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Where does gravitational potential energy reside?

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Abstract

In order to lift a body, energy is required. The potential energy increases. However, where exactly does this energy reside once the body is lifted? This is a natural question and it seems to be a reasonable question. The textbooks deal with it in different ways. Some say that the potential energy is located in the body that has been lifted. According to other sources it is localized in the system earth-body. In a third category of books, it is confirmed that the energy is stored, but nothing is said about the place where it is stored. Finally, there are books, especially for the university, in which the facts are formulated in such a way that it does not come to our mind to ask the question. A better answer is that the potential energy is stored within the gravitational field. However, the question arises as to where in the field the energy released by the lifted body can be found. The answer to this question is somewhat involved. One can only specify the extension of an area in which positive and negative energy density changes occur as the body is lifted.

Keywords: gravitation, potential energy, gravitoelectromagnetism

1. Introduction

The question in our title is a natural question. It is the question about the localization or localizability of the gravitational potential energy. Let us consider the process shown in figure 1.

The boy is lifting a crate using a rope. Thus energy flows from his muscles through the rope into the crate. What happens afterwards with this energy? Where does it end up?

Let us look in some reputed textbooks.

1. 'We call the sum of the weights times the heights *gravitational potential energy*—the energy which an object has because of its relationship in space, relative to the Earth' [1].
2. 'As m falls toward the Earth, practically all the potential energy of the system composed of the Earth and the mass m is transferred into the kinetic energy of the motion of m' [2].
3. 'It is not easy to say where this gained potential energy is localized, but its amount is definite enough, and the raised body clearly 'owns' it' [3].

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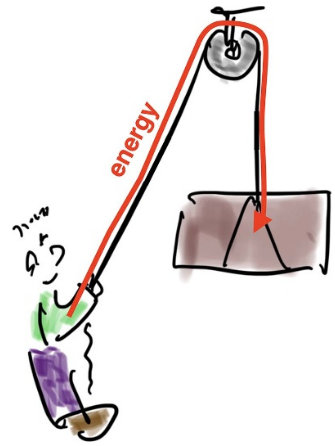

Figure 1. As the crate is lifted, energy flows from the muscles of the boy through the rope into the crate. But what happens to it thereafter?.

4. 'In addition to the kinetic energy, the potential energy V is defined as

$$V = E_{pot} = \int X \mathrm{d}x.$$

Here, X is the x-component of the force' [4].

5. 'For a conservative force F applies:

$$\nabla \times \mathbf{F} = 0.$$

Since the curl of a gradient always vanishes F must be the gradient of a scaler:

$$\mathbf{F} = -\nabla V.$$

Here, V is called potential energy' [5].

Here we have quoted internationally known works, some of which are older. However, more recent books for high schools and universities can also be easily classified into categories 1–5 [6–10]. Here, we briefly review our citations. They usually refer to two bodies, one of which is the Earth. Let us call the Earth L ('large'), the other body S ('small').

The first answer states that the potential energy is stored within body S.

According to the second answer, it resides in the system, which consists of S and L. However, it is not clear how it is distributed within the system.

In the third quotation, the question is explicitly acknowledged as justified; however, it is suggested that it cannot be answered.

The fourth quotation makes no assertions related to our question.

Finally, in the fifth, the potential energy is a magnitude that refers to a point. Therefore, an energy density and thus an energy distribution cannot be specified in principle.

It is obvious, that the statements do not agree with each other. Thus who wants to learn about the localization of the potential energy will be disappointed.

Might this mean that our question is not a sensible question?

In section 2, we discuss why it would be desirable for our teaching to provide an answer to the question of our title.

The reason for the confusion, which is manifested in our quotations lies in the historical development of the concept of energy and its localization. We recall this briefly in section 3 and conclude, that it is reasonable to attribute the gravitational potential energy to the gravitational field, just as we attribute the electrostatic potential energy to the electric field. We also discuss the question of what idea about a field we should convey in teaching.

However, one question still remains: How is the potential energy distributed within the field? In section 4 we will see that this question is somewhat tricky and that we must first clarify what exactly is meant by it.

Section 5 contains some comments on the consequences for our teaching.

2. The beneficial properties of extensive quantities

When we make a statement about a physical quantity, we employ specific verbalizations: nouns, verbs and adjectives. Thus we say that work is *done* or *realized*, a force is *exerted*, a voltage is *applied*. Students must not only acquire a physical understanding of the considered quantity; they must also learn the language that has to be employed.

However, there is a class of quantities that does not require a vocabulary of its own: the extensive quantities. A physical quantity is called an extensive quantity if its value refers to a region of space. Examples include mass, electric charge, entropy, momentum and amount of substance. We speak correctly about an extensive quantity in a physical sense, when we speak as if it were a substance. The cause of this liberty is that we can apply a model—the *substance model*. Therefore, extensive quantities are sometimes called substance-like or fluid-like quantities [11]. Using the substance model facilitates learning considerably, as students can develop a simple mental picture of the quantity.

For each substance-like quantity a density, a current intensity, and a current density can be specified as well as a rate of production, if necessary. This means that for each a local balance equation, also called the continuity equation, can be formulated.

It is well known for the electric charge:

$$\frac{\partial \varrho_Q}{\partial t} + \nabla \cdot \mathbf{j}_Q = 0. \tag{1}$$

Here ϱ_Q is the electric charge density and $\mathbf{j}_Q$ the electric current density.

However, a continuity equation can also be formulated for mass, momentum (Navier–Stokes equation), entropy [12], and also for energy:

$$\frac{\partial \varrho_\mathrm{E}}{\partial t} + \nabla \cdot \mathbf{j}_\mathrm{E} = 0. \tag{2}$$

Here ϱ_E is the energy density and $\mathbf{j}_\mathrm{E}$ the energy current density.

By specifying the density distribution, we localize the extensive quantity. With the current density we localize the corresponding current.

The fact that a quantity is extensive has beneficial consequences for teaching. However, as our quotations show, in the case of the energy one does not always make use of this advantageous property. This is because of the historical development of the concept of energy. We will briefly recall this in the following section.

3. How the entangled situation came about

Extensive literature exists on the history of the development of the energy concept. For the potential energy, see for example [13, 14].

We would like to take a closer look at two aspects of this development, that are relevant in the context of our question.

3.1. A quantity that does not measure any property

The appearance or ‘birth’ of the physical quantity energy was more problematic than that of most other physical quantities. This means that efforts to develop a consistent picture of the quantity for teaching purposes are still ongoing [15–17].

The story begins with a strange observation or discovery. When the values of certain variables change in a system A, the values of other variables in another system B also change, and the changes at A are related to those at B in a definite way. For example, if the altitude of a body A decreases by a certain value, the temperature of another system B increases in a specific manner. It turned out that one can establish a kind of balance if one defines so-called equivalents. It was an ingenious insight that one can describe such observations with the help of a new quantity, whose value remains constant during the process. It was called energy.

However, there was a problem with this new quantity. A ‘normal’ physical quantity is a measure of some perceptible or measurable property: mass is a measure of gravity and inertia, temperature measures the being warmer or colder of a body, pressure measures the tendency to expand. Energy, in contrast, manifests itself in a variety of ways, depending on the situation: sometimes in a high velocity, sometimes in a high temperature, sometimes in a high pressure, and so on. This made it difficult to deal with the new quantity in the same manner as with other physical quantities. One introduced the so-called forms of energy, as well as the concept of energy conversion [18].

It was only more than 50 years after energy was introduced as a physical quantity with its own unit of measurement that it turned out to have existed under a different name long before. The relationship

$$E = k \cdot m \tag{3}$$

where m is the quantity usually called relativistic mass [19], tells us that the mass and the new quantity energy differ only by a constant factor k, which is equal to the square of the terminal speed (also known as the speed of light) c. Thus, the flaw of energy, that it does not describe any property, was eliminated. Energy describes properties that we have always known to be described by mass: gravity and inertia. (The remaining oddity, namely that these are two properties at once, was finally resolved by the general theory of relativity: Gravity and inertia are manifestations of the same phenomenon in two different reference systems). In view of this fact, it is certainly inappropriate to talk about forms of energy and energy conversion. It is particularly misleading to say that mass is transformed into energy, or vice versa.

3.2. *The localization of the potential energy*

The fact that one can establish an energy balance (if its value in A decreases by a certain amount, it must increase in another system B by the same amount) suggested from the beginnings of the energy epoch, that one can deal with it like with a substance-like quantity, that is with mass or electric charge. This would significantly simplify its handling. However, for the time being, one was not able to do so. Although its value could be assigned to an object, body or 'physical system', it was not possible to specify the distributions of its density and current density. Today we would say that it was not yet possible to formulate a local balance equation.

Scientists in the second half of the 19th century were aware that this was a deficiency that had to be remedied. It was stated, for example by Max Planck in a booklet published in 1887 with the title *Das Prinzip der Erhaltung der Energie* (The principle of the conservation of the energy) [20]:

> 'Certainly it must be admitted that this (so to speak material) conception of energy as a stock of effects, the quantity of which is determined by the actual state of the material system, will possibly have done its service later on and will give place to another, more general and higher, conception; at present, at any rate, it is the concern of physical research to develop this conception as the most vivid and fruitful everywhere down to the last detail and to test its consequences on the basis of experience; in this direction, as we shall see later, many a new point of view can still be found.'

More clearly it was brought to the point in 1892 by Heinrich Hertz [21, p 234]:

> 'A greater concern seems to me to be the question of how far the localization of energy and its point-to-point tracing has any sense and meaning at all with our present knowledge of energy.'

Later, on p 293, Hertz describes the problem in very concrete terms [21, p 293]:

> 'If a steam engine drives a dynamo by means of a belt running back and forth, and the dynamo in turn feeds an arc lamp by means of a wire running back and forth, it is, however, a common and unobjectionable expression to say that the energy is transmitted from the steam engine through the belt to the dynamo, and from the dynamo to the lamp by means of the wire. However, does it have a clear physical sense to say that energy moves along the stretched belt against the direction of the belt movement from point to point? If not, can it have a clearer sense to say that the energy moves along the wires, or, according to Poynting, in the space between the two wires from point to point? The conceptual obscurities appearing here still need a lot of elucidation.'

We see that not only the problem of the localization of the potential gravitational energy had not yet been solved. The problem existed also for other 'forms of the energy'.

However, in the same year 1892, an article by Wien [22] appeared in which the localizability of energy was demonstrated, namely for hydrodynamics, for elastic solid bodies, for the electromagnetic field and for thermal phenomena:

> 'In our attempts to localize the energy, we have not come across anything that is incompatible with our previous concepts. In any event, in many cases this approach seems to allow simpler ideas than would be possible without this concept.'

However, gravitation is not addressed in the article.

In 1898, finally, a comprehensive publication by Mie [23] appeared, which represented a certain completion. However, Mie mentions gravitation only briefly in the very last paragraph.

One year after Hertz' and Wien's work, another paper on the subject was published by Oliver Heaviside in the magazine *The Electrician* which begins with the following sentence [24]:

> 'To form any notion at all of the flux of gravitational energy, we must first localise the energy.'

The article is entitled *A gravitational and electromagnetic analogy* in which the localization of energy is not only requested; the gravitational field energy is actually localized. Heaviside shows how to calculate the energy density and energy flux density in the gravitational field.

The energy density ϱ_{E} of a static gravitational field is

$$\varrho_{\mathrm{E}} = -\frac{1}{8\pi G}\mathbf{g}^2 \tag{4}$$

where $\mathbf{g}$ is the field strength of the gravitational field and the energy current density $\mathbf{j}_{\mathrm{E}}$ is

$$\mathbf{j}_{\mathrm{E}} = -\frac{c^2}{4\pi G}\mathbf{g}\times\mathbf{b} \tag{5}$$

where $\mathbf{b}$ is the strength of the gravitomagnetic field.

Sometimes $\mathbf{g}$ is called the gravistatic field strength and $\mathbf{b}$ the gravimetric field strength [25].

Equation (4) has the same structure as that of the energy density of the electrostatic field, which shortly before was formulated by Maxwell. It is easy to convince oneself of the validity of equation (4). Equation (5) is the gravitational analogue of Poynting's formula for the energy flow density within an electromagnetic field [26].

In summary, we can state the following: Only about 50 years after the introduction of the energy, it was possible to specify the energy density and current density for the gravitational field.

However, the quotations in our introduction reveal nothing of the view or insight that energy can be localized in the gravitational field and that energy flows can be specified in the gravitational field.

This does not mean that it has been completely ignored, see for example the articles by Poon [27], Hilborn [28], Tort [29] or Vasyliūnas [30].

The fact that there are still acceptance problems with the idea of energy as a locally balanceable quantity may also be due to a certain understanding or interpretation of the term physical quantity. We will briefly address this issue.

3.3. Physical quantity and physical system, and two concepts of a field

In the context of the discussion of potential gravitational energy, but also of potential electromagnetic energy, one occasionally hears arguments that we believe are inappropriate.

For example, energy is often not only discussed as a substance (in the sense of the substance model), but it is suggested that energy is actually a kind of substance. These ideas have long traditions. For example, in the past it was thought that there was an electric and a magnetic fluid, and in thermodynamics it was believed that we were dealing with a heat substance, the phlogiston, or later, Lavoisier's calorique.

Energy is not a substance, it is a physical quantity. This means that it is a variable in a theory, and therefore, a mathematical object [31]. We can talk about it as we talk about a substance, but we should keep in mind that we use a model when doing so. From this point of view, it seems inappropriate to claim that potential energy is 'not real', that it is not 'directly observable' or that it is a 'purely theoretical concept' [14]. It is no more or no less real or theoretical than any other physical quantity. Finally, it would be consequent not to call it 'potential' energy. It is simply the energy of the field.

We encounter a similar problem in the context of the term 'field'. The designation field is used in physics in two different meanings.

On the one hand, the term refers to a mathematical concept. A field is the distribution of a local quantity in space. For example, the spatial distribution of the temperature may be called a temperature field. Similarly, the distribution of the electric field strength is a field.

On the other hand, the term field is used as the name of a physical system, for example when discussing the electromagnetic or the gravitational fields.

When simply referring to the 'electric field', it is often not clear what exactly is meant: the physical system in the vicinity of an electrically charged body or the distribution of the quantity **E** in space.

The physical system field exists regardless of whether there are humans who are interested in its properties or not. This system can be described by the physical quantity field strength. However, the field strength is not the only quantity that can be used to characterize it. For the electromagnetic field, not only does the field strength have a non-zero value, but also the other standard physical quantities: energy, momentum, energy and momentum current, and in certain states also temperature, entropy and chemical potential.

Therefore, we recommend that the quantities **E** and **g** are not referred to as fields, but as field strengths, and that the distributions of this quantities in space are referred to as field strength distributions.

4. An inappropriate and an appropriate question

4.1. An inappropriate question

Let us now ask the question in the title again, but in a more explicit form. We slightly lift a body S, which is located in Earth's gravitational field. In doing so, we supply energy to it, say eight joules. These eight joules do not remain within the body, but pass over into the field. However, the field is large and we would like to know more precisely, where in the field are our eight joules to be found?

If we phrase our question this way, it may still sound reasonable. But in fact, it has become ambiguous.

To clarify this issue, let us compare the situation with another one. We consider a large lake and pump another cubic meter of water into it, and we ask: Where has this pumped-in water gone? Our spontaneous answer is simple: it is near the location where we pumped it in. We can even imagine that we have marked the added water in colour, for example by adding ink. This makes it easy to recognize where it is.

We know where the water has gone, because water is a substance. Each portion of this substance can be tracked individually and has a well-defined position and velocity at any given moment.

If we use the substance model in connection with an extensive quantity, we can determine the distribution of its density and current density. However, in general a current density cannot be associated with a velocity. This is only possible if we deal with a convective transport of the quantity. For the electrical charge transported by electrons in a conductor, we can write

$$\mathbf{j}_Q = \varrho_Q \mathbf{v} \quad (6)$$

where **v** is the drift velocity of the free electrons.

We can also assign a velocity to an energy transport if the transport is convective; for example, when we are dealing with the transport of kinetic and internal energy by a moving body. Then the following applies

$$\mathbf{j}_{\mathrm{E}} = \varrho_{\mathrm{E}} \mathbf{v}. \quad (7)$$

However, it is not always possible to meaningfully define a velocity for the current of a physical quantity. A corresponding example of the electric current is the displacement current, and an example of such an energy current is the energy current in the electromagnetic field or in the gravitational field.

There have been attempts to assign a velocity to these flows, namely by using equation (6) as the defining equation of the velocity [32]. However, in contrast to a convection velocity, the velocity defined in this manner cannot be measured independently.

But then what about our question regarding the fate of the potential energy? Apparently, there is no answer of the kind we had initially expected. However, we do not want to conclude that the question is senseless, but rather try to interpret it in a somewhat different way.

4.2. A somewhat better question

In fact, the question could have been understood differently, as suggested by the balance equation (2). The equation makes a clear statement: there are places where the energy density decreases and others where it increases, which are connected by streamlines. If we ask where the energy goes, it might seem an acceptable answer if we were told where the sinks of the streamline field were located, that is, where the energy density in the field increased.

If we apply this method to our lake, we can also obtain a clear answer. The flow of water ends at the surface of the entire lake.

Therefore, let us reformulate our question: where are the sinks of the energy current field?

However, we must realize that also the answer to this reformulated question is not what we may expect.

This is because, in addition to the places where the energy density increases, there are others where it decreases, even though energy is added to the field as a whole. This is because when body S is moved, energy is not only transferred to the field, so that the field energy increases everywhere. Energy is also redistributed within the field. It is easy to understand that this must occur. Even if body S is moved in the absence of body L, field energy must be shifted in space, which means that the energy density decreases in some places and increases accordingly in others [33]. This effect overlaps with what we are actually interested in. Therefore, if we notice that the energy density increases at one point, we cannot conclude that this increase corresponds to the energy that supplied to the body S, that is the potential energy.

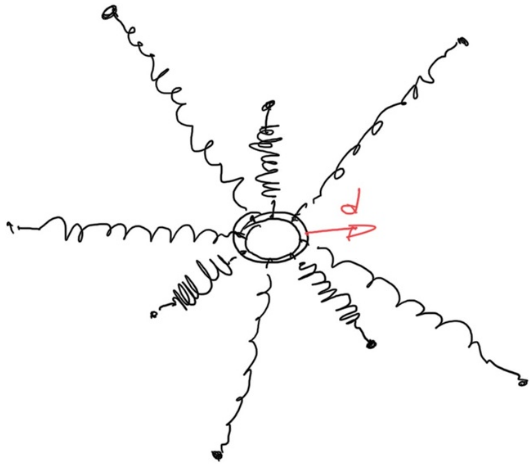

Figure 2. We displace the ring in the middle by a small distance d. In doing so, we have to supply energy to the arrangement. During this process, the energy of some springs increases and that of others decreases.

By the way, this is by no means a special phenomenon of fields. We can also observe it in more concrete systems. Figure 2 shows an arrangement of eight springs. One end of each spring is fixed, the other ends are attached to a ring in the middle. Some of the springs are under tension, whereas others under compression. First, we assume that the entire system is in equilibrium. Then, we displace the ring by a small distance d. In doing so, energy must be supplied to the ring. Here, too, the question may arise: Where does this energy go? And we have the same dilemma as with the gravitational field. When we shift the ring, energy is stored in some of the springs, whereas the others release energy. We cannot say where the energy we have supplied has gone. A joule is not an individual that we can follow on its path.

We therefore have to accept that also our new question does not lead to the answer we had hoped for. Should we give up after all and conclude that a question about the whereabouts of the energy is senseless?

We do not need to go that far. Despite all the objections, we can establish one thing: there is a region of space around the body S (without a sharp boundary) in which changes of the energy density occur as we lift the body. It is the area of space that absorbs the energy emitted by the body and in which energy is redistributed while the body is being lifted.

Finally, we are in a position to formulate an answerable question: How far does this area extend?

4.3. The question that can be answered and the answer

We consider the energy density of the common field of S and L:

$$\begin{aligned} \varrho_{\mathrm{E}} &= -\frac{1}{8\pi G}(\mathbf{g}_{\mathrm{L}} + \mathbf{g}_{\mathrm{S}})^2 \\ &= -\frac{1}{8\pi G}\left(\mathbf{g}_{\mathrm{L}}^2 + 2\mathbf{g}_{\mathrm{L}} \cdot \mathbf{g}_{\mathrm{S}} + \mathbf{g}_{\mathrm{S}}^2\right). \end{aligned} \tag{8}$$

The first term on the right-hand side of the equation is the energy density of the field of L alone and the third term is that of the field of S alone. If S is shifted (in the reference frame of L), the energy density distribution changes for two reasons:

First, because the third term in brackets on the right-hand side of equation (8) changes. This is the contribution to the energy density change that would also be present if S were moving in empty space. It only describes the fact that the field energy is shifted or displaced. As the magnitude of this energy density decreases with $1/r_{\mathrm{S}}^4$ (r_{S} = distance from the centre of S), this process takes place in an area around S that is small compared to the distance to the centre of L, that is the centre of the Earth. We are not interested in this contribution to ϱ_{E}.

The second term in the brackets (8) is responsible for storing the energy supplied to the field. This is called the interaction energy. When integrated, we obtain the energy supplied by the field when bringing the two bodies from infinity to their actual distance.

As discussed earlier, this does not mean that its change in time during the energy supply is positive everywhere.

This is why we have to modify our question again: How large is the area in which the greatest part of the interaction energy is located? To answer this question, we make a rough estimation.

We have already noticed that the magnitude of the energy density of S alone decreases with $1/r_{\mathrm{S}}^4$. For comparison, we now consider the decrease of the magnitude of the interaction energy term $1/\left(r_{\mathrm{S}}^2 \cdot r_{\mathrm{L}}^2\right)$. On one hand, it is decreasing with $1/r_{\mathrm{S}}^2$. However, this decrease is compensated for by the fact that the volume of a spherical shell of thickness dr increases with r_{S}^2. Therefore, the decay of the interaction energy density is determined by the second factor $1/r_{\mathrm{L}}^2$. The larger r_{L}, the further the region of the interaction energy is extended. In the limiting case of an exactly homogeneous field (instead of the field of L), that is, when $r_{\mathrm{L}} = \infty$, the interaction energy is distributed over an infinite region of space. We can again make a comparison with the lake into which we are pumping water. If the lake extends to infinity, the water level rises everywhere by an infinitely small amount.

The previous considerations did not take into account that the interaction energy density is calculated as the scalar product of two vectors and that it therefore depends on the direction. However, a more detailed analysis would not provide further insight with regard to our original question.

A discussion of the distribution of the (positive and negative) interaction energy density in the case of two point charges can be found in [34].

5. Conclusions

We can determine the energy density at any given point in a gravitational field. We can also specify how the energy density changes at any point when the body is moving, and calculate the energy current density distribution within the field. We can indicate the sources and sinks of the energy flow in the gravitational field. However, we cannot say where in the field the portion of energy that is supplied to the field when lifting a body is stored. All we can do is specify a region over which the supplied energy spreads. Thus, we have answered the question in our title as far as possible.

However, could not it be argued that this distribution is not really interesting? The answer is not suitable for solving any practical problem. Neither for mechanical processes near the Earth's surface, nor for processes related to satellites and space stations, we need to know the energy distribution in the gravitational field and how it changes.

However, the same objection could be raised in the case of electromagnetism. In the lecture, we introduce the formulas for the energy density and the energy current density, although electrical engineers do not benefit from the corresponding results. Why do we introduce them anyway? We do it to create an understanding of electromagnetism. We want to integrate electromagnetism into the rest of physics so that energy balances can be extended to the realm of fields. This makes physics coherent and understandable. With the preceding discussion of gravitation, we pursued the same goal.

Our arguments were somewhat involved. However, the results can now be summarized in two simple statements:

1. Talk about energy in such a way that students realize that it is a substance-like quantity. In the case of gravitation, when a body is lifted, energy flows into the body, and this energy leaves the body again and is stored in the gravitational field.
2. The energy is stored in a spatial area of the same order of magnitude as the distance between the centres of the gravitating bodies; in the case of a small body at the Earth's surface it is an area characterized by the Earth's radius.

Data availability statement

No new data were created or analysed in this study.

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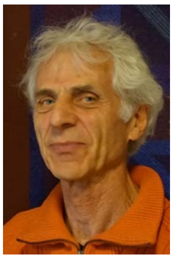

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