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Prediction of heat transfer to supercritical pressure fluid at rough surface☆

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ABSTRACT

The highly corrosive environment in supercritical water-cooled reactors (SCWR) is a big challenge for the fuel rod cladding materials. Within a refueling cycle, which is aimed to be at least 2 years long, an oxide layer grows on the outer surface of the fuel rod. Thus, roughness on the heat transfer surface is increased over time. To predict the heat transfer to fluids at supercritical pressure conditions is challenging, especially in the vicinity of the pseudocritical point, i.e.., when the fluid close to the wall already exceeds the pseudocritical temperature and the fluid in the core of the subchannels is still sub-critical. In the literature, many correlations exist, taking the variations of the thermophysical properties of the fluid into account. Further, effects such as local laminarization near the wall due to buoyancy or bulk flow acceleration are considered in many correlations. However, the rough wall induces another level of complexity to heat transfer to a fluid under supercritical pressure conditions. This work aims to develop a heat transfer correlation, which can take the effect of the wall roughness on the heat transfer to a fluid under supercritical pressure conditions into account. For this purpose, previously obtained experimental data, which has been obtained in a smooth reference tube and in a test tube, which has been artificially roughed at its inner surface, are available. The experiments have been conducted with the surrogate fluid R134a in the KIT Model Fluid Facility (KIMOF) at the Karlsruhe Institute of Technology (KIT), Germany.

1. Introduction

Due to the strong variation of the thermophysical properties of a fluid at supercritical pressure, especially in the vicinity of the pseudo-critical temperature, prediction of the heat transfer coefficient is challenging. There have been many attempts to develop heat transfer correlations from experimental data and an extensive review can be found in the state-of-the-art report of Vasic et at (2023), in which 194 correlations for heat transfer to a fluid at supercritical pressure conditions are reported. However, among those, only one correlation considers the surface roughness directly as a variable. As the environment in SCWR is highly corrosive and the surface roughness is expected to change with time, it is necessary to develop a heat transfer correlation that can predict the surface roughness effects on the heat transfer. Generally, the existing correlations can be divided into two types: Type 1 directly considers the roughness as a variable, and type 2 indirectly considers the surface roughness via the skin friction coefficient. Although for most of these correlations, a calculation of the skin friction coefficient only based on the Reynolds number is suggested, it can also be calculated as a function of the surface roughness, e.g. using the Colebrook equation (Colebrook, 1939). The number of correlations of the first type, directly considering the surface roughness is small. In the state-of-the-art report of Vasic et al (2023), only the correlation proposed by Cook (1984) includes the surface roughness:

$$Nu = \frac{0.4\frac{c_f}{8}Re_b Pr_b}{1+\sqrt{\frac{c_f}{8}}\left[5.19k_s^{0.2}Pr_b^{0.44}-8.5\right]} \tag{1}$$

where k_s is the equivalent sand grain roughness. Apparently, only the fluid properties at bulk temperature are considered and no further correction, which takes the strong variation of the thermophysical properties into account is applied here. Thus, it can not be expected that this correlation performs well in the vicinity of the pseudo-critical point. Another correlation, in which the surface roughness is included was proposed by McCarthy et al. (1968):

$$Nu = 0.025Re_b^{0.8}Pr_b^{0.4}\left(\frac{T_w}{T_b}\right)^{-0.55}\left[1+1000\left(\frac{R_a}{D}\right)\left(log_{10}Re_b^{-5.652}\right)\right] \tag{2}$$

Here, the mean roughness height R_a is considered and further, the variation of the thermophysical properties is reflected in the ratio of the wall temperature and the bulk temperature, which is applied as a

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Nomenclature

c_f	Darcy friction factor$(-)$
c_p	isobaric specific heat capacity (J/kgK)
D	tube diameter (m)
F_{VP}	correction factor for variation of fluid properties $(-)$
g	gravitational constant (m/s^2)
h	specific enthalpy (J/kg)
k_s	equivalent sand grain roughness (m)
$k_s^+ = \frac{k_s \rho_w u_\tau}{\mu_w}$	non-dimensional eq. sand grain roughness $(-)$
$\dot{q}_w$	wall heat flux (W/m^2)
R	radius of the tube (m)
R_a	mean roughness height(m)
u_τ	friction velocity(m/s)
$\alpha = \frac{\dot{q}_w}{T_w - T_b}$	heat transfer coefficient (W/m^2K)
λ	thermal conductivity (W/mK)
μ	dynamic viscosity $(kg/m{\cdot}s)$
ρ	density (kg/m^3)
τ_w	wall shear stress (N/m^2)
Subscripts	
b	bulk
pc	pseudo-critical
w	wall

Table 1
Different correction factors for the variation of the thermophysical properties.

Author	F_{VP}
Petukhov and Kirillov (Petukhov & Kirillov, 1958)	$\left(\frac{\mu_w}{\mu_b}\right)^{0.11}$
Petukhov et al. (Petukhov, Krasnoshchekov, & Protopopov, 1961, Aug. 28 – Sept. 1)	$\left(\frac{\mu_w}{\mu_b}\right)^{-0.11}\left(\frac{\lambda_w}{\lambda_b}\right)^{0.33}\left(\frac{\bar{c}_p}{c_{p,b}}\right)^{0.35}$
Krasnoshchekov et al. (Krasnoshchekov & Protopopov, 1966)	$\left(\frac{\rho_w}{\rho_b}\right)^{0.3}\left(\frac{\bar{c}_p}{c_{p,b}}\right)^{n}$
Razumovskiy et al. (Razumovskiy, Ornatskiy, & Mayevskiy, 1990)	$\left(\frac{\bar{c}_p}{c_{p,b}}\right)^{0.35}$

correction factor. More recently Chen et al. (2022) performed a systematic experimental study of the influence of the surface roughness on the heat transfer to supercritical pressure CO_2. In the experimental investigation, tubes with five different inner surface roughness have been used, which are reported as 1.5 μm, as a smooth reference and 10 μm, 20 μmm 30 μm and 40 μm as different rough tubes. Unfortunately, it was not reported which roughness parameter exactly was used, so it could be either the equivalent sand grain roughness k_s, the mean roughness height R_a, or even other parameters, such as the maximum distance from the largest peak to the lowest valley R_z or something else.

$$Nu = 0.017 Re_b^{0.838} Pr_b^{0.815}\left(\frac{\rho_w}{\rho_b}\right)^{0.176}\left(\frac{\bar{c}_p}{c_{p,b}}\right)^{0.18}\left(\frac{\mu_w}{\mu_b}\right)^{1.03}\left(\frac{\lambda_w}{\lambda_b}\right)^{-0.455}\left(1+\frac{S_z}{S_{z,0}}\right)^{0.135} \quad (3)$$

In this equation, S_z is the not further specified roughness parameter, $S_{z,0}$ is the reference roughness of 1.5 μm, corresponding to the roughness of the smooth tube in the experiment. This correlation is taking the variation of the density ρ, the specific heat c_p, the dynamic viscosity μ and of the thermal conductivity λ into account. The application of that large number of correction factors can result in a very good fitting to the applied experimental data, but on the other hand, the problem of no– or multiple solutions in the required iteratively solving procedure may be more likely to appear, especially when the correlation is applied to another fluid.

Table 2
Values of $\varphi(k^*)$ for the correlation of Kirillov et al. (1990), Pioro et al. (2003).

k^*	0.01	0.02	0.04	0.06	0.08	0.1	0.2	0.4
$\varphi(k^*)$	1.0	0.88	0.74	0.64	0.65	0.65	0.74	1.0

All other correlations applicable to heat transfer to supercritical pressure fluid at a rough surface are only indirectly considering the surface roughness. Several authors are using the same general formulation, which is given by (Petukhov & Kirillov, 1958; Petukhov et al., 1961; Krasnoshchekov & Protopopov, 1966; Razumovskiy et al., 1990):

$$Nu = \frac{\frac{c_f}{8} Re_b Pr_b}{1.07 + 12.7\sqrt{\frac{c_f}{8}}\left(Pr_b^{2/3} - 1\right)} F_{VP} \quad (4)$$

Here, F_{VP} is the correction factor for the variation of the thermophysical properties, which is different for different authors and c_f is the skin friction coefficient. Table 1 gives several examples. It must be pointed out, that the authors of the correlations mentioned here, all suggested the Filonenko equation (Filonenko, 1954) for the skin friction coefficient c_f, which only considers the Reynolds number and not the surface roughness. However, another equation, such as the Colebrook equation (Colebrook, 1939) can be applied to introduce the surface roughness:

$$\frac{1}{\sqrt{c_f}} = -2log\left(\frac{k_s}{3.71D} + \frac{2.51}{Re\sqrt{c_f}}\right)^{-2} \quad (5)$$

In the correlation proposed by Razumovskiy et al. (1990), the variation of the fluid properties is also considered for the friction coefficient c_f:

$$c_f = c_{f,0}\left(\frac{\mu_w}{\mu_b}\frac{\rho_w}{\rho_b}\right)^{0.18} \quad (6)$$

where $c_{f,0}$ is the friction coefficient calculated by the Colebrook equation. A similar formulation was proposed by Kirillov et al. (1990), Pioro et al. (2003), and by Kurganov (1998):

$$Nu = \frac{\frac{c_f}{8} Re_b \overline{Pr}}{1 + \left(\frac{900}{Re}\right) 12.7\sqrt{\frac{c_f}{8}}\left(\overline{Pr}^{2/3} - 1\right)} F_{VP} \quad (7)$$

Compared to the correlation shown above, the variation of the thermophysical properties is already considered by the application of the averaged Prandtl number, and additionally a correction factor F_{VP} can be introduced. The averaged Prandl number is given by:

$$\overline{Pr} = \frac{\bar{c}_p \mu_b}{\lambda_b} \quad (8)$$

with

$$\bar{c}_p = \frac{h_w - h_b}{T_w - T_b} \quad (9)$$

In the correlation of Kurganov (1998), no additional correction factor F_{VP} is applied, but the variation of the fluid properties is considered in the calculation of the skin friction coefficient c_f:

$$c_f = c_{f,0}\left(\frac{\rho_w}{\rho_b}\right)^{0.4} \quad (10)$$

Kirillov at al. (1990), Pioro et al. (2003) did not correct the friction coefficient, but introduced the following correction factor F_{VP}:

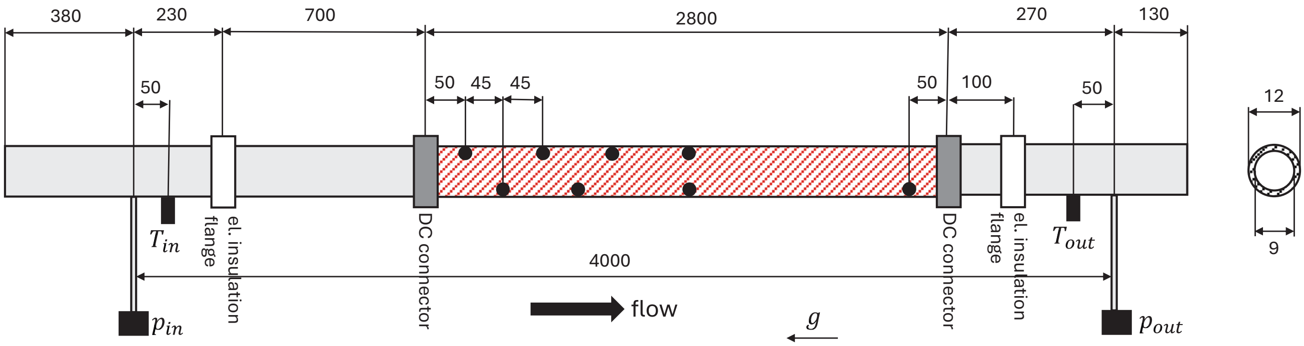

Fig. 1. Sketch of the test section.

Table 3
The parameter range of the experiments is given in Table 3.

$p[bar]$	T_{in} [°C]	$G[\frac{kg}{m^2 s}]$	$q[\frac{kW}{m^2}]$
43.5	50,	500	20 – 70
46	60,	750	30 – 75
	70,	1000	40 – 130
	80	1250	50 – 150
		1500	60 – 180
		1750	70 – 190
		2000	80 – 200

$$F_{VP} = \left(\frac{\rho_w}{\rho_b}\right)^m \left(\frac{\bar{c}_p}{c_{p,b}}\right)^{0.11} \varphi(k^*) \tag{11}$$

with

$$k^* = \left(1 - \frac{\rho_w}{\rho_b}\right) \frac{Gr}{Re^2} \tag{12}$$

and

$$Gr = \frac{g(\rho_b - \rho_w)D^3}{\upsilon_b^2} \tag{13}$$

The function $\varphi(k^*)$ is given in tabulated form in Table 2 and depicts the effect of the heat transfer deterioration (HTD) because of the buoyancy, which is caused by density gradient from the wall to the bulk flow.

Another correlation, which was proposed by Gnielinski (1975) is furthermore considering inlet effects, and reads as:

$$Nu = \frac{\frac{c_f}{8}(Re_b - 1000)Pr_b}{1 + 12.7\sqrt{\frac{c_f}{8}}\left(Pr_b^{2/3}\right) - 1} \left(\frac{Pr_b}{Pr_w}\right)^{0.11} \left[1 + \left(\frac{z}{D}\right)^{2/3}\right] \tag{14}$$

Here, the variation of the thermophysical properties is considered, using the ratio of the Prandtl number at wall and at bulk temperature, the inlet effect considered, using the distance z from the inlet, related to the tube diameter. The correlations introduced here will be used for comparison, to assess the performance of the new correlation, which will be introduced in Section 3.

2. Experimental data

Recently, an experimental database, consisting of reference data obtained in a smooth tube ($k_s = 0.95\mu m$) and data obtained in a tube with rough inner surface ($k_s = 10.78\mu m$) has been established Wiltschko and Cheng (2025), Wiltschko et al. (2024). The working fluid was the refrigerant R134a. The test section, which is shown in Fig. 1, consists of a simple tube, with 68 thermocouples mounted on the outer wall. Furthermore, the pressure as well as the fluid temperature are measured at the inlet and at the outlet of the test section. The test section was heated by electrical current, over a length of 2.8 m. The experimental conditions are summarized in Table 3, further details can be found in Wiltschko and Cheng (2025), Wiltschko et al. (2024).

3. New empirical correlation

The goal of this work is to derive a new empirical heat transfer correlation, which directly considers the wall roughness. This is done based on the dimensionless temperature, which can be described depending on the surface roughness for the turbulent layer by Apoix (2015):

$$T^+(y^+) = \frac{1}{\kappa_\Theta} ln(y^+) + C(Pr) - \Delta T^+\left(k_s^+\right) \tag{15}$$

The term $\Delta T^+(k^+)$ is called the roughness function. The coefficient $C(Pr)$ is given according to Schlichting & Gersten (2017) as:

$$C(Pr) = A^* Pr^m + B \tag{16}$$

Assuming that the bulk temperature is equal to the temperature at the edge of the turbulent layer, the non-dimensional wall temperature is expressed as:

$$T^+ = \frac{T_w - T_b}{T_\tau} \tag{17}$$

$$T_\tau = \frac{\dot{q}_w}{\rho c_p u_\tau} \tag{18}$$

where T_τ is the friction temperature. Combining Eqs. (17) and (18) and using the definition of the Nusselt number and the heat transfer coefficient, the following can be obtained:

Table 4
Accuracy of basic correlation for data with small variation of thermophysical properties.

Dataset	μ[%]	$\|\mu\|$[%]	σ[%]
Smooth	1.74	6.3	8.03
rough	0.88	7.83	10.16

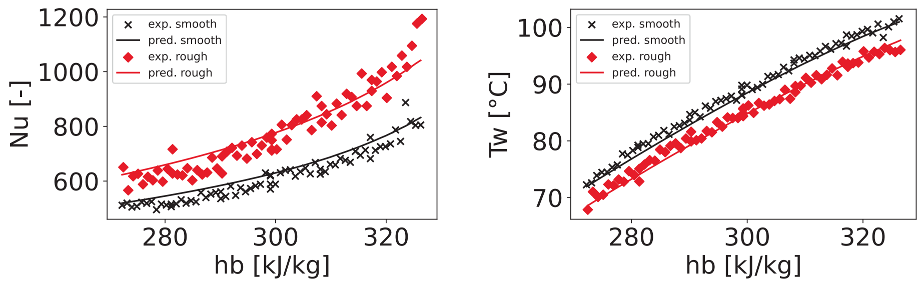

Fig. 2. Predicted Nusselt number (left) and predicted wall temperature (right) versus experimental data at p = 46 bar, G = 2000 kg/m^{2}s and q = 90 kW/m^2.

Table 5
Assessment of correlations in the rough tube.

Author	μ[%]	$\|\mu\|$[%]	σ[%]	1 sol.	3 sol.	5 sol.	0 sol.
New correlation	−12.28	19.73	23.3	43,416	19	0	0
Razumovskiy et al. (1990)	−3.43	21.22	29.61	43,435	0	0	0
Kurganov (1998)	−2.59	22.99	33.6	43,434	1	0	0
Krasnoshchekov & Protopopov (1966)	−5.34	26.63	34.33	43,432	3	0	0
Kirillov et al. (1990), Pioro et al. (2003)	−25.03	26.62	27.13	41,843	1591	1	0
Petukhov et al. (1961)	17.86	32.32	46.73	43,435	0	0	0
McCarthy et al. (1968)	−10.66	42.84	53.26	43,435	0	0	0
Petukhov & Kirillov (1958)	14.91	50.37	81.25	43,407	28	0	0
Gnielinski (1975)	40.99	59.64	97.76	43,263	172	0	0
Chen et al. (2022)	−9.99	35.9	39.72	39,145	4260	0	0
Cook (1984)	85.62	95.54	182.87	43,435	0	0	0

$$Nu = \frac{\alpha D}{\lambda} = \frac{\rho c_p u_\tau D}{\lambda T^+(y^+)} \tag{19}$$

With the definition of the friction velocity u_τ and the wall shear stress τ_w

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}} \tag{20}$$

$$\tau_w = \frac{c_f}{8}\rho u_b^2 \tag{21}$$

and introducing the Reynolds number and the Prandtl, the correlation turns into:

$$Nu = \frac{\rho c_p \sqrt{\frac{c_f}{8}} u_b D}{\lambda T^+(y^+)} = \frac{\mu c_p \sqrt{\frac{c_f}{8}} Re}{\lambda T^+(y^+)} = \frac{\sqrt{\frac{c_f}{8}} RePr}{T^+(y^+)} = \frac{\sqrt{\frac{c_f}{8}} RePr}{\frac{1}{\kappa_\Theta} ln(y^+) + C(Pr) - \Delta T^+(k_s^+)} \tag{22}$$

Now, the roughness function is included in the equation for the Nusselt number. The non-dimensional wall distance y^+ is considered to range from the heated wall to the center of the tube. Using the radius R of the tube yields:

$$R^+ = \frac{\rho R u_\tau}{\mu} = \frac{\rho R \sqrt{\tau_w}}{\mu \sqrt{\rho}} = \frac{\rho D \sqrt{\frac{c_f}{8}}}{2\mu} = \frac{\sqrt{c_f} Re}{2\sqrt{8}} \tag{23}$$

Finally, the coefficient $C(Pr)$ is simplified to $C(Pr) = A^*Pr^m$ and the roughness function is expressed as a simple linear function $\Delta T^+(k_s^+) = B^*k_s^+$ and the constant κ_Θ is 0.47. Thus, the basic form of the new correlation reads as:

$$Nu = \frac{\sqrt{\frac{c_f}{8}} RePr}{\frac{1}{\kappa_\Theta} ln\left(\frac{\sqrt{c_f} Re}{2\sqrt{8}}\right) + A^*Pr^m - B^*k_s^+} \tag{24}$$

Now, in the first step, the coefficients A, B and the exponent m must be determined. This is done, using data from the normal heat transfer regime. The data is screened for deteriorated heat transfer, which is detected by temperature peaks and further, all data with considerable effect of buoyancy ($Bu > 1.0{\cdot}10^{-5}$) have been excluded. The buoyancy parameter Bu is calculated according to Jackson (2013):

Table 6
Assessment of correlations in the smooth tube.

Author	μ[%]	$\|\mu\|$[%]	σ[%]	1 sol.	3 sol.	5 sol.	0 sol.
New correlation	−18.95	25.76	28.13	43,599	14	0	0
Kurganov (1998)	−10.74	22.4	29.25	43,613	0	0	0
Razumovskiy et al. (1990)	−9.73	23.42	31.08	43,613	0	0	0
Krasnoshchekov & Protopopov (1966)	−12.4	25.1	30.16	43,610	3	0	0
Petukhov et al. (1961)	14.14	29.23	41.6	43,613	0	0	0
Kirillov et al. (1990), Pioro et al. (2003)	−35.83	36.64	25.07	42,122	1489	2	0
McCarthy et al. (1968)	5.53	38.46	57.08	43,613	0	0	0
Petukhov and Kirillov (1958)	8.81	43.3	68.55	43,584	29	0	0
Gnielinski (1975)	36.43	52.16	82.1	43,442	171	0	0
Chen et al. (2022)	−46.56	50.3	28.72	39,409	4189	0	15
Cook (1984)	56.26	70.97	132.76	43,613	0	0	0

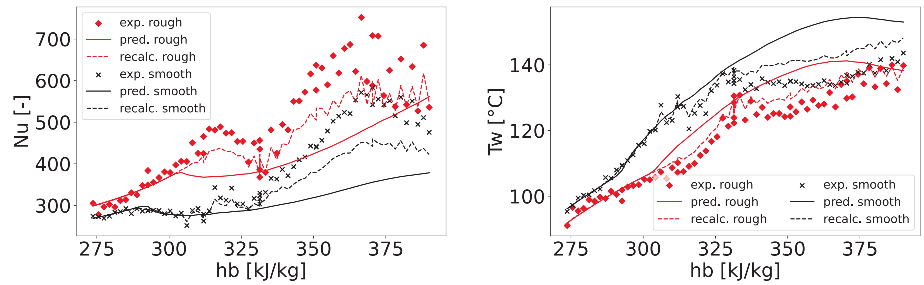

Fig. 3. Predicted and recalculated Nusselt number (left) and wall temperature (right) versus experimental data at p = 46 bar, G = 1000 kg/m^{2}s and q = 100 kW/m^2.

Table 7
Assessment of new correlation with data of Chen et al. (2022).

Correlation	μ[%]	$\lvert\mu\rvert$[%]	σ[%]
New correlation (predicted)	54.83	57.22	65.1
New correlation (recalculated)	−0.66	18.93	23.74
Chen et al. (2022) (predicted)	10.51	26.32	43.85
Chen et al. (2022) (recalculated)	2.6	9.76	13.62

$$Bu = \begin{cases} \dfrac{\overline{Gr_b}}{Re_b^{2.7}Pr_b^{0.5}} < 10^{-5} for T_w \leq T_{pc} \\ \dfrac{\overline{Gr_b}}{Re_b^{2.7}} < 10^{-5} for T_w > T_{pc} \end{cases} \tag{25}$$

After sorting out data points with measurement uncertainties for the Nusselt number of more than 20 %, 5092 data points were selected from the rough tube and 5097 data points from the smooth tube. Since even in the selected cases, some variation of the thermophysical properties is present, the Prandtl number is the average Prandtl number $\overline{Pr}$. Also, the density in the friction velocity u_τ is considered at wall temperature, which is especially necessary for the application of the correlation to conditions with a strong variation of the density near the wall, in the next step. The fitting yields $A = 10.63$, $B = 0.17$ and $m = 0.46$. The newly obtained correlation was assessed, using the same data with small variation of the thermophysical properties, which has been used for the fitting. The results are shown in Table 4. Further, the predicted Nusselt number as well as the predicted wall temperature are shown in comparison to the experimentally obtained values in Fig. 2. The correlation is capable to capture the effect of the roughness on the heat transfer, for the given conditions.

In the next step, the correlation needs to be adjusted for cases with strong variations of the thermophysical properties. This is achieved by multiplication of a correction factor, considering the ratio of the fluid properties at the wall and at the bulk temperature. Due to the nature of the fluid properties, the ratios of ρ, μ and λ follow a rather similar trend, and when the property at the wall temperature is in the numerator and the property at bulk temperature is in the denominator of the correction factor, only a deterioration of the heat transfer can be addressed, since this ratio would always be smaller than one. On the other hand, the ratio of the specific heat can yield a correction factor larger than 1, or also smaller than 1. To keep the correlation as simple as possible, only the density ratio and the specific heat ratio are considered for a systematic study. Analysis of the experimental data also shows that the roughness enhances the heat transfer more efficiently, when the density ratio from the wall to the bulk is small. For example, when the wall temperature exceeds the pseudo-critical temperature, a sudden decrease in the effect of the wall roughness is observed. The momentum induced by the roughness elements to the fluid can be transferred less efficiently from the low density near wall fluid towards the bulk, where fluid with larger density is present. Thus, the density ratio is considered in the roughness function $\Delta T(k_s^+)$, with an exponent which is to be fitted. Furthermore, previous assessments (Wiltschko et al. 2024) of correlations from the literature have shown that correlations that apply a correction factor for the property variation to the skin friction coefficient perform best. Thus, the correction factors in equations (6) and (10) have also been considered in the systematic analysis. The exponents in equations (6) and (10) are, as well the exponents for the other correction factors, fitted, using the whole available experimental dataset, excluding data points with measurement uncertainties of the Nusselt number of more than 20 %. 43,435 data points from the rough tube and 43,613 data points from the smooth tube fulfill that condition. All possible combinations of either no correction factor at all, the density ratio, the specific heat ratio, or both of them, as well as no additional correction factor for the skin friction coefficient, or equation (6) or equation (10), as well as density ratio or no correction for the roughness function have been tested, by first fitting the available exponents and then predicting the Nusselt number. Based on the accuracy, the best combination has been identified. The skin friction coefficient is corrected by equation (27) and the final correlation is given by:

$$Nu = \frac{\sqrt{\frac{c_f}{8}}Re\overline{Pr}}{\frac{1}{\kappa_\Theta}ln\left(\frac{\sqrt{c_f}Re}{2\sqrt{8}}\right) + 10.78\overline{Pr}^{0.46} - 0.17\left(\frac{\rho_w}{\rho_b}\right)^{0.68}k_s^+}\left(\frac{\rho_w}{\rho_b}\right)^{0.18} \tag{26}$$

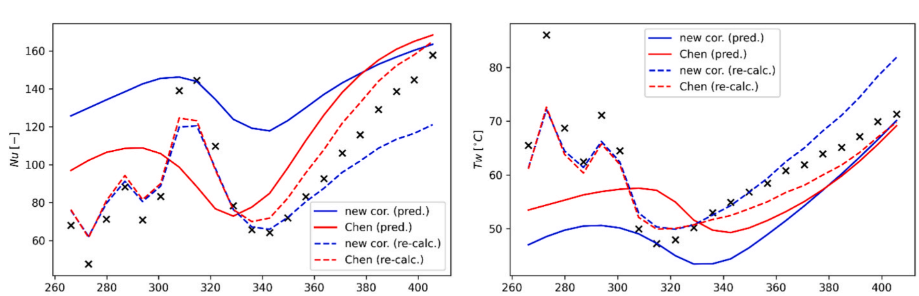

Fig. 4. Predicted and recalculated Nusselt number (left) and wall temperature (right) versus experimental data of Chen at al. (2022) at p = 7.5 MPa, G = 300 kg/m^{2}s and q = 50 kW/m^2.

$$c_f = c_{f,0}\left(\frac{\rho_w}{\rho_b}\frac{\mu_w}{\mu_b}\right)^{0.29} \tag{27}$$

4. Assessment of the new correlation

The newly obtained correlation is assessed with the available experimental data and the results are compared to the accuracy of the predictions of the correlations in the literature. Due to the iterative solving procedure, no– or multiple solutions are possible to appear. This can be understood, looking at the following equation:

$$Nu_{corr}(T_b, T_w) = \frac{\dot{q}D}{\lambda(T_w - T_b)} \tag{28}$$

The empirical correlations are considering the fluid properties at bulk and at wall temperature, thus are a function of both temperatures. The definition of the Nusselt number in the right-hand side of equation (28) is also a function of the wall and the bulk temperature. For a given bulk temperature and a given heat flux, it is possible to find one, multiple or no wall temperatures, which solve equation (28). Thus, the number of data points for which this problem appears is a criterion to assess the correlation's performance. To assess the accuracy of the correlations, the mean deviation μ, the mean absolute deviation $|\mu|$ and the standard deviation σ is calculated as:

$$\mu = \frac{1}{n}\sum_{i=0}^{n}\frac{Nu_{pred,i} - Nu_{exp,i}}{Nu_{exp,i}}\cdot 100 \tag{29}$$

$$|\mu| = \frac{1}{n}\sum_{i=0}^{n}\frac{\left|Nu_{pred,i} - Nu_{exp,i}\right|}{Nu_{exp,i}}\cdot 100 \tag{30}$$

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=0}^{n}\left(\frac{Nu_{pred,i} - Nu_{exp,i}}{Nu_{exp,i}}\cdot 100 - \mu\right)^2} \tag{31}$$

In the correlation proposed by Chen et al (2022), the roughness parameter R_z, which is the maximum peak to valley height of the roughness profile, is applied instead of the equivalent sand grain roughness k_s. In the smooth tube R_z is 1.91 μm, in the rough tube, R_z is 22.49 μm. Table 5 shows the results for the rough tube and Table 6 shows the results for the smooth tube. For the rough tube, the new correlation yields the most accurate results in terms of the absolute deviation $|\mu|$ and in terms of the standard deviation σ. On the other hand, this is not the case for the smooth tube, but the prediction accuracy is at least comparable to the accuracy of the best of the other correlations. This shows that it is hard to find a correlation which is valid for smooth and for rough wall at the same time.

In Fig. 3, the predicted results of the new correlation, for the Nusselt number as well as for the wall temperature are shown for p = 46 bar, G = 1000 kg/m^{2}s and q = 100 kW/m^2. Besides the predictions, which are obtained in an iterative procedure, without using the experimental wall temperature, also the results with application and re-calculation of the experimental wall temperature are shown. Note, that if for a given bulk temperature T_b and for a given heat flux $\dot{q}_w$, the experimentally obtained wall temperature T_w is additionally inserted into the Nusselt correlation, equation (28) is no longer satisfied. The experimental wall temperature can be re-calculated quite well in the rough tube, even though HTD is present. The re-calculated wall temperature and Nusselt number agrees well with the experimental wall temperature in the smooth tube.

In addition, the new correlation is validated against experimental data for supercritical pressure CO_2 published by Chen et al. (2022) A total of 2268 data points were digitized from this publication, covering pressures from 7.5 to 10 MPa, mass fluxes from 300 to 500 kg/m^{2}s, heat fluxes from 30 to 70 kW/m^2, and inlet temperatures from 15 to 25 °C. The experiments were conducted in tubes with an inner diameter of 4.57 mm, a heated length of 1 m, and different surface roughness values S_z = 1.5, 10, 20, 30 and 40 μm. Since no further description of the roughness parameter is provided, it is assumed that S_z represents the maximum peak-to-valley height of the roughness profile, commonly denoted as R_z. According to Adams et al. (2012), the equivalent sand grain roughness k_s is approximately equal to R_z. Therefore, the given values of S_z are used in the new correlation. The new correlation is used to predict the experimental Nusselt numbers digitized from Chen et al. (2022) and the results are shown in Table 7. For comparison, the predictions from the correlation proposed by Chen et al. (2022) are also included in Table 7. Additionally, the experimentally measured wall temperature is directly used in both correlations to calculate the Nusselt number.

First, the new correlation performs poorly in predicting the heat transfer for the Chen et al. CO_2 data Chen et al. (2022). The same applies to the correlation proposed by Chen et al., when used to predict the R134a data – it yields even worse results (see Tables 5 and 6). However, when the experimentally measured wall temperature is used in the new correlation, reasonable results are achieved. Fig. 4 shows trends of (left) Nusselt number and (right) wall temperature, together with predictions and recalculations of the new correlation, as well as the Chen correlation, which has been developed using this CO_2 dataset. Although heat transfer deterioration is difficult to predict, both correlations can reproduce the temperature peak. On the other hand, the predicted trends are quite similar for both correlations.

5. Conclusions

Based on a previously generated experimental database for heat transfer to a fluid at supercritical pressure conditions at a rough wall, a new heat transfer correlation, based on the non-dimensional temperature distribution in the radial direction of the tube is derived, accounting for the effect of wall roughness. The new correlation is evaluated using available experimental data, and the performance is compared with existing correlations applicable to such conditions. For rough tubes, the new correlation outperforms the existing ones. For smooth tubes, several correlations yield lower mean absolute percentage errors; however, the new correlation shows the smallest standard deviation of the errors. Therefore, it appears appropriate to use different correlations depending on the surface roughness conditions. Furthermore, applying the new correlation to CO_2 data from the literature reveals that it is limited to R134a. Similarly, the correlation developed using CO_2 cannot be applied to the new R134a data. This highlights the need for further improvement in empirical modelling of heat transfer under supercritical pressure conditions. Experiments using water would be of great interest, as the present work clearly demonstrates the challenges in applying existing correlations to different fluids.

CRediT authorship contribution statement

Fabian Wiltschko: Writing – original draft, Validation, Investigation, Formal analysis, Data curation, Conceptualization. **Ivan Otic:** Writing – review & editing, Supervision. **Xu Cheng:** Writing – review & editing, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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