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Probabilistic Reliability Analysis for Bolted Joints Considering Corrosion-Induced Failures

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Abstract

Bolted joints are essential in mechanical systems, but corrosion significantly impacts their reliability, leading to premature failure and increased maintenance costs. This This study introduces a systematic framework for reliability analysis by integrating tensile, fatigue, and corrosion-induced failures within a unified model. A combination of analytical and probabilistic methods, including FORM, Monte Carlo simulations, and Markov models, are employed to estimate failure probabilities, capture system degradation, and address uncertainties in loading conditions and material properties. Sensitivity analysis highlights the critical influence of corrosion rate on joint reliability. This approach supports failure prediction, maintenance strategies, reducing resource consumption, and extending the service life of components, thus enhancing sustainability and contributing to circular economy. Future work will focus on experimental validation and Bayesian modeling to refine predictions and enhance accuracy adaptability.

Keywords: reliability, markov model, monte carlo, remanufacturing, sustainability, circular economy

1. Introduction

Bolted joints are among the most commonly used machine elements, fundamental to mechanical and mechatronic systems' structural integrity and functionality [1]. These joints are ubiquitous, and found in industries ranging from aerospace and automotive to structural engineering. In the current era of circular economy and sustainability, predicting the state of bolted joints is crucial for early maintenance, timely repairs, and effective end-of-life strategies, collectively known as the R-strategies [2]. To achieve these goals, accurate system analysis is essential. Building on the importance of system analysis, reliability analysis offers critical insights into potential failure modes and predicts conditions that could compromise performance [3-5]. By identifying weak points early, reliability analysis supports predictive maintenance, minimizing downtime, reducing repair costs, and extending service life [4]. In a circular economy, reliability analysis is particularly valuable as it estimates the reliability metrics like, remaining useful life (RUL) [6,7], Mean Time to Failure (MTTF) and Mean Time Between Failure (MBTF) [8], of components in returned products [9], guiding decisions within the R-strategies [10]. Accurately predicting RUL ensures that bolted joints are optimally reintegrated or properly disposed of, promoting resource efficiency and sustainability [8].

There has been a growing focus on the reliability of mechanical systems, emphasizing models to predict component life under various operational stresses [11]. Research on bolted joint reliability, in particular, has primarily focused on optimization techniques [12], and failure mechanisms such as tensile [13], fatigue [14], self-loosening [15], and corrosion [16,17] as well as predictive models [18]. Optimization efforts, including bolt layout, material selection, and tightening sequences [12], aim to enhance load distribution and reliability, particularly in high-stress applications like aerospace. Furthermore, Tensile failures occur when loads exceed material strength [13], while fatigue results from cyclic loading [14], gradually weakening the material, and corrosion progresses over time due to environmental factors, often remaining unnoticed until substantial damage occurs [16,17]. Despite valuable insights, existing research typically addresses these failure modes separately, highlighting a gap for a unified, practical framework that integrates multiple degradation mechanisms to better support predictive maintenance and

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extend the operational life of bolted joints across diverse conditions.

In addition, in the reliability analysis research further probabilistic methods, such as Weibull analysis, Monte Carlo simulations [8], First Order Reliability Method, Second Order Reliability Method [19], Markov Model, and Bayesian Model [20] have been widely applied to predict failure rate, reliability Index, transition probability, failure probabilities, MTTF, MTBF and RUL. While these models are well-developed for complex systems [5,8,19,20], there remains a research gap in simplified and systematic reliability models specifically tailored for bolted connection.

This study presents a flexible and extensible framework to tackle the research gap in analyzing the reliability of bolted joints by integrating degradation with multiple failure modes. With a comprehensive analysis that also addresses uncertainty differently. The approach models the bolted joints strength and preload force based on different distributions for both tensile and fatigue loads while incorporating the degradation rate in both cases to model the effect of corrosion along with each failure mode. The modeled data are then used for the reliability model using FORM to approximate the first state function. The uncertainty of the model is then addressed from different perspectives by using Monte Carlo for bolt model data, Markov model to handle uncertainty in the reliability model as well as between failure modes, and finally a Sensitivity Analysis to address the overall uncertainty in the analysis. This comprehensive methodology is intended to improve the accuracy and adaptability of bolted joint reliability analysis.

2. Methodology

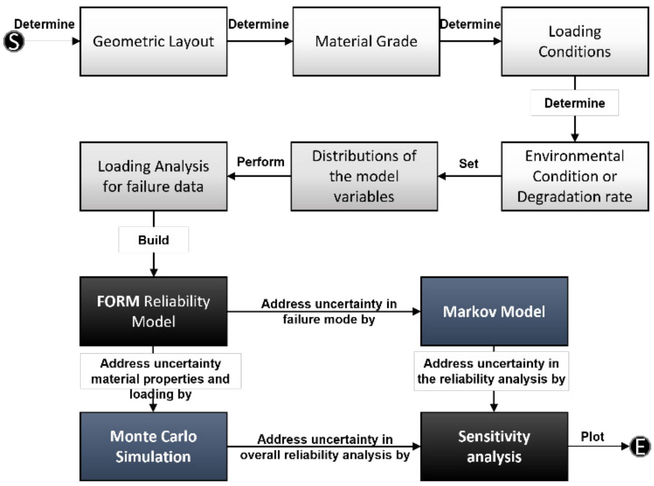


Fig. 1 Stepwise reliability analysis workflow addressing model uncertainties.

This study presents a comprehensive reliability analysis of bolted connections, focusing on a system reliability model that incorporates tensile and fatigue, along with corrosion-induced failures. The proposed methodology employs a combination of analytical and probabilistic methods to account for component failure modes, time-dependent reliability, and system-level interactions. This work addresses different sources of uncertainties through a comprehensive analysis. An overview of the methodology used is shown in Fig. (1).

2.1. Geometric Layout:

This study implements a comprehensive reliability modeling framework for an M10 8.8 bolted joint system seen in Fig. (2), consisting of an M10 hex bolt [21], two square steel plates (3 mm each) [22], one M10 washer [23], and one M10 hex nut [24] all made from blank steel. The chosen layout and material class facilitates standardized measurements and loading conditions and allows for the application of well-established models and historical data from the literature.

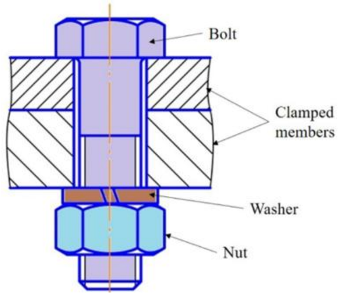


Fig. 2 Geometric Layout of the used Bolted Joint [12]

2.2. Initial Data Modeling for Bolted Joint

The aim of this data modeling process is to generate synthetic datasets that simulate real-world conditions for bolted connections, serving as a foundation for further reliability analysis. The synthetic data is grounded in analytical equations and historical norms [21-25], allowing for near-realistic representation of material and loading conditions. Material properties and loading conditions were defined based on distributions from historical data and standards, as outlined in Tables 1 and 2. The modeling was conducted to reflect two primary failure scenarios: bolted joints under tensile stress and fatigue under cyclic loading both with corrosion effects.

Table 1. Distributions of model's material properties variables.

Material Properties	Sampling Distribution	Distribution Parameters	
		μ	σ
$\sigma_{ultimate}$	Normal	800 MPa	5%
σ_{yield}	Normal	650 MPa	5%
$\sigma_{fatigue}$	Uniform	0.5 – 0.8	
Safety Factor	Uniform	1.5 - 2.0	

Table 2. Distributions of model's Loading Conditions.

Material Properties	Sampling Distribution	Distribution Parameters
$F_{preload}$	Uniform	12 – 30 KN
σ_{cyclic}	Uniform	25 – 200 MPA

To incorporate the effects of corrosion in tensile stress data, we used a corrosion rate provided by [26] to calculate material degradation over time. First, the material loss due to corrosion was estimated. This material loss was then used to adjust the bolt's diameter, simulating the effect of corrosion on cross-sectional area. With this adjusted area, we determined the tensile stress under corrosion by dividing the total tensile force, preload and external force, by the calculated area. Similarly, to incorporate corrosion effects in fatigue data under cyclic loading, the original fatigue limit was adjusted to account for cumulative material degradation. This adjustment was made by reducing the original fatigue limit based on the corrosion factor, yielding a corrosion-adjusted fatigue limit. Using this adjusted fatigue limit, synthetic data were generated for cyclic stress based on the distributions provided in Table 2. The corrosion-adjusted fatigue limit was applied in each instance of cyclic loading. This data modeling approach generates realistic, analytically grounded datasets for assessing the reliability of bolted connections under tensile and fatigue conditions with corrosion, supporting accurate reliability analysis under varying loads and environmental conditions.

2.3. Reliability Model using First Order Reliability Method

The First Order Reliability Method (FORM) is used to approximate failure probabilities by linearizing a complex limit-state function, $g(X)$, which separates safe and failure states, using a first-order Taylor expansion at the most probable point (MPP) of failure [19]. The reliability index β, seen in Eq. (2), calculated as the minimum distance from the origin in transformed space to the limit-state surface, quantifies system reliability, where μ_g and σ_g represent the mean and standard deviation of the limit-state function $g(X)$. The probability of failure P_f is then calculated using Eq. (3), where Φ is the standard normal cumulative distribution function [27-29].

$$\beta = \frac{\mu_g}{\sigma_g} \qquad (1)$$

$$P_f = \Phi(-\beta) \qquad (2)$$

In this study, FORM evaluates the reliability of bolted joints with the aforementioned geometric layout subject to tensile, fatigue, alongside corrosion-induced failures. Distributions of material properties, preload forces, and cyclic stresses represent uncertainty in input data, and corrosion is modeled as a time-dependent degradation factor. For corrosion-induced tension failure, the limit-state function is seen in Eq. (4), where the $\sigma_{applied}$, and $\sigma_{corrosion}$ represent tensile preload stress, and corrosion-induced degradation, respectively. For fatigue-corrosion interactions, cyclic stress and corrosion rate influence the endurance limit, as shown in Eq. (5), where k is an empirical factor, and CR is the corrosion rate.

$$g(X) = \sigma_{yield} - \sigma_{applied} - \sigma_{corrosion} \qquad (3)$$

$$g(X) = \sigma_{fatigue} - \sigma_{cyclic}(1 - k \cdot CR) \qquad (4)$$

$$MTTF = \int_0^\infty t \cdot f(t)\, dt \qquad (5)$$

FORM also allows estimation of MTTF through Eq. (6), integrating the failure density function f(t). RUL can be calculated as the difference between the critical time and current time. Applying FORM to these functions enables the assessment of failure probabilities under varying preload and cyclic stress levels, considering corrosion's progressive effect on reliability and expected service life.

2.4. Monte Carlo Simulation

To further address uncertainties in material properties and loading conditions, a Monte Carlo simulation is applied. Monte Carlo simulation is a statistical technique used to model and analyze complex systems by performing a large number of random samplings from probability distributions that represent uncertain parameters [30,31]. In reliability analysis, Monte Carlo simulation addresses the inherent uncertainties in input variables by generating numerous possible scenarios based on given distributions. In the context of bolted joint, it samples from distributions material properties and loading conditions in Table 1 and 2.

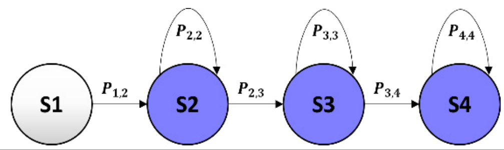
The Monte Carlo generates random time-to-failure values t_i for each mode using the inverse transform sampling method, derived as Eq. (7); where, U is a uniformly distributed random variable between 0 and 1. And t_i represents the simulated time-to-failure for each instance. By iterating this process across multiple trials, Monte Carlo simulations enable the aggregation of failure times into a distribution that reflects the variability inherent in the system [8]. The cumulative reliability function from the simulations is represented as Eq. (8). This approach yields a probabilistic estimate of the system's reliability and time-to-failure profile.

$$t_i = \eta(-\ln(1-U))^{1/\beta} \qquad (6)$$

$$R_{system}(t) = \frac{No.\ of\ simulations\ with\ no\ failure\ by\ time}{Total\ number\ of\ simulations} \qquad (7)$$

2.5. Markov Model

Furthermore, the Markov model is employed to further refine reliability analysis by offering a probabilistic, time-dependent framework to address uncertainties in failure mode progression. The Markov model is a stochastic process that describes a system's evolution through a series of discrete states, where transitions between states occur based on defined probabilities rather than deterministic rules [31]. Each state transition is characterized by a probability that depends only on the current state, making it well-suited to model systems with progressive failure mechanisms and time-dependent uncertainties. In reliability analysis, Markov models quantify the likelihood of moving between degradation stages, providing insights into both failure probability and progression through intermediate states. This study uses the Markov model to address uncertainties in the bolted joint system by modeling two primary failure pathways: tensile failure under corrosion and corrosion fatigue. For tensile-corrosion, the model defines



State	S1	S2	S3	S4
Tensile Corrosion	Safe	Mild Deformation	Deformation	Failure
Fatigue Corrosion	Safe	Crack Initiation	Crack Propagation	Failure

Fig. 3 Markov Chain for Failure Transitions

three failure stages as shown in Fig. 3 that reflect progressive degradation as corrosion weakens tensile strength. Similarly, for corrosion fatigue, three stages model crack growth under cyclic loads combined with corrosion as shown in Fig. 3. This Markov-based approach generates a transition probability matrix seen in Table 3 and 4 for each failure mode, respectively, capturing the joint's likelihood of progressing through or remaining in each degradation stage.

Table 3. Transition Probability Matrix for Tensile-Corrosion Failure

State	Mild Deformation	Deformation	Failure
Mild Deformation	$1-\frac{\sigma(t)}{\sigma_{yield}}$	$\frac{\sigma(t)}{\sigma_{yield}}$	0
Deformation	0	$P = \frac{\sigma(t)_{ultimate} - \sigma(t)}{\sigma(t)_{ultimate} - \sigma_{yield}(t)}$	1-P
Failure	0	0	1

Table 4. Transition Probability Matrix for Fatigue-Corrosion Failure

State	Safe - Initiation	Crack Initiation - Propagation	Failure
Safe -Initiation	$1-\frac{n}{N(t)}$	$\frac{n}{N(t)}$	0
Crack Initiation-Propagation	0	$1-\frac{a_n}{a_c}$	$\frac{a_n}{a_c}$
Failure	0	0	1

2.6. Sensitivity Analysis

The final stage applies Sensitivity Analysis to quantify the influence of individual input parameters on system reliability. Sensitivity Analysis is essential in reliability studies, as it examines the responsiveness of the model's outputs to variations in key parameters, thereby identifying those that most critically affect failure probability [32]. By systematically varying factors such as corrosion rate. This analysis captures how each input impacts the probability of failure, enabling a focused evaluation of parameter sensitivity. The sensitivity S of the system reliability R with respect to a parameter x can be calculated using Eq. (9) [32].

$$S = \left(\frac{\partial R_{system}}{\partial x}\right) \times \left(\frac{x}{R_{system}}\right) \quad (8)$$

In this study, Sensitivity Analysis is used to address the uncertainty inherent in reliability modeling by narrowing down the parameters most relevant to the bolted joint's performance. Specifically, it identifies the parameters that exhibit the highest correlation with failure likelihood under corrosion-tensile and corrosion-fatigue conditions.

3. Results

In this section, we present the results of the reliability analysis for two failure modes: tensile-corrosion failure and fatigue-corrosion failure. Both modes incorporate the effects of corrosion alongside tensile and fatigue loading to provide a comprehensive view of bolted joint degradation over time. The analysis combines results from the FORM, Monte Carlo simulations, Markov transition modeling, and Sensitivity Analysis to assess the reliability and predict failure behavior under varying conditions.

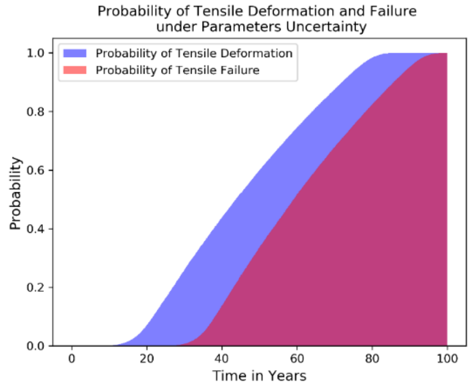


Fig. 4 Monte Carlo Simulation of Tensile-Corrosion

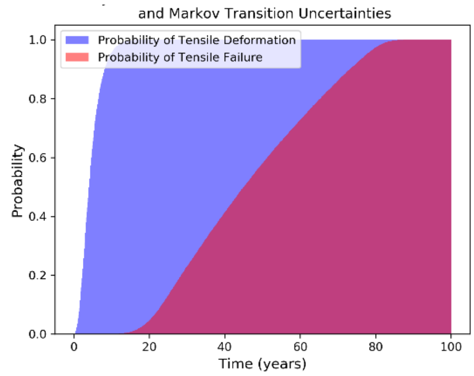


Fig. 5 Markov Model of Tensile-Corrosion Failure

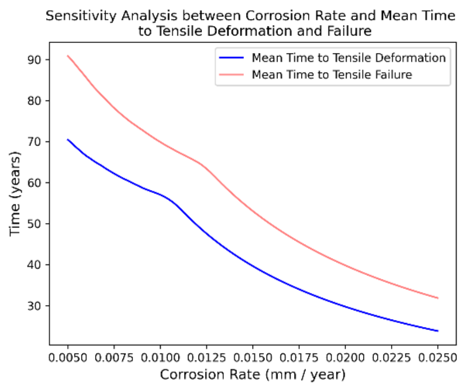


Fig. 6 Sensitivity Analysis of Tensile-Corrosion

The probability of tensile deformation and failure over time as derived from Monte Carlo is illustrated in Fig. (4). The probability of tensile deformation increases steadily, reaching near certainty around 50 years, while tensile failure probability follows a delayed trajectory. This progression indicates a clear sequence from deformation to eventual failure under combined tensile loading and corrosion effects, highlighting the gradual degradation process captured by the simulation. Fig. (5) displays the Markov model outcomes, illustrating transition probabilities between tensile deformation and failure states under uncertainty. Initially, the probability of remaining in the deformation state is dominant, but over time, the probability of transitioning to failure increases, surpassing deformation. This transition pattern reflects the Markov model's ability to capture the progressive nature of failure, driven by both parameter and transition uncertainties. Results also demonstrates that Markov has more realistic results in comparison to Monte Carlo. The results of the Sensitivity Analysis are presented in Fig. (6). It is showing the relationship between corrosion rate and mean time to both tensile deformation and failure. The analysis reveals a strong inverse relationship: as corrosion rate increases, the mean time to deformation and failure decreases significantly. This result underscores the corrosion rate as a highly influential factor, with even minor increases in corrosion substantially shortening the bolted joint's service life.

Building on the previous results for tensile-corrosion failure,

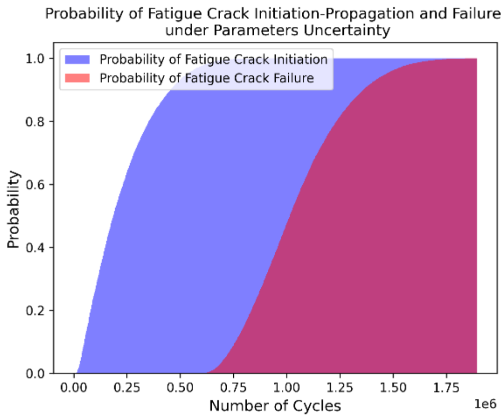


Fig. 7 Monte Carlo Simulation of Fatigue-Corrosion Failure

the following analysis focuses on the fatigue-corrosion failure mode, exploring how cyclic loading combined with corrosion affects the probability of failure and sensitivity to corrosion rate. These results provide further insights into the degradation behavior of the bolted joint under fatigue conditions. Fig. (7) illustrates the probability of failure over time under fatigue-corrosion conditions. The probability of fatigue stress failure initially rises slowly, reflecting the gradual accumulation of damage. However, the cumulative damage failure probability shows a more rapid increase. This progression indicates that cumulative damage due to cyclic loading and corrosion

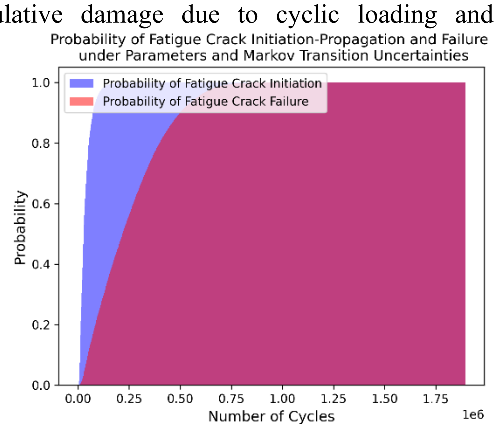


Fig. 8 Markov Model of Fatigue-Corrosion Failure

significantly accelerates failure. The cumulative probability of fatigue crack initiation and progression to failure over the number of cycles seen in Fig. (8), incorporating the effects of corrosion rate on the degradation process. The blue region represents the probability of crack initiation, which grows significantly in the initial cycles due to accelerated corrosion effects that reduce the material's resistance to fatigue. The red region illustrates the cumulative probability of crack propagation leading to failure, showing a steep increase as cycles progress. This combined analysis underscores the impact of corrosion in accelerating both crack initiation and progression, resulting in a higher likelihood of failure within a shorter fatigue life under corrosive conditions

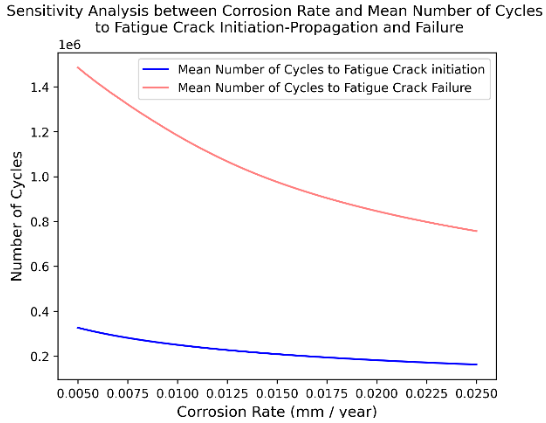


Fig. 9 Sensitivity Analysis of Fatigue-Corrosion Failure

Fig. (9) presents the Sensitivity Analysis of the mean time to fatigue stress and cumulative damage failure with respect to the corrosion rate. As the corrosion rate increases, both the mean time to stress failure and cumulative damage failure decrease notably. The sensitivity curves demonstrate that higher corrosion rates drastically reduce fatigue life, with cumulative damage failure being especially sensitive to even slight increases in corrosion. This finding reinforces the critical influence of corrosion rate on the durability of the joint under cyclic loading, where even minor corrosion rates lead to a marked decrease in expected service life.

4. Discussion

This study provides a comprehensive approach to reliability analysis for bolted connections by examining the combined impact of multiple failure modes, specifically tensile and fatigue failures, while incorporating corrosion as a common degradation factor. One significant advantage of this approach is the robust data modeling that underpins the reliability analysis. Without access to experimental data, data modeling based on analytical equations and historical norms enabled the construction of realistic distributions for material properties and loading conditions. In addition, based on the model's high level of uncertainty, given the approximations inherent in synthetic data, Monte Carlo simulations address it effectively. The implementation of a Markov model further strengthens the analysis by introducing stochastic transition probabilities between failure states, rather than assuming deterministic outcomes, which tackles another level of uncertainty in the model. This probabilistic approach captures the progressive nature of failure and offers a more nuanced view of system degradation over time, especially under combined failure modes. Finally, Sensitivity Analysis adds another layer of uncertainty analysis, revealing how variations in corrosion rate and other parameters impact system reliability. This is particularly valuable for understanding the relative influence of corrosion on overall failure risk and informing targeted maintenance strategies.

However, several limitations exist in this methodology. Relying on data modeling, rather than experimental data, introduces a dependency on the accuracy of historical norms and assumed distributions. This dependency could potentially limit the precision of the reliability estimates. Additionally, while Monte Carlo simulation and the Markov model address uncertainty, they may require extensive computational resources, especially when modeling complex systems with numerous variables. The Sensitivity Analysis highlights the importance of corrosion rate, but this approach does not fully capture all environmental conditions that could affect corrosion progression, such as temperature or humidity.

5. Conclusion

This study proposes a reliability analysis framework for bolted joints, addressing tensile, fatigue, and corrosion-induced failures. Synthetic data, derived from historical norms, served as a substitute for experimental data, enabling the analysis. Failure probabilities and reliability indices were estimated using FORM, while Monte Carlo simulations accounted for uncertainties in material properties and loading conditions. The Markov model provided a dynamic perspective by capturing transition probabilities between failure modes, compensating for the limitations and uncertainties of deterministic analysis. Sensitivity analysis further addressed parameter uncertainties, identifying corrosion as a critical factor influencing reliability.

Future work should integrate experimental data and real-world environmental conditions to validate and enhance the proposed reliability framework. Extending the framework to include Bayesian methods represents a logical progression, as Bayesian modeling allows for the continuous updating of reliability estimates by integrating prior knowledge with new observations. This dynamic capability is suited for addressing uncertainties in degradation rates and failure progression under varying operational conditions. By complementing existing methods, such as FORM and Markov models, Bayesian approaches can enhance the framework's probabilistic assessment, providing more robust predictions, deeper insights into system-level interactions, and adaptive strategies for maintenance and reliability optimization.

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References

[1] Sauer, B. (2012). Grundlagen der Festigkeitsberechnung. Konstruktionselemente des Maschinenbaus 1: Grundlagen der Berechnung und Gestaltung von Maschinenelementen, 71-162.

[2] Matsumoto, M., Yang, S., Martinsen, K., & Kainuma, Y. (2016). Trends and research challenges in remanufacturing. International journal of precision engineering and manufacturing-green technology, 3, 129-142.

[3] Rausand, M., & Hoyland, A. (2003). System reliability theory: models, statistical methods, and applications (Vol. 396). John Wiley & Sons.

[4] Zhang, M., Amaitik, N., Wang, Z., Xu, Y., Maisuradze, A., Peschl, M., & Tzovaras, D. (2022). Predictive maintenance for remanufacturing based on hybrid-driven remaining useful life prediction. Applied Sciences.

[5] O'Connor, P., & Kleyner, A. (2012). Practical reliability engineering. John Wiley & Sons.

[6] Si, X. S., Wang, W., Hu, C. H., & Zhou, D. H. (2011). Remaining useful life estimation–a review on the statistical data driven approaches. European journal of operational research, 213(1), 1-14.

[7] Hu, Y., Liu, S., Lu, H., & Zhang, H. (2014). Remaining useful life assessment and its application in the decision for remanufacturing. Procedia Cirp, 15, 212-217.

[8] Modarres, M., Kaminskiy, M. P., & Krivtsov, V. (2016). Reliability engineering and risk analysis: a practical guide. CRC press.

[9] Grauberger, P., Dörr, M., Lanza, G., Kaiser, J.-P., Albers, A., Düser, T., Tusch, L., Seidler, M., Dietrich, S., Schulze, V., & Matthiesen, S. (2024). Enabling the vision of a perpetual innovative product – predicting function fulfillment of new product generations in a circular factory. At - Automatisierungstechnik, 72(9), 815–828.

[10] Ortegon, K., Nies, L., Sutherland, J.W. (2014). Reuse. In: Laperrière, L., Reinhart, G. (eds) CIRP Encyclopedia of Production Engineering. springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-642-20617-7_6614

[11] Jiang, S. H., Li, D. Q., Zhang, L. M., & Zhou, C. B. (2014). Time-dependent system reliability of anchored rock slopes considering rock bolt corrosion effect. Engineering Geology, 175, 1-8.

[12] Croccolo, D., De Agostinis, M., Fini, S., Khan, M. Y., Mele, M., & Olmi, G. (2023). Optimization of Bolted Joints: A Literature Review. Metals.

[13] Brown, K. H., Morrow, C. W., Durbin, S., & Baca, A. (2008). Guideline for bolted joint design and analysis: version 1.0 (No. SAND2008-0371). Sandia National Laboratories (SNL), United States.

[14] Toribio, J., González, B., Matos, J. C., & Ayaso, F. J. (2012). Fatigue behaviour of bolted joints. Metals and materials international, 18, 553-558.

[15] Zhao, Penghao, et al. "Study on Tightening, Anti-Loosening, and Fatigue Resistance Performances of Bolted Joints with Different Anti-Loosening Washers and Nuts." Applied Sciences 13.24 (2023): 13253.

[16] Shah, J. K., Majhi, S., Mukherjee, A., & Wang, H. (2024). Investigating corrosion-induced deterioration in bolted steel plate joints using guided wave ultrasonic inspection. Journal of Civil Structural Health Monitoring, [17] Solazzi, L., Scalmana, R., Gelfi, M., & La Vecchia, G. M. (2012). Effect of different corrosion levels on the mechanical behavior and failure of threaded elements. Journal of failure analysis and prevention.

[18] Fernández Ceniceros, Julio, et al. "Predictive models for bolted T-stub connections combining FEM with Intelligent Artificial techniques." COMPLAS XI: proceedings of the XI International Conference on Computational Plasticity: fundamentals and applications. CIMNE, 2011.

[19] Haldar, Achintya, and Sankaran Mahadevan. "First-order and second-order reliability methods." Probabilistic Structural Mechanics Handbook: theory and industrial applications. Boston, MA: Springer Us, 1995. 27-52.

[20] Ghahramani, Zoubin. "An introduction to hidden Markov models and Bayesian networks." International journal of pattern recognition and artificial intelligence 15.01 (2001): 9-42.

[21] EN ISO 4014: 2022 - 10, Fasteners – Hexagon head bolts – Product grades A and B (EN ISO 4014: 2022).

[22] DIN 436:2024-10 (D) Scheiben, vierkant, vorwiegend für Holzkon

[23] EN ISO 7089: 2000 11, Plain washers – Normal series, Product grade A

[24] EN ISO 4032: 2013-04, Hexagon regular nuts (style 1) – Product grades A and B (EN ISO 4032: 2012).

[25] VDI – Verein Deutscher Ingenieure 2230. Systematische berechnung hochbeanspruchter schraubenverbindungen zylindrische einschraubenverbindungen. Verein Deutscher Ingenieure (2015).

[26] International Organization for Standardization. ISO 9224:2006 - Corrosion of Metals and Alloys — Corrosivity of Atmospheres — Guiding alues for the Corrosivity Categories. Geneva: ISO, 2006.

[27] Haldar, A., & Mahadevan, S. (2000). Probability, reliability and statistical methods in engineering design.

[28] Haldar, A., & Mahadevan, S. (2000). Reliability assessment using stochastic finite element analysis. John Wiley & Sons.

[29] Ditlevsen, O., & Madsen, H. O. (1996). Structural reliability methods (Vol. 178). New York: Wiley.

[30] Rubinstein, R. Y., & Kroese, D. P. (2016). Simulation and the Monte Carlo method. John Wiley & Sons.

[31] Edition, F., Papoulis, A., & Pillai, S. U. (2002). Probability, random variables, and stochastic processes. McGraw-Hill Europe: New York, USA.

[32] Cacuci, D. G., Ionescu-Bujor, M., & Navon, I. M. (2005). Sensitivity and uncertainty analysis, volume II: applications to large-scale systems. CRC