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Analytic results for electroweak precision observables at NLO in SMEFT

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Abstract: We present analytic results for electroweak precision observables (EWPO) at next-to-leading order (NLO) in dimension-six SMEFT, with no assumptions on the flavour structure of SMEFT Wilson coefficients. The results are given in five different electroweak input schemes, thus offering a simple means, along with scale variations, of estimating theory uncertainties related to higher-order terms in the SMEFT expansion. Our results will be useful to assess the constraining power of existing and future lepton colliders for new physics scenarios.

Keywords: SMEFT, Electroweak Precision Physics

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1 Introduction

Standard Model Effective Field Theory (SMEFT) is a robust and widely used framework for describing small deviations from the predictions of the Standard Model (SM) of particle physics [1]. To increase the precision of SMEFT predictions, higher-order corrections in the SM couplings or, equivalently, loops need to be included. The calculation of next-to-leading order (NLO) SMEFT corrections is an active area of research: NLO QCD corrections have been fully automated for most Wilson coefficients [2, 3] and NLO electroweak as well as NNLO QCD corrections have been calculated for various processes on a case-by-case basis in [4–54]. As emphasised in [55], the perturbative convergence and pattern of SMEFT Wilson coefficients appearing in higher-order corrections is influenced by the choice of electroweak input scheme, so evaluating observables in different schemes gives a means of assessing theoretical uncertainties beyond scale variations alone.

In this paper, we present analytic NLO results in dimension-6 SMEFT for the so-called electroweak precision observables (EWPO) measured at LEP and the Tevatron [56]. In particular, we extend the NLO SMEFT predictions presented in [33, 47, 57] in a (mostly) numerical form in the $\{G_F, \alpha(0), M_Z\}$ input scheme to five different electroweak schemes involving M_Z and combinations of G_F, M_W, $\alpha(M_Z)$ and $\sin\theta^{\ell}_{\mathrm{eff}}$ as inputs, in each case providing fully analytic results with no flavour assumptions on the SMEFT Wilson coefficients. Given that the EWPO currently provide some of the most precise probes of new physics and

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M_H	$125.20 \pm 0.11\,\text{GeV}$	(9×10^{-4})
m_t	$172.56 \pm 0.31\,\text{GeV}$	(2×10^{-3})
$\alpha_s(M_Z)$	0.1190 ± 0.0009	(8×10^{-3})
M_Z	$91.1880 \pm 0.0020\,\text{GeV}$	(2×10^{-5})
M_W	$80.3692 \pm 0.0133\,\text{GeV}$	(2×10^{-4})
G_F	$(1.166378 \pm 0.000006) \times 10^{-5}\,\text{GeV}^{-2}$	(5×10^{-6})
$\alpha(M_Z)^{-1}$	128.917 ± 0.008	(6×10^{-5})
$\sin\theta^{\ell}_{\text{eff}}$	0.23149 ± 0.00013	(6×10^{-4})

Table 1. Experimental values and uncertainties on the input parameters [62]. The relative precision of the measurement is given in round brackets.

are expected to be measured with significantly increased precision at a future e^+e^- collider like FCC-ee or CEPC [58, 59], these results will be useful in assessing theory uncertainties and providing cross-checks on near-term and future global SMEFT fits.[1] While the methods used in obtaining our results follow previous work [55, 61], where the leptonic decay rates of the Z and W bosons were calculated, we have extended those calculations by computing decays into hadrons and neutrinos and taking into account the chiral structure needed to obtain left-right and forward-backward asymmetries.

The paper is structured as follows. First, in section 2, we give some calculational details, defining NLO SMEFT expansions of the EWPO and the different electroweak input schemes in which the calculations are performed. In section 3 we present our results, explaining the notation and contents of ancillary electronic files as well as how to evaluate them numerically for arbitrary input parameters, including uncertainty estimates. We conclude in section 4 and provide in appendices further details of our calculations: in appendix A we define the EWPO on the Z-pole, appendix B covers different flavour assumptions provided along with our results, appendix C compares with previous literature, and in appendix D we give simple analytic expressions needed to evaluate EWPO at LO in SMEFT in the five different input schemes. The analytical results at NLO calculated in this work are provided as ancillary files with the arXiv submission.

2 Calculational details

We write the dimension-six SMEFT Lagrangian as

$$\mathcal{L} = \mathcal{L}^{(4)} + \mathcal{L}^{(6)}; \quad \mathcal{L}^{(6)} = \sum_i C_i(\mu)\, Q_i(\mu)\,, \tag{2.1}$$

where $\mathcal{L}^{(4)}$ denotes the SM Lagrangian and $\mathcal{L}^{(6)}$ is the dimension-six Lagrangian with operators Q_i given in the Warsaw basis [64] and the corresponding Wilson coefficients $C_i(\mu) \equiv C_i = c_i/\Lambda^2$ are inherently suppressed by the new physics scale Λ. The 59 independent

[1] Indeed, they have already been used in the recent analysis of [60].

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scheme s	inputs	suffix in filenames
$v_\mu^{\rm eff}$	G_F, $\sin\theta_{\rm eff}^\ell$, M_Z	`GF_SWeff_MZ`
$v_\alpha^{\rm eff}$	$\alpha(M_Z)$, $\sin\theta_{\rm eff}^\ell$, M_Z	`aEW_SWeff_MZ`
α_μ	G_F, M_W, M_Z	`GF_MW_MZ`
α	$\alpha(M_Z)$, M_W, M_Z	`aEW_MW_MZ`
LEP	G_F, $\alpha(M_Z)$, M_Z	`GF_aEW_MZ`

Table 2. Nomenclature for the EW input schemes considered in this work.

dimension-six operators, which in general carry flavour indices, are listed and grouped into eight classes in table 4.[2] Throughout this work, we truncate the SMEFT expansion of a given quantity to linear order in the Wilson coefficients.

We assume that the CKM matrix is the unit matrix and that all fermions are massless except the top quark with mass m_t. Given the expansion to linear order in the Wilson coefficients, no flavour-violating SMEFT interactions contribute, as we rely on the interference with the corresponding SM diagrams. We make no assumptions on the flavour structure of the SMEFT interactions, but we provide replacement rules to obtain the results for a $\mathrm{U}(3)^5$ symmetry of the fermion fields as well as in minimal flavour violation in appendix B.

We consider the five different electroweak input schemes listed in table 2, which use as inputs a combination of three parameters in the following list: the Fermi constant G_F, the masses of the Z and W bosons M_Z, M_W, the leptonic effective mixing angle $\sin\theta_{\rm eff}^\ell$ and the electromagnetic coupling constant, for which we use the on-shell definition $\alpha(M_Z)$. This definition is closely related to the $\overline{\rm MS}$ definition $\bar{\alpha}(\mu)$ in five-flavour QED×QCD used in [55], where the electroweak scale contributions are included through decoupling constants as described in section 4.2 of [65]. Explicitly, the definitions are linked via the perturbative relation

$$\begin{aligned}\overline{\alpha}(\mu) &= \alpha(M_Z)\left[1+\frac{\alpha(M_Z)}{\pi}\sum_{f\neq t}\frac{N_c^f}{3}Q_f^2\left(\frac{5}{3}+\ln\frac{\mu^2}{M_Z^2}\right)\right]\\ &= \alpha(M_Z)\left[1+\frac{\alpha(M_Z)}{\pi}\left(\frac{100}{27}+\frac{20}{9}\ln\frac{\mu^2}{M_Z^2}\right)\right],\end{aligned} \tag{2.2}$$

where Q_f is the charge of the fermion and $N_c^f=3$ ($N_c^f=1$) for quarks (leptons). In practice, the numerical value of $\alpha(M_Z)$ can either be taken from a fit, or else calculated from $\alpha(0)\approx 1/137$ through the relation

$$\alpha(M_Z)=\frac{\alpha(0)}{1-\Delta\alpha(M_Z)}, \tag{2.3}$$

with $\Delta\alpha(M_Z)=\Delta\alpha_{\rm lep}+\Delta\alpha_{\rm had}=0.03142+0.02783$, where the leptonic contribution has been obtained by evaluating the one-loop contribution

$$\Delta\alpha_{\rm lep}=\frac{\alpha}{3\pi}\left(-\frac{15}{3}+\sum_{\ell=e,\mu,\tau}\log\frac{M_Z^2}{m_\ell^2}\right), \tag{2.4}$$

[2]We employ the symmetric basis for the Wilson coefficients. This means that for Wilson coefficients contributing to operators with two identical fermion bilinears, we define the Wilson coefficient of both flavour combinations and take into account their symmetry, e.g. $C_{\underset{1221}{ll}}+C_{\underset{2112}{ll}}=2C_{\underset{1221}{ll}}$.

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and the value of the hadronic contribution is taken from [62]. Expanding the EWPO in terms of $\alpha(M_Z)$ rather than $\alpha(0)$ absorbs the light-fermion contributions contained in $\Delta\alpha$ into the definition of the coupling, and our use of $\alpha(M_Z)$ rather than $\alpha(0)$ in the LEP scheme is one of the differences compared to the corresponding NLO SMEFT calculations in [33, 47, 57]; other differences are explained in appendix C.

To shorten analytic expressions and make clear the origin of certain terms, we make use of dependent parameters whose explicit expressions are input-scheme dependent. For instance, the sine and cosine of the Weinberg angle appearing in our equations depend on the electroweak input scheme s, as does the W-boson mass, according to the following:

$$(c_w^s)^2 = 1 - (s_w^s)^2 = \frac{(M_W^s)^2}{M_Z^2} = \begin{cases} \frac{M_W^2}{M_Z^2}\,, & s = \{\alpha, \alpha_\mu\} \\ 1 - \left(\sin\theta_{\text{eff}}^\ell\right)^2\,, & s = \{v_\alpha^{\text{eff}}, v_\mu^{\text{eff}}\} \\ \frac{1}{2}\left(1 + \sqrt{1 - \frac{4\pi\alpha(M_Z)v_\mu^2}{M_Z^2}}\right)\,, & s = \text{LEP} \end{cases}\,. \tag{2.5}$$

The LEP-scheme result in eq. (2.5) depends on the quantity v_μ, which is related to the vacuum expectation value (vev) of the Higgs field at LO in the SM. This follows the notation of [55, 61], which denotes the vev in scheme s by v_s, where in this case the explicit expressions are

$$\begin{aligned} v_{\alpha_\mu} &= v_{v_\mu^{\text{eff}}} = v_{\text{LEP}} \equiv v_\mu = \left(\sqrt{2}G_F\right)^{-\frac{1}{2}}\,, \\ v_\alpha &= \frac{2M_W s_w}{\sqrt{4\pi\alpha(M_Z)}}\,, \quad v_{v_\alpha^{\text{eff}}} = \frac{2M_Z c_w s_w}{\sqrt{4\pi\alpha(M_Z)}}\,. \end{aligned} \tag{2.6}$$

Note that we have dropped the superscript s in factors of c_w and s_w appearing in the v_α^{eff}-scheme result, which must be understood in the sense of eq. (2.5). Given that it is always clear from the context which scheme s is under consideration, we follow this convention in the remainder of the paper and the associated electronic files.

2.1 Observables and the NLO SMEFT expansion

The considered EWPO are based on partial Z and W boson decay rates. For the W boson, we provide its total decay rate, as well as the partial lepton decay rates

$$\Gamma_W,\, \Gamma_W^{\ell\nu}\,, \tag{2.7}$$

with $\ell = e,\, \mu,\, \tau$. On the Z pole, we consider the total Z-boson decay rate, the hadronic total cross section in the narrow width approximation, as well as ratios, left-right asymmetries and forward-backward asymmetries for decays into leptons, bottom, charm, and strange quarks

$$\Gamma_Z,\, \sigma_{\text{had}},\, R_\ell,\, R_q,\, A_\ell,\, A_q,\, A_{\text{FB}}^\ell,\, A_{\text{FB}}^q\,, \tag{2.8}$$

with $q = s, c, b$. The definitions of these quantities are given in appendix A. In addition, we present predictions for

$$M_W,\, G_F,\, \alpha \tag{2.9}$$

in those schemes in which they are not an input parameter. Note that $\sin\theta_{\text{eff}}^\ell$ is not included in the list eq. (2.9), because as shown in eq. (A.3) it is equivalent to A_ℓ. For this reason, in input schemes involving $\sin\theta^{\ell}_{\rm eff}$, one must exclude $A_{\ell=L_{\rm eff}}$ from the list of independent observables, where $L_{\rm eff}$ is the reference lepton used in defining the effective Weinberg angle.

We expand all observables to linear/first order in the EFT and loop expansion and consistently drop all partial higher-order corrections. For an observable O we define LO and NLO results in terms of SMEFT expansion coefficients in scheme s as

$$
\begin{aligned}
O_s^{\rm LO} &= O_s^{(4,0)} + v_s^2\, O_s^{(6,0)}\,, \\
O_s^{\rm NLO} &= O_s^{\rm LO} + \frac{1}{v_s^2} O_s^{(4,1)} + O_s^{(6,1)}\,,
\end{aligned}
\tag{2.10}
$$

where the superscripts l and k in $O_s^{(l,k)}$ label the operator dimension and the number of loops ($k=0$ for tree-level and $k=1$ for one-loop), respectively. The expansion above makes the dependence on the vev explicit, as electroweak loop contributions are proportional to two powers of the coupling $g^2 \sim 1/v_s^2$.

2.2 Tools and conventions

SMEFT Feynman rules have been obtained using an in-house `FeynRules` [66] implementation of the dimension-six SMEFT Lagrangian, and cross checked with `SMEFTsim` [67, 68]. In contrast to our previous works [55, 61], we define the covariant derivative with a plus sign to match the conventions of `SMEFTsim` [67, 68][3] so that, for instance,

$$
D_\mu q = \left[\partial_\mu + i g_s T^a G^a_\mu + i\frac{g_W}{2}\sigma^i W^i_\mu + i\mathbf{y}_q g_1 B_\mu\right] q\,. \tag{2.11}
$$

Matrix elements were computed using `FeynArts` and `FormCalc` [69–71] and analytic results for Feynman integrals were extracted from `PackageX` [72]. We express the Passarino-Veltmann (PV) integrals in the notation of `LoopTools` [70]. In schemes in which the W-boson mass is not an input parameter, we have consistently expanded the M_W-dependent PV integrals for the SM one-loop contributions to linear order in the SMEFT. Phase-space integrals arising from the real emission of photons and gluons were calculated analytically using standard methods.

2.3 Treatment of divergences and cross-checks

UV and IR divergences appearing in the NLO calculation are treated in dimensional regularization in $d = 4 - 2\epsilon$ space-time dimensions, where ϵ is the dimensional regulator. IR divergences in the ϵ expansion cancel between virtual and real emission corrections, while UV divergences are cancelled by adding appropriate counterterms. Tadpoles are treated in the FJ tadpole scheme [73] and cancel out of results for physical observables. The SMEFT Wilson coefficients are renormalised in the $\overline{\rm MS}$ scheme, i.e. we relate the bare parameters, denoted with a subscript 0, to the renormalised quantities via the equation

$$
C_{i,0} = C_i + \delta C_i, \qquad\qquad \delta C_i \equiv \frac{1}{2\epsilon}\dot{C}_i \equiv \frac{1}{2\epsilon}\frac{{\rm d}C_i}{{\rm d}\log\mu}\,. \tag{2.12}
$$

[3] This flips the sign of the predictions for the Wilson coefficients C_W and those of the dipole operators.

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The $\dot{C}_i$ have been calculated at one loop in [74–76][4] and when written in terms of mass-basis parameters take the form

$$\dot{C}_i = \frac{1}{v_s^2}\gamma_{ij}^{(4,1)} C_j \,, \tag{2.13}$$

which makes clear that changing μ mixes the Wilson coefficients in a way determined by the one-loop anomalous dimension matrix $\gamma_{ij}^{(4,1)}$.

The masses M_W and M_Z are renormalised on-shell, relating the renormalised parameters to the bare ones according to

$$M_V^2 = M_{0,V}^2 - \Pi_{VV}(M_V^2) \,, \tag{2.14}$$

where $\Pi_{VV}(M_V^2)$ is the one-loop two-point function for the Z or W boson. G_F is renormalised by requiring that the relation $v_\mu = (\sqrt{2}G_F)^{-1/2}$ holds to all orders in perturbation theory, see appendix A of [55]. For the effective leptonic mixing angle, the renormalisation condition is $\sin\theta_{\rm eff}^{\ell} = s_w$ and the necessary counterterms are listed explicitly in [61]. Finally, counterterms from electric charge renormalisation are obtained by renormalising the photon-fermion vertex as described in [65].

As internal cross-checks on our calculation we have verified the cancellation of all UV and IR divergences and arrived at identical results in unitary and Feynman gauge. We have also checked that the μ-dependence in SMEFT corrections is as dictated by a renormalisation-group analysis. In particular, while the LO expansion coefficients of a given observable contain only implicit μ-dependence through the Wilson coefficients, $O_s^{(6,0)} = O_s^{(6,0)}(C_i(\mu))$, the NLO corrections also contain explicit μ-dependence through the UV-renormalised matrix elements. The fact that the NLO decay rate is independent of μ to that order then requires that the NLO coefficients take the form

$$O_s^{(6,1)}(C_i(\mu),\mu) = O_s^{(6,1)}(C_i(\mu), M_Z) - v_s^2 O_s^{(6,0)}(\dot{C}_i(\mu)) \ln\frac{\mu}{M_Z} \,, \tag{2.15}$$

and we have checked that our results indeed satisfy this equation.

3 Results and uncertainty estimates

The main results of this work are analytic expressions for the EWPO listed in eqs. (2.7)–(2.9). Results for the EWPO at LO in SMEFT in the different EW input schemes can be constructed from partial decay widths of the W and Z boson into left- and right-handed fermions, which are given in appendix D. Moreover, in the limit of vanishing quark masses, the NLO QCD corrections appearing in W and Z decays into quarks are a universal correction to the LO SMEFT results.[5] Explicitly,

$$\frac{\Gamma^{\rm QCD}(Z\to q\bar{q})}{\Gamma^{\rm LO}(Z\to q\bar{q})} = \frac{\Gamma^{\rm QCD}(W\to ud)}{\Gamma^{\rm LO}(W\to ud)} = \frac{3}{4}\frac{C_F\alpha_s(\mu_R)}{\pi} \,, \tag{3.1}$$

[4]We use the electronic implementation in `DsixTools` [77, 78] as the $\dot{C}_i$ typically depend on a large number of Wilson coefficients.

[5]The leading corrections to this limit scale as m_q^2/M_Z^2, with m_q the quark mass, and are thus expected to be at the per mille level even for b-quarks.

where $C_F = 4/3$ in QCD. This universality implies that NLO QCD corrections cancel out of EWPO based on ratios of quark decays; the set of such EWPO considered in this work is $R_b, R_c, A_b, A_c, A_{\rm FB}^b, A_{\rm FB}^c$.

The NLO electroweak corrections are rather lengthy and are not reproduced here. Instead, the full NLO results (including the QCD corrections just discussed) are provided along with the LO ones in ancillary computer files with the arXiv submission of this work, and can be evaluated numerically using the included Mathematica notebook. The notation used for these ancillary files is as follows. We present each observable at LO and NLO in the form

$$\begin{aligned} O_s^{\rm LO} &= O_s^{(4,0)} + v_s^2 \sum_i \texttt{Class[i]}\, O_{s,i}^{(6,0)} \,, \\ O_s^{\rm NLO} &= O_s^{(4,0)} + \frac{1}{v_s^2} O_s^{(4,1)} + \sum_i \texttt{Class[i]} \left(v_s^2 O_{s,i}^{(6,0)} + O_{s,i}^{(6,1)} \right) \,, \end{aligned} \tag{3.2}$$

where the summation index i runs over the eight operator classes of the Warsaw basis. The tags `Class[i]` (where here and below the font used in "`Object`" refers to a variable or file name in the electronic results) allow to separate contributions from specific operator classes or the SM. Predictions for the individual observables are given as separate files with the filename

`<observable>_SMEFT_<order>_<scheme>.m`,

where `order` is either `LO` or `NLO` and the `scheme` is referred to by a list of their input parameters, see table 2. Note again, that in contrast to [55, 61], we present our results for a covariant derivative defined with a plus sign to agree with the `SMEFTsim` conventions [68]. We list the variables used in our results in table 3. Wilson coefficient names are written in the conventions of `DsixTools` with the exception of C_W which we write as `CWw` to distinguish it from the cosine of the Weinberg angle written as `CW`. PV integrals are given in the notation of `LoopTools` [70], and only the finite part is to be used. These PV integrals can also be substituted by standard functions by applying the replacements `IntRep` in our ancillary notebook. An example is given in the notebook.

Realistic models of new physics typically do not match onto a set of Wilson coefficients with a completely arbitrary flavour structure. Instead, potential symmetries of the new physics model will manifest themselves in the structure and correlations of the SMEFT Wilson coefficients. The possibility that the SM Yukawa couplings are the only sources of the breaking of the flavour symmetry is known under the name of minimal flavour violation (MFV) [79–82]. Two lists of replacements in the notebook accompanying this publication allow to write the results in terms of the independent Wilson coefficients under the $U(3)^5$ and MFV assumptions. Details on the notation of the Wilson coefficients under these assumptions are given in appendix B.

In any phenomenological study, it is important to have a means of estimating uncertainties from uncalculated higher-order corrections in the SMEFT expansion. Within a given electroweak input scheme, a typical way to estimate higher-order perturbative corrections is to study the stability of results under variations of the SMEFT renormalisation scale μ around a particular default value, which for EWPO is typically chosen as $\mu^{\rm def} = M_Z$. Taking the Wilson

variable	definition	description
MX	M_X	mass of particle X
MX2	M_X^2	square of the mass of particle X
SW	s_w	sine of the Weinberg angle
CW	c_w	cosine of the Weinberg angle
SW2	s_w^2	sine of the Weinberg angle squared
CW2	c_w^2	cosine of the Weinberg angle squared
CWw	C_W	Wilson coefficient C_W (to distinguish it from c_w)
VMU	v_μ	vev in schemes using G_F as an input parameter
VALPHA	v_α	vev in the $\{\alpha, M_W, M_Z\}$ scheme
VALPHAEFF	$v_\alpha^{\rm eff}$	vev in the $\{\alpha, \sin\theta_{\rm eff}^{\ell}, M_Z\}$ scheme
Class[k]	tag	tag for SMEFT contributions from class k
Leff		Flavour index of the reference lepton
A0i, B0i, C0i		PV integrals in the notation of LoopTools

Table 3. Variables used in the ancillary files. $X = Z, W, H, T$ for the Z, W and Higgs boson and the top quark, respectively.

coefficients $C_i(M_Z)$ as unknown parameters, as is the case in a global fit, in order to perform μ-variations one uses renormalisation-group (RG) running to express coefficients at arbitrary μ in terms of those at the default choice M_Z. In the Mathematica notebook, we include the option to vary the scale μ using the fixed-order solution of the RG equations, namely

$$C_i(\mu) = C_i(M_Z) + \dot{C}_i(M_Z) \ln \frac{\mu}{M_Z} \, . \tag{3.3}$$

This fixed-order running is sufficient if $\mu \sim M_Z$, which is the case when the scale is varied up and down by the customary factors of two. More sophisticated implementations using the exact solutions to RG equations are also possible but are not considered here.

The simple form of eq. (3.3) and the fact that results depend only linearly on the Wilson coefficients allows to write explicit analytic expressions which make clear how variations of the SMEFT renormalisation scale μ estimate higher-order effects.[6] At LO, one has

$$O_s^{(6,0)}(C_i(\mu)) - O_s^{(6,0)}(C_i(M_Z)) = \ln \frac{\mu}{M_Z} O_s^{(6,0)}(\dot{C}_i(M_Z)) \, , \tag{3.4}$$

where the term on the right-hand side is of one-loop order and taken as an indication of beyond-LO corrections. Similarly, at NLO one can use eqs. (2.13), (2.15) and eq. (3.3) to arrive at

$$\begin{aligned} O_s^{\rm NLO}(C_i(\mu), \mu) - O_s^{\rm NLO}(C_i(M_Z), M_Z) = \ln \frac{\mu}{M_Z} O_s^{(6,1)}(\dot{C}_i(M_Z), M_Z) \\ - \ln^2 \left(\frac{\mu}{M_Z} \right) O_s^{(6,0)}(\gamma_{ij}^{(4,1)} \dot{C}_j(M_Z)) \, , \end{aligned} \tag{3.5}$$

where in the last term on the right-hand side the vev dependence of the $(6,0)$ contribution has cancelled against the one we have pulled out of the definition of the anomalous dimension

[6] In the present discussion, we do not vary the QCD renormalisation scale appearing through $\alpha_s(\mu_R)$ in the NLO QCD corrections eq. (3.1); in the Mathematica notebook correlated variations $\mu_R = \mu$ are implemented, although independent variations of μ_R and μ are also possible.

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matrix, see eq. (2.13). In this case, the two terms on the right-hand side are both NNLO in the couplings and thus give an indication of the size of higher-order corrections to the dimension-6 results. On the other hand, these are both products of one-loop quantities and thus miss genuine two-loop effects in the SMEFT anomalous dimension, which are currently unknown.

The size of the NLO corrections, and whether they lie inside the uncertainty bands from scale variation, depends heavily on the structure of the observable, the Wilson coefficient under consideration, and the EW input scheme. Systematic studies for partial decay widths of W, Z and Higgs bosons into fermions have been presented in [55, 61], where the dominant corrections in SMEFT, which originate from virtual top-quark loops and are largely universal, were identified. The generic size of such corrections is set by

$$\frac{\Delta\rho_t^{(4,1)}}{v_\mu^2} = \frac{3}{16\pi^2}\frac{m_t^2}{v_\mu^2} \approx 1\%\,, \tag{3.6}$$

and the LO (NLO) scale uncertainties estimated through running of Wilson coefficients are proportional to (the square of) this quantity. For example,

$$v_\mu^2 \dot{C}_{HD} = 8\Delta\rho_t^{(4,1)}\left[C_{HD} + 2C_{HQ}^{(1)} - 2C_{\underset{33}{Hu}}\right] + \dots\,, \tag{3.7}$$

where the ... refer m_t-independent contributions. However, EWPO observables sensitive to Z couplings to fermions at LO display a process and Wilson-coefficient dependent polynomial dependence on the Weinberg angle, which receives enhanced and scale-independent corrections; for instance, in the α_μ or α schemes these are proportional to

$$-\frac{\Delta r_t^{(4,1)}}{v_\mu^2} = \frac{c_w^2}{s_w^2}\frac{\Delta\rho_t^{(4,1)}}{v_\mu^2} \approx 3.5\%\,. \tag{3.8}$$

For this reason, scale variations only give a rough proxy for the size of dominant higher-order corrections to EWPO on the Z-pole in many cases.

As an example not analysed in [55, 61], consider the b-quark left-right asymmetry in the α_μ scheme, whose dependence on the Weinberg angle at LO can be derived using the expressions given in appendix D. Evaluating numerically, and including uncertainties from scale variation as described above, one has at LO and NLO

$$\begin{aligned}
A_b^{\rm LO} &= 0.94 + v_\mu^2\Big\{0.77^{+2.5\%}_{-2.5\%}C_{Hd} + 0.25^{+3.4\%}_{-3.4\%}C_{HWB} \\
&\qquad + 0.24^{+6.8\%}_{-6.8\%}C_{HD} + 0.14^{+1.1\%}_{-1.1\%}C_{HQ}^{(3)} \\
&\qquad + 0.14^{+21\%}_{-21\%}C_{HQ}^{(1)} + 0.00^{+0.02}_{-0.02}C_{Ht} + \dots\Big\}\,, \\
A_b^{\rm NLO} &= 0.94(1-0.007) + v_\mu^2\Big\{\,(0.77\times 1.09)^{+0.1\%}_{-0.3\%}\,C_{Hd} + (0.25\times 1.14)^{+0.3\%}_{-0.5\%}\,C_{HWB} \\
&\qquad + (0.24\times 1.13)^{+0.3\%}_{-1.3\%}\,C_{HD} + (0.14\times 1.03)^{+1.8\%}_{-0.1\%}\,C_{HQ}^{(3)} \\
&\qquad + (0.14\times 1.28)^{+0.9\%}_{-4.7\%}\,C_{HQ}^{(1)} - 0.02^{+0.007}_{-0.001}C_{Ht} + \dots\Big\}\,,
\end{aligned} \tag{3.9}$$

where the ... refer to Wilson coefficients contributing with smaller central values than those displayed, and for simplicity we quoted results using the MFV flavour assumption in the notation of appendix B. Whereas the NLO correction in the SM is at the per mille level and does not depend on the scale, the LO scale uncertainties and NLO corrections in SMEFT range from a few to roughly 30%, while the NLO scale uncertainties are typically at the percent level. Furthermore, as anticipated above, while the LO scale uncertainties give a rough estimate of the size of the NLO corrections, they generally underestimate them.

Given the somewhat arbitrary nature of uncertainty estimates based on scale variations, and also the fact that apart from α_s the SM parameters appearing in EWPO are renormalised on-shell and thus contain no scale to vary, it is important to have additional means of estimating uncertainties. In both the SM and SMEFT, a simple way to do this is to calculate observables in several different electroweak input schemes, which are equivalent at a given order in the SMEFT expansion, but organise higher-order corrections in both operator-dimension and loops differently. It is precisely for this reason that we have given results in five different input schemes, and advocate their use in global SMEFT fits. One would expect LO fits of Wilson coefficients in the two schemes to differ roughly by the size of the NLO corrections discussed above, in other words at the 5-10% level depending on the coefficient, while NLO fits should be subject to smaller differences — this is indeed the general pattern observed within the methodology of [60].

Our analytic results also enable the extraction of the parametric uncertainties from the input parameters as well as their covariance, which will be useful for their inclusion in global analyses. The average parameteric uncertainty is 0.04% in the $\{G_F, \alpha(M_Z), M_Z\}$ scheme, 0.1% in schemes including $\sin\theta^{\ell}_{\rm eff}$ and 0.3% schemes including M_W. In the latter, predictions for individual observables (σ_Z, Γ_Z, R_ℓ) and Wilson coefficients (C_{HD}, C_{HWB}, $C_{\underset{1221}{ll}}$) exceed 1%. Parametric uncertainties can thus be comparable in size with uncertainties from scale variation at NLO, although the latter will typically still dominate.

4 Conclusions

We have presented analytic results for EWPO to NLO in dimension-six SMEFT in the five different EW input schemes listed in table 2. These results will be useful for SMEFT analyses of data from current and future lepton colliders, which play an important role in global fits. A Mathematica notebook for the numerical evaluation of the results is provided with the arXiv submission of the paper, in which numerical inputs can be adjusted as needed to facilitate the combination of EWPO analyses with other observables. While our results have made no assumptions on the flavour structure of SMEFT Wilson coefficients, we have included in the notebook options to implement $\mathrm{U}(3)^5$ and MFV assumptions. Furthermore, theory uncertainties and their covariances for specific observables or in global fits can be estimated through scale variations, calculating in different EW input schemes, or preferably both.

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A EWPO on the Z pole

In this appendix, we define the EWPO on the Z pole in terms of the Z-boson partial decay rates. The ratios R_x are defined as

$$\begin{aligned} R_\ell &= \frac{\sum_q \Gamma(Z \to q\bar{q})}{\Gamma(Z \to \ell\ell)}\,, \\ R_{q_i} &= \frac{\Gamma(Z \to q_i\bar{q}_i)}{\sum_q \Gamma(Z \to q\bar{q})}\,, \end{aligned} \tag{A.1}$$

with $\ell = e, \mu, \tau$ and $q_i = s, c, b$ and the sum runs over $q = u, d, c, s, b$. The left-right asymmetries A_x are defined as

$$A_x = \frac{\Gamma(Z \to x_L\bar{x}_L) - \Gamma(Z \to x_R\bar{x}_R)}{\Gamma(Z \to x\bar{x})}\,. \tag{A.2}$$

The asymmetry A_ℓ is directly related to the effective weak mixing angle $\sin\theta^\ell_{\rm eff}$ (using the same reference lepton ℓ) defined as

$$\sin\theta^\ell_{\rm eff} = \frac{1}{2}\left(\frac{\Gamma(Z \to \ell_L\bar{\ell}_L)}{\Gamma(Z \to \ell_L\bar{\ell}_L) - \Gamma(Z \to \ell_R\bar{\ell}_R)}\right) = \frac{1}{4A_\ell}\left(-1 + A_\ell + \sqrt{1 - A_\ell^2}\right). \tag{A.3}$$

Therefore, only one of the quantities A_ℓ and $\sin\theta^\ell_{\rm eff}$ is an independent quantity. The forward-backward asymmetries $A^x_{\rm FB}$ are defined as

$$A^x_{\rm FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4}A_x A_\ell\,, \tag{A.4}$$

where σ_F and σ_B are defined by the angle θ between the incoming lepton ℓ^- and the outgoing anti-fermion $\bar{f}$ being within $\theta \in [0,\, \pi/2]$ and $\theta \in [\pi/2,\, \pi]$, respectively. The hadronic total cross section for the process $e^+e^- \to$ hadrons can be parametrised in the narrow width approximation as

$$\sigma_{\rm had} = \sum_{q=u,d,c,s,b} \frac{12\pi}{M_Z}\frac{\Gamma_e\Gamma_q}{\Gamma_Z^2}\,. \tag{A.5}$$

B Flavour assumptions

To facilitate the interpretation of our results under the assumption of common flavour assumptions, the replacement lists `flavU35` and `flavMFV` in our ancillary notebook allow to express the results under the $\mathrm{U}(3)^5$ and MFV assumption. A nice summary of the relevant simplifications, based on refs. [83, 84], is given in sections II B and II C of [47]. Instead of reproducing the necessary equations here, we only clarify the notation in which we present the Wilson coefficients.

B.1 $\mathrm{U}(3)^5$ symmetry

A rather strict requirement on the Wilson coefficients of the SMEFT is the assumption of a $\mathrm{U}(3)$ symmetry for all fermion fields

$$\mathrm{U}(3)^5 = \mathrm{U}(3)_\ell \times \mathrm{U}(3)_q \times \mathrm{U}(3)_e \times \mathrm{U}(3)_u \times \mathrm{U}(3)_d, \tag{B.1}$$

where $\{\ell, q, e, u, d\}$ represent the SM fermions. Under this assumption, there are 41+6 independent CP-even+CP-odd SMEFT operators [83]. Operators with two flavour indices have a single independent Wilson coefficient and we hence drop the flavour indices for these coefficients. Note that dipole operators like Q_{uW} and Q_{uB}, which generally contribute to EWPO are completely forbidden under a $\mathrm{U}(3)^5$ symmetry. Four-fermion operators with two different fermion bilinears again have a single independent Wilson coefficient, for which we thus also drop the flavour indices.

Four-fermion operator structures with two fermion currents of the same chirality, explicitly $Q_{qq}^{(1)}$, $Q_{qq}^{(3)}$, Q_{ll}, Q_{dd} and Q_{uu}, generally have two independent $\mathrm{U}(3)^5$ singlets. We refer to these two independent structures with unprimed and primed Wilson coefficients, corresponding to the operators contracting the flavour indices within the fermion bilinears or between the two different fermion bilinears, respectively. As an example, the operator $\underset{ijkl}{Q_{ll}} = (\bar{l}_i \gamma_\mu l_j)(\bar{l}_k \gamma^\mu l_l)$ has two flavour-symmetric contractions

$$C_{ll}\, \delta_{ij}\delta_{lk} \qquad \text{and} \qquad C'_{ll}\, \delta_{ik}\delta_{jl}\,. \tag{B.2}$$

In our numerical results, we refer to the primed coefficients by adding the letter `p` to the Wilson coefficient names, for instance we replace the different flavour combinations of C_{ll} as

$$\underset{1122}{C_{ll}} \to \texttt{Cll}\,, \qquad\qquad \underset{1221}{C_{ll}} \to \texttt{Cllp}\,. \tag{B.3}$$

For the operator Q_{ee}, a Fierz identity implies a single independent coefficient C_{ee}.

B.2 Minimal flavour violation

A more general flavour symmetry for the Wilson coefficients is given by MFV, which assumes the SM Yukawa couplings are the only sources of the breaking of the $\mathrm{U}(3)^5$ flavour symmetry. MFV is thus an expansion in powers of the Yukawa couplings. As we are assuming that all fermions except the top are massless, right-handed down-type quarks still retain a U(3) symmetry under our implementation of MFV. Wilson coefficients involving left-handed third-generation quark couplings or right-handed up-type quark couplings are independent of those of the first/second generation. Instead of keeping the flavour indices, we use the subscripts (`Q`, `t`) for the third generation and (`q`, `u`) for the first/second generation in the Wilson coefficients. For Wilson coefficients with two indices, we write, for instance

$$\underset{22}{C_{Hq}^{(3)}} \to \texttt{CHq3}\,, \qquad\qquad \underset{33}{C_{Hq}^{(3)}} \to \texttt{CHQ3}\,. \tag{B.4}$$

The four-fermion Wilson coefficients with two equal fermion bilinears simplify as

$$\begin{aligned} &\underset{1122}{C_{uu}} \to \texttt{Cuu}\,, \qquad \underset{1133}{C_{uu}} \to \texttt{Cut}\,, \qquad \underset{1221}{C_{uu}} \to \texttt{Cuup} \\ &\underset{1331}{C_{uu}} \to \texttt{Cutp}\,, \qquad \underset{3333}{C_{uu}} \to 2\,(\texttt{Cut} + \texttt{Cutp}) - \texttt{Cuu} - \texttt{Cuup}\,. \end{aligned} \tag{B.5}$$

There are four independent coefficients for operators with four up-type quarks, and two coefficients for those with two up-type quarks. Other four-fermion operators simplify in the same way as under a $\mathrm{U}(3)^5$ symmetry.

C Comparison with previous work

Numerical results for EWPO in an input-scheme involving $\{G_F, \alpha, M_Z\}$ have been previously published in [33, 47, 57]. These are closely related to our results in the LEP scheme, but differ in the following ways:

a) Definition of the electromagnetic coupling constant α: While we use the on-shell value at the Z-boson mass, $\alpha(M_Z)$, as an input, [33, 47, 57] employs $\alpha(0)$. The two choices are related by eq. (2.3) and the light fermion contributions to α are included in the NLO expansion coefficients in [33, 47, 57], rather than being absorbed into the definition of $\alpha(M_Z)$ as in our LEP scheme.

b) Partial higher-order corrections are included in [33, 47, 57], see eq. (58) of [47], through the numerical evaluation of the W-boson mass at its best theory prediction in LO contributions and at its NLO value in NLO contributions to observables.

c) Due to a different sign convention in the covariant derivative, our predictions have opposite signs for operators including an odd number of field strengths, specifically C_W, C_{uB} and C_{uW}.

Comparing numerically, we exactly agree on all LO predictions after switching to the numerical values used in [33, 47, 57] (including replacing $\alpha(M_Z)$ by $\alpha(0)$). At NLO, we find good numerical agreement for the decay rates Γ_W and Γ_Z despite the different choices for α and differences in the expansions. For observables based on ratios or differences of two similar-size contributions, individual Wilson coefficients (appearing first at NLO) experience larger differences.

D Analytic results for EWPO at LO in SMEFT

The EWPO in eqs. (2.7), (2.8) can be derived from W and Z-boson decays into left- and right-handed fermions. In what follows, we give compact results for such partial decay widths in the five EW input schemes used in this work at LO in SMEFT. These expressions involve products of functions which receive both SM and dimension-6 contributions; it is understood that to obtain the decay rates and consequently EWPO one must expand out the products and retain only up to linear corrections in the SMEFT Wilson coefficients. Results for the additional observables eq. (2.9) are given at the end of the section.

For Z decay, we can write

$$\Gamma(Z \to f\bar{f}) = M_Z \frac{N_c^f}{24\pi} |\mathcal{N}_Z|^2 \left(|\mathcal{Z}_L^f|^2 + |\mathcal{Z}_R^f|^2 \right) , \tag{D.1}$$

where N_c^f was defined after eq. (2.2). The normalisation factor is

$$\mathcal{N}_Z = \frac{M_Z}{v_s} \left[1 - v_s^2 \left(\frac{1}{4} C_{HD} + \frac{1}{2} \Delta v_s^{(6,0)} \right) \right] , \tag{D.2}$$

where the Δv_s depend on the scheme s and are listed in eq. (D.12) below. We define SMEFT expansion coefficients for decay into left-handed fermions as

$$\mathcal{Z}_L^f = \mathcal{Z}_L^{f(4,0)} + v_s^2 \mathcal{Z}_L^{f(6,0)} \tag{D.3}$$

and similarly for right-handed decays. In the SM, one has

$$\mathcal{Z}_L^{f(4,0)} = 2T_3^f - 2s_w^2 Q_f\,, \qquad \mathcal{Z}_R^{f(4,0)} = -2s_w^2 Q_f\,, \tag{D.4}$$

where Q_f is the charge and T_3^f is the third component of the weak isospin of fermion f.[7] The result in SMEFT can be written as

$$\mathcal{Z}_{L/R}^{f(6,0)} = Q_f\left(-G^{(6,0)} + 4c_w^2\Delta_W^s\right) + g_{L/R}^{f(6,0)}\,. \tag{D.5}$$

The above results contain, first off, the fermion-species independent function

$$G^{(6,0)} = -c_w^2 C_{HD} - 2c_w s_w C_{HWB}\,, \tag{D.6}$$

and secondly fermion-specific functions, which for charged leptons ℓ_i and neutrinos ν_i are

$$\begin{aligned} g_L^{\ell(6,0)} &= -C^{(1)}_{\underset{ii}{Hl}} - C^{(3)}_{\underset{ii}{Hl}}\,, & g_R^{\ell(6,0)} &= -C_{\underset{ii}{He}}\,, \\ g_L^{\nu(6,0)} &= -C^{(1)}_{\underset{ii}{Hl}} + C^{(3)}_{\underset{ii}{Hl}}\,, & g_R^{\nu(6,0)} &= 0\,, \end{aligned} \tag{D.7}$$

while for up and down-type quarks

$$\begin{aligned} g_L^{d(6,0)} &= -C^{(1)}_{\underset{ii}{Hq}} - C^{(3)}_{\underset{ii}{Hq}}\,, & g_R^{d(6,0)} &= -C_{\underset{ii}{Hd}}\,, \\ g_L^{u(6,0)} &= -C^{(1)}_{\underset{ii}{Hq}} + C^{(3)}_{\underset{ii}{Hq}}\,, & g_R^{u(6,0)} &= -C_{\underset{ii}{Hu}}\,. \end{aligned} \tag{D.8}$$

Finally, in schemes s where M_W is not an input, the quantity Δ_W^s appears in the following LO SMEFT relation between the on-shell and derived masses:

$$M_W^{\rm LO} = M_Z c_w\left(1 + v_s^2\Delta_W^{s(6,0)}\right)\,; \qquad s \in \{v_\alpha^{\rm eff}, v_\mu^{\rm eff}, {\rm LEP}\}\,. \tag{D.9}$$

The explicit results are

$$\Delta_W^{{\rm LEP}(6,0)} = -\frac{s_w^2}{2c_{2w}}\left[\Delta v_\mu^{(6,0)} - \Delta v_\alpha^{(6,0)}\right]\,, \tag{D.10}$$

$$\Delta_W^{v_\sigma^{\rm eff}(6,0)} = \frac{1}{4c_w^2}\left[G^{(6,0)} + 2s_w^2 g_L^{\ell(6,0)} + c_{2w}g_R^{\ell(6,0)}\right]\,, \tag{D.11}$$

where $\sigma \in \{\alpha, \mu\}$ and $c_{2w} \equiv 1 - 2s_w^2$. Here and in eq. (D.2), the tree-level vev shifts are

$$\begin{aligned} \Delta v_\mu^{(6,0)} &= C^{(3)}_{\underset{11}{Hl}} + C^{(3)}_{\underset{22}{Hl}} - C_{\underset{1221}{ll}}\,, \\ \Delta v_\alpha^{(6,0)} &= -2\frac{c_w}{s_w}\left[C_{HWB} + \frac{c_w}{4s_w}C_{HD}\right]\,, \\ \Delta v_\alpha^{\rm eff} &= -\frac{1}{2}C_{HD} - \frac{1}{c_w s_w}C_{HWB} - \frac{c_{2w}}{c_w^2}\left(g_L^{\ell(6,0)} + \frac{c_{2w}}{2s_w^2}g_R^{\ell(6,0)}\right)\,, \end{aligned} \tag{D.12}$$

where one is to use Δv_μ for $s \in \{\alpha_\mu, v_\mu^{\rm eff}, {\rm LEP}\}$, while for $s = \alpha$ or $s = v_\alpha^{\rm eff}$ one uses Δv_α or $\Delta v_\alpha^{\rm eff}$, respectively.

[7] Our convention is such that $Q_u = 2/3, Q_e = -1$, while $T_3^u = 1/2$, and so on. Here and below the definitions of M_W and s_w depend on the scheme s and should be understood as written in eq. (2.5).

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The results for $W \to ff'$ are considerably more compact. In this case we can write

$$\Gamma(W \to ff') = M_W \left(1 + v_s^2 \Delta_W^{s(6,0)}\right) \frac{N_c^f}{24\pi} |\mathcal{N}_W|^2 \left|1 + v_s^2\, \mathcal{W}^{ff'(6,0)}\right|^2 , \tag{D.13}$$

where the normalisation factor is

$$\mathcal{N}_W = \frac{\sqrt{2} M_W}{v_s} \left[1 + v_s^2 \left(\Delta_W^{s(6,0)} - \frac{1}{2} \Delta v_s^{(6,0)}\right)\right] , \tag{D.14}$$

and the fermion-specific functions for leptonic and hadronic decays are given by

$$\mathcal{W}^{\ell\nu(6,0)} = C_{\substack{Hl \\ ii}}^{(3)} , \qquad \mathcal{W}^{ud(6,0)} = C_{\substack{Hq \\ ii}}^{(3)} . \tag{D.15}$$

In schemes where they are not inputs, the predictions for M_W, G_F and $\alpha(M_Z)$ provide additional EWPO. LO results for the on-shell W boson mass in such schemes have already been given in eq. (D.9). For G_F, one has

$$G_F^{\rm LO} = \frac{1}{\sqrt{2} v_s^2} \left[1 - v_s^2 \left(\Delta v_s^{(6,0)} - \Delta v_\mu^{(6,0)}\right)\right] ; \qquad s \in \{\alpha, v_\alpha^{\rm eff}\} , \tag{D.16}$$

while for $\alpha(M_Z)$, one has instead

$$\alpha(M_Z)^{\rm LO} = \frac{M_Z^2 c_w^2 s_w^2}{\pi v_\mu^2} \left[1 + v_\mu^2 \left(\Delta v_\mu^{(6,0)} - \Delta v_{\bar{s}}^{(6,0)}\right)\right] ; \quad \bar{s} \in \{\alpha \text{ for } s = \alpha_\mu, v_\alpha^{\rm eff} \text{ for } s = v_\mu^{\rm eff}\} , \tag{D.17}$$

where $\bar{s}$ refers to the scheme which shares two inputs with s, but differs by using $\alpha(M_Z)$ instead of G_F.

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1 : X^3	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$
$Q_{\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$

2 : H^6	
Q_H	$(H^\dagger H)^3$

3 : H^4D^2	
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)$

5 : ψ^2H^3 + h.c.	
Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$

4 : X^2H^2	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu}B^{\mu\nu}$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu}B^{\mu\nu}$
Q_{HWB}	$H^\dagger\sigma^I H\, W_{\mu\nu}^I B^{\mu\nu}$
$Q_{H\widetilde{W}B}$	$H^\dagger\sigma^I H\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$

6 : ψ^2XH + h.c.	
Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\sigma^I H W_{\mu\nu}^I$
Q_{eB}	$(\bar{l}_p\sigma^{\mu\nu}e_r)H B_{\mu\nu}$
Q_{uG}	$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{H}\, G_{\mu\nu}^A$
Q_{uW}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\sigma^I \widetilde{H}\, W_{\mu\nu}^I$
Q_{uB}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{H}\, B_{\mu\nu}$
Q_{dG}	$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)H\, G_{\mu\nu}^A$
Q_{dW}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\sigma^I H\, W_{\mu\nu}^I$
Q_{dB}	$(\bar{q}_p\sigma^{\mu\nu}d_r)H\, B_{\mu\nu}$

7 : ψ^2H^2D	
$Q_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}_p\gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{l}_p\sigma^I\gamma^\mu l_r)$
Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p\gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}_p\gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{q}_p\sigma^I\gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_p\gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}_p\gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p\gamma^\mu d_r)$

8 : $(\bar{L}L)(\bar{L}L)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\sigma^I q_r)(\bar{q}_s\gamma^\mu\sigma^I q_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\sigma^I l_r)(\bar{q}_s\gamma^\mu\sigma^I q_t)$

8 : $(\bar{R}R)(\bar{R}R)$	
Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$
Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$
Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$
Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$
Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$

8 : $(\bar{L}L)(\bar{R}R)$	
Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

8 : $(\bar{L}R)(\bar{R}L)$ + h.c.	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$

8 : $(\bar{L}R)(\bar{L}R)$ + h.c.	
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$

Table 4. The 59 independent baryon number conserving dimension-six operators built from Standard Model fields, in the notation of [74]. The subscripts p, r, s, t are flavour indices, and σ^I are Pauli matrices.

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