



$b \rightarrow c$ semileptonic sum rule: extension to angular observables

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Abstract Lepton flavor universality is a key prediction of the Standard Model of particle physics, and any violation of it immediately indicates the existence of new physics. Given the recent discrepancy of more than 4σ in charged current semileptonic B meson decays and the absence of evident signals at the Large Hadron Collider, independent cross-checks become invaluable. In this context, $b \rightarrow c$ semileptonic sum rules based on heavy quark symmetry are interesting, since they allow us to check the consistency of experimental results. In this paper, we report newly found sum rules among angular observables of mesonic and baryonic $b \rightarrow c\ell\bar{\nu}$ decays holding exactly in the large mass limit of heavy quarks. Moreover, we investigate corrections to the sum rule in realistic situations and discuss phenomenological implications.

1 Introduction

The $b \rightarrow c$ semileptonic sum rule for $R_{H_c} = \text{BR}(H_b \rightarrow H_c \tau \bar{\nu}) / \text{BR}(H_b \rightarrow H_c \ell \bar{\nu})$ with $\ell = e, \mu$ is shown as [1–4]

$$\frac{R_{\Lambda_c}}{R_{\Lambda_c}^{\text{SM}}} - \alpha_R \frac{R_D}{R_D^{\text{SM}}} - \beta_R \frac{R_{D^*}}{R_{D^*}^{\text{SM}}} = \delta_R, \quad (1)$$

where the coefficients satisfy $\alpha_R + \beta_R = 1$ and are independent of new physics (NP) contributions. δ_R is found to be small compared to current experimental uncertainties, enabling us to apply it as a robust consistency check of experimental results. Note that B (D) and B^* (D^*) form

the lowest-lying beauty (charming) meson heavy quark doublet, and Λ_b (Λ_c) corresponds to the lowest-lying beauty (charming) baryon. Hence, the sum rule is satisfied among ground-state to ground-state transitions. Recently, based on the heavy quark effective theory (HQET) [5, 6], another relation has been derived for mesonic and baryonic differential decay rates as [7]

$$\frac{\kappa_{\Lambda_c}^w}{\zeta(w)^2} - \frac{2(\kappa_D^w + \kappa_{D^*}^w)}{(1+w)\xi(w)^2} \approx 0, \quad (2)$$

where $\kappa_{H_c}^w = d\Gamma^{H_c}/dw$ with $\Gamma^{H_c} = \Gamma(H_b \rightarrow H_c \tau \bar{\nu})$ are defined for $w = (m_{H_b}^2 + m_{H_c}^2 - q^2)/(2m_{H_b}m_{H_c})$, and q^2 is the invariant mass of the leptons. Equation (2) holds exactly in the large mass limit of heavy quarks, that is, $m_{b,c} \gg \Lambda_{\text{QCD}}$. Also, Eq. (2) can be rewritten as

$$\frac{\kappa_{\Lambda_c}^w}{\kappa_{\Lambda_c}^{w,\text{SM}}} - \alpha \frac{\kappa_D^w}{\kappa_D^{w,\text{SM}}} - \beta \frac{\kappa_{D^*}^w}{\kappa_{D^*}^{w,\text{SM}}} \approx 0, \quad (3)$$

again satisfying $\alpha + \beta = 1$. Then, Eq. (1) is obtained by integrating numerators and denominators over w and by normalizing with the decay widths of light-lepton modes. In other words, the HQET is explicitly shown to be a pillar of the robust $b \rightarrow c$ semileptonic sum rule for R_{H_c} .

Given the discrepancy of more than 4σ between the Standard Model (SM) prediction and the experimental value of R_D and R_{D^*} [8], a tremendous number of NP interpretations have been explored, and the relations which enable us to cross-check the consistency of the experimental results become more important. In this paper, we provide sum rules involving angular observables, e.g., the forward-backward asymmetry of the charged lepton,

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$$A_{\text{FB}}^{H_c l} = \left(\Gamma_{c_{\theta_l} > 0}^{H_c} - \Gamma_{c_{\theta_l} < 0}^{H_c} \right) / \left(\Gamma_{c_{\theta_l} > 0}^{H_c} + \Gamma_{c_{\theta_l} < 0}^{H_c} \right), \quad (4)$$

where $\Gamma_{c_{\theta_l} \gtrless 0}^{H_c}$ denote the decay rates satisfying $\cos \theta_l \gtrless 0$, with c_{θ_l} being short for $\cos \theta_l$ and θ_l being the angle between the charmed-hadron and the charged lepton in the bottomed-hadron rest frame, as well as the charged-lepton polarization observable,

$$P_l^{H_c} = \left(\Gamma_{\lambda_l = 1/2}^{H_c} - \Gamma_{\lambda_l = -1/2}^{H_c} \right) / \left(\Gamma_{\lambda_l = 1/2}^{H_c} + \Gamma_{\lambda_l = -1/2}^{H_c} \right), \quad (5)$$

where $\Gamma_{\lambda_l}^{H_c}$ is the rate with λ_l denoting a charged-lepton polarization. In the case of $l = \tau$, only the experimental measurement of $P_{\tau}^{D^*}$ is currently available, and its uncertainty is still as large as $\mathcal{O}(100)\%$ [9]. In the future, the Belle II experiment could determine P_{τ}^D at the 3% level and $P_{\tau}^{D^*}$ at about 15% [10–12]. Baryon observables measured at ongoing and future colliders [13–16] can be compared with the predictions via sum rules with upcoming Belle II input [11].

The outline of this letter is as follows: In Sect. 2, we introduce our framework and present the new sum rule involving double-differential decay rates. In Sect. 3, we investigate corrections to the sum rule and discuss phenomenological implications. Section 4 is devoted to the conclusion.

2 $b \rightarrow c$ semileptonic sum rule of angular observables

We assume that the NP contributes to the $b \rightarrow c l \bar{\nu}_l$ transitions in the following form:

$$\mathcal{L}_{\text{eff}} = -\frac{4G_F V_{cb}}{\sqrt{2}} \left[(1 + C_{V_L}^l) O_{V_L}^l + C_{S_L}^l O_{S_L}^l + C_{S_R}^l O_{S_R}^l + C_T^l O_T^l \right]. \quad (6)$$

The effective operators are defined at $\mu = \mu_b$ as

$$\begin{aligned} O_{V_L}^l &= (\bar{c} \gamma^\mu P_L b) (\bar{l} \gamma_\mu P_L \nu_l), & O_{S_L}^l &= (\bar{c} P_L b) (\bar{l} P_L \nu_l), \\ O_{S_R}^l &= (\bar{c} P_R b) (\bar{l} P_L \nu_l), & O_T^l &= (\bar{c} \sigma^{\mu\nu} P_L b) (\bar{l} \sigma_{\mu\nu} P_L \nu_l), \end{aligned} \quad (7)$$

where $l = e, \mu, \tau$, $P_{L(R)} = (1 \mp \gamma_5)/2$, and $\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu]$ with $\sigma^{\mu\nu} \gamma_5 = -(i/2)\epsilon^{\mu\nu\rho\sigma} \sigma_{\rho\sigma}$.¹ NP contributions are encoded into the Wilson coefficients (WCs)

¹ We assumed that the charged mediator of $b \rightarrow c l \bar{\nu}$ transition is much heavier than the typical energy scale of bottomed-hadron decays. Lepton flavor is assumed to be conserved. Also, all light neutrinos are assumed to be left-handed. For studies including right-handed neutrinos see Refs. [17–23].

C_X^l with $X = V_L, S_{L,R}$, and T . The SM case corresponds to $C_X^l = 0$. In the following, motivated by the discrepancy in $R_{D^{(*)}}$, we will focus on the tauonic NP scenario, that is, $l = \tau$ [3, 24, 25].

Let us provide a new sum rule holding among the double-differential decay rates:

$$\frac{\kappa_{\Lambda_c}^{wc_{\theta_l}, \lambda_l}}{\zeta(w)^2} - \frac{2 \left(\kappa_D^{wc_{\theta_l}, \lambda_l} + \kappa_{D^*}^{wc_{\theta_l}, \lambda_l} \right)}{(1+w)\xi(w)^2} = 0, \quad (8)$$

where $\kappa_{H_c}^{wc_{\theta_l}, \lambda_l} = d^2 \Gamma_{\lambda_l}^{H_c} / dw dc_{\theta_l}$,² and the leading-order Isgur–Wise (IW) functions, $\zeta(w)$ and $\xi(w)$, are introduced for $\Lambda_b \rightarrow \Lambda_c$ and $B \rightarrow D^{(*)}$, respectively. The relation holds for any c_{θ_l} and each λ_l as well as any w under the heavy quark symmetry, i.e., at the leading order of the IW expansion and the limit of heavy quark hadron masses satisfying $m_b \approx m_B \approx m_{\Lambda_b}$ and $m_c \approx m_D \approx m_{D^*} \approx m_{\Lambda_c}$. Summing up both lepton polarizations and integrating over c_{θ_l} reproduces the single-differential decay rate sum rule, Eq. (2). This finding leads us to propose two new sum rules involving A_{FB}^τ and P_τ for each to be tested experimentally:³

- Sum rule for the forward-backward asymmetry of the charged lepton: $(A_{\text{FB}}^{H_c \tau} / A_{\text{FB}}^{H_c \tau, \text{SM}})(R_{H_c} / R_{H_c}^{\text{SM}})$
- Sum rule for the τ -polarization difference: $(P_\tau^{H_c} / P_\tau^{H_c, \text{SM}})(R_{H_c} / R_{H_c}^{\text{SM}})$

Their explicit forms are shown in Eqs. (10) and (11) including corrections. See Appendix C for the construction in the heavy quark limit. It is noted that the tauonic total decay width in the denominator of Eqs. (4) and (5) is canceled with that in the numerator of R_{H_c} . P_τ^D is interesting since it is known to be useful to distinguish NP models which explain the $R_{D^{(*)}}$ anomaly [1, 26–30]. Therefore, confirming experimental results with the one involving $P_\tau^{H_c}$ provides an important cross-check.

3 Corrections to the sum rule and implication

Let us check how large corrections exist in the sum rules. In reality, the heavy quark symmetry is violated by higher-order corrections in the heavy quark expansion. The heavy flavored hadron masses include corrections as well as the heavy quark mass. In the HQET, they are expressed as [31–33]

² See Appendix A for details. Also, the sum rule holds for light-lepton modes as well.

³ Uncertainties from both form factor and experimental measurements will be reduced by normalizing with light-lepton decay widths.

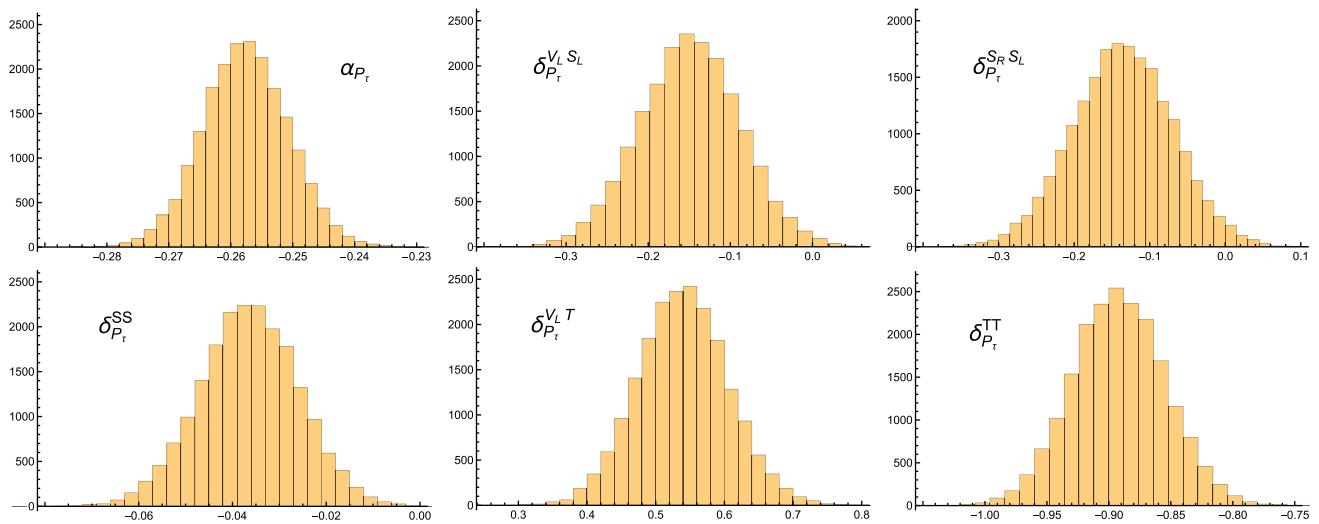


Fig. 1 Probability distribution of α_{P_τ} and the corrections $\delta_{P_\tau}^{ij}$

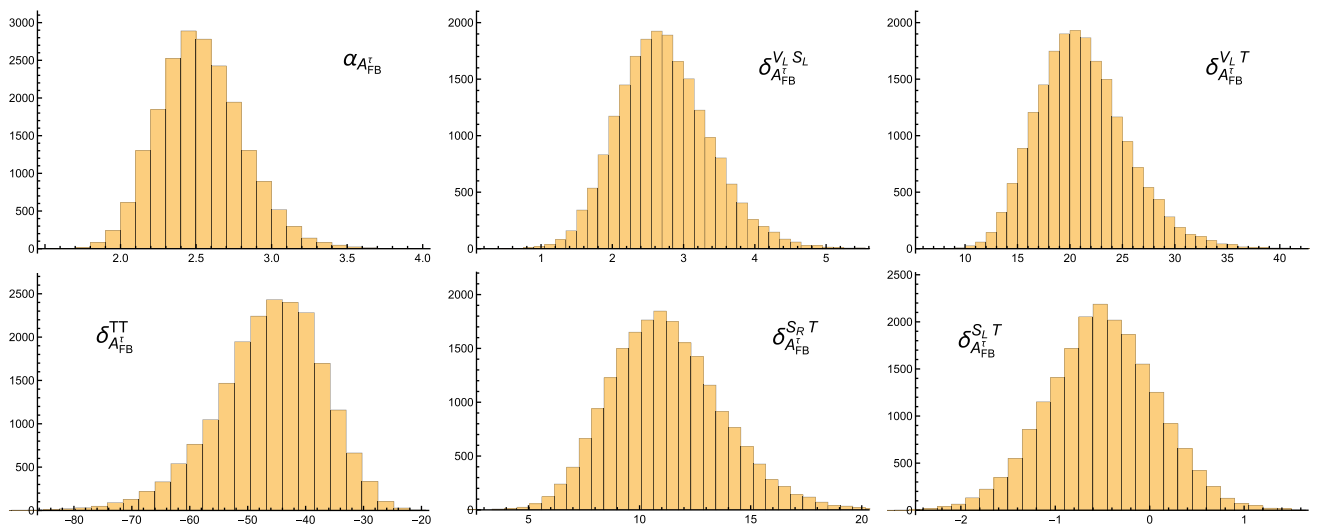


Fig. 2 Probability distribution of $\alpha_{A_{FB}^\tau}$ and the corrections $\delta_{A_{FB}^\tau}^{ij}$

Table 1 Corrections to the sum rules, δ_{P_τ} and $\delta_{A_{FB}^\tau}$, in the single operator (S_R , S_L , T) and single leptoquark scenarios (R_2 , S_1 , U_1). The first column represents the scenario whose relevant WCs are listed in the second column. For U_1 LQ, we consider the $U(2)$ -flavored scenario, satisfying $C_{S_R}^\tau = -3.7e^{i\phi}C_{V_L}^\tau$. See Ref. [35] for the detail. The best-fit

values of the WCs at the μ_b scale are shown in the third column, and the fit quality is expressed by the pull in the fourth column, whose definition is found in Ref. [35]. The last two columns provide the central values of δ_{P_τ} and $\delta_{A_{FB}^\tau}$ in each scenario

Scenario	Parameter	Value	Pull	δ_{P_τ}	$\delta_{A_{FB}^\tau}$
S_R	$C_{S_R}^\tau$	0.18	3.9	-0.001	0
S_L	$C_{S_L}^\tau$	$-0.57 \pm 0.86i$	4.3	0.05	-1.6
T	C_T^τ	$0.02 \pm 0.13i$	3.8	-0.004	-0.38
R_2	$C_{S_L}^\tau = 8.4C_T^\tau$	$-0.09 \pm 0.56i$	4.4	-0.008	-0.7
S_1	$C_{S_L}^\tau = -8.9C_T^\tau$	0.18	4.1	-0.04	0.05
U_1	$C_{V_L}^\tau, \phi$	$0.075, \pm 0.466\pi$	4.4	-0.003	0

$$m_{H_Q} \simeq m_Q \left(1 + \frac{\bar{\Lambda}}{m_Q} + \frac{\Delta m^2}{2m_Q^2} \right), \quad (9)$$

where m_Q is a heavy quark mass parameter. Also, $\bar{\Lambda}$ and Δm parameterize a light quark contribution and the heavy quark kinetic energy in a hadron, respectively, and are of the order of the quantum chromodynamics (QCD) scale Λ_{QCD} . Moreover, higher-order corrections to the IW functions of $\mathcal{O}(\Lambda_{\text{QCD}}/2m_Q)$ and $\mathcal{O}(\Lambda_{\text{QCD}}^2/4m_Q^2)$ enter in the $H_b \rightarrow H_c$ transition form factors. We will use the HQET form factor input from Refs. [33,34].⁴ Then, numerical formulae of $(P_\tau^{H_c}/P_\tau^{H_c,\text{SM}})(R_{H_c}/R_{H_c}^{\text{SM}})$ and $(A_{\text{FB}}^{H_c\tau}/A_{\text{FB}}^{H_c\tau,\text{SM}})(R_{H_c}/R_{H_c}^{\text{SM}})$ are obtained for mesonic and baryonic decays (see Eqs. (B3)–(B8) in Appendix B). Combining three relations where the coefficients α and β are determined such that $|1 + C_{V_L}^\tau|^2$ and $\text{Re}[(1 + C_{V_L}^\tau)C_{S_R}^{\tau*}]$ terms are vanishing (see, e.g., Refs. [1–4] for such a prescription), the sum rules are derived as

$$\frac{P_\tau^{\Lambda_c}}{P_\tau^{\Lambda_c,\text{SM}}} \frac{R_{\Lambda_c}}{R_{\Lambda_c}^{\text{SM}}} - \alpha_{P_\tau} \frac{P_\tau^D}{P_\tau^{D,\text{SM}}} \frac{R_D}{R_D^{\text{SM}}} - \beta_{P_\tau} \frac{P_\tau^{D^*}}{P_\tau^{D^*,\text{SM}}} \frac{R_{D^*}}{R_{D^*}^{\text{SM}}} = \delta_{P_\tau}, \quad (10)$$

$$\begin{aligned} & \frac{A_{\text{FB}}^{\Lambda_c\tau}}{A_{\text{FB}}^{\Lambda_c\tau,\text{SM}}} \frac{R_{\Lambda_c}}{R_{\Lambda_c}^{\text{SM}}} - \alpha_{A_{\text{FB}}} \frac{A_{\text{FB}}^{D\tau}}{A_{\text{FB}}^{D\tau,\text{SM}}} \frac{R_D}{R_D^{\text{SM}}} - \beta_{A_{\text{FB}}} \frac{A_{\text{FB}}^{D^*\tau}}{A_{\text{FB}}^{D^*\tau,\text{SM}}} \frac{R_{D^*}}{R_{D^*}^{\text{SM}}} \\ & = \delta_{A_{\text{FB}}}. \end{aligned} \quad (11)$$

If the corrections, δ_{P_τ} and $\delta_{A_{\text{FB}}}$, are small enough (even when their uncertainties are taken into account) compared to the experimental uncertainties, the sum rules are useful to check the consistency of the experimental results simply by neglecting the correction terms. Even when the experimental values are determined so precisely that δ_{P_τ} and $\delta_{A_{\text{FB}}}$ are not negligible, the sum rules could be applied to discriminate the NP scenarios. The corrections, δ_{P_τ} and $\delta_{A_{\text{FB}}}$, consist of bilinears of the WCs as $\delta_Y = \sum_{ij} C_i^\tau C_j^{\tau*} \delta_Y^{ij}$ with $Y = P_\tau, A_{\text{FB}}$, and are approximately expressed, ignoring uncertainties, as

$$\begin{aligned} \delta_{P_\tau} & \simeq -0.149\text{Re}[(1 + C_{V_L}^\tau)C_{S_L}^{\tau*}] - 0.036(|C_{S_L}^\tau|^2 + |C_{S_R}^\tau|^2) \\ & \quad - 0.136\text{Re}[C_{S_L}^\tau C_{S_R}^{\tau*}] + 0.540\text{Re}[(1 + C_{V_L}^\tau)C_T^{\tau*}] \\ & \quad - 0.892|C_T^\tau|^2, \end{aligned} \quad (12)$$

$$\begin{aligned} \delta_{A_{\text{FB}}} & \simeq +2.73\text{Re}[(1 + C_{V_L}^\tau)C_{S_L}^{\tau*}] + 21.2\text{Re}[(1 + C_{V_L}^\tau)C_T^{\tau*}] \\ & \quad - 46.2|C_T^\tau|^2 - 0.503\text{Re}[C_{S_L}^\tau C_T^{\tau*}] + 11.2\text{Re}[C_{S_R}^\tau C_T^{\tau*}], \end{aligned} \quad (13)$$

with $\alpha_{P_\tau} = 1 - \beta_{P_\tau} \simeq -0.258$, $\alpha_{A_{\text{FB}}} = 1 - \beta_{A_{\text{FB}}} \simeq 2.53$. In the absence of the NP contribution, i.e., within the SM, the corrections vanish as $\delta_{P_\tau} = \delta_{A_{\text{FB}}} = 0$. Also, we obtain

$P_\tau^{H_c,\text{SM}} R_{H_c}^{\text{SM}} \simeq 0.10, -0.12, -0.10$ and $A_{\text{FB}}^{H_c\tau,\text{SM}} R_{H_c}^{\text{SM}} \simeq 0.11, -0.0094, -0.0082$ for $H_c = D, D^*, \Lambda_c$, respectively. In Appendix D, Figs. 1 and 2 respectively show the probability distributions of α_Y and δ_Y^{ij} stemming from the form factor inputs.

For the central values, the coefficients involving the tensor operator in $\delta_{P_\tau}^{ij}$ are larger than the others, e.g., $\delta_{P_\tau}^{TT} \simeq -0.9$, while the scalar coefficients are smaller. In addition, the tensor coefficients are $\mathcal{O}(10)$ in $\delta_{A_{\text{FB}}}^\tau$, while $\delta_{A_{\text{FB}}}^{V_L S_L}$ is about $2 \sim 3$. The former mainly comes from mismatches between $(A_{\text{FB}}^{D^*\tau}/A_{\text{FB}}^{D^*\tau,\text{SM}})$ and $(A_{\text{FB}}^{\Lambda_c\tau}/A_{\text{FB}}^{\Lambda_c\tau,\text{SM}})$ as seen from Eqs. (B7) and (B8). In these numerical formulae, the coefficients are large because $A_{\text{FB}}^{H_c\tau,\text{SM}}$ is suppressed for $H_c = D^*, \Lambda_c$.

As seen in Figs. 1 and 2, the uncertainty of α_{P_τ} is less than 3%, while $\alpha_{A_{\text{FB}}}$ has $\sim 10\%$ uncertainty. Regarding δ_{P_τ} , the uncertainties of the scalar coefficients are 30–40%, while those of the tensor are $\sim 10\%$ for $ij = V_L T$ and $\sim 4\%$ for TT . Similarly, in $\delta_{A_{\text{FB}}}$, the uncertainty is $\sim 20\%$, except for the $ij = S_L T$ case, where we have $\sim 100\%$ uncertainty.⁵

Let us estimate δ_{P_τ} and $\delta_{A_{\text{FB}}}$ with the WCs which are determined by performing a global fit to the current experimental values of $R_{D^{(*)}}$ and $F_L^{D^*}$ [35]. The results are summarized in Table 1. We consider three “single-operator” scenarios and “three single-leptoquark (LQ)” scenarios. In Appendix D, Figs. 3 and 4 respectively show δ_{P_τ} and $\delta_{A_{\text{FB}}}$ at each benchmark point. We define the central value by fitting the probabilities to Gaussian distributions. Out of three single-operator scenarios, the S_L case predicts $\delta_{P_\tau} \sim 0.05$, while the others predict $\delta_{P_\tau} = \mathcal{O}(10^{-3})$. Among three LQ scenarios, only the S_1 LQ scenario yields $\delta_{P_\tau} \sim -0.04$, while the other two scenarios predict at most a 1% level in δ_{P_τ} . On the other hand, $\delta_{A_{\text{FB}}}$ can significantly deviate from 0 and become as large as $\mathcal{O}(-1)$ in the S_L and R_2 LQ scenarios. The T and S_1 LQ scenarios predict smaller corrections. It is noted that the S_R and U_1 LQ scenarios predict $\delta_{A_{\text{FB}}} = 0$, and hence they are removed from the figure.

In summary, the correction to the sum rule δ_Y can be large for $Y = A_{\text{FB}}^\tau$ and at most $\sim 5\%$ for $Y = P_\tau$. This means that the sum rule involving A_{FB}^τ is more sensitive to the NP effect and can be useful to discriminate the scenarios if the experimental uncertainties are comparable. Checking the experimental consistency with the τ -polarization sum rule will be important too. Currently, future experimental prospects are available only for P_τ^D and $P_\tau^{D^*}$. Although estimating the uncertainty of the product of observables, e.g., $P_\tau^{H_c} R_{H_c}$, needs careful study, as some of the uncertainties in each observable should correlate to each other, it is highly encouraged in light of the new angular sum rules.

⁴ To be precise, we used the fit result of the z210 scenario for the $B \rightarrow D^{(*)}$ transition and the $\hat{b}_{1,2} \neq 0$ scenario for the $\Lambda_b \rightarrow \Lambda_c$ transition.

⁵ In the $S_L T$ case, the relative uncertainty is large, because the central value is suppressed by a cancellation among the contributions.

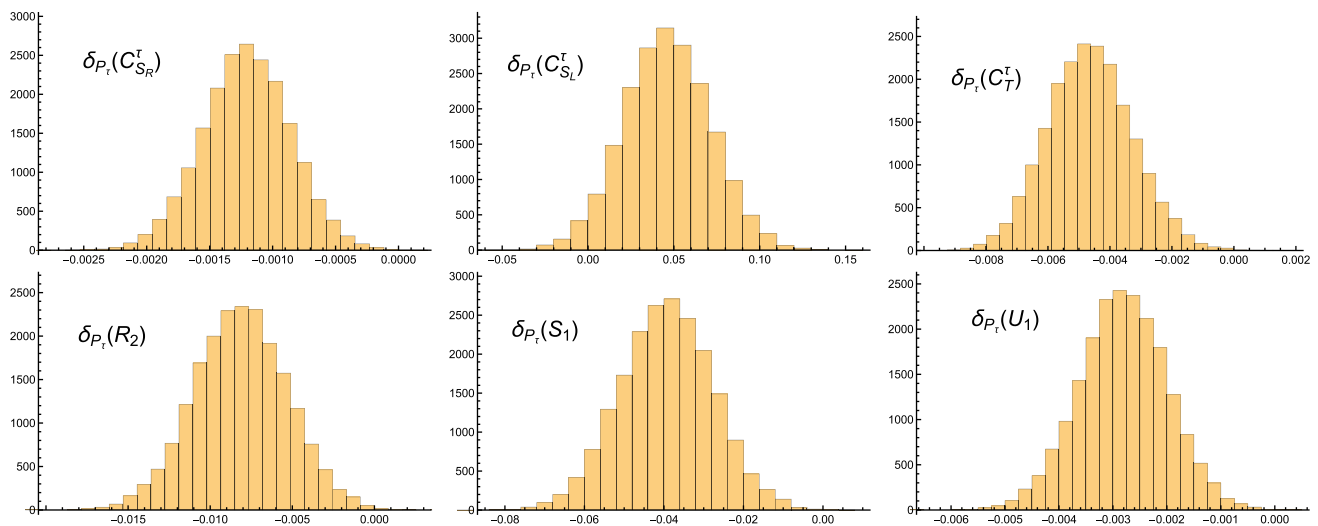


Fig. 3 Probability distribution of the correction δ_{P_i} based on the benchmark NP scenarios shown in Table 1

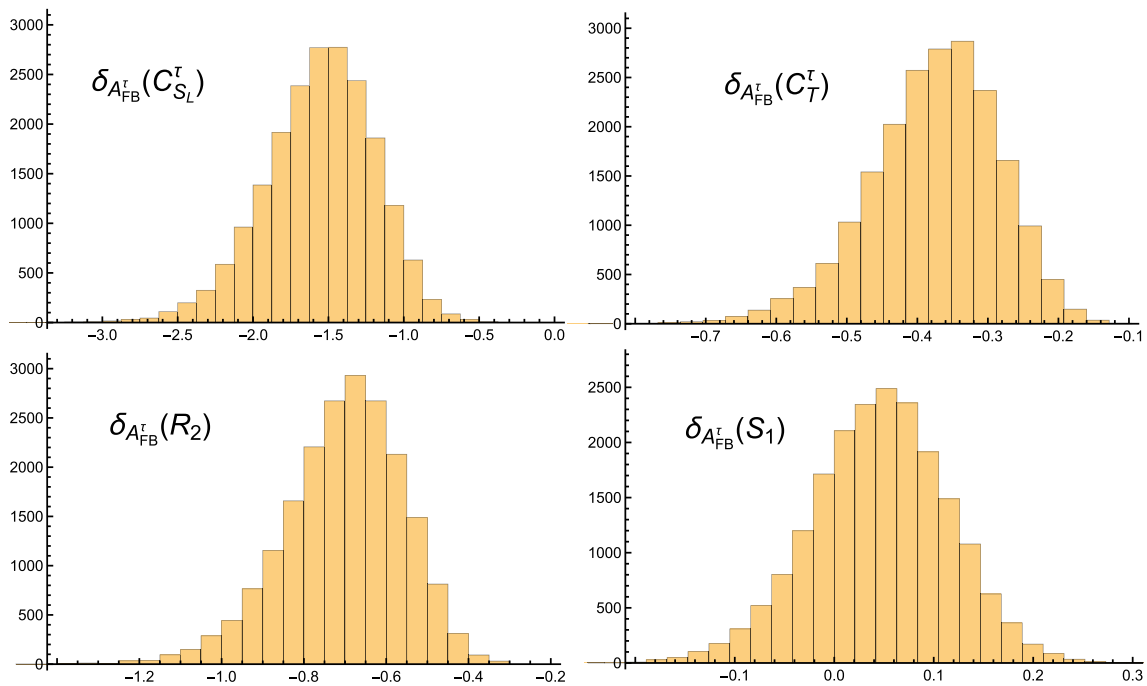


Fig. 4 Probability distribution of the correction $\delta_{A_{FB}^{\tau}}$ based on the benchmark NP scenarios shown in Table 1

4 Conclusion

In this paper, we extended the previous studies of the $b \rightarrow c$ semileptonic sum rule for the single-differential decay rates and found a new relation for the double-differential decay rates, which holds exactly in the heavy quark limit. We then derived two sum rules involving the τ -polarization observable, P_{τ} , and the charged lepton forward-backward asymmetry, A_{FB}^{τ} . In reality, the heavy quark symmetry is violated, and the sum rules receive corrections from higher-order effects in the heavy quark expansion, i.e., the realistic mass spec-

trum and the inclusion of higher-order terms of the IW functions. We also evaluated these corrections numerically. Using HQET-based form factors, we demonstrated that the sum rule coefficients $\alpha_{P_{\tau}}$ and $\alpha_{A_{FB}^{\tau}}$ can currently be determined with precision of approximately 3% and 11%, respectively. To reduce the uncertainty of these coefficients, experimental input of $\Lambda_b \rightarrow \Lambda_c \mu \bar{\nu}$ and lattice calculations are important. Furthermore, we estimated the corrections in the NP scenarios motivated by the $R_{D^{(*)}}$ anomaly. We found that these corrections are at most $\sim 5\%$ for the τ -polarization sum rule, Eq. (10), while they can reach around -100% for the A_{FB}^{τ}

case, Eq. (11), depending on the NP scenarios. These new sum rules will provide independent cross-checks of experimental results. By testing both angular sum rules, we can better explore and distinguish NP scenarios. These results encourage the feasibility study at ongoing and future experiments [11, 13–16].

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Code Availability Statement Code/software will be made available on reasonable request. [Authors' comment: The code/software generated during and/or analysed during the current study is available from the corresponding author on reasonable request.].

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Appendix A: Differential decay rate

Here, let us write the double-differential decay rates of $B \rightarrow D^{(*)}l\bar{\nu}$ and $\Lambda_b \rightarrow \Lambda_c l\bar{\nu}$ for each l -lepton polarization such that

$$\frac{d^2\Gamma^{\lambda_l}(B \rightarrow Dl\bar{\nu})}{dq^2 d\cos\theta_l} = \frac{G_F^2 |V_{cb}|^2 \eta_{EW} \sqrt{Q_+^D Q_-^D}}{256\pi^3 m_B^3} q^2 \left(1 - \frac{m_l^2}{q^2}\right)^2 \times \left(\mathcal{A}_0^{\lambda_l} + \mathcal{A}_1^{\lambda_l} \cos\theta_l + \mathcal{A}_2^{\lambda_l} \cos^2\theta_l\right), \quad (\text{A1})$$

$$\frac{d^2\Gamma^{\lambda_l}(B \rightarrow D^*l\bar{\nu})}{dq^2 d\cos\theta_l} = \frac{G_F^2 |V_{cb}|^2 \eta_{EW} \sqrt{Q_+^{D^*} Q_-^{D^*}}}{512\pi^3 m_B^3} q^2 \left(1 - \frac{m_l^2}{q^2}\right)^2 \times \left(\mathcal{B}_0^{\lambda_l} + \mathcal{B}_1^{\lambda_l} \cos\theta_l + \mathcal{B}_2^{\lambda_l} \cos^2\theta_l\right), \quad (\text{A2})$$

$$\frac{d^2\Gamma^{\lambda_l}(\Lambda_b \rightarrow \Lambda_c l\bar{\nu})}{dq^2 d\cos\theta_l} = \frac{G_F^2 |V_{cb}|^2 \eta_{EW} \sqrt{Q_+^{\Lambda_c} Q_-^{\Lambda_c}}}{512\pi^3 m_{\Lambda_b}^3} q^2 \left(1 - \frac{m_l^2}{q^2}\right)^2 \times \left(C_0^{\lambda_l} + C_1^{\lambda_l} \cos\theta_l + C_2^{\lambda_l} \cos^2\theta_l\right), \quad (\text{A3})$$

with η_{EW} being an electroweak correction, and

$$Q_{\pm}^{H_c} = (m_{H_b} \pm m_{H_c})^2 - q^2. \quad (\text{A4})$$

The squared amplitudes for $B \rightarrow Dl\bar{\nu}$ are given by

$$\mathcal{A}_0^{+1/2} = |1 + C_{V_L}^l|^2 \frac{m_l^2}{q^2} (H_{V_l}^s)^2 + |C_{S_L}^l + C_{S_R}^l|^2 (H_S^s)^2 + 2\text{Re}\left[(1 + C_{V_L}^l)(C_{S_L}^l + C_{S_R}^l)^*\right] \frac{m_l}{\sqrt{q^2}} H_{V_l}^s H_S^s, \quad (\text{A5})$$

$$\begin{aligned} \mathcal{A}_1^{+1/2} = & 2|1 + C_{V_L}^l|^2 \frac{m_l^2}{q^2} H_{V_l}^s H_{V_0}^s \\ & - 8\text{Re}\left[(1 + C_{V_L}^l)C_T^{l*}\right] \frac{m_l}{\sqrt{q^2}} H_{V_l}^s H_T^s \\ & + 2\text{Re}\left[(1 + C_{V_L}^l)(C_{S_L}^l + C_{S_R}^l)^*\right] \frac{m_l}{\sqrt{q^2}} H_{V_0}^s H_S^s \\ & - 8\text{Re}\left[(C_{S_L}^l + C_{S_R}^l)C_T^{l*}\right] H_S^s H_T^s, \end{aligned} \quad (\text{A6})$$

$$\begin{aligned} \mathcal{A}_2^{+1/2} = & |1 + C_{V_L}^l|^2 \frac{m_l^2}{q^2} (H_{V_0}^s)^2 + 16|C_T^l|^2 (H_T^s)^2 \\ & - 8\text{Re}\left[(1 + C_{V_L}^l)C_T^{l*}\right] \frac{m_l}{\sqrt{q^2}} H_{V_0}^s H_T^s, \end{aligned} \quad (\text{A7})$$

$$\begin{aligned} \mathcal{A}_0^{-1/2} = -\mathcal{A}_2^{-1/2} = & |1 + C_{V_L}^l|^2 (H_{V_0}^s)^2 + 16|C_T^l|^2 \frac{m_l^2}{q^2} (H_T^s)^2 \\ & - 8\text{Re}\left[(1 + C_{V_L}^l)C_T^{l*}\right] \frac{m_l}{\sqrt{q^2}} H_{V_0}^s H_T^s, \end{aligned} \quad (\text{A8})$$

$$\mathcal{A}_1^{-1/2} = 0. \quad (\text{A9})$$

For $B \rightarrow D^*l\bar{\nu}$, they are given by

$$\begin{aligned} \mathcal{B}_0^{+1/2} = & |1 + C_{V_L}^l|^2 \frac{m_l^2}{q^2} ((H_{V_+})^2 + (H_{V_-})^2 + 2(H_{V_l})^2) \\ & + 2|C_{S_L}^l - C_{S_R}^l|^2 H_S^2 + 16|C_T^l|^2 ((H_{T_+})^2 + (H_{T_-})^2) \\ & - 4\text{Re}[(1 + C_{V_L}^l)(C_{S_L}^l - C_{S_R}^l)^*] \frac{m_l}{\sqrt{q^2}} H_{V_l} H_S \\ & - 8\text{Re}[(1 + C_{V_L}^l)C_T^{l*}] \frac{m_l}{\sqrt{q^2}} (H_{V_+} H_{T_+} - H_{V_-} H_{T_-}), \end{aligned} \quad (\text{A10})$$

$$\begin{aligned} \mathcal{B}_1^{+1/2} = & 4|1 + C_{V_L}^l|^2 \frac{m_l^2}{q^2} H_{V_0} H_{V_l} - 4\text{Re}[(1 + C_{V_L}^l) \\ & \times (C_{S_L}^l - C_{S_R}^l)^*] \frac{m_l}{\sqrt{q^2}} H_{V_0} H_S \\ & - 16\text{Re}[(1 + C_{V_L}^l)C_T^{l*}] \frac{m_l}{\sqrt{q^2}} H_{V_l} H_{T_0} \\ & + 16\text{Re}[(C_{S_L}^l - C_{S_R}^l)C_T^{l*}] H_S H_{T_0}, \end{aligned} \quad (\text{A11})$$

$$\begin{aligned} \mathcal{B}_2^{+1/2} = & |1 + C_{V_L}^l|^2 \frac{m_l^2}{q^2} (2(H_{V_0})^2 - (H_{V_+})^2 - (H_{V_-})^2) \\ & + 16|C_T^l|^2 (2(H_{T_0})^2 - (H_{T_+})^2 - (H_{T_-})^2) \\ & - 8\text{Re}[(1 + C_{V_L}^l)C_T^{l*}] \frac{m_l}{\sqrt{q^2}} (2H_{V_0}H_{T_0} \\ & - H_{V_+}H_{T_+} + H_{V_-}H_{T_-}), \end{aligned} \quad (\text{A12})$$

$$\begin{aligned} \mathcal{B}_0^{-1/2} = & |1 + C_{V_L}^l|^2 (2(H_{V_0})^2 + (H_{V_+})^2 + (H_{V_-})^2) \\ & + 16|C_T^l|^2 \frac{m_l^2}{q^2} (2(H_{T_0})^2 + (H_{T_+})^2 + (H_{T_-})^2) \\ & - 8\text{Re}[(1 + C_{V_L}^l)C_T^{l*}] \frac{m_l}{\sqrt{q^2}} (2H_{V_0}H_{T_0} \\ & + H_{V_+}H_{T_+} - H_{V_-}H_{T_-}), \end{aligned} \quad (\text{A13})$$

$$\begin{aligned} \mathcal{B}_1^{-1/2} = & 2|1 + C_{V_L}^l|^2 ((H_{V_+})^2 - (H_{V_-})^2) + 32|C_T^l|^2 \\ & \times \frac{m_l^2}{q^2} ((H_{T_+})^2 - (H_{T_-})^2) \\ & - 16\text{Re}[(1 + C_{V_L}^l)C_T^{l*}] \frac{m_l}{\sqrt{q^2}} (H_{V_+}H_{T_+} + H_{V_-}H_{T_-}), \end{aligned} \quad (\text{A14})$$

$$\begin{aligned} \mathcal{B}_2^{-1/2} = & |1 + C_{V_L}^l|^2 (-2(H_{V_0})^2 + (H_{V_+})^2 + (H_{V_-})^2) \\ & + 16|C_T^l|^2 \frac{m_l^2}{q^2} (-2(H_{T_0})^2 + (H_{T_+})^2 + (H_{T_-})^2) \\ & + 8\text{Re}[(1 + C_{V_L}^l)C_T^{l*}] \frac{m_l}{\sqrt{q^2}} (2H_{V_0}H_{T_0} \\ & - H_{V_+}H_{T_+} + H_{V_-}H_{T_-}). \end{aligned} \quad (\text{A15})$$

The squared amplitudes for $\Lambda_b \rightarrow \Lambda_c l \bar{\nu}$ are given by

$$\begin{aligned} \mathcal{C}_0^{+1/2} = & |1 + C_{V_L}^l|^2 \frac{m_l^2}{q^2} ((H_{V_\perp}^{H+})^2 + (H_{V_\perp}^{H-})^2 + (H_{V_0}^{H+})^2 \\ & + (H_{V_0}^{H-})^2) + 16|C_T^l|^2 ((H_{T_\perp}^{H+})^2 + (H_{T_\perp}^{H-})^2) \\ & + (|C_{S_L}^l|^2 + |C_{S_R}^l|^2) ((H_S^{H+})^2 + (H_S^{H-})^2) \\ & + 4\text{Re}[C_{S_L}^l C_{S_R}^{l*}] H_S^{H+} H_S^{H-} \\ & + 2\text{Re}[(1 + C_{V_L}^l)C_{S_L}^{l*}] \frac{m_l}{\sqrt{q^2}} (H_{V_0}^{H+} H_S^{H+} + H_{V_0}^{H-} H_S^{H-}) \\ & + 2\text{Re}[(1 + C_{V_L}^l)C_{S_R}^{l*}] \frac{m_l}{\sqrt{q^2}} (H_{V_0}^{H+} H_S^{H-} + H_{V_0}^{H-} H_S^{H+}) \\ & + 8\text{Re}[(1 + C_{V_L}^l)C_T^{l*}] \frac{m_l}{\sqrt{q^2}} (H_{V_\perp}^{H+} H_{T_\perp}^{H+} + H_{V_\perp}^{H-} H_{T_\perp}^{H-}), \end{aligned} \quad (\text{A16})$$

$$\begin{aligned} \mathcal{C}_1^{+1/2} = & 2|1 + C_{V_L}^l|^2 \frac{m_l^2}{q^2} (H_{V_0}^{H+} H_{V_+}^{H+} + H_{V_0}^{H-} H_{V_+}^{H-}) \\ & + 2\text{Re}[(1 + C_{V_L}^l)C_{S_L}^{l*}] \frac{m_l}{\sqrt{q^2}} (H_{V_+}^{H+} H_S^{H+} + H_{V_+}^{H-} H_S^{H-}) \\ & + 2\text{Re}[(1 + C_{V_L}^l)C_{S_R}^{l*}] \frac{m_l}{\sqrt{q^2}} (H_{V_+}^{H+} H_S^{H-} + H_{V_+}^{H-} H_S^{H+}) \end{aligned}$$

$$\begin{aligned} & + 8\text{Re}[(1 + C_{V_L}^l)C_T^{l*}] \frac{m_l}{\sqrt{q^2}} (H_{V_0}^{H+} H_{T_+}^{H+} + H_{V_0}^{H-} H_{T_+}^{H-}) \\ & + 8\text{Re}[C_{S_L}^l C_T^{l*}] (H_{T_+}^{H+} H_S^{H+} + H_{T_+}^{H-} H_S^{H-}) \\ & + 8\text{Re}[C_{S_R}^l C_T^{l*}] (H_{T_+}^{H+} H_S^{H-} + H_{T_+}^{H-} H_S^{H+}), \end{aligned} \quad (\text{A17})$$

$$\begin{aligned} \mathcal{C}_2^{+1/2} = & |1 + C_{V_L}^l|^2 \frac{m_l^2}{q^2} ((H_{V_+}^{H+})^2 + (H_{V_+}^{H-})^2 - (H_{V_\perp}^{H+})^2 \\ & - (H_{V_\perp}^{H-})^2) + 16|C_T^l|^2 ((H_{T_+}^{H+})^2 + (H_{T_+}^{H-})^2 \\ & - (H_{V_\perp}^{H+})^2 - (H_{V_\perp}^{H-})^2) + 8\text{Re}[(1 + C_{V_L}^l)C_T^{l*}] \\ & \times \frac{m_l}{\sqrt{q^2}} (H_{V_+}^{H+} H_{T_+}^{H+} + H_{V_+}^{H-} H_{T_+}^{H-} - H_{V_\perp}^{H+} H_{T_\perp}^{H+} \\ & - H_{V_\perp}^{H-} H_{T_\perp}^{H-}), \end{aligned} \quad (\text{A18})$$

$$\begin{aligned} \mathcal{C}_0^{-1/2} = & |1 + C_{V_L}^l|^2 ((H_{V_+}^{H+})^2 + (H_{V_+}^{H-})^2 + (H_{V_\perp}^{H+})^2 \\ & + (H_{V_\perp}^{H-})^2) + 16|C_T^l|^2 \frac{m_l^2}{q^2} ((H_{T_+}^{H+})^2 + (H_{T_+}^{H-})^2 \\ & + (H_{T_\perp}^{H+})^2 + (H_{T_\perp}^{H-})^2) + 8\text{Re}[(1 + C_{V_L}^l)C_T^{l*}] \\ & \times \frac{m_l}{\sqrt{q^2}} (H_{V_+}^{H+} H_{T_+}^{H+} + H_{V_+}^{H-} H_{T_+}^{H-} + H_{V_\perp}^{H+} H_{T_\perp}^{H+} \\ & + H_{V_\perp}^{H-} H_{T_\perp}^{H-}), \end{aligned} \quad (\text{A19})$$

$$\begin{aligned} \mathcal{C}_1^{-1/2} = & 2|1 + C_{V_L}^l|^2 ((H_{V_\perp}^{H+})^2 - (H_{V_\perp}^{H-})^2) \\ & + 32|C_T^l|^2 \frac{m_l^2}{q^2} ((H_{T_\perp}^{H+})^2 - (H_{T_\perp}^{H-})^2) \\ & + 16\text{Re}[(1 + C_{V_L}^l)C_T^{l*}] \frac{m_l}{\sqrt{q^2}} \\ & \times (H_{V_\perp}^{H+} H_{T_\perp}^{H+} - H_{V_\perp}^{H-} H_{T_\perp}^{H-}), \end{aligned} \quad (\text{A20})$$

$$\begin{aligned} \mathcal{C}_2^{-1/2} = & |1 + C_{V_L}^l|^2 ((H_{V_\perp}^{H+})^2 + (H_{V_\perp}^{H-})^2 - (H_{V_+}^{H+})^2 \\ & - (H_{V_+}^{H-})^2) + 16|C_T^l|^2 \frac{m_l^2}{q^2} ((H_{T_\perp}^{H+})^2 + (H_{T_\perp}^{H-})^2 \\ & - (H_{T_+}^{H+})^2 - (H_{T_+}^{H-})^2) + 8\text{Re}[(1 + C_{V_L}^l)C_T^{l*}] \frac{m_l}{\sqrt{q^2}} \\ & \times (H_{V_\perp}^{H+} H_{T_\perp}^{H+} + H_{V_\perp}^{H-} H_{T_\perp}^{H-} - H_{V_+}^{H+} H_{T_+}^{H+} \\ & - H_{V_+}^{H-} H_{T_+}^{H-}). \end{aligned} \quad (\text{A21})$$

The hadronic helicity amplitudes, H_X^s and H_X , for $B \rightarrow D^{(*)} l \bar{\nu}$ are given in the HQET description as [29,36]

$$\begin{aligned} H_{V_0}^s &= m_B \sqrt{\frac{r_D(w^2 - 1)}{\hat{q}_D^2}} [(1 + r_D)h_+ - (1 - r_D)h_-], \\ H_{V_i}^s &= m_B \sqrt{\frac{r_D}{\hat{q}_D^2}} [(1 - r_D)(w + 1)h_+ - (1 + r_D)(w - 1)h_-], \\ H_S^s &= m_B \sqrt{r_D}(w + 1)h_S, \\ H_T^s &= -m_B \sqrt{r_D}(w^2 - 1)h_T, \end{aligned} \quad (\text{A22})$$

and

$$\begin{aligned}
 H_{V_{\pm}} &= m_B \sqrt{r_{D^*}} \left[(w+1)h_{A_1} \mp \sqrt{w^2-1}h_V \right], \\
 H_{V_0} &= m_B \sqrt{\frac{r_{D^*}}{\hat{q}_{D^*}^2}} (w+1) \left[(r_{D^*}-w)h_{A_1} \right. \\
 &\quad \left. + (w-1)(r_{D^*}h_{A_2} + h_{A_3}) \right], \\
 H_{V_t} &= -m_B \sqrt{\frac{r_{D^*}(w^2-1)}{\hat{q}_{D^*}^2}} \left[(w+1)h_{A_1} + (r_{D^*}w-1)h_{A_2} \right. \\
 &\quad \left. + (r_{D^*}-w)h_{A_3} \right], \\
 H_S &= -m_B \sqrt{r_{D^*}(w^2-1)}h_P, \\
 H_{T_{\pm}} &= \pm m_B \sqrt{\frac{r_{D^*}}{\hat{q}_{D^*}^2}} \left[1 - r_{D^*}(w \mp \sqrt{w^2-1}) \right] \\
 &\quad \times \left[h_{T_1} + h_{T_2} + (w \pm \sqrt{w^2-1})(h_{T_1} - h_{T_2}) \right], \\
 H_{T_0} &= -m_B \sqrt{r_{D^*}} \left[(w+1)h_{T_1} + (w-1)h_{T_2} \right. \\
 &\quad \left. + 2(w^2-1)h_{T_3} \right], \tag{A23}
 \end{aligned}$$

where $\hat{q}_{H_c}^2 = q^2/m_{H_b}^2 = 1 - 2r_{H_c}w + r_{H_c}^2$ and $r_{H_c} = m_{H_c}/m_{H_b}$ are defined. Regarding $\Lambda_b \rightarrow \Lambda_c l \bar{\nu}$, they are summarized as [33,37]

$$\begin{aligned}
 H_{V_{+}}^{H_{\pm}} &= m_{\Lambda_b} \sqrt{\frac{2r_{\Lambda}}{\hat{q}_{\Lambda}^2}} \left\{ \sqrt{w-1} \left[(1+r_{\Lambda})f_1 \right. \right. \\
 &\quad \left. \left. + (w+1)(f_2r_{\Lambda} + f_3) \right] \mp \sqrt{w+1} \left[(1-r_{\Lambda})g_1 \right. \right. \\
 &\quad \left. \left. - (w-1)(g_2r_{\Lambda} + g_3) \right] \right\}, \\
 H_{V_{\perp}}^{H_{\pm}} &= m_{\Lambda_b} \sqrt{2r_{\Lambda}} \left[\sqrt{w-1}f_1 \mp \sqrt{w+1}g_1 \right], \\
 H_{V_0}^{H_{\pm}} &= m_{\Lambda_b} \sqrt{\frac{2r_{\Lambda}}{\hat{q}_{\Lambda}^2}} \left\{ \sqrt{w+1} \left[(1-r_{\Lambda})f_1 + f_2(1-wr_{\Lambda}) \right. \right. \\
 &\quad \left. \left. + f_3(w-r_{\Lambda}) \right] \mp \sqrt{w-1} \left[(1+r_{\Lambda})g_1 \right. \right. \\
 &\quad \left. \left. - g_2(1-wr_{\Lambda}) - g_3(w-r_{\Lambda}) \right] \right\}, \\
 H_S^{H_{\pm}} &= m_{\Lambda_b} \sqrt{2r_{\Lambda}} \left(\sqrt{w+1}h'_S \pm \sqrt{w-1}h'_P \right), \\
 H_{T_{+}}^{H_{\pm}} &= m_{\Lambda_b} \sqrt{2r_{\Lambda}} \left\{ \sqrt{w-1} \left[h_1 - h_2 + h_3 - (w+1)h_4 \right] \right. \\
 &\quad \left. \pm \sqrt{w+1}h_1 \right\}, \\
 H_{T_{\perp}}^{H_{\pm}} &= m_{\Lambda_b} \sqrt{\frac{2r_{\Lambda}}{\hat{q}_{\Lambda}^2}} \left\{ \sqrt{w-1} \left[(1+r_{\Lambda})h_1 - (1-wr_{\Lambda})h_2 \right. \right. \\
 &\quad \left. \left. - (w-r_{\Lambda})h_3 \right] \pm \sqrt{w+1} \left[(1-r_{\Lambda})h_1 \right. \right. \\
 &\quad \left. \left. - (w-1)(h_2r_{\Lambda} + h_3) \right] \right\}. \tag{A24}
 \end{aligned}$$

To be self-complete, we introduce the HQET form factors as

$$\begin{aligned}
 \langle D | \bar{c} \gamma^{\mu} b | B \rangle &= \sqrt{m_B m_D} [h_+(v+v')^{\mu} + h_-(v-v')^{\mu}], \\
 \langle D | \bar{c} b | B \rangle &= \sqrt{m_B m_D} (w+1)h_S, \\
 \langle D | \bar{c} \gamma^{\mu} \gamma_5 b | B \rangle &= \langle D | \bar{c} \gamma_5 b | B \rangle = 0, \\
 \langle D | \bar{c} \sigma^{\mu\nu} b | B \rangle &= -i\sqrt{m_B m_D} h_T (v^{\mu}v'^{\nu} - v'^{\mu}v^{\nu}) \tag{A25}
 \end{aligned}$$

for the $B \rightarrow D$ transitions, and

$$\begin{aligned}
 \langle D^* | \bar{c} \gamma^{\mu} b | B \rangle &= i\sqrt{m_B m_{D^*}} h_V \varepsilon^{\mu\nu\rho\sigma} \epsilon_{\nu}^* v'_{\rho} v_{\sigma}, \\
 \langle D^* | \bar{c} \gamma^{\mu} \gamma_5 b | B \rangle &= \sqrt{m_B m_{D^*}} [h_{A_1}(w+1)\epsilon^{*\mu} - (\epsilon^* \cdot v) \\
 &\quad \times (h_{A_2}v^{\mu} + h_{A_3}v'^{\mu})], \\
 \langle D^* | \bar{c} \gamma_5 b | B \rangle &= -\sqrt{m_B m_{D^*}} (\epsilon^* \cdot v) h_P, \\
 \langle D^* | \bar{c} b | B \rangle &= 0, \\
 \langle D^* | \bar{c} \sigma^{\mu\nu} b | B \rangle &= -\sqrt{m_B m_{D^*}} \varepsilon^{\mu\nu\rho\sigma} [h_{T_1} \epsilon_{\rho}^* (v+v')_{\sigma} \\
 &\quad + h_{T_2} \epsilon_{\rho}^* (v-v')_{\sigma} + h_{T_3} (\epsilon^* \cdot v) \\
 &\quad \times (v+v')_{\rho} (v-v')_{\sigma}] \tag{A26}
 \end{aligned}$$

for $B \rightarrow D^*$. For $\Lambda_b \rightarrow \Lambda_b l \bar{\nu}$, they are given as

$$\begin{aligned}
 \langle \Lambda_c | \bar{c} \gamma_{\mu} b | \Lambda_b \rangle &= \bar{u}(p', s') [f_1 \gamma_{\mu} + f_2 v_{\mu} + f_3 v'_{\mu}] u(p, s), \\
 \langle \Lambda_c | \bar{c} \gamma_{\mu} \gamma_5 b | \Lambda_b \rangle &= \bar{u}(p', s') [g_1 \gamma_{\mu} + g_2 v_{\mu} + g_3 v'_{\mu}] \gamma_5 u(p, s), \\
 \langle \Lambda_c | \bar{c} b | \Lambda_b \rangle &= h'_S \bar{u}(p', s') u(p, s), \\
 \langle \Lambda_c | \bar{c} \gamma_5 b | \Lambda_b \rangle &= h'_P \bar{u}(p', s') \gamma_5 u(p, s), \\
 \langle \Lambda_c | \bar{c} \sigma_{\mu\nu} b | \Lambda_b \rangle &= \bar{u}(p', s') [h_1 \sigma_{\mu\nu} + i h_2 (v_{\mu} \gamma_{\nu} - v_{\nu} \gamma_{\mu}) \\
 &\quad + i h_3 (v'_{\mu} \gamma_{\nu} - v'_{\nu} \gamma_{\mu}) \\
 &\quad + i h_4 (v_{\mu} v'_{\nu} - v_{\nu} v'_{\mu})] u(p, s), \tag{A27}
 \end{aligned}$$

where $u(p, s)$ are spinors with momentum p and spin s . Moreover, $v = p/m_{H_b}$ and $v' = p'/m_{H_c}$ satisfying $v \cdot v' = w = (m_{H_b}^2 + m_{H_c}^2 - q^2)/(2m_{H_b}m_{H_c})$ are introduced. The form factors f_i , g_i , and h_i are functions of w and expressed in the heavy quark limit as [5,38]

$$\begin{aligned}
 h_+ &= h_V = h_{A_1} = h_{A_3} = h_S = h_P = h_T = h_{T_1} = \xi(w), \\
 h_- &= h_{A_2} = h_{T_2} = h_{T_3} = 0, \\
 f_1 &= g_1 = h'_S = h'_P = h_1 = \zeta(w), \\
 f_2 &= f_3 = g_2 = g_3 = h_2 = h_3 = h_4 = 0, \tag{A28}
 \end{aligned}$$

where $\xi(w)$ and $\zeta(w)$ are the leading-order IW functions for ground-state mesons and ground-state baryons, respectively, satisfying $\xi(1) = \zeta(1) = 1$. For the mesonic transition, we take corrections of the heavy quark expansion into account at $\mathcal{O}(\alpha_s, \bar{\Lambda}/m_{b,c}, \bar{\Lambda}^2/m_c^2)$ [34]. For the baryonic transition, by following Refs. [33,39], we take corrections of heavy quark expansion into account at $\mathcal{O}(\alpha_s, \bar{\Lambda}/m_{b,c}, \alpha_s \bar{\Lambda}/m_{b,c}, \bar{\Lambda}^2/m_c^2)$.

Appendix B: Generic formula

In this appendix, we provide a set of numerical formulae of the charged-lepton polarization and the forward-backward asymmetry. They are defined by

$$P_l^{H_c} = \frac{\Gamma_{\lambda_l=1/2}^{H_c} - \Gamma_{\lambda_l=-1/2}^{H_c}}{\Gamma_{\lambda_l=1/2}^{H_c} + \Gamma_{\lambda_l=-1/2}^{H_c}}, \quad \Gamma_{\lambda_l}^{H_c} = \int_1^{w_{H_c, \max}} dw \int_{-1}^1 dc_{\theta_l} \frac{d^2 \Gamma^{\lambda_l}(H_b \rightarrow H_c l \bar{\nu})}{dw dc_{\theta_l}}, \quad (B1)$$

$$A_{\text{FB}}^{H_c l} = \frac{\Gamma_{c_{\theta_l} > 0}^{H_c} - \Gamma_{c_{\theta_l} < 0}^{H_c}}{\Gamma_{c_{\theta_l} > 0}^{H_c} + \Gamma_{c_{\theta_l} < 0}^{H_c}}, \quad \Gamma_{c_{\theta_l} > (<) 0}^{H_c} = \sum_{\lambda_l = \pm 1/2} \int_1^{w_{H_c, \max}} dw \int_{0(-1)}^{1(0)} dc_{\theta_l} \frac{d^2 \Gamma^{\lambda_l}(H_b \rightarrow H_c l \bar{\nu})}{dw dc_{\theta_l}}, \quad (B2)$$

with $w_{H_c, \max} = (m_{H_b}^2 + m_{H_c}^2 - m_l^2)/(2m_{H_b}m_{H_c})$. Then, the generic formulae of $(P_\tau^{H_c}/P_\tau^{H_c, \text{SM}})(R_{H_c}/R_{H_c}^{\text{SM}})$ and $(A_{\text{FB}}^{H_c \tau}/A_{\text{FB}}^{H_c \tau, \text{SM}})(R_{H_c}/R_{H_c}^{\text{SM}})$ are obtained as

$$\frac{P_\tau^D}{P_\tau^{\text{SM}}} = \left(\frac{R_D}{R_D^{\text{SM}}} \right)^{-1} \times \left(|1 + C_{V_L}^\tau|^2 + 3.02(0.01)|C_{S_L}^\tau + C_{S_R}^\tau|^2 + 0.16(0.01)|C_T^\tau|^2 + 4.47(0.02)\text{Re}[(1 + C_{V_L}^\tau)(C_{S_L}^\tau + C_{S_R}^\tau)^*] - 1.06(0.03)\text{Re}[(1 + C_{V_L}^\tau)C_T^{\tau*}] \right), \quad (B3)$$

$$\frac{P_\tau^{D^*}}{P_\tau^{\text{SM}}} = \left(\frac{R_{D^*}}{R_{D^*}^{\text{SM}}} \right)^{-1} \times \left(|1 + C_{V_L}^\tau|^2 - 0.09(0.01)|C_{S_R}^\tau - C_{S_L}^\tau|^2 - 1.88(0.02)|C_T^\tau|^2 + 0.26(0.02)\text{Re}[(1 + C_{V_L}^\tau)(C_{S_L}^\tau - C_{S_R}^\tau)^*] - 3.50(0.04)\text{Re}[(1 + C_{V_L}^\tau)C_T^{\tau*}] \right), \quad (B4)$$

$$\frac{P_\tau^{\Lambda_c}}{P_\tau^{\Lambda_c, \text{SM}}} = \left(\frac{R_{\Lambda_c}}{R_{\Lambda_c}^{\text{SM}}} \right)^{-1} \times \left(|1 + C_{V_L}^\tau|^2 - 1.48(0.01) \times \text{Re}[(1 + C_{V_L}^\tau)C_{S_R}^{\tau*}] - 0.97(0.01)\text{Re}[(1 + C_{V_L}^\tau)C_{S_L}^{\tau*}] - 1.47(0.02) \times \text{Re}[C_{S_L}^\tau C_{S_R}^{\tau*}] - 0.92(0.01)(|C_{S_L}^\tau|^2 + |C_{S_R}^\tau|^2) - 3.59(0.05) \times \text{Re}[(1 + C_{V_L}^\tau)C_T^{\tau*}] - 3.30(0.03)|C_T^\tau|^2 \right), \quad (B5)$$

$$\frac{A_{\text{FB}}^{D\tau}}{A_{\text{FB}}^{D\tau, \text{SM}}} = \left(\frac{R_D}{R_D^{\text{SM}}} \right)^{-1} \times \left(|1 + C_{V_L}^\tau|^2 + 1.19(0.01) \times \text{Re}[(1 + C_{V_L}^\tau)(C_{S_R}^\tau + C_{S_L}^\tau)^*] + 2.40(0.03) \times \text{Re}[(1 + C_{V_L}^\tau)C_T^{\tau*}] + 3.04(0.04) \times \text{Re}[(C_{S_R}^\tau + C_{S_L}^\tau)C_T^{\tau*}] \right), \quad (B6)$$

$$\frac{A_{\text{FB}}^{D^*\tau}}{A_{\text{FB}}^{D^*\tau, \text{SM}}} = \left(\frac{R_{D^*}}{R_{D^*}^{\text{SM}}} \right)^{-1} \times \left(|1 + C_{V_L}^\tau|^2 - 2.61(0.60) \right.$$

$$\times \text{Re}[(1 + C_{V_L}^\tau)(C_{S_R}^\tau - C_{S_L}^\tau)^*] + 31.4(5.9)\text{Re}[(1 + C_{V_L}^\tau)C_T^{\tau*}] - 107(17)|C_T^\tau|^2 + 12.7(2.9)\text{Re}[(C_{S_R}^\tau - C_{S_L}^\tau)C_T^{\tau*}] \Big), \quad (B7)$$

$$\frac{A_{\text{FB}}^{\Lambda_c \tau}}{A_{\text{FB}}^{\Lambda_c \tau, \text{SM}}} = \left(\frac{R_{\Lambda_c}}{R_{\Lambda_c}^{\text{SM}}} \right)^{-1} \times \left(|1 + C_{V_L}^\tau|^2 + 6.87(0.63) \times \text{Re}[(1 + C_{V_L}^\tau)C_{S_R}^{\tau*}] + 1.87(0.12)\text{Re}[(1 + C_{V_L}^\tau)C_{S_L}^{\tau*}] - 19.6(1.2)\text{Re}[(1 + C_{V_L}^\tau)C_T^{\tau*}] + 115(10)|C_T^\tau|^2 + 26.1(2.6)\text{Re}[C_{S_R}^\tau C_T^{\tau*}] + 0.020(0.008)\text{Re}[C_{S_L}^\tau C_T^{\tau*}] \right). \quad (B8)$$

The number in parentheses denotes the uncertainty, e.g., 3.02(0.01) means 3.02 ± 0.01 that comes from the form factor input and is approximated by Gaussian distributions. The result agrees with Ref. [35] within uncertainty.

Appendix C: More on the sum rule in the heavy quark limit

In the heavy quark limit, the sum rule for the double-differential decay rates of Eq. (8) can be transformed as

$$\frac{\kappa_{\Lambda_c}^{w, +1/2} - \kappa_{\Lambda_c}^{w, -1/2}}{\kappa_{\Lambda_c \text{ SM}}^{w, +1/2} - \kappa_{\Lambda_c \text{ SM}}^{w, -1/2}} = \alpha_{\text{HQL}}^{P_\tau} \frac{\kappa_D^{w, +1/2} - \kappa_D^{w, -1/2}}{\kappa_{D \text{ SM}}^{w, +1/2} - \kappa_{D \text{ SM}}^{w, -1/2}} + \beta_{\text{HQL}}^{P_\tau} \frac{\kappa_{D^*}^{w, +1/2} - \kappa_{D^*}^{w, -1/2}}{\kappa_{D^* \text{ SM}}^{w, +1/2} - \kappa_{D^* \text{ SM}}^{w, -1/2}}, \quad (C1)$$

$$\frac{\kappa_{\Lambda_c+}^w - \kappa_{\Lambda_c-}^w}{\kappa_{\Lambda_c+ \text{ SM}}^w - \kappa_{\Lambda_c- \text{ SM}}^w} = \alpha_{\text{HQL}}^{A_{\text{FB}}} \frac{\kappa_{D+}^w - \kappa_{D-}^w}{\kappa_{D+ \text{ SM}}^w - \kappa_{D- \text{ SM}}^w} + \beta_{\text{HQL}}^{A_{\text{FB}}} \frac{\kappa_{D^*+}^w - \kappa_{D^*-}^w}{\kappa_{D^*+ \text{ SM}}^w - \kappa_{D^*- \text{ SM}}^w}, \quad (C2)$$

where the differential decay rates in denominators are the SM ones, i.e., $C_X^\tau = 0$. Also, $\kappa_{H_c}^{w, \lambda_\tau} = \frac{d\Gamma_{H_c}^{\lambda_\tau}}{dw}$, $\kappa_{H_c \pm}^w = \pm \int_0^{\pm 1} dc_{\theta_\tau} \frac{d^2 \Gamma_{H_c}^w}{dw dc_{\theta_\tau}}$, and $\frac{d^2 \Gamma_{H_c}^w}{dw dc_{\theta_\tau}} = \frac{d^2 \Gamma_{H_c}^{+1/2}}{dw dc_{\theta_\tau}} + \frac{d^2 \Gamma_{H_c}^{-1/2}}{dw dc_{\theta_\tau}}$ are defined. The coefficients are given by

$$\alpha_{\text{HQL}}^{P_\tau} = \frac{2}{1+w} \frac{\xi(w)^2}{\xi(w)^2} \frac{\kappa_{D \text{ SM}}^{w, +1/2} - \kappa_{D \text{ SM}}^{w, -1/2}}{\kappa_{\Lambda_c \text{ SM}}^{w, +1/2} - \kappa_{\Lambda_c \text{ SM}}^{w, -1/2}} = \frac{(1+r_D)^2(1+\rho_\tau^2 - w(1-2\rho_\tau^2)) - 6r_D\rho_\tau^2(1+w)}{4r_D(1-2\rho_\tau^2 + w^2(2-\rho_\tau^2)) - 6w(1-\rho_\tau^2)(1+r_D^2)},$$

$$\alpha_{\text{HQL}}^{A_{\text{FB}}} = \frac{2}{1+w} \frac{\xi(w)^2}{\xi(w)^2} \frac{\kappa_{D+ \text{ SM}}^w - \kappa_{D- \text{ SM}}^w}{\kappa_{\Lambda_c+ \text{ SM}}^w - \kappa_{\Lambda_c- \text{ SM}}^w} = \frac{-(1-r_D^2)\rho_\tau^2}{2(1-\rho_\tau^2 + r_D^2(1+\rho_\tau^2) - 2r_D w)}, \quad (C3)$$

where $\rho_\tau = m_\tau/\sqrt{q^2}$ and $r_i = m_i/m_B$ are defined. They are independent of the leading-order IW functions $\zeta(w)$ and

$\xi(w)$, and satisfy $\alpha_{\text{HQL}} + \beta_{\text{HQL}} = 1$, as the sum rule should hold in the SM limit. α_{HQL} and β_{HQL} are independent of NP contributions C_X , and hence the sum rule holds in any NP model captured in Eq. (6). Similar to the construction of the sum rule for R_{H_c} , the sum rules involving angular observables, Eqs. (10) and (11), are obtained by integrating numerators and denominators over w and by normalizing with the decay widths of light-lepton modes. It is noted that $\Gamma(H_b \rightarrow H_c \tau \bar{\nu})$ dependence is canceled in the combinations of $P_{\tau}^{H_c} R_{H_c}$ and $A_{\text{FB}}^{H_c} R_{H_c}$ for each.

Appendix D: Figures

In this appendix, we show the plot of probability distributions. In each plot, we generated 20,000 Monte Carlo samples assuming the form factor parameters have Gaussian probability distributions. Figure 1 (Fig. 2) shows the probability distribution of $\alpha_{P_{\tau}} (\alpha_{A_{\text{FB}}})$ and coefficients in the corrections to the sum rule, $\delta_{P_{\tau}}^{ij} (\delta_{A_{\text{FB}}}^{ij})$. Here, the results are not normalized. Also, correlations exist among the distributions, but they are not shown. Figure 3 (Fig. 4) shows the probability distribution of the correction $\delta_{P_{\tau}} (\delta_{A_{\text{FB}}})$ for the benchmark scenarios in Table 1. The correction is zero in the S_R and U_1 LQ scenarios, and hence is omitted.

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