8a)$$

where η is a fixed value (9.81×10^{-3}) that includes gravity's acceleration, water's density, and a unit energy conversion factor. ρ_i is the efficiency of the reservoir i when converting turbined water into electricity. The net water-head h_{it}^{net} can be stated as follow;

$$h_{it}^{net} = h_{it}^f(v) - h_{it}^{tail}(y) - h_{it}^{loss} \quad (9a)$$

where h_{it}^f indicates the forebay level that relies on volume v_{it}, while h_{it}^{tail} is the tailrace level, depending on the reservoir i and water spillage y_{it}; h_{it}^{loss} refers to penstock head loss. The objective (7a) minimizes the total cost of power supply. Equalities (7b) enforce water balance and constraints (7c) represent demand balance. (7d)–(7f) establish the bounds for reservoirs level, water discharge of reservoirs and power generation of thermal plants. The dual value (μ_{it}) associated with (7b) are the water values for each stage t and each reservoir i. We want to point out that most HTSP models consider a linear approximation of the hydro production function:

$$f_{it}(x,h) = g_i x_{it} \quad (10a)$$

with parameter g_i specific to hydro production for hydro plant i.

For an in-depth discussion of the mathematical models proposed for reservoir management and operations in this period, we refer the reader to the review paper by Yeh (1985). Since the 1970s, stochastic dynamic programming has been utilized to solve HTSP (Grygier & Stedinger, 1985; Yakowitz, 1982). However, several curses-of-dimensionalities (due to the exponential growth of the number of inflow scenarios with the considered stages T and correlated reservoirs H) make solving large-scale problems intractable with stochastic dynamic programming. As a general guide, systems with more than 10 reservoirs can no longer be solved efficiently with stochastic dynamic programming methods. To mitigate the scalability issues of stochastic dynamic programming, Benders decomposition was proposed to solve multi-stage HTSP (Morton, 1996; Pereira & Pinto, 1985). SDDP was introduced by Pereira and Pinto (1991) as a sampled version of Nested Benders Decomposition (NBD). Finally, SDDP was the game changer to solve large-scale systems, even with more than 100 hydro plants. Since the 1990s, SDDP is the method-of-choice for solving large HTSPs. As mentioned, the hydro production function leads to a nonlinear hydro-thermal problem. Studies and modeling approaches are proposed in the literature to address the nonlinear hydro-thermal problem and we review these in Section 3.3.4.

2.4. Operations

2.4.1. Optimal power flow—NLP

The optimal power flow (OPF) is a classical optimization tool in power systems. Over the past fifty years, OPF has been an important and extensively researched nonlinear and non-convex optimization problem. It can be used effectively for decision making over a wide range of planning horizons from long-term planning to real-time adjustments (Wood et al., 2013). OPF extends the economic dispatch problem by considering power flows in the power system. The aim of the OPF problem is to determine active and reactive power output of each generator subject to system and power flow constraints (Conejo & Baringo, 2018).

The formulation of the classical optimal power flow problem dates back to the 1960s (Carpentier, 1962). It models Ohm's and Kirchoff's laws for a system in steady-state. We denote N as a set of buses, L as a set of branches, and G as a set of generator units. The OPF model can be formulated as follows

$$(O) \quad \min \sum_{i\in G} C_i\left(p_i^G\right) \tag{11a}$$

$$\text{s.t.} \quad P_i(V,\delta) = p_i^G - p_i^L \qquad \forall i \in N, \tag{11b}$$

$$Q_i(V,\delta) = q_i^G - q_i^L \qquad \forall i \in N, \tag{11c}$$

$$0 \le P_i^{G,min} \le p_i^G \le P_i^{G,max} \qquad \forall i \in G, \tag{11d}$$

$$Q_i^{G,min} \le q_i^G \le Q_i^{G,max} \qquad \forall i \in G, \tag{11e}$$

$$0 \le V_i^{G,min} \le V_i^G \le V_i^{G,max} \qquad \forall i \in N, \tag{11f}$$

$$\delta_i^{G,min} \le \delta_i^G \le \delta_i^{G,max} \qquad \forall i \in N. \tag{11g}$$

The aim of OPF model (11a)–(11g) is to minimize the total generation cost. The non-negative variable p_i^G is the active power output of generator i, while q_i^G is the reactive power output of generator i. Constraints (11b) are the power flow equations of active power, also called real power (P), that refers the actual power flows, moving from generators to loads. Constraints (11c) are the power flow equation of reactive power (Q) which is related to voltage levels. Constraints (11d)–(11g) identify bounds for the capacity of the generators, the voltage range, and the power flow to ensure a balanced operation of the power system. Fig. 3 presents a small example OPF problem, illustrating the bus, branch, and load. This OPF model (11a)–(11g), including alternating current (AC) power flow Eqs. (11b)–(11c), is a nonlinear, non-convex continuous program, depending on the cost function $C_i(\cdot)$. AC power flow equations can be represented equivalently in both polar and rectangular forms. The polar form is connected to the voltage magnitude V, voltage phase angle δ, admittance Y and angle θ of admittance. Constraints (11b)–(11c) can be formulated in the polar form as below (Frank et al., 2012a)

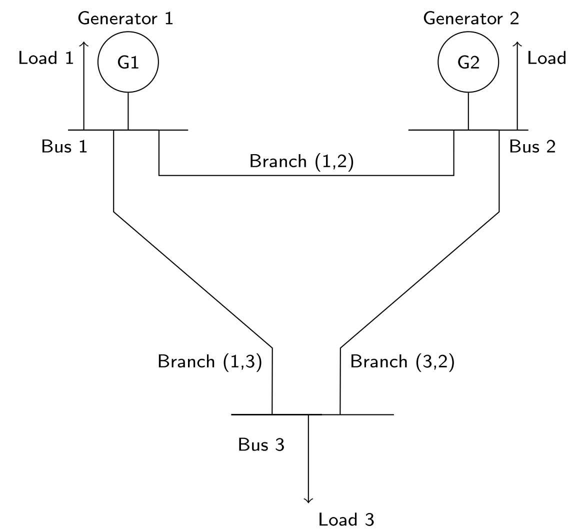

Fig. 3. 3-bus system (Lesieutre et al., 2011).

$$P_i(V,\delta) = V_i \sum_{k=1}^{N} V_k Y_{ik} \cos\left(\delta_i - \delta_k - \theta_{ik}\right) \qquad \forall i \in N, \tag{12a}$$

$$Q_i(V,\delta) = V_i \sum_{k=1}^{N} V_k Y_{ik} \sin\left(\delta_i - \delta_k - \theta_{ik}\right) \qquad \forall i \in N. \tag{12b}$$

Let us express voltage, $\tilde{V}_i = E_i + jF_i$, and admittance, $\tilde{Y}_{ik} = G_{ik} + jB_{ik}$, in rectangular coordinates. The real (E_i and G_{ik}) and imaginary (F_i and B_{ik}) components of power flow equations are separated using the imaginary unit j. Then, constraints (11b)–(11c) can be rewritten in the rectangular form as follows (Frank & Rebennack, 2016)

$$P_i(E,F) = \sum_{k=1}^{N} G_{ik}\left(E_iE_k + F_iF_k\right) + B_{ik}\left(F_iE_k - E_iF_k\right) \qquad \forall i \in N, \tag{13a}$$

$$Q_i(E,F) = \sum_{k=1}^{N} G_{ik}\left(F_iE_k - E_iF_k\right) - B_{ik}\left(E_iE_k + F_iF_k\right) \qquad \forall i \in N. \tag{13b}$$

The resulting formulation (11a)–(11g) with (13a)–(13b) and a quadratic cost function $C_i(\cdot)$ is a non-convex quadratically constrained quadratic program. The OPF model may incorporate additional factors such as contingency and security constraints, reserve requirements, renewable policy prerequisites, and stability constraints. Broadly speaking, these extensions incorporate additional variables and some of them are discrete in nature, resulting in mixed-integer nonlinear problems (Skolfield & Escobedo, 2022).

The OPF problem has been widely addressed in the literature with numerous classic optimization methods to solve it (Frank et al., 2012a, 2012b). The proposed approaches during this period (early years to the 1990s) do not guarantee global optimal solutions. The Reduced Gradient Method that is a gradient-based optimization method for solving large-scale OPF problems, uses the Jacobian matrix to compute the direction of the search at each iteration (Fernandes et al., 1980). Newton's method (Sasson et al., 1973), which involves computing the Hessian matrix, and Quasi-Newton method (Housos & Irisarri, 1982), that utilizes the approximation of Hessian matrix, were studied. Interior point methods (Momoh et al., 1994), Sequential Linear Programming (Alsac et al., 1990), Sequential Quadratic Programming (Burchett et al., 1984) were also used (Table 2). In recent years, significant efforts (*e.g.* convex relaxations) have been investigated to address and find global optimal solutions at least for special cases, which we discuss in Section 3.

Table 2
A selection of solution methodologies for solving optimal power flow problems up to the 1990s.

Method	Reference	Capabilities	Limitations
Gradient descent methods	Fernandes et al. (1980)	Jacobian matrix is used to compute direction of the iteration search.	It is computationally expensive due to slow convergence.
Newton's method	Sasson et al. (1973)	It converges faster than gradient descent and can handle nonlinear constraints more efficiently.	It requires computing the Hessian matrix that can be computationally expensive for large systems.
Quasi-Newton methods	Housos and Irisarri (1982)	The approximation of Hessian matrix is used.	The approximation of Hessian matrix can be unable to provide effective search directions.
Sequential Linear Program	Alsac et al. (1990)	Solving nonlinear problems with a sequence of linear subproblems.	It can find a local optima and lead to oscillation.
Sequential Quadratic Programming	Burchett et al. (1984)	Providing better computational performance than SLP.	It can find a local optima and lead to oscillation.
Interior Point Methods	Momoh et al. (1994)	Compared to traditional methods, it is an efficient method with less iterations and solution time.	It is dependent on initial values.

2.4.2. Economic dispatch—LP, NLP

The economic dispatch (ED) problem aims at minimizing the cost of generating units by determining the generation outputs. The solution of the unit commitment problem (see Section 2.4.3) — which identifies the generation units that are online — serves as an input to the ED problem to determine the optimal output of each unit during the planning horizon. The ED problem was first formulated in the early 1920s when the need of new methods for optimizing the economic allocation of power generation became apparent (Happ, 1977). The optimal dynamic dispatch problem, as an extension of static ED, was also developed to determine the optimal power outputs for a particular period of time. Ramp rate limits (Bechert & Kwatny, 1972) and reserve constraints (Wood, 1982) were considered in the ED problem. Because of increased environmental concerns, the ED models incorporated environmental constraints (Bernow et al., 1991). Given that the power output is generally considered to be a continuous variable, the ED problem is often expressed as a linear program. However, the ED problem was also studied as a nonlinear optimization problem. Nonlinearity in ED has been observed from several factors. Quadratic cost functions for the thermal plants (Reid & Hasdorff, 1973) and nonlinear expressions for the hydro production function (Luo et al., 1989) resulted in a nonlinear optimization problem. A short-term operation planning for a combined thermal and hydroelectric power system with transmission losses was investigated (Chandler et al., 1953). Since network-constraint ED determines the output of generating units while conforming to power flow constraints on transmission lines, AC power flow equations also introduce nonlinearity. To linearize the power flow equations, network-constrained ED and dynamic ED were frequently presented with a DC power flow formulation (Pereira & Pinto, 1982; Podmore, 1974). The DC power flow is a simplified linear approximation of the AC power flow equations, assuming constant voltage magnitudes and phase angles. That is, it is effective in analyzing power flows in large transmission networks.

The objective of the ED problem is to determine the actual power output of each generator to fulfill demands at minimum cost, while satisfying the operational and technical constraints of the generating units. The ED problem is formulated (as an LP model) as below (Conejo & Baringo, 2018):

$$(E) \quad \min \sum_{t\in T}\sum_{i\in G} C_i^V p_{it} \tag{14a}$$

$$\text{s.t.} \sum_{i\in G} p_{it} = P_t^D \qquad \forall t\in T, \tag{14b}$$

$$0 \le P_i^{min} \le p_{it} \le P_i^{max} \qquad \forall i\in G, t\in T, \tag{14c}$$

where we consider a power system with G generation units. The non-negative decision variable p_{it} is the actual power production of generator i for each hour t and the parameter P_t^D represent the load for each hour t. The objective of the model (14a) is to minimize the generation cost while meeting balance Eqs. (14b) and bounds (14c).

Several linear optimization methods were used to solve the ED problem, such as the simplex method (Wang & Shahidehpour, 1994), interior-point methods (Irisarri et al., 1998), Lagrangian relaxation methods (Hindi & Ghani, 1991) and Dantzig–Wolfe decomposition (Pereira & Pinto, 1982; Quintana et al., 1981).

2.4.3. Unit commitment—MILP

The unit commitment (UC) problem is a practically important and widely studied optimization problem in power system operations. It aims at determining the optimal schedule of generating units. In order to supply the aggregate demand during a certain planning horizon, typically one day, it seeks to identify which generation units should be scheduled to produce electricity, while satisfying their technical constraints (Anjos & Conejo, 2017). The UC problem emerged in the 1940s as a result of the growth of the power industry (Li et al., 1997). Since the 1970s, the proliferation of computer technology and the escalating intricacy of power systems have led to the evolution of more complex UC problem models that integrated additional constraints, such as transmission constraints (Ma & Shahidehpour, 1998), fuel constraints (Cohen & Wan, 1987) and environmental constraints (Kuloor et al., 1992). Hence, UC models are becoming more sophisticated to ensure the reliability, efficiency, and sustainability of power system operations. Therefore, the stochastic UC problem was extensively studied (Zheng et al., 2014). Deterministic UC focuses on fixed inputs for next-day scheduling, while stochastic models encompass also uncertainties within the same time frame (Bouffard et al., 2005).

The UC problem is typically formulated as a MILP model. The UC problem uses binary variables to represent the on/off status of units. To obtain a mixed-integer linear program, certain aspects of the formulation may be relaxed, for instance by using linearized cost functions. We consider a power system with G generation units and a forecasted electricity demand P_t^D for each hour t of the planning horizon T. The objective of the UC problem is to minimize the total operating cost of the generation units while satisfying the demand and the system constraints.

The deterministic UC problem can be formulated (as a MILP model) as follows (Conejo & Baringo, 2018):

$$(U) \quad \min \sum_{t\in T}\sum_{i\in G}\left(C_i^F u_{it} + C_i^V p_{it} + C_i^{SU} y_{it} + C_i^{SD} z_{it}\right) \tag{15a}$$

$$\text{s.t.} \quad y_{it} - z_{it} = u_{it} - u_{i,t-1} \qquad \forall i\in G, t\in T, \tag{15b}$$

$$y_{it} + z_{it} \le 1 \qquad \forall i\in G, t\in T, \tag{15c}$$

$$P_i^{min} u_{it} \le p_{it} \le P_i^{max} u_{it} \qquad \forall i\in G, t\in T, \tag{15d}$$

$$p_{it} - p_{i,t-1} \le R_i^U u_{i,t-1} + R_i^{SU} y_{it} \qquad \forall i\in G, t\in T, \tag{15e}$$

Table 3
A selection of solution methodologies for solving unit commitment problems up to the 1990s.

Method	Reference	Capabilities	Limitations
Mixed-integer Linear Programming	Dillon et al. (1978)	It guarantees an exact solution. It increases modeling capability.	Inadequate computational performance against the combinatorial nature and physical and operational constraints.
Exhaustive Enumeration	Kerr et al. (1966)	It guarantees an exact solution for various optimization problems.	Impractical for large-scale systems.
Priority Listing	Shoults et al. (1980)	Simple, straightforward method that can be easily implemented.	It is dependent on the priority criteria and the initial schedule.
Dynamic Programming	Pang et al. (1981)	It is used to solve subproblems in the decomposition scheme.	It has restrictions due to computational and memory needs.
Benders Decomposition	Turgeon (1978)	Decomposing the original problem into easier-to-solve subproblems.	Lack of performance for large cases due to slow convergence.
Lagrangian Relaxation	Zhuang and Galiana (1988)	Decomposing the original problem and relaxing certain constraints.	Dependent on the formulation of problem and the multipliers selection.

$$p_{i,t-1} - p_{it} \le R_i^D u_{it} + R_i^{SD} z_{it} \qquad \forall i \in G, t \in T, \quad (15f)$$

$$\sum_{i \in G} p_{it} = P_t^D \qquad \forall t \in T, \quad (15g)$$

$$p_{it} \ge 0 \qquad \forall i \in G, t \in T, \quad (15h)$$

$$u_{it}, y_{it}, z_{it} \in \{0, 1\} \qquad \forall i \in G, t \in T, \quad (15i)$$

with the decision variables u_{it}, p_{it}, y_{it} and z_{it}. Here, the binary variable u_{it} indicates whether a generating unit i is online in time period t. The non-negative decision variable p_{it} is the power output of generating unit i during time period t. y_{it} and z_{it} are binary variables to represent start-up and shut-down decisions, respectively. The parameters R_i^U, R_i^D, R_i^{SU} and R_i^{SD} are ramping limits. These limits correspond to the generating units' ramping up, ramping down, start up and shut down, respectively. The objective function (15a) aims to minimize the total cost, including fix (C_i^F), variable (C_i^V), start-up (C_i^{SU}), and shut-down (C_i^{SD}) costs. Constraints (15b)–(15c) impose that shutdown is only possible for online thermal generating units, while startup is only permissible for offline units. Constraints (15d) enforce the bounds on the power output of generating units, and constraints (15e)–(15f) refer to ramping limits. Equality constraints (15g) ensure that the output of generating units meets the total demand at each time period.

Numerous algorithms have been developed in the past five decades to solve the UC problem, including exact methods, (meta-)heuristic and hybrid algorithms. In the late 1970s and early 1980s, dynamic programming emerged as a tool for solving the UC problem. Dynamic programming methods can handle complex constraints and allowed for the inclusion of multiple operating conditions and system states (Pang et al., 1981). In response to the computational limitations "curse-of-dimensionality" of dynamic programming, from the early 1980's, the Lagrangian relaxation (Zhuang & Galiana, 1988), Benders decomposition (Turgeon, 1978) and branch-and-bound (Turgeon, 1977) algorithms were proposed. These methods reduce the computational burden by decomposing the original problem into subproblems and solving them sequentially. The efficacy of Lagrangian relaxation technique largely depends on the selection of appropriate updating rules for the Lagrange multipliers and on the formulation of the problem itself (Virmani et al., 1989). Within a Lagrangian decomposition scheme, dynamic programming was also utilized to solve each individual unit problem (with relaxed demand balance). In the 1990s, MILP approaches were proposed (Dillon et al., 1978). Table 3 presents a selection of methods to solving UC problems up until the 1990s.

As an important extension to UC, the network-constrained unit commitment (NCUC) problem was proposed in the 1980s. It extends the UC problem by incorporating the transmission network constraints. A NCUC problem can be seen as the combination of the UC and the ED problems. The objective of the NCUC problem is to obtain the optimal generation schedule that minimizes the total generation cost subject to the transmission network constraints. Given the presence of AC constraints, currently, convex relaxation techniques like semidefinite and second-order cone formulations, along with decomposition approaches, are predominantly employed to solve the NCUC and security-constrained unit commitment (SCUC) problems, as discussed in Section 3.4.

2.5. Control

Power systems need to keep the frequency and voltages at specified operational levels. Power system control is a dynamic process that optimizes and coordinates various control mechanisms for ensuring the stability of the entire power system (Gomez-Exposito et al., 2018). This section provides an overview of three critical aspects of power system control.

2.5.1. Frequency and reactive (optimal) control

In power systems, loads can fluctuate unpredictably, leading to undesirable deviations in the system frequency. These deviations may have a significant impact on power operations. Optimal frequency control aims to maintain the power system frequency within specified limits (Bevrani et al., 2021). The secondary frequency control, also known as Load Frequency Control (LFC), regulates the balance between generation and demand to address the frequency deviations. In the 1970s, optimal control theory was employed for LFC. The control scheme takes into account the state variable representation of the model to provide feedback (Fosha & Elgerd, 1970). The integration of frequency control constraints into ED problem was initially suggested in the literature and limits (16a)–(16b) on reserve r_i are ensured as follow (Cardozo et al., 2017)

$$\sum_{i \in G} r_i \ge R^{min} \quad (16a)$$

$$r_i \le R_i^{max} \qquad \forall i \in G. \quad (16b)$$

The optimal reactive power control and the OPF problem are related concepts in power systems optimization. The reactive power is essential to keep voltage levels within acceptable limits and to provide the reliable operation. Optimal reactive control involves regulating the output of reactive power to minimize power losses and maintain voltage stability while satisfying demand and operational constraints. Typically, optimal reactive power control is a nonlinear optimization problem that can be formulated as follow (Khan et al., 2016)

$$(C) \qquad \min \ P_{loss} + \lambda_V \sum_{i=1}^{N_{V^*}} (V_i - V_i^*) + \lambda_Q \sum_{i=1}^{N_{Q^*}} (q_i^G - q_i^{G*}) \quad (17a)$$

$$\text{s.t.} \ \ p_i^G - p_i^L - V_i \sum_{k=1}^{N} V_k Y_{ik} cos\left(\delta_i - \delta_k - \theta_{ik}\right) = 0 \qquad \forall i \in N, \quad (17b)$$

$$q_i^G - q_i^L - V_i \sum_{k=1}^{N} V_k Y_{ik} sin\left(\delta_i - \delta_k - \theta_{ik}\right) = 0 \qquad \forall i \in N, \tag{17c}$$

(11d)–(11g). (17d)

We note that this model is similar to the OPF model (see Section 2.4.1). The first term of the objective function refers to power loss. With the following formulation,

$$P_{loss} = \sum_{i=1}^{N} p_i^G - \sum_{i=1}^{N} p_i^L,$$

the power loss can be obtained. The second term aims to maintain voltage stability by considering the reference voltage (V^*) and penalty cost (λ_V), while the third term is to regulate reactive power by using the reference reactive power (Q^*) and penalty cost (λ_Q). Eqs. (17b)–(17c) are balance constraints. In the context of centralized framework, linear program, gradient method and interior point method are most commonly used for optimal reactive power control in the literature.

2.5.2. State estimation—NLP

State estimation seeks to identify the most likely "state" of the system using redundant measurements. The outcome of the state estimator provides real-time information for various purposes such as security, control, and economic dispatch. The data for the state estimator include power flows, injections, and voltage measurements.

Since state estimation is a nonlinear programming problem, various techniques, including linear approximations, are employed. The weighted least squares (WLS) algorithm was introduced in the late 1960s for static state estimation (Schweppe & Wildes, 1970). The primary concept in WLS is to enhance accuracy by minimizing the difference between predicted and actual values. Alternative methods have also been introduced, such as least absolute value, quadratic-constant, quadratic-linear, least median of squares, and least trimmed of squares. Given its computational effectiveness and stability, WLS is predominantly used as a state estimator in the literature. Considering the concept of WLS, the state estimation problem is formulated as (Conejo & Baringo, 2018):

$$(S) \qquad \min \quad \sum_{k\in K} W_k \left(H_k(x_1,\dots,x_S) - Z_k\right)^2, \tag{18a}$$

where K is the set of measurements. H_k is the function of the state variables, $x_1, \dots, x_S$, that are determined based on measurement $k \in K$. W_k represents the weight of measurement k. Z_k are the measurements, including power flows, injections, and voltages. Decoupled algorithms were proposed (Garcia et al., 1979). An initial practical algorithm for the state estimator was provided (Allemong et al., 1982).

2.6. Forecasting

In a centralized framework, due to the absence of market dynamics and the availability of limited renewable technologies, forecasting methods primarily focus on electricity demand and commodity prices. This section focuses on demand and price forecasting that are crucial for optimizing power generation, facilitating load management, and aiding in system planning and operation.

2.6.1. Demand and price forecasting—Time series and others

Knowing accurately the future demand plays a key role in making decisions, such as managing power generation, distributing load and other infrastructure. Demand forecasting is categorized into various horizons; short-term, medium-term and long-term. In a centralized framework, forecasting power demand mainly relied on historical load data and customer consumption patterns. Until the 1990s, power demand forecasting mainly used linear approaches, with the assumption that past load patterns and relationships would continue to progress in a linear manner. The choice of linear models was influenced by their simplicity and suitability for calculations (Hernandez et al., 2014). Demand forecasting is primarily centered on traditional methods relying on time series analysis. Additionally, qualitative techniques, such as expert assessment, were utilized. Multiple regression is an extension of linear regression, considering multiple independent variables, that aims at determining the connection between power load and independent variables such as weather conditions (Papalexopoulos & Hesterberg, 1990). Exponential smoothing was employed as well for load forecasting (Christiaanse, 1971). Several time series models, such as auto-regressive (AR), auto-regressive moving-average (ARMA), and auto-regressive-integrated-moving average (ARIMA) were commonly used for demand forecasting depending on the characteristics of the specific dataset and the underlying patterns in the time series. The ARIMA model is formulated based on three key components: autoregressive (AR), differencing, and moving average (MA). In the ARIMA model, forecasting values involves a mix of linear forms of previous values and historical errors. An ARIMA model can be expressed as follows (Box et al., 2015):

$$Y_t = \phi_0 + \varepsilon_t + \sum_{p\in P} \left(\phi_p Y_{t-p} - \theta_p \varepsilon_{t-p}\right), \tag{19a}$$

where p is the order lag of the time series. Y_t is the observed value at time t, ε_t is a random error term at time t, and ϕ are the autoregressive parameters, while θ are the moving average parameters. The goal here is to predict value Y_t from the past observations $Y_{t-1}, Y_{t-2}, \dots, Y_{t-p}$.

Since the 1970s, the forecasting of fuel prices has been important for power producers that rely on a fuel source for power production. Until the 1990s, oil, coal, and natural gas were the most heavily used fuels. The forecasting of fuel price was primarily conducted using traditional statistical methods. In particular, time series analysis models such as moving averages, and autoregressive models have been used.

3. Recent times—Decentralized (and centralized) operations and planning (1990-date)

3.1. Overview

In this section, we focus on the decentralized operations and planning from 1990 to the present. Additionally, we present recent methodologies for centralized operations and planning during the same time frame. From the 1990s onward, significant changes have occurred in many power systems around the world, including a transition to a liberalized market environment, a substantial increase in weather-dependent electricity production, and advancements in energy storage and control systems (Conejo & Sioshansi, 2019; Parker et al., 2019). These changes increase the necessity for discrete decisions, nonlinearity, uncertain parameters and a decentralized structure in the optimization models. With the developments in operations research as well as advancements in computation capabilities, the utilization of MILP and MINLP has increased to solve complex problems (Achterberg & Wunderling, 2013). EPEC, MPEC and decentralized algorithms have been developed in response to the needs of a decentralized framework. Regarding the modeling of uncertainty, factors such as demand levels, generation outputs, electricity and commodity prices have been considered uncertain during this period. Several methods, including stochastic optimization, robust optimization and fuzzy logic, are being explored (Roald et al., 2023; Weber, 2006). In particular, as computing machinery and software tools have evolved, certain approaches for multi-stage problems and advanced algorithms have being studied to consider uncertainty and risk factors simultaneously. Furthermore, advancements in optimization software, artificial intelligence and machine learning techniques played a pivotal role in ensuring the accuracy and efficiency of power system models. This section focuses on the main power systems problems — investment, operation planning, operations, control, and forecasting — in order to examine this period.

3.2. Investment

The expansion planning in the centralized structure aims typically to either minimize the total cost or maximize social welfare. However, in the context of a decentralized framework, the objective of power companies is to maximize their profits via investment decisions, considering expected future prices and earnings from potential investments (Chuang et al., 2001). Additionally, compared to early times (1970–1990s), the expansion planning — starting in the 1990s — encounters greater uncertainties, encompassing deviations in renewable power generation, demand growth, electricity and commodities prices (Botterud et al., 2005).

3.2.1. Investment in generation facilities by producers—MPEC, EPEC

In a decentralized structure, power producers participate in an energy market to maximize their profits by making optimal investment decisions. Given this objective, it aims to use market-clearing outcomes as short-term operation decisions to its advantage. Generation expansion problems in the decentralized structure commonly use a bi-level model, where the upper level seeks investment decisions to optimize its profit and lower-level problems are to capture diverse market clearing scenarios (Conejo et al., 2016). Converting this bi-level model into an MPEC involves substituting lower-level problems by their optimality conditions (Kazempour et al., 2013). With the formulation of MPEC, each agent's profitability is optimized while equilibrium constraints ensure the optimality conditions. EPEC is a broader formulation of investment problem that considers a series of connected MPECs to involve multiple power producers in a decision-making process (Conejo et al., 2020).

An MPEC of the generation expansion problem is formulated below (Gabriel et al., 2012),

$$(I) \quad \min \sum_{h\in H}\Big[\sum_{i\in G} C_i^E p_{ih}^E + \sum_{n\in N} C_n^N p_{nh}^N\Big] + \sum_{n\in N} I_n^N p_n^{N^{max}} \tag{20a}$$

$$\text{s.t.} \quad 0 \le p_n^{N^{max}} \le \bar{P}_n^{N^{max}} \qquad \forall n \in N, \tag{20b}$$

$$\left\{ \sum_{i\in G} p_{ih}^E + \sum_{n\in N} p_{nh}^C = \sum_{d\in J} P_{dh}^D \right. \tag{20c}$$

$$0 \le p_{ih}^E \le P_i^{E^{max}} \qquad \forall i \in G, \tag{20d}$$

$$0 \le p_{nh}^N \le p_n^{N^{max}} \qquad \forall n \in N, \tag{20e}$$

$$C_i^E - \lambda_h + \mu_{ih}^{E^{max}} \ge 0 \qquad \forall i \in G, \tag{20f}$$

$$C_i^N - \lambda_h + \mu_{nh}^{N^{max}} \ge 0 \qquad \forall n \in N, \tag{20g}$$

$$\mu_{ih}^{E^{max}} \ge 0 \qquad \forall i \in G, \tag{20h}$$

$$\mu_{nh}^{N^{max}} \ge 0 \qquad \forall n \in N, \tag{20i}$$

$$\left. \sum_{i\in G} C_i^E p_{ih}^E + \sum_{n\in N} C_n^N p_{nh}^N = \lambda_h \sum_{d\in J} P_{dh}^D - \sum_{i\in G} \mu_{ih}^{E^{max}} P_i^{E^{max}} - \sum_{n\in N} \mu_{nh}^{N^{max}} p_n^{N^{max}} \right\} \qquad \forall h \in H. \tag{20j}$$

The GEP is the nested problem that can be decomposed into an upper-level (investment problem) and lower-level (market clearing problem) problem. The primal–dual formulation is used for optimality conditions when developing an MPEC as single-level. Thus, we substitute the market-clearing problem with its primal constraints, dual constraints, and strong duality equality to develop an MPEC. $p_n^{N^{max}}$ is the investment decision that correspond to the upper-level problem. p_{ih}^E and p_{nh}^N are the operational decision in the lower-level problem for each market condition at hour h. C_i^E is the production cost of existing generating unit i and C_n^N is the production cost of new generating unit n. P_{dh}^D is the load of demand d for each hour h. Eqs. (20j) are strong duality equalities, requiring that the optimal values of the primal and dual objective functions for the market clearing problem are identical. Constraints (20c)–(20e) are primal constraints, while constraints (20f)–(20i) are dual constraints of the market clearing problem. We note that λ_h is the dual value of the equality constraint (20c), also known as the marginal market prices. μ_{ih}^E and μ_{nh}^N are indicated for the dual value of the inequality constraints (20d)–(20e).

Several methods for market-oriented GEP have been thoroughly investigated (Table 4). Moreover, there are advancements in handling uncertainty in both centralized and decentralized structures. Baringo and Conejo (2011b) proposed MPEC for wind power investment decisions within the decentralized environment. Baringo and Conejo (2011a) introduced a Benders decomposition algorithm for considered MPEC to solve the wind power investment problem. A generation investment model under uncertainty was proposed to solve stochastic MPECs by utilizing Benders decomposition (Kazempour & Conejo, 2011). An EPEC model was proposed for investment equilibria in electricity and gas markets (Chen et al., 2020). Further models (MPEC, EPEC) can be found in Kazempour et al. (2010). Game theory (Lucas & Taylor, 1993) and agent-based simulation (Gnansounou et al., 2004) were also used for generation expansion problems in the decentralized structure. Within this period, there has been some development on a centralized-based approach. In Rebennack (2014), Benders decomposition was utilized, where the subproblems are multi-stage stochastic linear optimization problems solved by SDDP, in order to solve the hydro-thermal expansion planning problem. Bundle methods (Sagastizábal & Solodov, 2012), robust optimization (Dehghan et al., 2013), generalized Benders decomposition (Liu et al., 2024), nested Benders decomposition (Lara et al., 2018; Yagi & Sioshansi, 2024) and stochastic dual dynamic integer programming (SDDiP) (Lara et al., 2020) were used for generation expansion problems (Table 4). Hinojosa and Velásquez (2016) provided a multi-stage generating capacity expansion planning problem with DC-based security constraints.

With the goal of capturing associated operational costs and constraints, operational problems are usually considered in investment problems. This results in large-scale models with short time resolution, increasing computational burden. There are some modeling efforts in the literature to handle investment problems with long planning horizon and short time resolution. A multi-horizon modeling approach was proposed for expansion planning problem to reduce model size when compared to standard multi-stage formulations (Zhang et al., 2024). This approach is also capable of accounting for different levels of uncertainty. In order to examine the short-term effects and uncertainties of operational problems, the representation of system operations was simplified. As a result, representative time periods such as representative days (Lara et al., 2018) and representative weeks (De Sisternes et al., 2016) were utilized in the investment problems instead of taking into account all time steps. Additionally, Merrick (2016) investigated how time-dependent variables are represented in power expansion planning models and assessed if utilizing representation time periods can mislead technology capacities. Ueckerdt et al. (2015) utilized residual load duration curves to integrate short-term fluctuations into long-term planning frameworks. Moreover, Kotzur et al. (2021) review various approaches to reduce the complexity of the power system models.

3.2.2. Investment in transmission and distribution facilities—MILP, MINLP

In a decentralized framework, an independent agent manages the operation and expansion of the transmission facilities. From this standpoint, transmission expansion planning (TEP) is typically centralized, marking a key distinction between TEP and GEP in the decentralized framework. The aim of TEP is to minimize the total generation cost while increasing the reliability (Conejo et al., 2016). TEP is commonly formulated as MILP or MINLP. Research on decomposition methods has been studied in the literature. Garcés et al. (2009) proposed a TEP model as bi-level problem that is transformed to MILP problem by using duality theory. Escobar et al. (2008) introduced a TEP framework tailored for a competitive electricity market, solved through the utilization of a genetic algorithm. Tor et al. (2008) introduced a TEP model that incorporates transmission congestion and the influence of investment decisions; Benders decomposition was employed to solve the model. Roh et al. (2007) involved incorporating the influence of

Table 4
A selection of solution methodologies for solving expansion problems.

Method	Reference	GEP	TEP	DEP	Comments
Mixed-integer Nonlinear Programming	Alizadeh and Jadid (2015)	Yes	Yes	No	Insufficient computational performance against the long planning horizon and renewable constraints.
Mathematical Programs with Equilibrium Constraints	Baringo and Conejo (2011b) Jenabi et al. (2013)	Yes No	No Yes	No No	It considers equilibrium conditions and interactions in power systems. It is computational challenging.
Equilibrium programs with Equilibrium Constraints	Jin and Ryan (2013) Chen et al. (2020)	No Yes	Yes No	No No	It considers equilibrium conditions and interactions in power systems. It includes computational challenges.
Benders Decomposition	Tor et al. (2008) Kazempour and Conejo (2011)	No Yes	Yes No	No No	It guarantees an exact solution however it lacks performance for multi-stage stochastic problems.
Progressive Hedge	Munoz and Watson (2015)	Yes	Yes	No	Decomposing the original problem but no guarantee for exact solution.
Nested Benders Decomposition (NBD)	Lohmann and Rebennack (2017) Lara et al. (2018)	Yes Yes	Yes No	No No	Decomposing the original problem. Linear cutting planes are used. Curse-of -dimensionality is partially broken.
Stochastic Dual Dynamic Programming (SDDP)	Rebennack (2014) Hole et al. (2025)	Yes Yes	No Yes	No No	It is a sampling variant of NBD. Linear cutting-planes are used. Statistical upper bound is obtained.

transmission security into generation planning. Lagrangian relaxation and Benders decomposition techniques are effectively utilized to break down the main problem into subproblems. Shortle et al. (2013) focused on minimizing blackout probabilities.

Although GEP aims to maximize the profit of each power producer, it can be solved jointly with TEP from a centralized perspective to achieve the coordinated expansion of both transmission and generation facilities. Regarding transmission and generation expansion planning, Sharan and Balasubramanian (2012) proposed a comprehensive model, and demonstrated the benefits of integrated generation and transmission expansion planning. Jenabi et al. (2013) explored bi-level programming for TEP, considering the behavior of power producers. Jin and Ryan (2013) introduced a tri-level model as EPEC problem for transmission and generation expansion planning. Munoz and Watson (2015) presented stochastic transmission and generation investment planning problems using Progressive Hedging. SDDP, with the first stage including expansion decisions, is used to solve the generation and transmission expansion planning problem (Hole et al., 2025).

3.3. Operation planning

In the context of a decentralized framework, conflicting interest between power producers and market operators exist. Whereas power producers increase their own profit, the market operator aims to maintain a reliable and secure system (Conejo et al., 2005). This nature has an impact on maintenance and hydro scheduling strategies, necessitating a revision in terms of formulation and solution methodologies. Additionally, in operation planning, power producers and consumers seek mid-term trading in future markets to mitigate their risks. In this section, maintenance and hydro-thermal scheduling, energy trading models in future markets are reviewed.

3.3.1. Generation and transmission maintenance planning—MILP, EPEC

The maintenance scheduling is essential for ensuring the smooth operations of the power system. Maintenance scheduling for the GMS and TMS must be carefully designed to ensure the security and reliability of the power system, considering the conflicting interests among power producers. GMS can be addressed either independently or along with the TMS. The decentralized structure typically incorporates the same constraints as the centralized structure. Additionally, market-based constraints such as policies, market requirements, and environmental constraints including emission level are encompassed in the decentralized structure (Froger et al., 2016). Mainly, the maintenance planning focuses on economic objectives, aiming to maximize profit. Furthermore, environmental and reliability objectives can be considered.

A MILP approach is commonly employed to formulate the GMS problem. The coordination between power producers and the market operator for a generation maintenance plan should be provided to optimize producer profit and ensure reliability (Conejo et al., 2005). Benders decomposition with Lagrangian relaxation was used iteratively to solve the GMS problem, providing coordination and communication between power producers and the market operator (Geetha & Swarup, 2009). Pandzic et al. (2012) solved yearly GMS in a market environment, formulated as an EPEC, with each producer's problem expressed as an MPEC. Mazidi et al. (2018) developed a formulation for solving non-cooperative GMS using game theory, while addressing conflicts through bi-level optimization and Karush–Kuhn–Tucker conditions.

Marwali and Shahidehpour (1999b) introduced a decomposition method, based on a duality theory, for short-term scheduling of transmission line maintenance. Generation and transmission scheduling for long-term were coordinated by utilizing Benders decomposition. Additional constraints like fuel, network and emission constraints were incorporated (Marwali & Shahidehpour, 1999a). By using both dual and Benders decomposition, the large-scale problem was solved to coordinate generation and transmission maintenance with hourly security-constrained unit commitment (Fu et al., 2007). Benders decomposition with Lagrangian relaxation was used for both generation and transmission maintenance scheduling (Geetha & Swarup, 2009). Yearly TMS has been proposed as a bi-level model that is formulated into an MPEC (Pandzic et al., 2011).

3.3.2. Models for producers in futures markets (risk control)—SP

In the market environment, where electricity price serves as the primary determinant of supply and demand equilibrium, there are financial risks for both seller/buyer. Risk adjustments in real options are generally made by considering probability distributions through the incorporation of market price data (Nadarajah & Secomandi, 2023). In the power system, two distinct markets are considered for energy trading: a pool and a futures market. The pool includes a day-ahead market as well as shorter time scales, while the futures market enables electricity trading for longer periods (up to years). The futures market, characterized by lower volatility, may offer a lower average price for the producer. The pool price may be more favorable (higher or lower depending on the market positions), but it exhibits high volatility.

Therefore, the future market mitigates the impact of pool price volatility by hedging against the risks (Conejo et al., 2008). To determine the optimal hedging strategy considering price risk, various methods have been developed in the literature. Kaye et al. (1990) utilized forward contracts as a strategy to mitigate the risk associated with profit volatility. Conejo et al. (2008) explored a power producer's optimal engagement in the futures electricity market to hedge against pool price volatility, employing CVaR methodology for accurate risk modeling. Bruno et al. (2016) presented investment planning for renewable energy under uncertainty using a forward contract as a hedging tool. A risk-averse multi-stage stochastic integer program was formulated, and solved by using SDDP. Peura and Bunn (2021) used game theory to examine the impact of renewable energy production on electricity prices with participation in the future market.

3.3.3. Consumer energy procurement (risk control)—SP

In the electricity markets, large consumers aim to minimize costs by utilizing power market, contracts, self-generation, renewables such as wind and solar technologies, energy storage systems and demand response for cost reduction (Nojavan et al., 2019). In particular, energy procurement strategies from electricity markets were studied to manage costs and hedge against price risks (Oum & Oren, 2010). In the literature, stochastic programming and robust optimization are mainly used to determine optimal energy procurement strategies against the price risk. The electricity procurement decision problem for a large consumer was addressed as a stochastic programming problem, incorporating risk aversion through CVaR methodology (Carrión et al., 2007). Zhang et al. (2018) introduced a multi-stage stochastic model for long-term electricity procurement and production planning, addressing uncertainty in product demand, and applied the progressive hedging algorithm to solve the model. Nojavan et al. (2019) suggested robust optimization to address electricity price uncertainty in solving an energy procurement problem formulated through MILP.

3.3.4. Hydro-thermal scheduling—SP

The transition to the market environment imposed an additional uncertainty to the HTSP, next to the inflow uncertainty: the electricity market prices. This poses the conceptual difficulty that the (water) value functions have a saddle-shape in the prices and water inflows. Thus, Benders-type cuts, as employed in SDDP, can no longer be directly applied. The earliest work in this context applied a discretization of the prices, leading to concave value functions (Gjelsvik et al., 2010); other approaches utilized saddle-cuts (Downward et al., 2020). The SDDP algorithm has been extended algorithmically in various different ways, for example, to deal with stage-wise dependence (Infanger & Morton, 1996; Lohmann et al., 2016), risk-aversion (Shapiro, 2011), CO_2 emissions quotas (Rebennack et al., 2011), scenario trees (Rebennack, 2016), nonlinearities (Cerisola et al., 2012; Steeger & Rebennack, 2017), integer variables (Zou et al., 2019), non-convexities (Füllner & Rebennack, 2022), parallel schemes (Machado et al., 2021) and distributionally robust multistage problems using the Wasserstein distance (Duque & Morton, 2020). Many of these extensions concern both centralized and decentralized systems. For an extensive discussion on SDDP and its many extensions, we refer to Füllner and Rebennack (2025).

Hydro plants participating in electricity markets have to decide on their optimal bidding strategies (Steeger et al., 2014). The presence of large power companies in combination with very significant hydro-reservoir capacity leads to market power. In such a setting, the market bid can influence the market price (Flach et al., 2010; Steeger et al., 2018; Steeger & Rebennack, 2017). If more than one price-maker is present in the system, then game-theoretic models are needed (Barroso et al., 2006; Steeger & Rebennack, 2015). SDDP has been the state-of-the-art in solving HTSPs in both centralized and decentralized markets since the 1990s (Maceiral et al., 2018).

The hydro production function is a nonlinear function of the water discharge and net water-head effect. This results in a nonlinear hydro-thermal problem to provide more realistic representation of hydroelectric power generation features. MILP models were developed using a linearization technique to consider the water-head effect (Borghetti et al., 2008). Nonlinear program (Catalao et al., 2008) and a multi-dimensional piecewise linear model were also proposed to account for the water-head effect (Diniz & Maceira, 2008). In addition, the incorporation of additional renewable energy sources, such solar and wind power, into the hydro-thermal scheduling problem was studied in the literature. Since variable energy sources produce power intermittently, dependent on the wind and sunshine, these technologies can perfectly be combined with large storage like hydro plants. Several works were conducted on the integration of wind (Khodayar et al., 2013) and solar (Mari & Nabona, 2014) with hydro-thermal systems. For a more thorough examination of methods in recent decades, we refer to the review paper by Favereau et al. (2024).

3.4. Operations

Power markets are typically structured with a futures market for long-term trading and a pool for short-term trading. The pool comprises the day-ahead market, intraday markets (within some regions), and the real-time market. Additionally, a reserve market maintains energy balance with its future generation and consumption power, can also be considered as a day-ahead market. The optimization problems related to power operations discussed in this section predominantly relate to the short-term trading in the pool. Market clearing auctions are particularly applicable in European electricity markets, while UC and ED problems are more commonly used in the USA markets (Conejo et al., 2016).

3.4.1. Market clearing tools (unit commitment (in the US) and auctions (in the EU))—MILP, SP

In the European context, the market clearing mechanism operates in a decentralized framework. Power producers compete to sell the electricity they generate, while power consumers submit bids to purchase electricity. By assessing offers from power producers and bids from power consumers, the market operator determines the output for each producer, demand for each consumer, and the market clearing price while ensuring secure and reliable markets (Froger et al., 2016). The market-clearing auctions are utilized by market operators for clearing the market (Galiana & Conejo, 2008). Typically, the following types of auctions are considered in the literature: single-period auctions, multi-period auctions, network-constrained single auctions, network-constrained multi-period auctions, and stochastic auctions.

Let us formulate a single-period auction (as an LP model) as follow (Conejo et al., 2016)

$$(A) \qquad \max \sum_{j\in D} C_j^D p_j^D - \sum_{i\in G} C_i^G p_i^G \tag{21a}$$

$$\text{s.t.} \sum_{j\in J} p_j^D = \sum_{i\in G} p_i^G \qquad : \lambda, \tag{21b}$$

$$0 \le p_j^D \le P_j^{D,max} \qquad \forall j \in D, \tag{21c}$$

$$0 \le p_i^G \le P_i^{G,max} \qquad \forall i \in G, \tag{21d}$$

where the non-negative decision variable p_i^G represents the power output of power producer i and the non-negative decision variable p_j^D denotes the load of power consumer j. Power producer i has $P_i^{G,max}$ capacity and C_i^G marginal cost, while power consumer j has $P_j^{D,max}$ capacity and C_j^D utility cost. The objective function (21a) maximizes social welfare delineated by the shaded area in Fig. 4 (between the accepted consumption bids — blue line — and the accepted production offers—red line). Constraints (21c) and (21d) impose bounds on demand and generation, respectively. Finally, constraint (21b) ensures

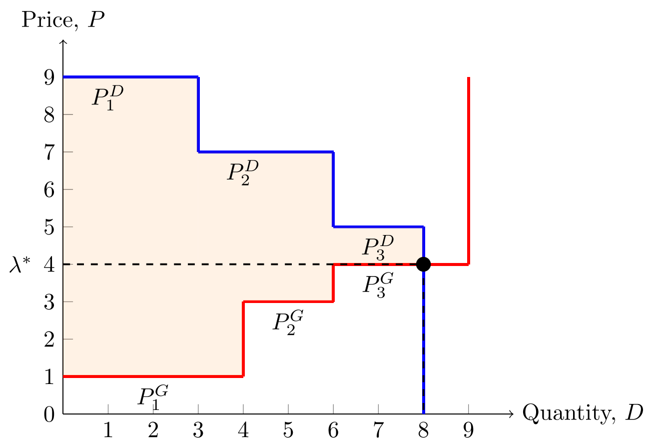

Fig. 4. Market Clearing Graph.

power balance. The market-clearing price (Fig. 4), corresponding to the dual value of constraint (21b), is denoted by λ.

A MILP has been proposed to optimize the clearing process efficiently, that is a day-ahead multi-period auction, while taking into account thermal plants technical restrictions (Arroyo & Conejo, 2002). Motto et al. (2002) presented a MILP model that incorporates transmission constraints, losses, and temporal operational limitations. Alternating direction method of multipliers (ADMM) (Zhang & Giannakis, 2015), dual decomposition (Okawa & Namerikawa, 2017), heuristic methods (Huse et al., 1999), bi-level programming (Fernandez-Blanco et al., 2011; Verma et al., 2024), interior-point algorithm (Rehfeldt et al., 2022), as well as Benders decomposition (Liang et al., 2017) are primarily utilized as solution methodologies for market clearing formulations. Market clearing mechanisms under uncertainty are solved by using stochastic programming (Kazempour et al., 2018; Pritchard et al., 2010; Zavala et al., 2017) and robust optimization (Zugno & Conejo, 2015). Simulation and equilibrium models have also been utilized in electricity market applications, as detailed in Ventosa et al. (2005). Agent-based simulation is a well-suited method for capturing the dynamics of energy markets and clearing the market. Numerous agents can be incorporated to simulate local flexibility and power trading (Nunna & Srinivasan, 2017). Möst and Genoese (2009) used agent-based simulation to examine how market power is applied. Game theory approaches can analyze the interactions and strategic decision-making of multiple players with the possibility of cooperation and competing objectives (Wang et al., 2014). Metzler et al. (2003) developed a Nash-Cournot equilibrium for the electricity market on a linearized DC network and investigated its properties. Moreover, an analysis of several approaches — perfect competition, Cournot model, bi-level approach — for modeling the electricity market is provided in Koschker and Möst (2016).

In the USA, unit schedules, generation dispatches and clearing prices are determined by solving optimization problems. That is, UC, ED, and security analysis are pivotal problems in the electricity markets across the USA. These problems can be integrated in various combinations to address diverse challenges (Litvinov et al., 2019). Further details on their classical approaches are discussed in Section 2.4. However, in this section, we review recent methodologies for OPF, UC and ED that have been well-adapted to current needs and uncertain environment since the 1990s.

Given the recent developments, the OPF problem has become increasingly complex due to the incorporation of additional constraints, such as involvement of reliability and security, the utilization of energy storage systems as well as the integration of advanced control devices and smart grid connection. The utilization of relaxation techniques and sparse matrix methods in conjunction with advanced algorithms have facilitated the solution of increasingly complex OPF (Gomez-Exposito et al., 2018). The direct current (DC) power flow algorithms utilize a linearization of the alternating current (AC) power flow and assumes the constant voltage magnitudes and phase angles by utilizing linear constraints (Alsac et al., 1990). Although a rapid solution might be provided by DC OPF, it has been noted that a global optimal solution might not be found in this approach (Kocuk et al., 2017). In order to obtain (approximate) global optimal solutions, various convex relaxations were investigated in the literature (Table 5). Semidefinite program (SDP) provides global optimal solutions for OPF problems under guaranteed exactness for special cases (Jabr, 2006). However, the exactness of SDP depends on the formulation and the network schedule; SDP imposes a computational burden for solving large-scale problems. Second-order cone program (Farivar et al., 2011) has been studied and has become one of the integral methods due to its lower computational cost compared to SDP. Several methods in the literature such as valid inequalities (Kocuk et al., 2016), cutting planes and convex envelope were proposed to have strong SOC relaxation (Zohrizadeh et al., 2020). Convex quadratic relaxation (Hijazi et al., 2017) and LP relaxations (Bienstock & Munoz, 2014) were also proposed and it is noted that they can provide reduced computational cost over SDP. With due consideration of adequate conditions and the context of distribution networks, the exactness of convex relaxations continues to be under study (Kocuk et al., 2015; Low, 2014).

Recently, enhancing the reliability of electricity systems and integrating the dynamic structure of the smart grid are trending challenges faced by UC (Skolfield & Escobedo, 2022). SCUC problem extends the unit commitment problem. SCUC is an optimization problem that schedules power generating units within a planning horizon to fulfill projected demand, while ensuring system security and compliance with transmission limitations. That is, the addition of scheduling constraints and limits imposed on system voltages and branch flows necessitates a power flow analysis for each time period. The existing studies on SCUC increases efforts to address the challenges posed by environmental considerations, advanced control devices, smart grids and the integration of multiple energy systems, including power, natural gas, and heating (Badakhshan et al., 2019; Basu, 2013; Kargarian et al., 2015; Liu et al., 2019, 2009; Yang et al., 2021).

The security-constrained AC unit commitment. An ideal SCUC model encompasses AC constraints for an accurate system representation. However, integration of AC constraints increases the complexity and difficulty of solving the SCUC model (Yang et al., 2021). There has been notable progress on SCUC solution methodologies. Because there are no efficient global optimization methods and due to the strength of the best relaxation methods, the current emphasis is on employing semidefinite program (Bai & Wei, 2009) and second-order conic program (Quan et al., 2014). A significant limitation of these methods is their inability to ensure both optimality and AC feasibility (Constante-Flores et al., 2021). Decomposition techniques such as Benders decomposition (Constante-Flores et al., 2021; Fu et al., 2005b) were utilized to deal with large-scale problems.

The security-constrained DC unit commitment. Security assessment often involves using a simplified DC model in the SCUC problem, converting it into a MILP problem for computational efficiency. However it lacks explicit data on bus voltages and reactive power, and offers limited accuracy for power flow computations (Fu et al., 2013). In order to solve SCUC model with DC constraints, several optimization methods were proposed including Lagrangian relaxation (Merlin & Sandrin, 1983), Benders decomposition (Gupta et al., 2019), Dantzig–Wolfe decomposition (Fu et al., 2005a), contingency-filtering scheme (Xavier et al., 2019) and a sequential method (Lee et al., 1994).

Stochastic unit commitment. Numerous research studies address the stochastic UC problem (Bouffard et al., 2005; Papavasiliou & Oren,

Table 5
A selection of solution methodologies for solving optimal power flow problems.

Method	Reference	Capabilities	Limitations
Semidefinite program	Jabr (2006) Bai and Wei (2009)	It can model a wide range of of convex optimization problems. A positive semidefinite constraint.	It can become computationally expensive for large-scale problems.
Second-order cone program	Farivar et al. (2011) Quan et al. (2014) Kocuk et al. (2016)	It includes both linear and second-order cone constraints and handles nonlinear constraints.	It can still have limitations in handling highly non-convex problems.
Convex quadratic relaxation	Hijazi et al. (2017)	Approximating a non-convex problem for various optimization problems.	Solution can be local for the original non-convex problem.

2013). In general, two types of stochastic models, the two-stage stochastic UC model and the multi-stage stochastic UC model, have been investigated. Wang et al. (2008) formulated the stochastic UC problem as a two-stage stochastic mixed-integer linear program, being solved by Benders decomposition. In order to solve multi-stage stochastic UC problems, stochastic dual dynamic integer programming (SDDiP) (Zou et al., 2018) was proposed. Although they show that the suggested approach may effectively tackle the multi-stage stochastic UC problems with a large number of scenarios, it is still impractical in real-world stochastic UC problems. Robust optimization (Guan & Wang, 2013) and stochastic dynamic programming (Analui & Scaglione, 2017) were also proposed in the literature. Xiong et al. (2016) introduced a distributionally robust optimization model to solve UC problem with uncertainty in wind power generation. Singh et al. (2020) developed a chance-constrained stochastic unit commitment model to deal with uncertainty in renewable energy output. Esteban-Pérez and Morales (2023) proposed a distributionally robust chance-constrained OPF model.

3.4.2. Offering/bidding algorithms for producers/consumers—LP

Power producers and consumers as principal agents are participating in the market environment. Power producers determine production offers for each production unit, while power consumers decide their bids for their intended consumptions. Both agents in the electricity market submit their offers and bids for trading the energy. These offers and bids are assessed by market operator to clear the market, enabling the determination of the market clearing price (Section 3.4.1). Power producers struggle with the complicated task of identifying optimal offering strategies in the energy market. In the literature, the self-scheduling problem was typically used. It can be formulated for power producer i (as a MILP model) as follow (Conejo & Baringo, 2018)

$$(B) \qquad \max \sum_{t\in T}\left[\lambda_t p_{it} - \left(C_i^F u_{it} + C_i^V p_{it} + C_i^{SU} y_{it} + C_i^{SD} z_{it}\right)\right] \tag{22a}$$

$$\text{s.t.} \quad y_{it} - z_{it} = u_{it} - u_{i,t-1} \qquad \forall t \in T, \tag{22b}$$

$$y_{it} + z_{it} \leq 1 \qquad \forall t \in T, \tag{22c}$$

$$p_{it} - p_{i,t-1} \leq R_i^U u_{i,t-1} + R_i^{SU} y_{it} \qquad \forall t \in T, \tag{22d}$$

$$p_{i,t-1} - p_{it} \leq R_i^D u_{it} + R_i^{SD} z_{it} \qquad \forall t \in T, \tag{22e}$$

$$P_i^{min} u_{it} \leq p_{it} \leq P_i^{max} u_{it} \qquad \forall t \in T, \tag{22f}$$

$$u_{it}, y_{it}, z_{it} \in \{0,1\} \qquad \forall t \in T. \tag{22g}$$

We note that model (22a)–(22g) is similar to the one explained in Section 2.4.3. The main distinction is found within the objective function. The aim of power producer i is to maximize its profit in the power market by obtaining optimal offers. In the objective function (22a), the first term, $\lambda_t p_{it}$, is the total revenue of power producer i, while the second term refers to its total cost.

Several methods have been used for optimal offering strategies in the literature. A MILP was proposed for optimal offering strategy of power producers (Simoglou et al., 2010). In the 1990s, a dynamic programming (David, 1993) and an analytical approach (Gross & Finlay, 1996) were employed to solve optimal offering problems. Genetic algorithm (Richter & Sheblé, 1998), heuristic methods (Huse et al., 1999), game theory (Ferrero et al., 1997), Lagrangian relaxation (Zhang et al., 1999) were utilized to derive optimal offering strategies. Since the 2000s, optimal offering strategies in the pool-based markets have been explored under uncertainty, particularly with the penetration of renewable technologies and the energy storage systems. Given renewable technologies and storage, the optimal offering was formulated for the day-ahead (DA) market (Kim & Powell, 2011). A two-stage stochastic MILP model was used for a virtual power plant (VPP) to trade in the DA and balancing markets (Pandžić et al., 2013). A multi-stage stochastic model was considered for VPP to determine optimal offering in the discrete Intraday (ID) market (Wozabal & Rameseder, 2020). In particular, offering strategies became an integral point for hydro producers (Fleten et al., 1997; Steeger & Rebennack, 2015). Löhndorf et al. (2013) formulated the offering problem within the ID market as a multi-stage stochastic program, encompassing trading decisions and hydro storage operations. Optimal offering strategies of a hydro power producer in the DA market have been studied (Fleten & Kristoffersen, 2007). Kaya et al. (2024) developed a multi-stage stochastic mixed-integer formulation for renewable energy providers to participate in the reserve market and maximize their profits by utilizing intraday trading and batteries as hedging instruments.

Limited studies are found in the literature that addresses bidding strategies specifically for consumers. Various demand-side bidding structures within the electricity market are examined to explore its impact on all factors such as total cost or marginal prices (Strbac et al., 1996). A demand bid generation method was proposed for buyers based on optimal allocation and the forecast price (Liu & Guan, 2003). A Monte Carlo-based algorithm was presented to address the challenges faced by consumers, having price sensitive demand, in determining optimal bidding curves within the day-ahead energy market (Menniti et al., 2009). A stochastic model was introduced to capture the strategic behavior of a large consumer and determine its strategic offers, and it was solved by a proposed heuristic approach to enhance computational efficiency (Kazempour et al., 2014).

3.5. Control

Power control is an important tool in the power systems to maintain the frequency and voltage within accepted limits. The decentralized framework, utilization of renewable technologies and storage systems and the development in the topology system led to changes in models as well as methodologies for frequency control and state estimation.

3.5.1. Frequency and reactive (optimal) control

The increased integration of renewables (specifically PV and wind technologies) in a power system has a significant impact on the traditional frequency control mechanism, typically modified by updating power generation to maintain the load stability. Renewable sources, specifically photovoltaic (PV) and wind technologies, can employ Energy Storage Systems (ESS) to provide additional active power in situations of an imbalance (Fernández-Guillamón et al., 2019). Concerning frequency control in wind technology, various methods exist, such as inertial response and de-loading technique. The de-loading technique involves de-loading wind turbines, with pitch angle control (Wilches-Bernal et al., 2015) and speed control being the primary mechanisms. Inertia response typically involves the injection or extraction of power from the system to address frequency deviations (Fernández-Guillamón et al., 2019). In order to ensure reliable and secure operations, frequency constraints have been incorporated into operation problems (Cardozo et al., 2017). ED problem with frequency control constraints (Doherty et al., 2005) and SCUC with frequency response constraints (Ahmadi & Ghasemi, 2014) have been proposed.

3.5.2. State estimation—NLP

Given the advancements in the topology technology and observation of power systems, the complexity of the state estimation problem has increased, resulting in a rise in nonlinear functions. Convex relaxation methods in the state estimation problem have primarily been employed to address these challenges since the 2000s. Furthermore, due to the transition to a decentralized structure, the utilization of decomposition algorithms are being increased. To address nonlinear functions, Gauss Newton method and Newton Raphson method were employed as initial methods. However, these methods do not guarantee a globally optimal solution. SDP relaxation was used in state estimation problem for nonlinear AC power system (Zhu & Giannakis, 2014). To address the non-convex state estimation problem, both SDP and SOCP relaxations were utilized with penalty terms (Zhang et al., 2017). A distributed state estimator was determined by solving a decomposition algorithm using the Auxiliary Problem Principle (Ebrahimian & Baldick, 2000). A decentralized state estimation method was introduced for multi-area power system, relying solely on border data exchange (Conejo et al., 2007). Lagrangian relaxation decomposition technique was proposed for a decentralized state estimation (Caro et al., 2011).

3.6. Forecasting

Since the 1990s, with the transition to market-oriented systems and the growing adoption of renewable technologies in the power systems, demand forecasting methods have evolved to meet the changing needs of the market environment. Additionally, the significance of price and renewable production forecasting has increased for power producers and consumers.

3.6.1. Demand forecasting—Time series and others

The relevance of demand forecasting has grown in power markets. Power producers require reliable estimates of energy demand to formulate their offers effectively. Inaccurate forecasting leads to either increased operational costs or diminished resource utilization. Time series models, including ARMA, ARIMA maintain their significance in demand forecasting for capturing historical patterns and seasonality in demand data. However, considering recent advancements such as demand response, market structure, and environmental considerations, nonlinear functions are mainly emerged in demand forecasting. To address these challenges, novel methods are being utilized for demand forecasting. Support vector machines have been proposed for data classification and regression (Chen et al., 2004). Artificial neural networks (ANN), fuzzy logic and evolutionary algorithms are emerging methods that have seen a growing utilization (Hahn et al., 2009).

3.6.2. Price forecasting—Time series and others

Price forecasting, in particular electricity and commodities such as CO2, has become a significant factor in the electricity markets for power producers and consumers (Möst & Keles, 2010). Market clearing prices are determined in the spot market by considering the submitted bids and offers from agents. Additionally, forward contracts are used to manage the risks associated with price changes. Price forecasting is essential for market participants to formulate their respective strategies. For the purpose of price forecasting, it is crucial to identify the price drivers, which can be categorized into the demand side and the supply side of the electricity market. An initial approach of price forecasting involves the use of linear regression. Day-ahead electricity prices are forecasted using the Bayesian approach (Kostrzewski & Kostrzewska, 2019). Lasso Estimated AutoRegressive (LEAR) (Uniejewski et al., 2016) and Deep Neural Network (DNN) (Lago et al., 2018) are considered as recent methods. LEAR is considered among machine learning and autoregressive techniques, while DNN is a deep learning method and use Bayesian optimization (Lago et al., 2021). Several methods were also introduced, including dynamic and fuzzy regression (Carrión et al., 2007), Artificial Neural Networks (ANN) (Keles et al., 2016), and Fourier and Hartley transforms (Nogales et al., 2002). Ferrari et al. (2021) presented that dynamic sparse factor model outperforms machine learning methods in providing more accurate forecasts. Fundamental models were also proposed in the literature with the goal of considering the physical and economic relationships to forecast price curves over a longer time horizon (*i.e.*, in the order of years). Data availability and the integration of stochastic processes of the fundamental drivers are considered the main challenges (Weron, 2014). Howison and Coulon (2009) developed a fundamental model for electricity prices with a bid stack model, considering stochastic processes for the main factors that are responsible for determining the spot power prices. A stochastic bid stack model was utilized to transform power demand and fuel prices into electricity prices (Carmona et al., 2013). Using a piecewise linear bid stack, Kallabis et al. (2016) developed a parsimonious fundamental model and found that emission prices surpass renewable penetration in impacting electricity prices.

3.6.3. Renewable production forecasting—Time series and others

The shares of renewable technologies have witnessed notable growth in recent decades and solar and wind technologies have emerged as main components of the renewable energy sources in addition to hydro plants. Due to the increase of renewable penetration, the impact of output from renewable energy sources on the market clearing prices have become increasingly apparent (Ringkjøb et al., 2018). With respect to this, the accurate forecasting of solar and wind power is crucial for ensuring reliable and effective operations. Given that renewable energy sources are primarily weather-dependent technologies, the forecasting of renewable power generation heavily relies on meteorological variables. Numerous methods are available for forecasting solar and wind energy. These are categorized into four groups such as physical, statistical, computational and hybrid models (Hodge et al., 2018). Physical methods in renewable energy forecasting rely on physical data such as wind speed and solar radiation. By utilizing numerical data and satellite images, physical methods forecast renewable energy generation. Traditional statistical methods, leveraging historical data, encompass AR, ARMA, and ARIMA models, as discussed in Section 2.5.1. With the advancements in computational methods, artificial neural network and support vector machine are being used.

3.6.4. Hydro inflow forecasting—Time series and others

The inflow of a hydro plants is a stochastic process that is uncertain and dependent on the weather conditions of the reservoir. Time series models such ARIMA, AR, and ARMA were employed for inflow forecasting. Importantly, periodic autoregressive (PAR) (Maceira & Damázio, 2006), spatial PAR (Lohmann et al., 2016) and seasonal ARIMA (Bender & Simonovic, 1994) were proposed in the literature to deal with shorter time resolutions and time-dependency. Machine learning methods such as support vector machine and artificial neural network (Yang et al., 2017) and deep learning methods (da Silva et al., 2024) have also been used. We note that forecasting and scenario reduction are related, since scenario reduction involves reducing the range of potential future scenarios that may result from a forecast. Hence, scenario reduction approaches are needed to reduce the number of scenarios while maintaining as much the stochastic information as possible. Several approaches exist for scenario reduction—used in stochastic

Table 6
List of abbreviations.

Abbreviations	Definition	Abbreviations	Definition
AC	Alternating current	MISOC	Mixed-integer second-order cone
ANN	Artificial neural networks	MPEC	Mathematical program with equilibrium constraints
AR	Autoregressive	NBD	Nested Benders decomposition
ARIMA	Autoregressive integrated moving average	NCUC	Network-constrained unit commitment
ARMA	Autoregressive moving average	NLP	Nonlinear programming
DC	Direct current	OPF	Optimal power flow
DEP	Distribution expansion planning	PAR	Periodic autoregressive
ED	Economic dispatch	SCUC	Security-constrained unit commitment
EPEC	Equilibrium problems with equilibrium constraints	SDDP	Stochastic dual dynamic programming
ESS	Energy storage system	SDDiP	Stochastic dual dynamic integer programming
GEP	Generation expansion planning	SDP	Stochastic dynamic programming
GMS	Generator maintenance scheduling	SDP	Semidefinite program
HTSP	Hydro-thermal scheduling problem	SOC	Second-order cone
LEAR	Lasso estimated autoregressive	SP	Stochastic programming
LFC	Load frequency control	TEP	Transmission expansion planning
LP	Linear programming	TMS	Transmission maintenance scheduling
MILP	Mixed-integer linear programming	UC	Unit commitment
MINLP	Mixed-integer nonlinear programming	WLS	Weighted least squares

programming. A forward selection approach for scenario reduction that uses discrete probability distributions was proposed (Dupačová et al., 2003). Wasserstein distance-based scenario reduction (Rujeerapaiboon et al., 2022), importance sampling method (Papavasiliou & Oren, 2013) and backward selection approach (Heitsch & Römisch, 2003) were also introduced. Moreover, the periodic structure of scenarios in certain multi-stage stochastic problems has been used to reduce the number of stages, particularly in hydrothermal generation planning problems (Shapiro & Ding, 2020).

4. Future research directions

In recent years, operations research has seen notable progress in addressing non-convexity (Zohrizadeh et al., 2020). The AC power flow equations are one prominent example for non-convex, nonlinear models in power systems. Other examples are nonlinear head effects of hydro turbines, valve-point-effects of gas turbines, nonlinear charging, discharging and degradation of batteries as well as nonlinear emission curves. The utilization of discrete variables in combination with nonlinear functions leads to MINLPs, and MISOC models. This naturally occurs when incorporating OPF into MILP models, such as the UC model. The application of such nonconvex models for large-scale problems are of increasing interest. Consequently, efforts to enhance relaxation techniques and develop exact and heuristic (decomposition) algorithms are underway for getting reliable and robust solutions. However, solving real-world systems — which are of very large-scale — to proven global optimality for general systems are still a major research challenge.

The liberalized market structure introduces a challenging environment characterized by the market clearing process and the establishment of pricing mechanisms. This framework leads to separate objectives for each power producer and consumer, thereby leading to conflicting interests within the market. The studies on market environment have underscored the needs for handling a decentralized structure. Addressing these needs, EPEC and MPEC have been developed. However, there has been an increased necessity on employing decentralized and decomposition algorithms as a solution methodology in the recent literature.

The total costs in the electricity market including fixed cost and nonlinear cost function exhibit non-convex characteristics. Establishing optimal prices poses a significant challenge in the non-convex markets. Given marginal pricing mechanisms, power producers may not cover their costs in the non-convex markets, leading to the so-called "missing money" problem. Several price mechanisms have been proposed, focusing on the short-term. There is a potential research area in integrating long-term capacity expansion with various pricing mechanisms (Byers & Hug, 2023).

An increased penetration of intermittent suppliers into electricity markets with marginal prices causes increased problems. We are already observing times with negative prices, for example, in the German market. However, also marginal prices of zero will become the standard, when intermittent supplies — such as wind and solar — are having a large installed capacity and bidding at marginal cost of zero. Therefore, new market models are required. The new market models are expected to bring new challenging operations research problems.

Since the 1990s, the need for power systems optimization under uncertainty has risen. Increased penetration of renewables, managing uncertainty and risk factors, especially with detailed temporal and spatial information, has introduced complexity to the solution of large-scale problems. In order to obtain robust and adaptive decision-making processes under uncertainty, the combination of different uncertainty models has recently emerged as a potential research area. For instance, methods from robust optimization, stochastic optimization, distributionally robust optimization and fuzziness have to be combined in unique ways to better capture the different nature of the uncertainties present. Next to the modeling challenge, solution algorithms have to be adjusted to cope with the new model structure.

The coupling of the electricity sector with other sectors, such as transportation (through electric vehicles), heating and cooling, as well as hydrogen (for storage) is also an emerging research area. The challenges are both on the modeling side as well as the solution algorithms. Real-world models are of massive size, when considering detailed models for multiple sectors, combined in one framework.

Power system problems are often characterized by repeated decisions over a long-time horizon. For example, in investment problems, the operational problems can have hourly or even quarter-hourly resolution. This leads to tens of thousands of small models which are rather loosely coupled, for example, through ramping constraints or battery/water levels. Other examples of loosely coupled models are multi-area models, such as interconnected electricity grids of several areas. Algorithms, exploiting this loose coupling, are the most promising class of methods to tackle the real-world power system problems. Multi-horizon modeling is an efficient approach that decomposes the original problem into interstage and intrastage components, addressing different level uncertainties (Abgottspon, 2015). Algorithms using this modeling approach can produce significantly reduced model sizes compared to multi-stage stochastic programming approaches.

5. Conclusion

This paper reviews the evolution of power systems optimization over the last fifty years and focuses on the mainstream models and their solution methodologies. We conduct a review of two distinct periods from 1970 to 1990 and from 1990 to the present. Both periods are discussed by considering power system problems such as control, operations, operation planning, investment and forecasting. The first period covers classical mathematical models and early solution methods for a centralized framework. The second period features recent optimization models and solution methodologies for both centralized and decentralized structures. The evolution of the power system has undergone significant changes over the past fifty years, particularly in terms of technology, regulatory framework, and environmental considerations. In addition to the power system, important progress has occurred in the field of operations research and we observe how these developments influence the evolution of power systems optimization models. Finally, we present promising opportunities for the development and enhancement of power systems optimization methods.

CRediT authorship contribution statement

Anil Kaya: Writing – review & editing, Writing – original draft, Visualization, Methodology, Investigation, Conceptualization. **Antonio J. Conejo:** Writing – review & editing, Supervision, Methodology, Investigation, Conceptualization. **Steffen Rebennack:** Writing – review & editing, Supervision, Methodology, Investigation, Conceptualization.

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