Karlsruhe Institute of Technology

Sonification and Haptic Feedback for Mathematical Graph Accessibility

State of the art and future directions in assistive technologies and educational applications for users with visual impairments

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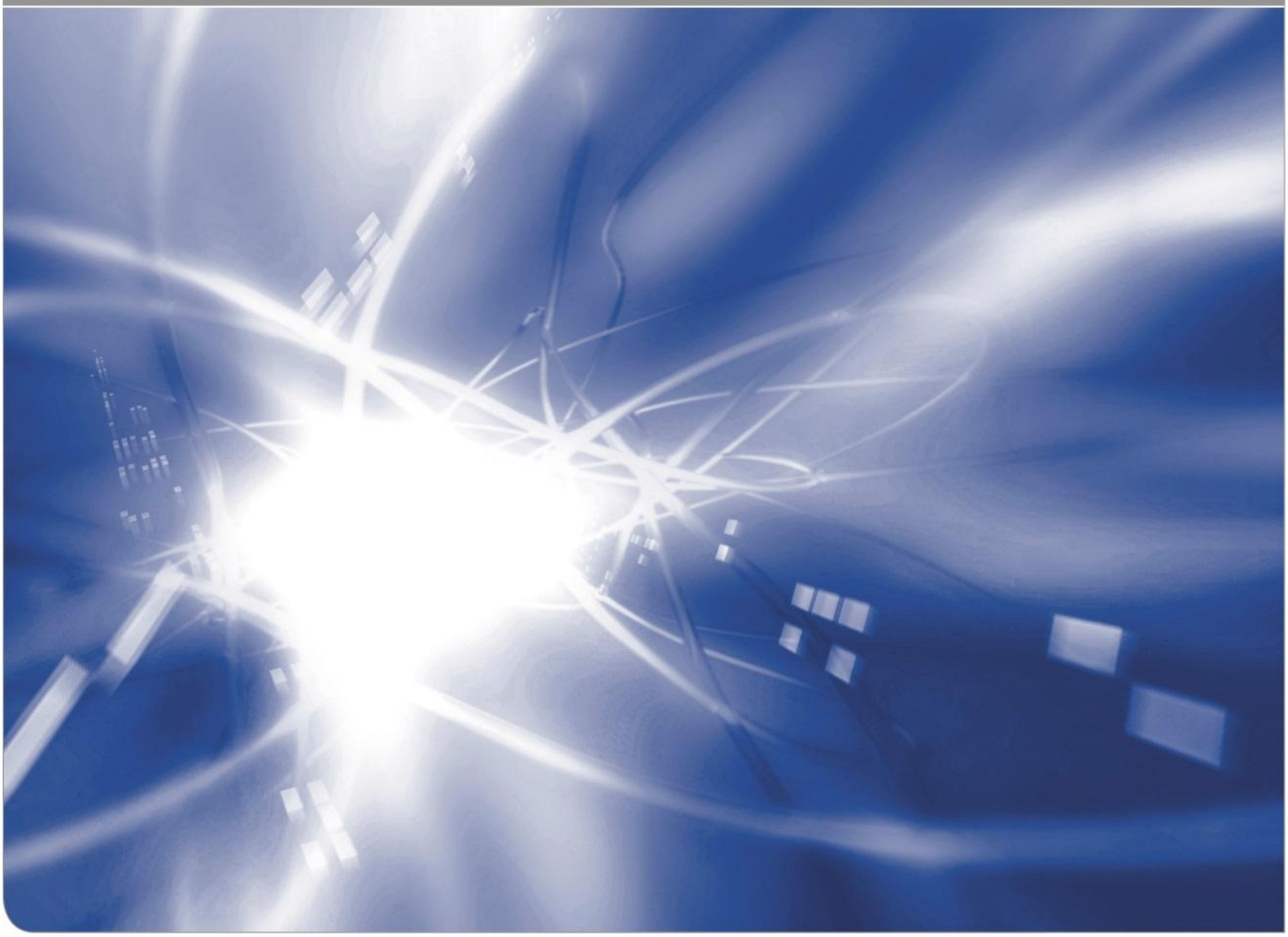

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Abstract

This report provides a comprehensive analysis of sonification and haptic feedback technologies for making mathematical function graphs accessible to visually impaired users. It examines existing tools across three main categories: audio-based sonification applications, tactile devices, and multimodal systems combining both approaches. The analysis categorizes research on this topic by target audience, sonification methods, topics, feedback interfaces, and teaching intentionality. The report also includes an in-depth examination of educational aspects specifically related to the teaching and learning of function graphs for visually impaired students, reviewing educational papers that explore mathematical learning barriers and investigate how sonification addresses these challenges. Despite promising results, critical gaps have been identified, including the absence of standardized frameworks for sonification and difficulties in mapping sound elements to certain graphical properties, highlighting the need for further research in technological and pedagogical implementation.

Zusammenfassung

Dieser Bericht bietet eine umfassende Analyse aktueller Technologien, die verwendet werden, um mathematische Funktionsgraphen für sehbeeinträchtigte Nutzende hör- und fühlbar zu machen. Dabei werden bestehende Werkzeuge der folgenden drei Hauptkategorien untersucht: Sonifikationsanwendungen, taktile Geräte und multimodale Systeme, die audio und taktile Ansätze kombinieren. Die Ansätze werden nach Zielgruppe, Sonifikationsmethoden, Themen, Feedback-Schnittstellen und didaktischer Intentionalität ausgewertet. Außerdem enthält der Bericht auch eine eingehende Untersuchung pädagogischer Aspekte, die sich speziell auf das Lehren und Lernen von Funktionsgraphen für sehbeeinträchtigte Schülerinnen und Schüler beziehen. Dafür werden pädagogische Arbeiten ausgewertet, die Barrieren beim Mathematiklernen erforschen und die untersuchen, wie Sonifikation diesen Herausforderungen begegnet. Trotz vielversprechender Ergebnisse wurden verschiedene Forschungslücken identifiziert, darunter das Fehlen standardisierter Konzepte für Sonifikation und Schwierigkeiten bei der Abbildung von Klängen auf bestimmte grafische Merkmale, was den Bedarf an weiterer Forschung in der technologischen und pädagogischen Umsetzung unterstreicht.

Contents

Introduction

This report intends to offer a comprehensive analysis of the existing literature on the application of sonification and haptic-tactile feedback in developing assistive technologies for users with visual impairments and for teaching and learning mathematical function graphs. Currently, haptic exploration represents the most widely implemented process for teaching and learning function graphs to visually impaired students. However, these tactile approaches face inherent limitations, being very expensive and static in nature. This has led researchers to explore alternative or complementary approaches, particularly sonification techniques. **Sonification**, as defined by Hermann et al. (2011), is "the technique of rendering sound in response to data and interactions" (p. 1). As a valuable technique for auditory visualisation, it enhances the interpretation and communication of information via non-speech audio representations. Therefore, this report explores existing tools utilizing sonification technology as well as those employing tactile interaction, providing a thorough overview of current assistive technologies in this field.

The report springs from the SONAIRGRAPH[1] project's objective to investigate the educational potential and challenges of using sonification techniques for the conceptualisation of function graphs for all users and as a way of providing access to this mathematical content for people with visual impairments through a **multimodal**, **embodied**, and **inclusive** approach. The project also aims to develop a scientific method to test sonification methods for the representation of mathematical function graphs. The report will serve not only as a valuable resource for the scientific community working on the development of assistive technologies for teaching and learning mathematics, but will also provide the foundation for the development and testing of an innovative digital application called **Audiofunctions**, which applies sonification for representing function graphs, using auditory feedback and proprioceptive techniques to create an interactive learning experience.

The report is divided into three main sections:

1. An analysis of existing tools - applications and technologies used to study mathematical functions by visually impaired users.
2. A literature review of papers addressing sonification and accessibility in mathematics, particularly functions, in the field of Human-Computer Interaction.
3. A short review on the role of sonification in teaching and learning mathematics, particularly functions, in the field of Education.

The two reviews, while sharing the focus on sonification of mathematical function and their accessibility for visually impaired users, present distinct yet complementary approaches and objectives. The first review, oriented towards Human-Computer Interaction, examines technical and design aspects of sonified interfaces, analysing usability, sonification algorithms,

[1] "SONAIRGRAPH - SONification for Accessible and Inclusive Representation of GRAPHs in Education", is a European project within the ERASMUS+ programme, in response to Call 2024 Round 1 KA2 - KA220-HED - Cooperation partnerships in higher education, CUP B63C24000960006. For more information, visit: https://sonairgraph.unito.it/

assistive devices, and user experience testing methodologies, with particular attention to digital accessibility standards and audio rendering techniques. The second review, in the Education field, will instead adopt a didactic perspective, examining learning effectiveness through sonification, and the impact on students' educational outcomes.

Analysis of existing tools

The analysis of existing tools for accessing mathematical function graphs for users with visual impairments is motivated by the need for instruments that assist students, teachers, or both in the teaching-learning process of this content. Some tools have emerged as the result of scientific research, others as assistive technologies, and some as developer libraries designed to perform tasks completely unrelated to the study of mathematical function graphs. However, most of them have been recognized as valuable within the SONAIRGRAPH project, either as complete tools or for specific components or concepts they introduce. Our research focuses on tools specifically designed to represent mathematical function graphs, tools created to represent data series, and tactile devices capable of displaying a limited set of points from an image. This selection is based on the fact that in digital environments all representations are discretized, including continuous function graphs, which must be approximated by a finite set of points derived from their analytical form.

Analytical categories

In the following sections, each analysed tool is categorized. Our aim is to characterize the selected instruments by analysing them according to several criteria (categories), which are detailed in subsequent sections of this report. The categories were defined a priori based on their significance in the field of accessibility, as well as differences that emerged during the SONAIRGRAPH team's research process. The rationale behind the choice of these categories is twofold: to provide an overview of each tool from different practical perspectives and to highlight the main features related to human-computer interaction.

Human-computer interaction (HCI) is the discipline that studies the best strategies for a person to interact with a system in a given context. Its primary objective is to enhance system usability, which is defined in terms of efficacy, efficiency, and user satisfaction. The interface serves as the primary touchpoint between the user and the system, facilitating communication and enabling users to perceive system responses. We analysed this communication through two categories: "**Depending on feedback**" and "**Interactivity**", with specific attention to interface design and feedback mechanisms to support persons with visual impairments.

Regarding the overview of each tool, we considered its nature to inform readers whether it is a software application, a web app, a physical device, or a programming library. This aspect is addressed in the "**Standalone app or library**" category. Additionally, given that our research explores tools for educational purposes, it is useful to assess whether studies have been conducted on their applicability in teaching. This aspect is examined in the "**Suitable or not for teaching**" category. Finally, we analysed how data is fed into each system. At the beginning of this introductory phase of the project, we recognized that we were considering systems with different natures and purposes, ranging from those designed to represent and analyse data series to those intended for exploring mathematical function graphs. In some tools, the underlying data remains transparent to the user, but we must consider whether a teacher needs to configure the system. This aspect is addressed in the "**Input data of the tool**" category.

Depending on feedback

A broad range of solutions has emerged to make data visualization more accessible through sonification, text descriptions, tactile feedback, or a combination of these methods. For instance, **Accessible Graphs**, **Chart Reader**, **Infosonics**, **MathTrax**, the **Sonify (R package)**, and **VoxLens** all provide ways to map numeric data onto audio signals. Some of these also generate additional explanatory layers: **MathTrax** uses a math solver and classifier to produce automatic text descriptions of functions (including information on minima, maxima, and zeros), which can then be read aloud by a screen reader; **Chart Reader** and **Infosonics** allow users to manually add brief introductions or annotations so that relevant insights are read out; and **VoxLens** offers a concise textual summary of the data. Textual overviews are likewise central to **Olli**, which creates a hierarchical structure that screen reader users can navigate to access either general or detailed information, depending on their preferred level of depth.

Several tools emphasize the synchronization of audio and visual components. **Audiographs** plays a tone whose pitch corresponds to a function's value, simultaneously highlighting the corresponding point on the chart. It can also identify points of interest—like minima or maxima—by naming them. **SenseMath** similarly maps pitch to the function's y-value while visually marking the current x-value, using earcons to flag significant points. The **Highcharts Sonification Studio** applies the same principle, highlighting the x-position whose y-value is being played; it can optionally highlight only the specific sonified point with details about its value. The **Desmos Graphing Calculator** lets users type a function and switch to "Audio trace" mode, where pitch denotes the y-value at the current x-point; negative values include static noise, and intersecting curves generate a distinctive "pop" sound. Beyond these user-facing tools, developer libraries for JavaScript (e.g., **Tonejs**) and Python (e.g., **Astronify**, **Strauss**, **Sonecules**, **SoniPy**, **Audio-plot-lib**) enable custom audio feedback. **Audio-plot-lib** extends typical sonification by incorporating a built-in text-to-speech feature that delivers initial spoken context—such as a reference pitch for the function's minimum—before playback begins.

To handle statistical data, the **SAS Graphics Accelerator** uses sequences of piano chords to represent discrete variable values, allowing users to perceive correlations through pitch changes. It also provides a data table and basic text descriptions so that screen readers can give an alternative overview. Meanwhile, tactile solutions focus on different types of 2D displays or printing technologies. **HyperBraille**, **DotPad**, **Orbit Graphiti**, **DotView DV-2**, and the **Monarch** are interactive tactile displays, where raised pins form a refreshable surface for exploring charts by touch. They vary in resolution (from 5 to 10 dpi, similar to braille), refresh speed (from fractions of a second to a few seconds), and functionality (for example, **DotPad** and **Monarch** cannot refresh areas currently being touched by the user, but **HyperBraille** allows dynamic updating even during interaction).

For users requiring permanent hard-copy material, **ViewPlus graphics embossers** and **swell paper printing** technologies offer static tactile graphics that can be labelled in braille. While these graphics are unchangeable once printed, they help users develop tactile reading skills gradually—from simple, uncluttered shapes to more complex diagrams containing overlapping elements. Finally, hybrid approaches combine physical paper-based charts with interactive electronics. **IVEO** (by ViewPlus), the **Tactonom Reader** (by Inventivio), and the **TPad** all use either embossed or swell paper for tactile exploration. These devices supplement tactile graphics with digital audio features, triggered by touching the paper (**IVEO**, **TPad**) or by pointing a camera-tracked finger (**Tactonom Reader**). Though more interactive than static prints, they typically lack dynamic features like zooming or real-time updates. By integrating audio descriptions, sonification, and tactile representations, each of these solutions addresses a different aspect of accessible data visualization, ensuring that visually impaired users can glean both high-level overviews and detailed insights from a wide variety of graphic content.

Interactivity

Many of these solutions vary in how much interactivity they provide to users. At one end of the spectrum, the R package **Sonify** offers little interactivity, focusing on straightforward sonification with minimal user controls. By contrast, tools such as **Accessible Graphs**, **Chart Reader**, **Infosonics**, **Olli**, and **VoxLens** let users navigate through their data via keyboard commands, allowing them to move step by step through different sections or data points. In addition, **MathTrax** functions as a standalone desktop application that supports both modifying the displayed function (or data) and interacting with menus and buttons to explore various features.

Audiographs offers a particularly high degree of interactivity for function exploration; users can navigate the data in batches, step by step, or jump directly to points of interest, all through keyboard commands. Conversely, **SenseMath** presents a form-based approach: the user specifies the function, the plot, and which features should be sonified during playback, but the playback itself is non-interactive—there is only the option to start and stop the audio. Similarly, the **Highchart Sonification Studio** enables simple playback controls (play, stop, pause) and primarily focuses on providing forms to customize sonification design parameters.

Looking at programming libraries, **Astronify**, **Strauss**, and **SoniPy** do not furnish interactive sonification out of the box. However, **Sonecules** supports interactions by enabling updates to parameters such as amplitude while the sonification is in progress, and **Audio-plot-lib** goes a step further by producing either a standalone .wav file or an interactive HTML page. In interactive mode, the user can manipulate a slider in the browser—either with a mouse or keyboard—to hear how pitch changes across the data. The **Tonejs** JavaScript library similarly supports real-time modifications to audio parameters, allowing users to alter sound characteristics during playback.

Turning to end-user applications once more, the **Desmos** graphing tool offers comprehensive keyboard navigation that gives screen reader users detailed information about zeros, intersection points, extrema, and any chosen coordinates. Finally, the **SAS Graphics Accelerator** provides keyboard shortcuts to quickly switch among different representations—such as audio playback of data series, tabular data views, or textual descriptions—and to adjust verbosity and other playback modes.

Standalone app or library

Many tools for accessible data visualization exist in a variety of formats and platforms. **MathTrax** is a Windows desktop application that depends on NASA's MDE, a Java-based framework, which also enables integration with other software. **Accessible Graphs** is a browser-based tool where users upload or input data to generate a new webpage for exploration.

Some solutions take the form of JavaScript libraries that require integration with other JavaScript plotting tools. These include **Chart Reader**, **Olli**, and **VoxLens**. Additionally, **Sonify** is a free R package that can be installed and used from within the R environment.

Focusing on standalone applications, **Audiographs** is a free Windows-only program, while **SenseMath** is a free iOS-only app available on the App Store. **Highchart Sonification Studio** is an online service accessible free of charge in any modern web browser.

For Python developers, the libraries **SoniPy**, **Audio-plot-lib**, **Astronify**, **Strauss**, and **Sonecules** are freely available through PyPI (via pip). In a similar vein, **Tonejs** is a JavaScript library available for free via npm.

Finally, some tools are fully online services: **Desmos** and the **SAS Graphics Accelerator** (after widget installation) can both be used without installation by visiting their respective websites, also at no cost.

Suitable or not for teaching

Many of these tools lend themselves to educational contexts, though each has its own strengths and requirements. **Desmos** stands out as a widely adopted classroom resource in U.S. schools, supporting synchronized visual and audio representations of functions, and featuring a large community of teachers who create and share pedagogical activities. Students can use **Desmos** during state-level assessments and college entrance exams, facilitating collaboration between blind and sighted learners.

Highchart Sonification Studio is also geared toward classroom or collaborative environments, allowing teachers and students to create, share, and explore sonification in a ready-to-use web interface. While it mainly works with data series rather than analytical functions, it can nevertheless aid students' understanding of mathematical principles. **SenseMath**, an iOS application, focuses on helping students study mathematical functions, harnessing iOS's built-in accessibility features to provide a comfortable learning experience for visually impaired users accustomed to Apple devices.

Other tools, though not explicitly designed for classroom instruction, could still be used in educational settings. **Audiographs** has an accessible interface and straightforward commands, suggesting potential classroom utility, though its documentation does not specifically reference educational use. **MathTrax**, despite no longer being actively maintained, offers text descriptions of graphs, sonification, and the ability to modify functions or data directly within the application—a set of features that can be valuable for teaching mathematics.

Several tools are less suitable for student use unless they possess certain technical skills. **Astronify**, **Strauss**, **Sonecules**, **SoniPy**, and **Audio-plot-lib**—all Python libraries—require basic knowledge of programming to set up sonification, making them less accessible to students without coding experience. Similarly, **Accessible Graphs**, **Chart Reader**, **Infosonics**, **Olli**, and **VoxLens** rely on instructor-provided data rather than user-entered functions, and most of them (except **Accessible Graphs**) need JavaScript familiarity to develop teaching materials. Finally, the R package **Sonify** can be effective if instructors are comfortable working with R, though it is not as straightforward for those lacking a background in statistical programming.

Input data of the tool

Many of these accessible visualization tools allow users to input functions or data in different ways. **MathTrax** accepts an expression of the function to be displayed and then generates both audio representations and textual descriptions of key features. Similarly, **Audiographs** lets users enter mathematical expressions via a text box following a C++-like syntax, while **SenseMath** provides an on-screen mathematical keyboard specifically designed for this purpose. The **Desmos Graphing Calculator** also focuses on function input: users typically type their expressions in a LaTeX-like notation, though a virtual keyboard is available for mathematical symbols, and there is an option to load teaching activities, which can be especially practical in classroom settings.

Other tools rely primarily on external data files. **Accessible Graphs**, **Chart Reader**, **Infosonics**, **Olli**, and **VoxLens** each work with JSON or spreadsheet data, and some also support additional descriptive layers. **Chart Reader** needs a file containing key data insights, and **Infosonics** uses introductory and annotation audio tracks, so both systems benefit from extra metadata that contextualizes the numerical content. The **Highcharts Sonification Studio** similarly reads data from an in-browser table or through file uploads, interpreting each column as a separate data series whose title appears in the column's first cell. Meanwhile, the **SAS Graphics Accelerator** requires data to be prepared in advance, usually by means of SAS Studio or other SAS software, so that a finalized series can be fed into the Accelerator for sonification.

For users working directly in coding environments, a range of libraries offers customizable input strategies. The **Sonify** R package allows users to type in either functions or data at the R prompt, whereas **Tonejs** in JavaScript focuses on sound parameters, such as triggering a piano note of a specific pitch for a given duration. **Astronify** accepts a two-column table—an x axis paired with a y axis—and uses a fixed mapping where pitch corresponds to y-values. The **Strauss**, **Sonecules**, **SoniPy**, and **Audio-plot-lib** libraries all take arrays of data and turn them into sound based on a selected sonification strategy; among them, **Audio-plot-lib** goes further by incorporating text-to-speech for conveying contextual details before or during playback.

Finally, tactile graphics often entail a different workflow. Static or dynamic tactile representations usually begin as images created in specialized software; however, because such software is rarely accessible to blind users, teachers or experts typically handle this editing phase. They must keep braille labelling and tactile perception limits in mind, ensuring the final product remains both readable and informative. Once the source figures are prepared, they can be embossed or printed on swell paper to yield static tactile graphics or integrated into interactive devices that add audio feedback. In any case, careful planning of the tactile image and thoughtful labelling are crucial for achieving a comprehensible reading experience.

List of tools examined

Tools for audio access (i.e., Sonification)

- **Aural**: plots polynomials of degree up to two; sonification when the user crosses the graph of the polynomial.
  Paper: https://doi.org/10.1109/TENSYMP54529.2022.9864411
  Source: https://github.com/t-ROY-coder/aural/
- **Accessible graphs**: aims to create accessible graphs for blind and visually impaired people. The data is introduced through a builder (not meant for function graphs). This builder then generates a separate web page with text and columns displaying the data. The user can navigate with keys or with a braille display.
  Documentation, tutorials and examples: https://accessiblegraphs.org
- **Chart Reader (Microsoft)**: web-accessibility engine for rendering accessible, interactive charts optimized for screen reader users. It offers sonification in combination with text insights into the relevant parts of the data plot. It works with d3.js.
  Paper: https://dl.acm.org/doi/10.1145/3544548.3581186
  Code: https://github.com/microsoft/chart-reader
  Examples and documentation: https://jrthomp.com/chart-reader/
- **Infosonics**: accessible infographics for people who are blind using sonification and voice. An introduction to the graph being displayed is added to the introduction track. Labels explaining relevant parts of the graph are added to the annotation track.
  Paper: https://dl.acm.org/doi/10.1145/3491102.3517465
  Example provided here: https://infosonics.surge.sh/
- **MathTrax**: based on NASA's Math Description Engine Software Development Kit (MDE SDK), this is a Windows application that plots, sonifies, and offers text descriptions of mathematical functions. It can also plot data.
  Available at: https://github.com/TerryHodgson/NASA-MathDescriptionEngine1.0.3
  Discontinued.
- **Olli**: A library for converting web visualizations into accessible text structures for blind and low-vision screen reader users. It produces a tree of text that can be used in conjunction with a screen reader; the data is plotted using other JavaScript libraries.
  Source: https://github.com/mitvis/olli
  Documentation, tutorial and demo: https://mitvis.github.io/olli/
- **Sonify (R package):** An alternative to R's plot function for sonifying data. It is invoked from the command line.
  Documentation: https://CRAN.R-project.org/package=sonify
- **VoxLens**: JavaScript library to make online data visualizations accessible to screen-reader users. A microphone can be used for voice commands; it offers a summary of the data displayed, and sonification of the data. It is used in combination with other data plot libraries.

Paper: http://dx.doi.org/10.21785/icad2023.1978
Source: https://github.com/athersharif/voxlens
Demos: https://athersharif.github.io/voxlens/playground/#/

- **Sonify (standalone app)**: Standalone Windows and macOS software used to explore the spectrum of molecular data. It allows users to import tabular data to create *musification* output—a combination of rhythm, melody, and harmony. It allows tagging and commenting on selected sections of the entire data series. It provides high customization of sound parameters. Each tag has a different sound preset.
  Paper: https://digital.wpi.edu/concern/etds/vd66w237p?locale=en
  Website: https://vjmanzo.com/wpi/sonify/
- **Audiographs:** Windows software for sonifying mathematical functions, created for blind and visually impaired users. It provides an accessible and screen reader compatible user interface and a rich set of keyboard shortcuts to change function parameters and interact with the function chart. The user can explore the function step by step or in batches and can choose to sonify the function as well as the first and second derivatives.
  Software: https://sourceforge.net/projects/audiographs/
- **SenseMath:** iOS app designed for students to present function graphs in audio form. SenseMath allows non-interactive playback of functions and sonifies different function key points such as maxima and minima using earcons. It also exploits a metronome during playback to provide audio context for the sonification—a rhythm that can be useful for providing periodicity combined with the actual sonification. SenseMath provides an accessible user interface and is compatible with VoiceOver. Using the VoiceOver rotor functionality, SenseMath allows quick customization of function properties (such as those that can be activated or deactivated) to speed up interaction and provide accessible shortcuts.
  Website: https://enviter.eu/sensemath-making-sense-of-math/
  App (iOS): https://apps.apple.com/nl/app/sensemath/id1546257766
- **Highcharts Sonification Studio:** Web application created to help students study mathematical functions and to help teachers with its ability to easily share sonification strategies. It's a sandbox where users can experiment by testing various sonification strategies, combining different audio features to sonify data series without any code.
  Paper: https://repository.gatech.edu/entities/publication/ad4383f6-4aad-48b4-9b30-16128fb591d4
  Tutorial: https://sonification.highcharts.com/#/tutorial
  Web app: https://sonification.highcharts.com/#/app
- **Astronify:** Python library created in the astronomy field to sonify data series using pitch as a feature. Astronify uses a discrete (point-wise) sonification technique and provides simple playback interaction with functions. It allows a limited set of parameterization choices such as note spacing and pitch range. The library is freely available via PIP.
  PIP package: https://astronify.readthedocs.io/en/latest/astronify/install.html
  Paper: https://arxiv.org/abs/2209.04465
- **Strauss:** Python library created to be integrated into data analysis workflows, especially in the field of astronomy. It targets data analysts and visually impaired students. It offers an object-oriented approach to sonification that allows even non-

technical users to use sonification. It sonifies data series and offers many customization options: continuous and discrete sonification, full spatial audio with up to 16 channels, and multiple audio sources. Freely available via PIP. Examples: https://data.ncl.ac.uk/articles/media/Trayford_2023_STRAUSS_ICAD_examples/22241182
Website: https://www.audiouniverse.org/research/strauss
Code: https://github.com/james-trayford/strauss
Paper: https://repository.gatech.edu/entities/publication/2849fc7a-99be-49df-90f1-4cf38114e2fb
PIP package: https://pypi.org/project/strauss/

- **Sonecules:** Python library for sonifying data series, which aims to bridge the gap between non-extendible GUI-based sonification tools and code-based ones that require significant expertise to design sonifications. Sonecules aims to facilitate sonification adoption and sharing by employing an object-oriented approach to provide ready-to-use sonification presets, while simultaneously allowing expert users to extend these classes (presets) to customize internal sonification properties. In Sonecules, the sonification also includes interaction capabilities (i.e., playback, stop) to fully define the "sonification strategy", not only in terms of sound design. Finally, Sonecules comes with a comprehensive set of presets covering all common sonification models, including mapping-based, model-based, and trigger-based approaches.
  Paper: https://pub.uni-bielefeld.de/download/2979095/2982363/ReinschHermann-ICAD2023-Sonecules.pdf
  Code: https://github.com/interactive-sonification/sonecules
- **SoniPy:** An open-source project aimed at becoming a collection of sonification modules for Python. Currently, there is only the possibility of sonifying discrete data series. However, users can use various parameters, such as sound duration, to influence the audio output.
  Website: https://www.sonification.com.au/sonipy/index.html
  Paper: https://www.sonification.com.au/sonipy/Worrall_SonipyICAD07.pdf
  Code: https://github.com/lockepatton/sonipy
  PIP package: https://pypi.org/project/sonipy/
- **Audio-plot-lib:** A Python library that makes it possible to sonify discrete data. The target group consists of visually impaired people with programming experience who are interested in data science or machine learning. By sonifying data series, the aim is to provide easier data handling for visually impaired users. It is also possible to combine multiple data series, for example, to display regression lines together with raw data. The sonification can be created as a playable (.wav file) or interactive (HTML page).
  Website: https://hassaku.github.io/DS-and-ML-with-screen-reader/
  Code: https://hassaku.github.io/DS-and-ML-with-screen-reader/
  PIP package: https://pypi.org/project/audio-plot-lib/
- **Tonejs:** JavaScript library for creating audio pipelines using an object-oriented paradigm. The library aims to bring Digital Audio Workstation functionality to the coding

domain, offering an easy-to-use interface with low-level audio devices and effects. The library is suitable for creating highly parameterizable and efficient audio pipelines.
Website: https://tonejs.github.io/

- **SAS Graphics Accelerator:** A widget that enables users with visual impairments or who are blind to explore data series visualizations prepared in SAS Studio or other SAS applications. Bar charts, bubble plots, histograms, pie charts, and other statistical graphs can be explored using sonification; users can listen to text descriptions of graphs or explore tabular data using keyboard input and screen readers.
  Website: https://support.sas.com/software/products/graphics-accelerator/
- **Desmos:** A common name for a group of web calculators such as Graphing, Scientific, Four Function, Matrix, Geometry, and 3D; the Desmos graphing calculator enables users to type in functions, plot their graphs, and explore them using sonification of curves in Audio trace mode; it is possible to use keyboard input to get additional information, navigate between points of interest, switch to graphs of other functions, or set up sonification properties; the software is accessible via screen readers on Windows, Mac, or Android devices.
  Website: https://www.desmos.com/
  Regarding accessibility: https://www.desmos.com/accessibility
  Examples: https://help.desmos.com/hc/en-us/articles/4406040715149-Getting-Started-Desmos-Graphing-Calculator

Tools for tactile access

- **ViewPlus Graphics Embossers:** embossers based on Tiger or TigerPlus technology for embossing multi-dot tactile graphics on sheets of standard paper; graphics can be created in any graphics software including specialized ones (TactileView or Tiger Software Suite); braille labels can be added in a special font.
  Website: https://viewplus.com/product-category/braille-embossers/
- **Swell Paper Tactile Graphics:** swell (or microcapsule) paper is a heat-reactive material designed for creating tactile graphics that can be drawn manually or prepared in any graphics software and then printed with a standard laser or ink printer; the sheet of swell paper is then heated so the black shapes become tactile; there are two types of heating devices and brands of swell paper available (ZyFuse Heater, Piaf Tactile Image maker).
  ZyFuse Heater website: https://zychemltd.com/swell-form-tactile-graphics-machine/
  Piaf Tactile Image maker website: https://int.harpo.com.pl/products/piaf/
- **Tactile Graphic Display DV-2:** this is a portable 2D braille display consisting of 1536 dots (32 x 48) and some other keys to control the displayed image; it can be connected to a computer using a USB interface and display the screen content in real time. The device is no longer available for purchase.
- **HyperBraille S/F:** A 2D tactile display with 104x60 dots/pins (10 dpi). Depending on the version, it can be connected to a computer via USB or Bluetooth and has 28 buttons with a 10-finger multi-touch function. The display has a high refresh rate and can also be updated during user interaction.
  Website: https://metec-ag.de/en/produkte-graphik-display.php

- **DotPad 320:** A 2D tactile display with 60x40 dots/pins (10 dpi). It can be connected to computers, smartphones, or tablets via USB or Bluetooth and has 6 buttons. However, the display can only be updated line by line and only in areas where it is not being touched. It is characterized by its low weight and comparatively low price.
  Website: https://pad.dotincorp.com/
- **Orbit Graphiti:** A tactile 2D display with 60x40 dots/pins (5 dpi). It can be connected to computers, smartphones, or tablets via USB or Bluetooth and offers 14 buttons and a touch function. The resolution is too low to display braille, but the pins can be positioned at 5 different heights.
  Website:
  https://www.orbitresearch.com/product/graphiti/?srsltid=AfmBOop9wrwXZ6vr199jd98LtySJARrfi8yTdlKx0Rz1CVYXL0QDLZBz
- **Monarch:** A tactile 2D display with 96x40 dots/pins (10 dpi). It has an Android-based operating system and can be used as a standalone device. The display is based on the technology of the DotPad 320, but it has been enhanced with touch functionality using infrared sensors.
  Website: https://www.aph.org/product/monarch/
- **Touch Haptic Device:** this is a force feedback device for 3-dimensional navigation of virtual 3D objects, which requires haptic programs that have to be developed by programmers; the device is connected to a computer using USB interface.
  Website: https://www.3dsystems.com/haptics-devices/touch
- **IVEO by Viewplus**: this uses sight, sound, and touch and allows users to understand tactile graphics in an interactive solution. It consists of content creation software and a touchpad for audio-tactile response.
  Website: https://viewplus.com/product/iveo-3-hands-on-learning-system/
- **Tactonom Reader by Inventivio**: The Tactonom Reader is an interactive graphics reader. It enables blind people to access tactile graphics independently. Elements that are touched are explained by audio information while the finger is on them.
  Website: https://www.tactonom.com/product/tactonomreader/
- **TPad by KIT**: The TPad is a prototype solution that uses standard hardware—the iPad Pro as a device—along with an app and digital files to interact with tactile graphics. However, it is not publicly available but shows how commonly available hardware can be utilized to allow audio-haptic interaction with graphic content.
  Website: https://doi.org/10.1145/3313831.3376508

Literature review of HCI papers

For the review of literature from the field of Human-Computer Interaction, 35 articles from journals and conferences were retrieved by searching the ACM database and via Google Scholar, as well as from the references of the retrieved papers, for terms such as sonification, assistive technologies, visual impairments, blindness, data management, Human-Computer Interaction, accessibility, mathematics, and functions. We focused on papers published during the last ten years from 2014-2024. However, we also included older papers if they contained fundamental research about making functions accessible for visually impaired individuals using sonification approaches.

We analysed the papers according to the following categories: target group (visually impaired (VI), low vision (LV) individuals, students), number of study participants, sonification model, function types, topic/field, the platform on which tests were performed, feedback interfaces, availability, teaching intentionality, main results, and additional aspects. A **brief summary of the papers reviewed**[2] can be found in the annex of this report (Annex 1).

In general, we observed increased attention in research papers regarding various sonification approaches during the last few years: in particular, the increasing interest in the astronomical field certainly helped this process. There are papers that combine audio, vocal, and tactile feedback for data analysis or for mathematical functions directly, although we found fewer articles about this topic compared to those focused on data analysis. In any case, the articles provide evidence that sonification can be beneficial for students in facilitating non-visual access to visual data and revealing patterns in data.

Target groups

Our analysis of the reviewed HCI papers showed that they address various target groups:

(1) **blind and visually impaired individuals in general** (28 papers),
(2) **students and the education sector**, e.g. students, education, teachers (16 papers),
(3) **screen reader users** (9 papers),
(4) **professionals and researchers**, e.g. BVI, space science researchers, artists; BVI, data analysts (students, researchers); research, art (12 papers),
(5) **general audience**, e.g. BLV general audience; visitors to planetarium shows; novice users in the field of sonification (6 papers), and
(6) **developers and engineers** such as developers and sound engineers (5 papers).

Some of the papers address several target groups; for instance, Sharif et al. (2022b) focuses on blind and visually impaired individuals who are using screen readers. Other papers such as Trayford & Harrison (2023) even focus on blind and visually impaired individuals who are professionals, researchers, students, educators, developers, or engineers.

[2] To access the summary of an article directly, click on the authors' names, while to access its bibliographic reference, click on the year of publication. For example, for Sharif et al. (2022b), clicking on 'Sharif et al.' will redirect you to the summary, while clicking on '2022b' will redirect you to the bibliographic reference.

1. Blind and Visually Impaired (BVI) Individuals

- Chundury et al. (2024)
- Dobosz & Hanak (2024)
- Enge et al. (2024)
- He & Yu (2024)
- Riga & Kouroupetroglou (2024)
- Zhang et al. (2024)
- Zhao et al. (2024)
- Fitzpatrick & Neff (2023)
- Hoque et al. (2023)
- Maćkowski et al. (2023)
- Ramôa et al. (2023)
- Thompson et al. (2023)
- Trayford & Harrison (2023)
- United Nations Office for Outer Space Affairs [UNOOSA] (2023)
- Fan et al. (2022)
- Holloway et al. (2022)
- Roy & Boppana (2022)
- Sharif et al. (2022a)
- Sharif et al. (2022b)
- Cantrell et al. (2021)
- Ohshiro et al. (2021)
- Ahmetovic et al. (2019)
- Kim et al. (2019)
- Vines et al. (2019)
- Strumillo et al. (2018)
- Tomlinson et al. (2017)
- Wörtwein et al. (2016)
- Grond et al. (2010)

2. Students and the Education Sector

- He & Yu (2024)
- Riga & Kouroupetroglou (2024)
- Zhang et al. (2024)
- Zhao et al. (2024)
- Fitzpatrick & Neff (2023)
- Hoque et al. (2023)
- Ramôa et al. (2023)
- Reinsch & Hermann (2023)
- Trayford & Harrison (2023)
- UNOOSA (2023)
- Roy & Boppana (2022)
- Cantrell et al. (2021)
- Ahmetovic et al. (2019)
- Vines et al. (2019)

- Tomlinson et al. (2017)
- Grond et al. (2010)

3. Screen Reader Users

- Riga & Kouroupetroglou (2024)
- Zhang et al. (2024)
- Zhao et al. (2024)
- Hoque et al. (2023)
- Holloway et al. (2022)
- Sharif et al. (2022a)
- Sharif et al. (2022b)
- Cantrell et al. (2021)
- Ahmetovic et al. (2019)

4. Professionals and Researchers

- Enge et al. (2024)
- Zhang et al. (2024)
- Zhao et al. (2024)
- Hoque et al. (2023)
- Reinsch & Hermann (2023)
- Trayford & Harrison (2023)
- UNOOSA (2023)
- Wang et al. (2022)
- Cantrell et al. (2021)
- Ahmetovic et al. (2019)
- Soiffer & Noble (2019)
- Tomlinson et al. (2017)

5. General Audience

- Hoque et al. (2023)
- UNOOSA (2023)
- Holloway et al. (2022)
- Ahmetovic et al. (2019)
- Strumillo et al. (2018)
- Tomlinson et al. (2017)

6. Developers and Engineers

- Reinsch & Hermann (2023)
- Trayford & Harrison (2023)
- Cantrell et al. (2021)
- Lindetorp & Falkenberg (2021)
- Soiffer & Noble (2019)

Sonification methods

Sonification is the process of mapping data to sound to enhance accessibility, perception, and interpretation of information. Various approaches to sonification exist, each tailored to different interaction styles, user needs, and data characteristics. This section explores seven key sonification methods, from discrete event-based techniques to complex model-driven approaches.

- **(1) Discrete Event-Based Sonification** provides immediate auditory feedback for specific data points or interactions, useful for stepwise exploration and event detection (11 papers).
  - **Characteristics:** Point-wise sounds triggered by specific interactions or data points.
  - **Examples of Audio Types:** Point, TTS, earcons, beeps.
  - **Sonic Mapping:** Pitch variation, stereo positioning, spatial comparison.
  - **Play Order:** Point-wise or stepwise interaction (e.g., "short beep when the mouse intersects the graph").
  - **Representative Methods:**
    - Earcons for boundaries.
    - Discrete sonification for stepwise interactions.
- **(2) Continuous Sonification** represents evolving data trends through uninterrupted sound streams, facilitating real-time monitoring of dynamic datasets (10 papers).
  - **Characteristics:** Continuous sound streams to represent ongoing changes in data.
  - **Examples of Audio Types:** Oscillator waveforms, synthesizers, spatial sound.
  - **Sonic Mapping:** Frequency modulation, amplitude modulation, panning.
  - **Play Order:** Continuous streams or gradual variations (e.g., "continuous oscillator").
  - **Representative Methods:**
    - Continuous oscillator.
    - Frequency mapping for the y-axis.
- **(3) Hybrid Sonification** blends discrete and continuous approaches, allowing for rich, multivariate auditory representations (11 papers).
  - **Characteristics:** A combination of point-based and continuous methods for rich representation.
  - **Examples of Audio Types:** TTS with auditory feedback, synthesized sounds, and earcons.
  - **Sonic Mapping:** Mixed mappings using frequency, timbre, and amplitude.
  - **Play Order:** Sequences of sounds and parallel play (e.g., "multivariate series are sonified simultaneously").
  - **Representative Methods:**
    - Text-to-speech with pitch adjustments.
    - Mapping and periodic region-based sonification.
- **(4) Spatial Sonification** enhances spatial awareness by positioning sound in stereo or 3D space to communicate data relationships (6 papers).

  - **Characteristics:** Position-based sound cues to enhance spatial awareness of data.
  - **Examples of Audio Types:** Spatialized sound effects, natural sounds, TTS.
  - **Sonic Mapping:** Stereo or 3D panning, volume modulation based on position.
  - **Play Order:** Sequential or parallel positioning cues (e.g., "different natural sounds to identify data categories").
  - **Representative Methods:**
    - Direction-based sonification.
    - Spatial comparison via stereo audio.
    - 3D virtual microphone mapping.
- **(5) Parameter Mapping Sonification** assigns data attributes to sound parameters such as pitch, timbre, and volume, enabling flexible and scalable representations (14 papers).
  - **Characteristics:** Direct mapping of data attributes to sound parameters.
  - **Examples of Audio Types:** Frequency modulation, pitch, volume scaling.
  - **Sonic Mapping:** Assigning pitch to data values, and timbre to object categories.
  - **Play Order:** Configurable (simultaneously or sequentially).
  - **Representative Methods:**
    - Configurable mappings (pitch, pan, volume).
- **(6) Contextual and Narrative Sonification** integrates auditory cues into storytelling or instructional content, improving comprehension and engagement (4 papers).
  - **Characteristics:** Sonification integrated into a broader narrative for accessibility.
  - **Examples of Audio Types:** Speech explanations, musical scales, aesthetic sounds.
  - **Sonic Mapping:** Explanatory TTS alongside musical representation.
  - **Play Order:** Descriptive, summary modes, question-answer interactions.
  - **Representative Methods:**
    - Summary mode with speech.
    - Sonification used in teaching contexts.
- **(7) Model-Based and Interactive Sonification** allows real-time data interaction through dynamic sonification models, enabling deeper exploration and discovery (1 paper).
  - **Characteristics:** Dynamic and customizable models allowing interaction.
  - **Examples of Audio Types:** Model-driven sonification, triggers, alarms.
  - **Sonic Mapping:** Customizable mapping frameworks.
  - **Play Order:** Real-time interaction based on user input.
  - **Representative Methods:**
    - Sonecules (mapping, interaction, purpose).
    - Score-based model sonification.
    - Buffer synthesis with real-time adjustments.

Each method offers unique advantages, making sonification a versatile tool for accessibility, scientific analysis, education, and interactive applications. The following sections are meant to show which articles are more relevant to each sonification category.

1. Discrete Event-Based Sonification

- Dobosz & Hanak (2024)
- Zhang et al. (2024)
- Zhao et al. (2024)
- Maćkowski et al. (2023)
- Reinsch & Hermann (2023)
- Thompson et al. (2023)
- Trayford & Harrison (2023)
- Roy & Boppana (2022)
- Sharif et al. (2022a)
- Sharif et al. (2022b)
- Kim et al. (2019)
- Grond et al. (2010)

2. Continuous Sonification

- Reinsch & Hermann (2023)
- Trayford & Harrison (2023)
- Sharif et al. (2022b)
- Cantrell et al. (2021)
- Ohshiro et al. (2021)
- Ahmetovic et al. (2019)
- Kim et al. (2019)
- Strumillo et al. (2018)
- Wörtwein et al. (2016)
- Grond et al. (2010)

3. Hybrid Sonification (Mixed Discrete & Continuous)

- Dobosz & Hanak (2024)
- Zhang et al. (2024)
- Ramôa et al. (2023)
- Reinsch & Hermann (2023)
- Trayford & Harrison (2023)
- Holloway et al. (2022)
- Sharif et al. (2022b)
- Wang et al. (2022)
- Ohshiro et al. (2021)
- Kim et al. (2019)
- Tomlinson et al. (2017)

4. Spatial Sonification

- Hoque et al. (2023)
- Reinsch & Hermann (2023)
- Thompson et al. (2023)

- Trayford & Harrison (2023)
- UNOOSA (2023)
- Ahmetovic et al. (2019)

5. Parameter Mapping Sonification

- Dobosz & Hanak (2024)
- He & Yu (2024)
- Zhang et al. (2024)
- Fitzpatrick & Neff (2023)
- Reinsch & Hermann (2023)
- Trayford & Harrison (2023)
- UNOOSA (2023)
- Fan et al. (2022)
- Cantrell et al. (2021)
- Lindetorp & Falkenberg (2021)
- Ahmetovic et al. (2019)
- Vines et al. (2019)
- Strumillo et al. (2018)
- Grond et al. (2010)

6. Contextual and Narrative Sonification

- He & Yu (2024)
- Ramôa et al. (2023)
- Holloway et al. (2022)
- Tomlinson et al. (2017)

7. Model-Based and Interactive Sonification

- Reinsch & Hermann (2023)

Topics

Providing access to non-visual data of any kind is essential for people with visual impairments. We have analysed the papers regarding what topics they deal with. The analysis showed that they cover the following eight different main topics ranging from STEM and astronomy to programming:

(1) **Data accessibility and visualization** such as data visualization, sonification and its distribution, information display models, visualization for blind users, infographics, exploring line graphics, floor plans, bar charts, universal design, Web Audio API, accessibility in visualization, visualization of any data (21 papers).
(2) **Assistive technology and accessibility** such as touchscreen experiences, smartphone and tablet usage, multimodal interaction, speech and audio-tactile

interaction, visual impairments and blindness, general accessibility improvement, first-hand access to data, participatory design (10 papers).
(3) **STEM education and learning**, e.g., mathematics in education, STEM education, distance learning, function graphs, graph exploration, usability in educational tools (14 papers).
(4) **Research and studies**, e.g., psychoacoustic research, sonification potential, research surveys, categorized tool surveys, effectiveness of sonification for trends, studies comparing interfaces (8 papers).
(5) **Programming and technical implementation**, e.g., programming library overviews, JavaScript libraries (D3.js, Chart.js, Google Charts), sonifier.js prototypes, technical details of libraries, implementation choices, web-based technologies (6 papers).
(6) **Astronomy and science applications**, e.g., astronomy applications, sonification in space science, scientific research accessibility, planetary and space data representation (3 papers).
(7) **Sound design and interaction**, e.g., sound design principles, gesture interaction with audio, earcons and auditory icons, parameter mapping for sound, interactive sonification techniques (7 papers).
(8) **Critique and evaluation**, e.g., critical evaluation of sonification methods, issues in current sonification approaches, testing effectiveness of frequency ranges, comparative studies (4 papers).

1. Data Accessibility and Visualization

- Chundury et al. (2024)
- He & Yu (2024)
- Zhang et al. (2024)
- Zhao et al. (2024)
- Fitzpatrick & Neff (2023)
- Hoque et al. (2023)
- Reinsch & Hermann (2023)
- Thompson et al. (2023)
- UNOOSA (2023)
- Fan et al. (2022)
- Holloway et al. (2022)
- Sharif et al. (2022a)
- Sharif et al. (2022b)
- Wang et al. (2022)
- Cantrell et al. (2021)
- Lindetorp & Falkenberg (2021)
- Soiffer & Noble (2019)
- Vines et al. (2019)
- Strumillo et al. (2018)
- Tomlinson et al. (2017)
- Wörtwein et al. (2016)

2. Assistive Technology and Accessibility

- Zhang et al. (2024)
- Zhao et al. (2024)
- Fitzpatrick & Neff (2023)
- Hoque et al. (2023)
- Ramôa et al. (2023)
- Holloway et al. (2022)
- Lindetorp & Falkenberg (2021)
- Ahmetovic et al. (2019)
- Soiffer & Noble (2019)
- Strumillo et al. (2018)

3. STEM Education and Learning

- Dobosz & Hanak (2024)
- He & Yu (2024)
- Zhang et al. (2024)
- Zhao et al. (2024)
- Hoque et al. (2023)
- Ramôa et al. (2023)
- Reinsch & Hermann (2023)
- Riga & Kouroupetroglou (2024)
- Cantrell et al. (2021)
- Lindetorp & Falkenberg (2021)
- Ahmetovic et al. (2019)
- Soiffer & Noble (2019)
- Vines et al. (2019)
- Grond et al. (2010)

4. Research and Studies

- Riga & Kouroupetroglou (2024)
- Fitzpatrick & Neff (2023)
- Hoque et al. (2023)
- Reinsch & Hermann (2023)
- UNOOSA (2023)
- Sharif et al. (2022b)
- Cantrell et al. (2021)
- Vines et al. (2019)

5. Programming and Technical Implementation

- Reinsch & Hermann (2023)
- Trayford & Harrison (2023)
- Sharif et al. (2022a)
- Sharif et al. (2022b)
- Cantrell et al. (2021)
- Lindetorp & Falkenberg (2021)

6. Astronomy and Science Applications

- Trayford & Harrison (2023)
- UNOOSA (2023)
- Tomlinson et al. (2017)

7. Sound Design and Interaction Techniques

- Zhang et al. (2024)
- Fitzpatrick & Neff (2023)
- Hoque et al. (2023)
- Reinsch & Hermann (2023)
- UNOOSA (2023)
- Cantrell et al. (2021)
- Grond et al. (2010)

8. Critique and Evaluation

- Enge et al. (2024)
- UNOOSA (2023)
- Sharif et al. (2022b)
- Wang et al. (2022)

Feedback interfaces

Feedback interfaces play a crucial role in enhancing user interaction with sonification and accessibility tools. These interfaces vary in modality, interactivity, and purpose, catering to different user needs and technological setups. Below are the key categories of feedback interfaces explored in this section:

- **(1) Auditory Feedback Interfaces** rely on sound-based interaction, using Text-to-Speech (TTS), real-time sonification, audification, and spatialized audio to convey data and summaries (23 papers).
    - **Characteristics:** Interfaces that primarily rely on sound-based feedback.
    - **Examples of Interfaces:**
        - TTS (Text-to-Speech)
        - Sonification (single data series, general data representation)
        - Audio (2D/3D, multi-channel)
        - Speech with beep sounds
        - Audification (real-time audio signals)
        - Audio descriptions (coordinates, points)
        - Sound-based summaries
- **(2) Haptic and Tactile Feedback Interfaces** provide touch-based interaction through vibrations, tactile graphics, and direct touch mapping, enhancing accessibility for users with visual impairments (4 papers).

  - **Characteristics:** Interfaces providing touch-based interaction through vibrations, tactile graphics, or physical elements.
  - **Examples of Interfaces:**
    - Haptic feedback
    - Vibration (for content leaving detection)
    - Tactile graphics (e.g., embossed paper)
    - Touch interaction (1 or 2 taps, virtual buttons)
    - Direct Touch Mapping (DTM)
- **(3) Multimodal Interfaces (Audio + Visual + Haptic)** integrate multiple sensory modalities such as screen readers, augmented reality (AR), virtual reality (VR), and graphical user interfaces (GUIs) for comprehensive data exploration (9 papers).
  - **Characteristics:** Systems combining multiple feedback types such as visual, audio, and touch interaction for enhanced accessibility and user experience.
  - **Examples of Interfaces:**
    - Audio-visual interfaces
    - AR (Augmented Reality) and VR (Virtual Reality)
    - Graphical User Interfaces (GUI) for checking and controlling sonification
    - Mobile apps with assistive technologies (screen reader integration)
    - UI for interactive exploration (desktop, touchscreen, head-mounted displays)
    - Code-based and audio-visual integrations
- **(4) Code-Based and Customizable Interfaces** offer technical control over feedback through coding environments, parameter adjustments, and user-configurable sonification tools (4 papers).
  - **Characteristics:** Systems that involve coding or technical control for feedback customization and exploration.
  - **Examples of Interfaces:**
    - UI for sonification control and configuration
    - Code-based feedback systems
    - Desktop environments with immersive elements
    - Parameter control and exploration GUIs
- **(5) Interactive Interfaces (User Input-Driven)** allow active engagement using keyboards, touchscreens, gestures, and mouse input, enabling real-time exploration and customization of data representations (15 papers).
  - **Characteristics:** Interfaces that allow users to actively engage through direct input devices such as keyboards, touchscreens, or mouse.
  - **Examples of Interfaces:**
    - Keyboard interactions
    - Mouse control for data exploration
    - Touchscreens for navigation and sonification control
    - Gesture-based interactions (touch, taps, swipes)
- **(6) Non-Interactive Passive Interfaces** deliver predefined, passive feedback such as automated sonifications or visual summaries, requiring no direct user input (3 papers).
  - **Characteristics:** Systems that provide passive feedback without requiring active user interaction.

- **Examples of Interfaces:**
    - Audio-only passive feedback
    - Predefined sonification processes without interaction
    - Non-interactive visual summaries

Each interface type enhances accessibility and user experience in different ways, making them essential for data interaction, assistive technology, and interactive applications. The following sections show which articles are more relevant to the different interaction modalities.

1. Auditory Feedback Interfaces

- Dobosz & Hanak (2024)
- Zhang et al. (2024)
- Zhao et al. (2024)
- Thompson et al. (2023)
- Fitzpatrick & Neff (2023)
- Hoque et al. (2023)
- Reinsch & Hermann (2023)
- Trayford & Harrison (2023)
- UNOOSA (2023)
- Holloway et al. (2022)
- Roy & Boppana (2022)
- Sharif et al. (2022a)
- Sharif et al. (2022b)
- Wang et al. (2022)
- Cantrell et al. (2021)
- Lindetorp & Falkenberg (2021)
- Ohshiro et al. (2021)
- Strumillo et al. (2018)
- Ahmetovic et al. (2019)
- Soiffer & Noble (2019)
- Vines et al. (2019)
- Wörtwein et al. (2016)
- Grond et al. (2010)

2. Haptic and Tactile Feedback Interfaces

- Zhang et al. (2024)
- Ramôa et al. (2023)
- Soiffer & Noble (2019)
- Wörtwein et al. (2016)

3. Multimodal Interfaces (Audio + Visual + Haptic)

- He & Yu (2024)
- Riga & Kouroupetroglou (2024)
- Zhang et al. (2024)
- Ramôa et al. (2023)

- UNOOSA (2023)
- Fan et al. (2022)
- Holloway et al. (2022)
- Cantrell et al. (2021)
- Ahmetovic et al. (2019)

4. Code-Based and Configurable Interfaces

- Reinsch & Hermann (2023)
- Trayford & Harrison (2023)
- Sharif et al. (2022b)
- Lindetorp & Falkenberg (2021)

5. Interactive Interfaces (User Input-Driven)

- Zhang et al. (2024)
- Zhao et al. (2024)
- Hoque et al. (2023)
- Maćkowski et al. (2023)
- Ramôa et al. (2023)
- Reinsch & Hermann (2023)
- UNOOSA (2023)
- Fan et al. (2022)
- Holloway et al. (2022)
- Sharif et al. (2022a)
- Lindetorp & Falkenberg (2021)
- Ahmetovic et al. (2019)
- Soiffer & Noble (2019)
- Strumillo et al. (2018)
- Grond et al. (2010)

6. Non-Interactive Passive Interfaces

- Reinsch & Hermann (2023)
- UNOOSA (2023)
- Tomlinson et al. (2017)

Teaching intentionality

We also analysed the papers regarding their teaching intentionality. Understanding and exploring mathematical concepts is the core aim of the SONAIRGRAPH project. In general, the authors pursue six different teaching intentionalities:

(1) **Understanding and exploring mathematical concepts**. The tools and methods are designed to help users explore, understand, and analyse mathematical functions, graphs, and curves; e.g., explore and study curves in the Cartesian plane, improving

the general perceptibility of mathematical graphs, reading and better understanding visualizations, accessibility of mathematical function graphs with sonification and proprioception, facilitating learning of mathematical functions and shapes, reading and better understanding visualizations of complex graphs using multimodal tools (TTS, sonification, vibro-tactile) (12 papers).

(2) **Sonification adoption and learning facilitation**. They describe efforts to introduce and promote sonification tools in educational environments, e.g., facilitating the adoption of sonification tools in classrooms by teachers, facilitating students in experimenting with sonification, facilitating the first approach to sonification and promoting its sharing in schools, facilitating the approach to sonification, and engaging with gamification elements to support learning (6 papers).

(3) **Outreach and STEM engagement**. Initiatives aimed at inspiring curiosity and broadening engagement in STEM (Science, Technology, Engineering, and Mathematics) through sonification and other sensory methods, e.g., outreach in science, inspiring curiosity in STEM, outreach and data analysis, outreach efforts to encourage exploration and learning (6 papers).

(4) **Data analysis and visualization**. Teaching intentionality focused on helping users analyse and interpret data using sonification and multimodal techniques, e.g., data analysis support, sonic mapping using natural sounds, reading and better understanding visualizations of data, enhancing the comprehension of complex visual data (16 papers).

(5) **Inclusive and accessible education**. Teaching methods that focus on providing accessible learning experiences for both sighted and visually impaired (BLV) students, e.g., distance learning, augmenting teaching materials, and inclusive tools, facilitating the understanding of mathematical functions for BLV through interactive sonification, promoting inclusive teaching materials for diverse learning needs (11 papers).

(6) **General or unspecified**. Papers where the exact teaching intentionality is not clearly defined (1 paper).

1. Understanding and Exploring Mathematical Concepts

- Dobosz & Hanak (2024)
- Zhang et al. (2024)
- Hoque et al. (2023)
- Maćkowski et al. (2023)
- Ramôa et al. (2023)
- Reinsch & Hermann (2023)
- Roy & Boppana (2022)
- Cantrell et al. (2021)
- Ohshiro et al. (2021)
- Ahmetovic et al. (2019)
- Wörtwein et al. (2016)
- Grond et al. (2010)

2. Sonification Adoption and Learning Facilitation

- Reinsch & Hermann (2023)

- UNOOSA (2023)
- Roy & Boppana (2022)
- Cantrell et al. (2021)
- Lindetorp & Falkenberg (2021)
- Ahmetovic et al. (2019)

3. Outreach and STEM Engagement

- Hoque et al. (2023)
- Thompson et al. (2023)
- Trayford & Harrison (2023)
- UNOOSA (2023)
- Cantrell et al. (2021)
- Ahmetovic et al. (2019)

4. Data Analysis and Visualization

- He & Yu (2024)
- Zhang et al. (2024)
- Zhao et al. (2024)
- Hoque et al. (2023)
- Reinsch & Hermann (2023)
- Thompson et al. (2023)
- Trayford & Harrison (2023)
- UNOOSA (2023)
- Fan et al. (2022)
- Holloway et al. (2022)
- Sharif et al. (2022a)
- Sharif et al. (2022b)
- Wang et al. (2022)
- Cantrell et al. (2021)
- Soiffer & Noble (2019)
- Vines et al. (2019)

5. Inclusive and Accessible Education

- Riga & Kouroupetroglou (2024)
- Zhang et al. (2024)
- Hoque et al. (2023)
- Ramôa et al. (2023)
- Reinsch & Hermann (2023)
- Holloway et al. (2022)
- Cantrell et al. (2021)
- Ahmetovic et al. (2019)
- Soiffer & Noble (2019)
- Vines et al. (2019)
- Tomlinson et al. (2017)

6. General or Unspecified Purpose

- Cantrell et al. (2021)

Function types

In the literature, there is a clear division between studies in data visualization or sonification and mathematical function exploration. In these categories, there are some studies conducted on particular types of functions or shapes of data. In the following categorization, we try to delve deeper into each category to determine whether an article addresses one or more specific data shapes. We were especially interested in mathematical functions such as straight lines, parabolas, and hyperbolas; linear, hyperbolic, parabolic, sine, square-root, and exponential functions; polynomial functions, and discontinuous functions; as well as mathematical function graphs in general.

Line Charts

- He & Yu (2024)
- Fitzpatrick & Neff (2023)
- Ramôa et al. (2023)
- Thompson et al. (2023)
- Fan et al. (2022)
- Sharif et al. (2022a)

Bar Charts

- He & Yu (2024)
- Thompson et al. (2023)
- Sharif et al. (2022a)

Scatter Plots

- He & Yu (2024)

Mathematical Functions

- Dobosz & Hanak (2024)
- Roy & Boppana (2022)
- Ohshiro et al. (2021)
- Kim et al. (2019)
- Wörtwein et al. (2016)
- Grond et al. (2010)

General Data Visualization

- Sharif et al. (2022a)
- Wang et al. (2022)
- Soiffer & Noble (2019)

Single Data Series

- Sharif et al. (2022a)
- Sharif et al. (2022b)

Short Review of Educational Papers

This section delves deeper into the educational aspects regarding the role of sonification in teaching and learning mathematics, particularly functions. We will first attempt to shed light on the educational barriers encountered by visually impaired students in learning mathematics in general, and functions specifically. This choice is related to the fact that the concept of function, and in particular the concept image (Tall & Vinner, 1981) possessed by a student, can be linked to various types of representation (symbolic, tabular, graphic, etc.) and examples, relating to both graphic and non-graphic aspects (Vinner & Dreyfus, 1989). It is therefore interesting to recall and analyse the possible obstacles that visually impaired students might encounter in various areas of mathematics (geometry, numbers, algebra, etc.). Subsequently, we will focus on how sonification might support the teaching and learning of functions for these individuals. For all these purposes, an initial overview was conducted using the WOS and SCOPUS databases, with keywords such as sonification, assistive technology, mathematics, mathematical graphs, and visual impairments. Then, the search was extended to other catalogues, such as Latindex, Dialnet, Google Scholar, and specialized journals, such as the British Journal of Visual impairment, the Journal of Visual Impairment and Blindness, and the Revista Brasileira de Educação Especial. In general, we observed that there are not many studies that analyse obstacles in the teaching and learning of mathematics with visually impaired individuals; the vast majority of these studies relate learning difficulties in mathematical concepts to the use of assistive tools employed for their instruction.

Mathematical Domains and Learning Challenges for Visually Impaired Students

Two comprehensive literature reviews by Mendes et al. (2021) and Minholi & Rocha (2024) examined academic research on mathematics education for students with visual impairments, revealing both convergent findings and complementary insights into instructional practices, materials, and challenges in this field.

1. Geometry

Both reviews identified *geometry* as the most extensively researched mathematical domain for visually impaired students. This prevalence likely reflects geometry's amenable nature to tactile manipulation. Common geometric topics across both reviews include plane and solid figures, spatial relationships, properties of shapes, area calculations, similarity of triangles, trigonometry, and spatial localization. Both reviews documented extensive use of manipulative materials. The geoboard (*Geoplano*) emerged as a fundamental tool, enabling tactile construction of figures using rubber bands stretched across pegs. The multiplane (*Multiplano*) appeared frequently for representing geometric shapes in relief and constructing tactile graphs. Materials included logical blocks, geometric figures made from cardboard, wood, or EVA foam, various types of geometric solids, and everyday packaging for exploring spatial concepts. Despite the variety of materials, all these approaches share a primary limitation: the difficulty of representing complex figures or those too large for complete tactile exploration within hand span. Simultaneous management of multiple geometric elements in constructions proves challenging due to sequential tactile exploration. Precision in representing curves, angles, and measurements is necessarily approximate in tactile materials. Some geometric constructions become cumbersome with many elements, and overlapping figures in the same representation can create confusion. Moreover, dynamic representations of rotations, transformations, and continuous variations are particularly difficult to achieve with static tactile materials. Beyond these technical difficulties, learning challenges also emerged: understanding spatial relationships and properties emerged as a primary difficulty in geometry across both studies. Blind students must construct mental spatial representations based exclusively on tactile and verbal information, requiring longer time and more complex cognitive processes than visual perception allows.

2. Numbers and Arithmetic

The *numbers* thematic unit distinguished itself through the greatest variety of materials used. Topics included fundamental arithmetic operations, understanding the positional number system, integer operations, fractions (both conceptual understanding and operations), and decimals. The *Soroban* (adapted Japanese abacus) emerged as fundamental across both reviews for calculations, place value understanding, and operations with integers and decimals. Golden bead material (*Material Dourado*) appeared frequently for place value, operations, and fractions. Fraction instruction was also approached using recyclable materials and educational toys for adolescents, and adapted *Cuisenaire* rods for working on unit and non-unit fractions. However, it has been observed that execution speed limitations affect practical classroom use. Besides, some devices have limited capacity for handling very large numbers or complex operations. When working with fractions, some materials proved too specific to support generalization of concepts. The Soroban, while effective, requires extensive practice to achieve fluency. Concrete materials for fractions may not easily translate to symbolic manipulation and formal operations. Both reviews noted difficulties with place value comprehension and understanding the positional nature of number systems. Visualization of column operations proved challenging, as students cannot simultaneously view entire written procedures and must maintain intermediate results in memory. Managing complex multi-step calculations presented difficulties. Abstract conceptualization of fractions as numbers rather than merely parts of concrete objects challenged students, as did understanding fraction equivalences requiring complex proportional thinking. Finally, moving from concrete fraction representations to algebraic operations with fractions created a significant conceptual leap.

3. Probability and Statistics

A significant gap identified in both reviews concerns *probability and statistics*, with only few studies including this topic. This limited coverage represents a notable curricular deficit given the importance of statistical reasoning in contemporary education. Topics addressed included data representation, construction and interpretation of graphs, measures of central tendency, and probability calculation. Tools included tactile graph simulators for creating accessible graphical representations, *Soroban* for calculations, *Cubaritmo*, *Monet Software* for accessible graphs, EVA materials for creating tactile representations, 3D pictograms providing three-dimensional data visualization, tactile models (*maquetes táteis*) for spatial data representation, playing cards for probability activities, and various manipulative materials. Representing large amounts of data tactilely becomes unwieldy and confusing. Complex graphs with multiple data series or overlapping information are particularly difficult to render accessibly. The conventional visual efficiency of graphs—conveying much information at a glance—cannot be replicated tactilely, where exploration is necessarily sequential and time-consuming. Three-dimensional tactile models, while informative, are limited in the complexity and data volume they can effectively convey. From a learning point of view, both reviews identified challenges in reading and interpreting graphs, traditionally very visual representations requiring translation to tactile form. Understanding abstract probabilistic concepts without visual supports proved difficult. Grasping relationships between data representations and underlying patterns challenged students when explored tactilely rather than visually.

4. Algebra and Functions

Algebra emerged as the second most researched area in both reviews. This core area also included functions and their graphical representations, here mentioned as one of the most studied concepts. Both reviews noted a concentration on polynomial functions (particularly quadratic), exponential functions, matrices and determinants, algebraic expressions, notable products, and equation solving. The multiplane appeared as the most versatile tool for algebra, particularly for graphical function representations. Other tools included expression boards (*tabuleiro das expressões*) for working with order of operations and algebraic expressions, the *Matrizmat* device specifically for matrices, adapted Rubik's cubes for algebraic patterns, specific kits for notable products, algebraic plates, golden bead material for algebraic concepts, metal Cartesian planes with magnetic elements for coordinate systems and function graphs, geoboards for introducing functions, and various concrete materials. *Monet* software was mentioned for creating accessible graphs with sound feedback. Relief grid paper (*malha quadriculada em relevo*) enabled tactile graphing, while cardboard boards with mesh (*prancheta de papelão com tela*) provided alternative graphing surfaces. An analysis of these materials and their didactic use revealed that tactile representations of functions are necessarily discrete and approximate, limiting a fine understanding of continuous variations. Relief graphs become confusing when multiple functions overlap, and representation scale significantly affects tactile readability. Functions with complex behaviours (asymptotes, discontinuities, oscillations) are particularly difficult to represent comprehensibly. Tools struggle to show dynamic function behaviour and continuous change. Representing very complex or nested algebraic expressions proves difficult, and some concrete materials become cumbersome when working with many terms. For matrices, large dimensions create challenges for effective tactile exploration. The abstract nature of algebraic symbolism, when translated into braille, can obscure structural relationships visible in standard notation. Both reviews also emphasized challenges in visualizing graphical representations of functions. Students must construct mental images of curves through sequential tactile exploration, making it difficult to perceive global trends, variations, and functional behaviours that are immediately apparent visually. Understanding the correspondence between algebraic expressions and graphical representations emerged as particularly complex. Simultaneous management of multiple operations in algebraic expressions and understanding the hierarchical structure when represented linearly in braille presented difficulties. The transition from concrete to abstract in algebra represents a particular challenge for students relying primarily on tactile perception. Comprehending the concept of equivalence and balancing equations proved difficult, as these operations require an overall view that is hard to obtain through sequential tactile exploration.

5. General Barriers in Mathematics Education

Beyond content-specific difficulties, these reviews identified several general barriers affecting mathematics learning for visually impaired students across all topics. Both reviews noted that the high degree of abstraction inherent in mathematics makes developing adequate concrete materials particularly challenging. Students with visual impairments require multisensory approaches and didactic materials that do not rely exclusively on vision as the sole learning channel. Mendes et al. (2021) emphasized the strong dependence on “active touch” and the necessity of utilizing other senses to capture and interpret information.

Barriers related to teacher training emerged as a critical systemic problem in both studies: many teachers do not know the means and places where they can obtain appropriate didactic resources. Mendes et al. (2021) documented specific cases of teachers who, despite using accessible materials, continued to act as if there were no visually impaired students in the classroom. Minholi & Rocha (2024) provided statistical context, noting that in 2019 only 5.8% of teachers with special education students had adequate continuing education.

Regarding specific tool limitations, both reviews identified that the Multiplano, while versatile across multiple mathematical domains, showed limitations for certain types of content representation, and the Geoplano proved less suitable for complex mathematical content. Both reviews noted that generic concrete materials required specific adaptations for each type of content and presented difficulties in availability and standardization.

Despite these criticisms, both reviews highlighted significant positive aspects. Minholi & Rocha (2024) mentioned several studies (Berbetz, 2019; Koepsel, 2017; Mamcasz-Viginheski, 2017) demonstrating how accessible materials designed according to Universal Design for Learning (UDL) principles can be effectively used by both sighted students and those with visual impairments. This review emphasized the importance of manipulative materials, with researchers defining them as “a bridge for the development and elaboration of meanings for all students” (Felipe et al., 2022, authors’ translation). Similarly, Mendes et al. (2021) documented positive students' testimonies, including reports that having materials similar to their classmates makes understanding mathematics much easier (Uliana, 2015).

The Role of Technology and Digital Resources

From what has emerged, it is therefore possible to affirm that most of the critical issues arise regarding how to support visually impaired students in studying increasingly advanced and complex mathematical content. To do so, according to DePountis et al. (2015), “for a classroom teacher who has a student who is visually impaired, the presence of technology in the classroom is not optional but necessary" (p. 266). Digital resources have become prevalent educational instruments among hands-on and modified learning materials. These tools function as catalysts in the educational experience by providing students with visual disabilities access to information that remained previously unavailable. Utilizing digital resources permits mathematical subject matter created for visually impaired pupils to be presented with extensive detail during instruction, enabling educators to overcome certain obstacles and more effectively address complex mathematical topics that might be challenging to convey through traditional methods alone (Maćkowski et al., 2020).

Recognizing the significant impact of technology in improving the education of visually impaired students must not, however, overlook some important issues highlighted by Ketema Dabi & Negassa Golga (2024). Their research identifies multiple impediments to effectively employing assistive technologies in mathematics education: accessibility issues, system complexity, misguided recommendations, financial constraints, and lack of ongoing expert assistance. On this last point, already highlighted in the first two literature reviews, the study by Zhou et al. (2011) attributes these deficits primarily to the inadequacy of university teacher preparation programs, many of which do not offer specific courses in assistive technologies or provide them at insufficient levels. This creates a vicious cycle in which teachers' lack of competencies limits students' access to technologies that could significantly improve their learning and educational participation.

DePountis et al. (2015) also reveal a significant gap in technology for real-time translation from braille to print format, limiting simultaneous communication between blind and sighted students. This directly corresponds to what Schweikhardt (2000) describes as a critical condition for successfully incorporating blind and visually impaired students into regular mathematics education settings - namely, the availability of mathematical notation that can be accessed equally by both braille users and print readers. Starting from this concept of "mathematical notation" common to all subjects, it is possible to broaden the horizon by shifting attention to the search for another “vehicle” for transmitting information, one that is accessible to everyone and that facilitates communication and discussion among all regarding even complex mathematical notions and properties: sound.

Sonification as an Accessible Communication Tool

Research in recent years has led to examining in detail the role of auditory perceptions, recognizing them as among the most effective mechanisms for helping people with visual impairments acquire knowledge and distinguish concepts that conventionally depend heavily on vision. Audio serves a crucial function for individuals with sight limitations by offering acoustic signals for movement, spatial understanding, and data processing. The use of auditory data representation creates opportunities for participation and transforms the social dynamics among visually impaired learners. Beyond promoting their independence, it provides a common platform for educational engagement, enabling cooperative learning experiences (Tomlinson et al., 2016). Therefore, we now examined how auditory representation of data could enhance the instruction and comprehension of mathematics for students who are blind or have low vision. Remembering the objective of this section, we will focus only on the use of sound for the graphic representation of mathematical functions, specifically using sonification methods. As reported in the introduction, *sonification* is "the technique of rendering sound in response to data and interactions" (Hermann et al., 2011, p. 1). In essence, sonification seeks to leverage auditory perceptual abilities to make data relationships understandable by representing them through sound (a data-dependent generation of sound).

Research concerning sonification and assistive technologies that make use of it has made giant strides. During the ‘90s, the research community began to investigate the aesthetics of sonification and “musifications”, i.e., the use of musical sounds rather than pure sine waves. Starting with short musical melodies (earcons) to provide information and feedback about computer entities (Brewster & Crease, 1997) and to develop telephone-based user interfaces (Brewster, 1997), researchers began to believe that incorporating music when presenting graphical information could decrease interpretation errors and help users develop a more meaningful cognition of the data (Alty & Rigas, 1998). Today, we have a deeper understanding of how to enhance graph comprehension for blind and low vision readers through sonification, incorporating technical guidelines such as “place data points between 50-70ms apart; only use MIDI notes within the range 35-100; and use stereo panning to separate data series” (Holloway et al., 2022, p. 3). Unfortunately, at least from our overview, it seems that studies on the use of sonification to facilitate the teaching and learning of functions with visually impaired individuals are more limited to conference contributions than to research articles in specialized journals (showing no continuity between conference proceedings and journal publications).

Moreover, the main drawback of sonification appears to be that there is no consolidated framework that can prescribe to designers how to create an audio chart from given data or convert a visualization into an audio chart (Damsma, 2024; Gatto et al., 2024). In the absence of a framework to guide the development of tools, designers must start with a list of goals and functionalities (Ali et al., 2020, *Sonify*; Gatto et al., 2024, *GraficiAccessibili*; Holloway et al., 2022, *Infosonic*; Roy & Boppana, 2022):

- the use of Android tablets or iPads;
- the use of a cross-platform framework to develop the application;
- the application must be fully accessible (to ensure that visually impaired students can independently understand and analyse graphs), that is, it is designed as a free web app (and thus accessible from any device with a web browser), it offers voice-driven navigation, keyboard navigation, and interactions with a (voice) assistant; when asked, it gives spoken directions to the students;
- the system allows users to personalize auditory settings to better suit students’ needs (customizability);
- potential for broader applications, particularly in education, and range of math content (impact);
- functionality (does it allow the listener to understand the data?);
- foster autonomy and interactivity, i.e., users can freely explore graphs and diagrams using input devices such as keyboards or touch screens;
- user engagement or pleasure (being engaging, aesthetically pleasing, or easy to understand; suitability of text labels; use of haptic feedback and sound to make points recognizable in a Cartesian plane, etc.).

In this regard, some articles have investigated which sonification methods are the most effective. Pitch and volume have been shown to be effective methods of sonification, and tempo less so (Huang et al., 2023). Along this line, researchers have been interested in studying the perceptual congruence between sonification encodings such as pitch, tempo, volume, and roughness and the data values they represent, as well as visually impaired listeners' perception of scale and polarity for audio line graphs using pitch (frequency) and tempo (Walker & Mauney, 2010).

Applications of Sonification for Function Graphs

The application of sonification specifically for representing function graphs shows encouraging potential, including its effectiveness in teaching visually impaired students. It has been used specifically for the following content:

- matching increasing/decreasing or concavity properties with visible properties of graphs (Hetzler & Tardiff, 2006; 2007; Vines et al., 2019; Salamonczyk et al., 2020);
- estimating locations of zeros and extreme values of functions (Hetzler & Tardiff, 2006; Grond et al., 2010; Salamonczyk et al., 2020);
- limits (of continuous functions) and derivatives (Hetzler & Tardiff, 2006; 2007);
- relative positions of a graph and the x-axis; differentiability points (the addition of speech information is required for recognizing differentiability) (Kim et al., 2019);
- discontinuity (experiments implemented using the JavaScript library p5.js; Ohshiro et al., 2021);
- Real-world applications (Koman et al., 2023).

Hetzler and Tardiff (2006, 2007) developed comprehensive sonification tools for calculus instruction as part of their "Rule of Five" approach, extending traditional mathematical representation beyond numerical, graphical, symbolic, and verbal modes to include auditory representation. Their research involved two main components: a web-based training tool teaching students to interpret sonification through multiple-choice exercises, and an Excel-based spreadsheet tool allowing students to create and explore auditory graphs. The sonification design mapped x-values to time and y-values logarithmically to frequencies, using major chords for pleasant sound quality, white noise to distinguish negative values, ping sounds for zero crossings, and regular pulses for horizontal position tracking. Student performance data from Fall 2005 and 2006 showed that calculus students could quickly learn to interpret these sonifications, with most students achieving good accuracy on basic tasks including estimating numerical values, matching visual and auditory graphs, and locating function extremes and zeros. In calculus applications, students demonstrated comparable or sometimes superior ability to identify increasing/decreasing behaviour and concavity through auditory graphs versus traditional visual or numerical methods, though concavity remained challenging across all representational modes. The research concluded that sonification can be effectively integrated into standard calculus courses without requiring specialized equipment or extensive training time.

Salamonczyk et al. (2020) tested two audio interfaces to enable blind students to explore mathematical function graphs on touchscreens. The first interface uses non-interactive panoramic sonification: graph points are converted into stereo sounds where frequency represents the y-value and position in the stereo panorama (from left to right) represents movement along the x-axis. The second interface offers interactive exploration: users slide their finger horizontally across the screen and the sound frequency varies according to the function value, allowing identification of changes in monotonicity. Seven blind students tested the system on linear, quadratic, and piecewise functions. Results show that students correctly answered questions about monotonicity and axis intersections, with response times between 5-45 seconds. The main limitations identified were: inability to precisely determine intersection point coordinates, difficulties with complex graphs (the piecewise function), and the need for a preliminary training period to familiarize users with the interfaces.

Vines et al. (2019) conducted a study with twelve participants (five sighted students and seven blind or visually impaired individuals) to explore the effectiveness of sonification as a teaching tool in distance learning STEM contexts, working particularly on recognizing characteristics of mathematical functions. For example, regarding the sonification of a sinusoidal function, participants effectively recognized the periodic nature of the function. After listening to the sonification, most accurately described the wave-like pattern, with comments such as "goes up and down three times" and "sweeping series of arcs going between 100 on the y-axis and 0" (p. 7). When compared with tactile or visual versions, participants confirmed that these matched the impressions built from the sonification. Qualitative results highlighted that, although many sighted students did not find sonification personally useful (being accustomed to visual interpretation), some recognized that "listening required thinking more about the shape of the graph than simply looking at it" and that sonification "corroborated understanding" (p. 15). The lack of training in interpreting sonification was identified as an important limitation, since interpreting graphs visually is a skill learned over time, whereas sonification was new to participants.

Grond et al. (2010) conducted an educational evaluation of *SonicFunction* with fourteen blind and partially sighted students who explored six test functions (including parabolas, linear functions, rational functions with singularities, and trigonometric functions). They were asked to navigate these functions and place markers to identify specific mathematical features: maxima, minima, zeros $f(x) = 0$, and the point $x = 0$. The didactic design embedded the first derivative $\frac{df(x)}{dx}$ into the sonification through amplitude modulation, where oscillation amplitude approached zero as the derivative approached zero, providing an acoustic cue for critical points where $\frac{df(x)}{dx} = 0$. The learning outcomes revealed that for extrema identification in the sinusoidal function $f(x) = sin((0.2x + 3)^2) \cdot 1.5$, students' precision correlated with curvature magnitude at extrema—higher curvature (creating greater acoustic contrast through rapid derivative changes) resulted in more accurate identification, while lower curvature led to less precise recognition. This demonstrates how perceptual salience could affect learning accuracy. More critically, the confusion observed in function $f(x) = \frac{1}{sin(x)}$ revealed a fundamental conceptual challenge: students sometimes identified "maxima" and "minima" based on absolute function values (associating "maximum" with higher pitch) rather than applying the proper mathematical definition involving the second derivative sign at critical points. In this function, function values at minima were higher in absolute terms than at maxima, leading some students to reverse their classifications. These findings indicate that sonification effectiveness depends on students' prior conceptual understanding. While the tool can effectively support extrema identification in well-behaved functions when mathematical concepts are established, it may inadvertently reinforce misconceptions if used without adequate theoretical preparation.

Kim et al. (2019) investigated how visually impaired users comprehend fundamental mathematical concepts through sonification in three educational experiments. For understanding spatial relationships with the x-axis, serial presentation facilitated learning of general graph structure, while parallel presentation supported the specific learning objective of identifying intersection points. The use of distinct timbres for different quadrants significantly improved students' ability to accurately draw and conceptualize coordinate plane positions. Most significantly for mathematical understanding, the study revealed that learners could perceive sonic differences between differentiable and non-differentiable points but could not interpret their mathematical meaning without explicit verbal instruction. This finding demonstrates that perceptual discrimination does not automatically translate to conceptual understanding, highlighting the need for scaffolding that bridges auditory perception with mathematical interpretation in educational contexts.

Ohshiro et al. (2021) developed sonification techniques specifically for teaching discontinuity concepts in calculus to visually impaired students in remote learning contexts. The researchers created three distinct sonification approaches for different types of discontinuity: removable discontinuity (represented by silence when y-values are undefined, or sudden pitch changes when distinguishable values exist), jump discontinuity (conveyed through abrupt pitch jumps corresponding to y-value changes), and infinite discontinuity (using rapidly rising or falling pitch that exceeds audible range). Learning outcomes showed that infinite discontinuity was easiest to interpret due to its distinctive sound signature, while the other two proved more challenging to distinguish conceptually. However, with detailed explanations of both the sounds and corresponding graphs, participants successfully understood the discontinuity concepts, suggesting that sonification combined with verbal descriptions can effectively introduce mathematical discontinuity in remote educational settings.

The study by Koman et al. (2023) employs data sonification as an innovative pedagogical tool to address real-world problems. Specifically, the sonification converts oceanographic measurements of water transport into audio representations where higher pitch corresponds directly to higher transport values. Each data point becomes an individual sound in the audio sequence, creating temporal patterns that students, including those with visual impairments, can perceive auditorily. This auditory approach allows students to identify trends, outliers, and patterns that might be less apparent in visual graphs, particularly helping them understand concepts of data bias and the importance of long-term sampling. The sonification serves as both an accessibility tool and an alternative analytical method for pattern recognition in time-series oceanographic data.

Synthesis and Future Directions

Drawing together the main findings presented so far, this short review reveals that sonification represents a promising technological frontier for educational inclusion of visually impaired students in mathematics learning, particularly for the study of functions. The combined use of audio-based and manipulative technologies could facilitate the learning of complex mathematical concepts, enhancing students' drive to learn, persistence, engagement, and inquisitiveness (Spinczyk et al., 2019). However, the field clearly requires a more systematic and integrated approach. As a result of this educational review, there is still a need to clarify the following issues:

- the intrinsic difficulties faced by blind or visually impaired students in learning mathematics (particularly functions), including the specific role of mathematical language as an obstacle, both from a cognitive and affective perspective;
- which specific content can be addressed with the available sonification tools, and which cannot, to identify how existing sonification applications can help and what developments or improvements would be necessary to address additional gaps;
- how these tools could be integrated into the mathematics classroom (the role of mathematical content, the teacher, and the student);
- the training required by teachers to effectively integrate sonification into the teaching of functions.

While the analysed research demonstrates encouraging results regarding the effectiveness of auditory representation of mathematical graphs, it reveals the absence of a consolidated framework to guide the development of standardized educational tools. The identified challenges—from insufficient teacher training to technical limitations of existing tools—demand coordinated effort among researchers, technology developers, and educators. Future research should focus not only on developing new technologies but also on their practical and sustainable implementation in real educational contexts, ensuring that technological innovation translates into effective improvement of the learning experience for all students. The path forward requires addressing both the technical aspects of sonification design and the pedagogical frameworks necessary for successful classroom integration, ultimately bridging the gap between research findings and educational practice.

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Annex 1 - Summary of HCI Articles of the Literature Review

- Ahmetovic, D., Bernareggi, C., Guerreiro, J., Mascetti, S., & Capietto, A. (2019, May). Audiofunctions. web: Multimodal exploration of mathematical function graphs. In *Proceedings of the 16th International Web for All Conference* (pp. 1-10).
  - https://doi.org/10.1145/3315002.3332438
  - The paper presents AudioFunctions.web, a web-based tool designed to enable blind and visually impaired (BVI) users to explore mathematical function graphs through sonification, auditory icons (earcons), and speech synthesis. Unlike existing solutions, this platform is accessible across multiple devices—including desktops, smartphones, touchscreens, keyboards, mice, and touchpads—and allows seamless access from digital teaching materials via hyperlinks. A user study involving 13 visually impaired participants (12 blind and 1 low-vision) demonstrated that all proposed interaction modalities were highly usable. Preferences varied based on user abilities and familiarity: touchscreen interaction was the most intuitive, closely resembling tactile graph exploration, while keyboard navigation was favored for its precision. However, touchpad interaction proved more challenging due to the lack of a consistent reference frame. The study confirmed that the dual sonification using frequency to represent function values and intensity to indicate the distance between interaction points and the function line—enhanced graph comprehension. Additionally, directional cues reinforced spatial positioning, and text-to-speech (TTS) provided key information about points of interest, such as minima, maxima, and axis intersections. Mathematical expertise played a crucial role in usability, with participants proficient in mathematics finding the system easier to learn and use. Future work aims to enhance personalization, expand multi-modal interactions, and improve sonification for arbitrary graphs.

- Cantrell, S. J., Walker, B. N., & Moseng, Ø. (2021, June). Highcharts sonification studio: an online, open-source, extensible, and accessible data sonification tool. In *Proceedings of the International Conference on Auditory Display* (Vol. 2).
  - https://doi.org/10.21785/icad2021.005
  - Highcharts Sonification Studio (HSS) is an online tool designed to make sonification more accessible, flexible, and useful for a variety of users, from researchers to students and accessibility advocates. Developed through a collaboration between Highsoft, the creators of Highcharts, and the Georgia Tech Sonification Lab, HSS builds upon the legacy of Sonification Sandbox, an older Java-based tool, but modernizes it by moving everything to a web-based platform. This shift eliminates installation barriers, making sonification available directly in the browser, with no need for specialized software. The tool allows users to input data in a spreadsheet format, which can then be converted into sound through an intuitive multimodal graphing system. By enabling both visual and auditory data representation, HSS provides a more inclusive approach to data analysis, particularly for blind and visually impaired (BVI) users. Unlike many traditional sonification tools that require programming expertise, HSS is designed to be user-friendly while still allowing for extensive customization. Users can adjust how data is mapped to sound by modifying pitch, tempo, volume, and panning, ensuring that sonifications are both meaningful and engaging. A major focus during development was accessibility, and HSS underwent multiple rounds of usability testing, including with BVI users and

accessibility experts. Early feedback highlighted issues with screen reader compatibility and user interface complexity, which led to improvements in navigation, labeling, and interactive controls. The platform also introduced features like real-time interaction, allowing users to play, pause, and loop sonifications while interacting with the visual representation of the data. One of the biggest strengths of HSS is its adaptability. It serves multiple purposes, from helping students in STEM fields understand complex data through sound to providing researchers with an alternative way to analyze multidimensional datasets. The tool also has potential applications in assistive technology, allowing BVI users to explore data independently, rather than relying solely on textual descriptions. Despite its advancements, challenges remain. The tool still requires refinement in screen reader integration, and some users found the interface overwhelming, prompting further simplifications and enhancements. However, with ongoing development and community contributions, HSS is positioned as an important step toward making sonification a mainstream tool in education, research, and accessibility. By offering an open-source, web-based solution, it lowers the barriers to entry and encourages broader adoption of sonification as a legitimate and valuable method for data analysis.

- Chundury, P., Thakkar, U., Reyazuddin, Y., Jordan, J. B., Elmqvist, N., & Lazar, J. (2024, October). Understanding the visualization and analytics needs of blind and low-vision professionals. In *Proceedings of the 26th International ACM SIGACCESS Conference on Computers and Accessibility* (pp. 1-5).
    - https://doi.org/10.1145/3663548.3688496
    - The paper reports on an online survey exploring the experiences and challenges faced by BLV professionals who analyze data. The authors found that BLV professionals faced accessibility issues at various points in the data analysis pipeline— ranging from data loading and transformation, availability and compatibility of data tools (mostly Excel, Python, SAS) with assistive technology, and visualization authoring. Furthermore, collaboration with sighted colleagues indicates compromised independence. Sonification was used with very simple linear relationships.

- Dobosz, K., & Hanak, D. (2024, July). Audible charts of mathematical functions. In the *International Conference on Computers Helping People with Special Needs* (pp. 211-216).
    - https://doi.org/10.1007/978-3-031-62846-7_25
    - The article describes the implementation of a prototype educational mobile application for visually impaired students for math education to explore the features of polynomial functions by audio feedback pointwise. The audio reflects the position of the indicator relative to the function graph line and the value of the nearest point on the graph. The sound waves are synthesized in real time with an amplitude that is a function of the distance of the indicator from the graph line. The following features can be analysed: type (sinusoidal, square, triangular, sawtoothed), base tone, sound sampling frequency, buffer size, amplitude decay threshold. The tool uses a Text-To-Speech engine to emit voice information about special points on the chart (e.g. minimum, maximum, zero places) and can be controlled by speech commands. They recommend automatically scaling a function to easily find areas with audible sounds, while not exceeding the maximum threshold in order to avoid excessive information noise.

- Enge, K., Elmquist, E., Caiola, V., Rönnberg, N., Rind, A., Iber, M., Lenzi, S., Lan, F., Höldrich, R., & Aigner, W. (2024, June). Open your ears and take a look: A state-of-the-art report on the integration of sonification and visualization. In *Computer Graphics Forum, 43*(3), e15114.
  - https://doi.org/10.1111/cgf.15114
  - The report explores the intersection of visualization and sonification, highlighting their combined potential for data representation. While these two fields have traditionally developed in parallel, this study examines how their integration can enhance data comprehension and accessibility. The research categorizes 57 academic publications from 2011 to 2023, analyzing their methodologies, dataset types, and evaluation frameworks. One of the key takeaways is that audiovisual integration is more than the sum of its parts—it leverages the strengths of both modalities, allowing users to detect patterns that may be difficult to discern through a single sensory channel. For instance, visual perception excels at spatial organization and structure recognition, while auditory perception is highly sensitive to temporal variations and subtle fluctuations in data. A classification system is introduced to organize existing works based on:
    - Reading level and dataset type (e.g., time-series, multivariate data)
    - Visualization and sonification techniques (e.g., parameter mapping, auditory icons, earcons)
    - Evaluation methods and user studies focusing on usability and effectiveness

    The study identifies three major adjacent fields where audiovisual integration has proven useful:
    - Audiovisual monitoring – Sonification supports real-time monitoring, such as in medical diagnostics and space science.
    - Accessibility – Sonification can improve data accessibility for visually impaired users by complementing screen readers and tactile graphics.
    - Audiovisual data art – Artistic projects explore sonification not just for function but also for aesthetics, engaging broader audiences.

    Another important aspect is the discussion of redundant and complementary mappings in audiovisual displays. Redundant mappings use both sound and visuals to reinforce the same information (e.g., a graph where higher values correspond to both a rising line and a higher pitch), while complementary mappings allocate different aspects of data to each modality.

- Fan, D., Siu, A. F., Law, W. S. A., Zhen, R. R., O'Modhrain, S., & Follmer, S. (2022, April). Slide-tone and tilt-tone: 1-DOF haptic techniques for conveying shape characteristics of graphs to blind users. In *Proceedings of the 2022 CHI Conference on Human Factors in Computing Systems* (pp. 1-19).
  - https://doi.org/10.1145/3491102.3517790
  - The research article is focused on access to data visualizations for blind and visually impaired users through different modalities. Authors designed two activities: (a) a formative workshop for three blind users that explored line graphs through 3D laser cutouts and standard tactile graphics; (b) evaluation by 8 BVI adults (all over 40) of a 1-DOF haptic interface that is the authors' own prototype to enable four modes of data visualization exploration: (1) interactive tactile graphics; (2) sonification only; (3) sonification with slide-tone (motorized slider moving vertically representing y-value); (4) sonification with tilt-tone (tilting platform to show inclination, i.e. angle of the tangent line at the point). All of these four modes were accompanied by speech output that served as an immediate possibility to request coordinates of the current point. Four datasets

visualizations were used in the form of line graphs. The participants of the study suggested using sonification with a simple haptic feedback in order to reinforce people's interaction and understanding of datasets visualizations.

- Fitzpatrick, J., & Neff, F. (2023). Perceptually congruent sonification of auditory line charts. *Journal on Multimodal User Interfaces, 17*(4), 285-300.
    - https://doi.org/10.1007/s12193-023-00413-w
    - The paper deals with the development of perceptually congruent sonification models for auditory line charts. The authors present the Perceptually Congruent Sonification (PerCS) model, which integrates psychoacoustic principles and insights from auditory scene analysis into the sonification process. The focus is on the investigation of different design parameters for auditory line charts, in particular the frequency range. The authors argue that a narrower frequency range of 400-1400 Hz could be advantageous, as it allows for more uniform loudness perception and offers other psychoacoustic benefits. A user study compares three sonification approaches with different frequency ranges: 65-1480 Hz without loudness compensation, 400-1400 Hz without compensation and 65-1480 Hz with compensation. The results show that the narrower range (400-1400 Hz) performs as well as the wider ranges, supporting the authors' hypothesis. The paper also discusses other important aspects such as presentation rate, tonality and spatial representation. The authors conclude that the simultaneous presentation of multiple sonified trends is problematic and recommend focusing on individual trends. The paper emphasises the need to consider perceptual constraints and proposes concrete design guidelines to improve the interpretability and efficiency of sonifications.

- Grond, F., Droßard, T., & Hermann, T. (2010). SonicFunction: experiments with a function browser for the visually impaired. In *Proceedings of the 16th international conference on auditory display*.
    - https://www.icad.org/Proceedings/2010/GrondDro%C3%9FardHermann2010.pdf
    - SonicFunction is an innovative tool designed to make mathematical functions accessible through interactive sonification, specifically for visually impaired users. Unlike traditional auditory graphing tools, it introduces a hybrid approach that combines both discrete and continuous sonification, allowing users to explore mathematical functions dynamically. The discrete mode provides immediate auditory feedback when the user interacts with the function step by step, while the continuous mode ensures a smooth auditory representation of function continuity. One of its most distinctive features is the integration of derivative-based sonification, where LFO (Low-Frequency Oscillator) modulation is applied to amplitude, creating a tremolo effect that provides a perceptual cue for local maxima and minima. This approach enhances the perception of turning points in a way that is more intuitive than simple pitch mapping alone. Another significant innovation is the method used for boundary detection. Instead of using traditional earcons, SonicFunction employs spatialized noise with a bandpass filter—where the center frequency indicates whether the user has exited the graph's upper or lower bounds, while panning effects signal whether they have moved beyond the left or right edges. This creates a more immersive and natural experience for users exploring a function. Inspired by MathTrax, a Java-based function sonification tool that is compatible with screen readers, SonicFunction also incorporates keyboard shortcuts for efficient exploration. Users can navigate functions using stepwise

adjustments, modifying step size and movement speed along the x-axis. User testing with 14 blind and visually impaired students evaluated the interface's learnability and effectiveness. Results showed that the combination of multi-parameter mapping and spatialized cues enabled users to develop mental representations of mathematical functions. The interface proved intuitive and responsive, though the complexity of simultaneous auditory information required some familiarization time.

- He, S., & Yu, L. (2024, May). Charting beyond sight with DataStory: Sensory substitution and storytelling in visual literacy education for visually impaired children. In *Extended Abstracts of the CHI Conference on Human Factors in Computing Systems* (pp. 1-8).
    - https://doi.org/10.1145/3613905.3650800
    - The article introduces results of the authors' work on the tool called DataStory that offers a tactile storybook with embedded visualizations and sonified data narratives. The main goal of the tool is to enhance visual literacy of young visually impaired children (under 12) by creating stories that could be combined with visual, tactile and audio format of simple charts (line charts, bar charts and scatter plots) demonstrating facts about objects that may be interesting and attractive to explore. These charts are prepared in a tactile format with braille labels but also sonified using natural sounds of river, animals and insects taken from Youtube Audio Library. The quantitative data are sonified using the loudness mapping technique. The prototype of the tool was further tested only by nine sighted and blindfolded children and a teacher of maths in a special education school. The authors plan to involve visually impaired individuals to help with designing the tool and expand the scope of accessible visualization types such as network diagrams or heat maps.

- Hoque, M. N., Ehtesham-Ul-Haque, M., Elmqvist, N., & Billah, S. M. (2023, April). Accessible data representation with natural sound. In *Proceedings of the 2023 CHI Conference on Human Factors in Computing Systems* (pp. 1-19).
    - https://doi.org/10.1145/3544548.3581087
    - This paper presents Susurrus, a novel open-source sonification tool designed to enhance data visualization accessibility, particularly for blind and low-vision (BLV) users. Unlike traditional sonification methods that rely on artificial tones and pitch variations, Susurrus utilizes natural sounds, such as rain or birdsong, to represent different data categories. By mapping data values to loudness and spatial positioning within a structured soundscape, it provides a more intuitive and immersive auditory experience. Additionally, synthesized speech is used to provide details on demand. A user study involving 12 BLV participants (with an additional 5 participants in a pilot study) demonstrated several advantages of natural sound-based sonification over conventional methods. Participants found that parallel sonification, which allows up to five types of sounds to be heard simultaneously, improved perception and differentiation of data points. The loudness-based mapping proved to be more accessible than pitch-based sonification, particularly for users without musical training. Moreover, the use of familiar, complex aperiodic sounds led to a more engaging and enjoyable experience, reducing cognitive load and increasing user comfort. The approach was also effective in representing trends across bar, line, multi-line, and scatter plots, making data exploration more efficient. Susurrus is accessible via a web browser and API, with keyboard shortcuts enabling interaction. It supports common data exploration actions, including gist extraction, navigation, situating within data, selection, and details on demand. Findings indicate that Susurrus

significantly improves users' ability to understand categorical data and is particularly beneficial for individuals without formal musical training. Given its success, future work will focus on expanding its soundscapes and refining interaction techniques to further enhance accessible data visualization. Actually, Susurrus is an open-source project available on GitHub.

- Holloway, L. M., Goncu, C., Ilsar, A., Butler, M., & Marriott, K. (2022, April). Infosonics: Accessible infographics for people who are blind using sonification and voice. In *Proceedings of the 2022 CHI Conference on Human Factors in Computing Systems* (pp. 1-13).
    - https://doi.org/10.1145/3491102.3517465
    - Infosonics is an innovative approach designed to provide audio equivalents of infographics, enhancing accessibility for blind and visually impaired (BVI) individuals. Unlike traditional sonification, which primarily converts data into sound, Infosonics integrates non-speech audio, speech annotations, and a structured auditory narrative to improve both understanding and engagement. The system is built on the idea that while sonification is effective in identifying patterns in data, it often lacks contextual clarity, making it difficult to interpret without additional information. By combining spoken explanations with structured auditory cues, Infosonics ensures that users can grasp both the high-level trends and the specific data points present in an infographic. Infosonics enhances data accessibility by introducing interactive sonification, where users can explore different aspects of a dataset through layered audio tracks. This method allows BVI users to independently analyse the information, overcoming the limitations of static text descriptions, which often provide only a mediated summary of data rather than direct exploration. Unlike traditional alternative text, which passively describes an infographic, Infosonics provides a parallel track of annotations that highlights key points, explains the sonication process, and introduces the dataset, ensuring that the auditory experience is both informative and engaging. A crucial component of Infosonics is the focus on aesthetic sonification, or "musification," which prioritizes pleasurable and meaningful auditory experiences. Traditional sonifications often prioritize function over form, resulting in harsh or unintuitive sounds that can be difficult to interpret. Infosonics challenges this notion by proposing that a pleasing auditory experience can enhance comprehension. Similar to how visual infographics invest in design aesthetics to improve clarity and engagement, Infosonics uses carefully chosen audio cues, musical structures, and spatial panning to present information in an intuitive and engaging manner. A practical example of Infosonics was its application to COVID-19 data visualization, where different data series (e.g., confirmed cases, hospitalizations, and deaths) were mapped to distinct sound properties. By using stereo panning and timbral differentiation, Infosonics successfully represented multiple data layers without overwhelming the listener. The system used a rhythmic mapping technique where each death was represented by a distinct note, and the rate of deaths determined the rhythm's intensity, creating a more immersive and emotionally engaging experience. A controlled user study with BVI participants demonstrated that Infosonics significantly improved data comprehension compared to standard sonification methods. Participants found that Infosonics helped them form a clearer mental image of data trends, while traditional sonification alone often felt overwhelming or confusing. However, there was no strong preference for Infosonics over text descriptions, suggesting that while Infosonics provides independent exploration, it is most effective when

combined with textual explanations. Key design insights from the study indicate that Infosonics should:

- Balance sonification with speech – Users benefit from spoken introductions and annotation tracks that explain how to interpret the sounds.
- Prioritize aesthetic and intuitive audio design – The choice of instruments, pitch mappings, and spatial placement can significantly impact engagement and understanding
- Enable interactive exploration – Users should be able to control which layers they hear, adjust playback speed, and explore data selectively.
- Ensure accessibility – The interface should be browser-based and easy to navigate, allowing casual users to engage with Infosonics without specialized software.

- Kim, J., Lee, Y., & Seo, I. (2019, May). Math graphs for the visually impaired: audio presentation of elements of mathematical graphs. In *Extended Abstracts of the 2019 CHI Conference on Human Factors in Computing Systems* (pp. 1-6).
  - https://doi.org/10.1145/3290607.3308452
  - The article describes experiments to investigate optimal methods for representing mathematical elements of graphs with sound. Kim et al. conducted three interaction design studies focusing on the x-axis, quadrants, and differentiability of mathematical graphs. The usability findings led to specific design guidelines for auditory displays: (1) serial presentation (x-axis and graph displayed sequentially) reduces cognitive load compared to parallel presentation, though parallel mode offers advantages for detecting intersection points; (2) timbre variation provides an effective perceptual cue for quadrant boundaries, particularly when navigating across all four quadrants; (3) pitch-based sonification alone creates ambiguous interfaces for differentiability, requiring supplementary speech feedback to disambiguate sonic patterns. These results establish interaction design principles for audio-based mathematical graph representations.

- Lindetorp, H. & Falkenberg, K. (2021). Sonification for everyone everywhere - Evaluating the WebAudioXML sonification toolkit for browsers. In *Proceedings of the 26th international conference on auditory display*.
  - https://repository.gatech.edu/entities/publication/ffbe5199-4339-4853-b5f9-e2cac6bebe32
  - The WebAudioXML Sonification Toolkit (WAST) was developed to make sonification more accessible, particularly for students and non-experts who might not have extensive technical backgrounds. The toolkit was evaluated in a pedagogical setting, where it was tested on ten student projects, demonstrating that it successfully helped students achieve learning outcomes. WAST simplifies the process of creating and sharing sonifications by using a web-based platform that eliminates the need for installations, making it highly accessible across different devices. One of the main advantages of WAST is its use of parameter mapping sonification, which remains the most widely used technique in sonification research. However, the broader challenge of distributing interactive sonifications has not been extensively studied. Typically, sonifications are distributed as pre-rendered audio files, which limits their interactivity. This is a significant drawback, especially for educational purposes, where interactive exploration is key to understanding data relationships. A major benefit of WAST is that it allows sonifications to be shared via a simple web link, which makes collaborative learning and data exploration easier. This

feature is crucial since educational activities involving sonification are often collaborative, interactive, and exploratory. Existing tools for sonification often require programming expertise or specialized software installations, whereas WAST simplifies this process through its WebAudioXML-based approach, which provides an XML configuration file for parameter mapping and audio processing. Historically, sonification toolkits have been developed along a spectrum between flexibility and accessibility. Tools like SuperCollider and PureData are extremely flexible but require coding skills, while applications like Sonification Sandbox offer accessibility but are limited in customization. WAST finds a middle ground by providing structured, configurable sonification without requiring direct JavaScript coding, making it more accessible than previous solutions. The toolkit integrates a graphical user interface for parameter mapping, where users can connect data variables to various sound parameters, such as pitch, volume, or filter settings. The UI is designed to be intuitive, even for beginners, while still offering advanced configuration options for more experienced users. The ability to visually explore datasets alongside the auditory representation enhances the overall learning experience. A key insight from the evaluation of WAST was that students often approached parameter mappings as a mixing tool, similar to a Digital Audio Workstation (DAW), positioning sounds in space rather than dynamically mapping them to data variables. This suggests that a sonification toolkit could benefit from built-in audio mixing features, such as equalization, compression, and reverb, to create a more polished auditory experience. Overall, WAST demonstrates the potential for web-based, interactive sonification to become more widely used in education and research. By lowering the technical barriers to entry, it makes sonification more approachable for students, educators, and researchers alike, encouraging greater adoption in non-technical fields.

- Maćkowski, M., Brzoza, P., & Spinczyk, D. (2023). An alternative method of audio-tactile presentation of graphical information in mathematics adapted to the needs of blind. *International Journal of Human-Computer Studies, 179*, 103122.
  - https://doi.org/10.1016/j.ijhcs.2023.103122
  - The authors present an alternative audio-tactile method supporting the ability to use tactile graphic information during solving mathematical exercises by students with blindness. The method uses a picture displayed on a tablet with a tactile print and assumes to explore the elements of the picture by using audio descriptions at varying levels of detail. The method allows users to explore and navigate graphics to learn mathematics. They evaluated the proposed method with 76 blind students at high school level using a set of 60 exercises that encompassed five areas of mathematics. Each exercise offered 3 levels of detailed description. In general, the assessment was made of the ability to correctly solve mathematical exercises, which is supported by the developed method. They evaluated the method using SUS for participant satisfaction and NASA-TLX to measure mental load. The method contributes to achieving the required level of learning effectiveness with various levels of efficiency with respect to the student's math skills.

- Ohshiro, K., Hurst, A., & DuBois, L. (2021, October). Making math graphs more accessible in remote learning: Using sonification to introduce discontinuity in calculus. In *Proceedings of the 23rd International ACM SIGACCESS Conference on Computers and Accessibility* (pp. 1-4).
  - https://doi.org/10.1145/3441852.3476533

  - The authors designed sonification techniques to represent mathematical discontinuities through non-speech sounds. The design was informed by remote interviews with 6 participants (2 teachers, 4 blind persons) to understand their experiences with math education using graphs. Usability evaluation revealed specific interface challenges: participants sometimes confused intentional silence (removable discontinuity) with technical glitches, and discrete pitch changes proved ambiguous across different discontinuity types. The study highlighted the importance of providing multimodal feedback (auditory and verbal) to disambiguate the interface.

- Ramôa, G., Moured, O., Schwarz, T., Müller, K., & Stiefelhagen, R. (2023, July). Enabling people with blindness to distinguish lines of mathematical charts with audio-tactile graphic readers. In *Proceedings of the 16th International Conference on PErvasive Technologies Related to Assistive Environments* (pp. 384-391).
  - https://doi.org/10.1145/3594806.3594818
  - This paper explores the capacity of blind and visually impaired individuals (BVI) to discern lines in mathematical charts through the utilisation of audio-tactile graphic readers. The authors have developed a novel, upgraded user interface for the Tactonom Reader that enhances and improves upon the standard interface. This enhanced user interface is distinguished by dynamic trigger sounds that are activated when the finger reaches a new line on the tactile chart. Furthermore, it facilitates the filtering of individual lines. By pressing a designated button, a selected line can be marked with a repeating beep sound, the pitch of which changes to indicate movement towards the top or bottom along the Y-axis. In a study involving four BVI participants, the efficacy of the new interface was compared to the standard interface of the Tactonom Reader. The results indicated that all participants found the new user interface to be more effective in retrieving detailed information. The paper concludes that the current 2D graphic reader user interface is not yet sufficiently sophisticated to distinguish lines in all line charts for individuals with BVI.

- Riga, P., & Kouroupetroglou, G. (2024). How blind individuals recall mathematical expressions in auditory, tactile, and auditory–tactile modalities. *Multimodal Technologies and Interaction, 8*(7), 57.
  - https://doi.org/10.3390/mti8070057
  - The research article is focused on different input modalities and how they affect the ability of blind readers when they try to recall mathematical expressions they have read before. The authors made an experiment with 16 blind adults aged between 15 to 28 years who are experienced users of screen readers and braille input, both embossed or displayed on refreshable braille displays. A set of 25 different mathematical expressions was selected and offered to participants of the experiment in auditory (math-to-speech), tactile (embossed braille according to the Nemeth code for mathematics braille encoding), and auditory-tactile format. During the experiment, participants first listened/read an expression and then tried to recall its content. Administrators of the experiment counted the number of recalling errors and categorized them. After performing the experiment with all the 16 participants, authors analysed the results and came to some conclusions. They confirmed that the increase in the number of recalling errors affected by the increase in complexity of mathematical expressions is significantly greater in Auditory modality (TTS) compared to Tactile only and Auditory-Tactile modalities.

- Reinsch, D., & Hermann, T. (2023, June). Sonecules: A Python sonification architecture. In *Proceedings of the 28th International Conference on Auditory Display* (pp. 62-69).
    - https://doi.org/10.21785/icad2023.5580
    - Sonecules is a Python-based framework designed to make sonification more flexible, accessible, and scalable. Built on mesonic, it bridges the gap between highly customizable but complex platforms like SuperCollider and user-friendly but limited GUI-based tools such as Highcharts Sonification Studio. The name "Sonecules" draws inspiration from molecules, highlighting the idea that sonification can be composed of small, interrelated elements that work together to form complex auditory structures. Just as molecules combine atoms to create something more significant, sonecules allow for modular and layered sound representations of data. Unlike traditional sonification tools, which often require deep expertise in sound synthesis, Sonecules integrates seamlessly into Python-based data workflows, making it accessible for both beginners and experienced users. The idea is to make sonification as common and easy to use as data visualization tools like Matplotlib. At its core, Sonecules structures sonification into four key components: data representation, interaction, algorithmic mapping, and application purpose. This ensures that sonification isn't just about transforming data into sound but meaningfully reflecting relationships between data points. One of its defining features is its modular object-oriented architecture, which allows users to combine multiple sonecules into a single auditory representation. These sonifications can be built around a shared time structure, much like a visualization graph, ensuring that different auditory elements remain cohesive and contextually relevant. Sonecules also introduces timeline-based interaction, allowing users to control sound elements dynamically, creating an interactive auditory experience rather than a static transformation. The framework supports different sonification techniques, including score-based sonification, where data is precomputed and mapped into a timeline (useful for functions and statistical models), buffer synthesis sonification, which varies harmonic amplitudes to represent multivariate data, and trigger-based sonification, where events are represented through structured auditory cues. These techniques ensure that Sonecules is adaptable to a wide range of data types and use cases, whether it's for scientific analysis, education, or artistic exploration. A unique aspect of Sonecules is its use of timbral sonification, where data variations are mapped to changes in sound texture rather than just pitch or volume. This approach makes complex datasets more intuitively perceivable, especially when dealing with multidimensional data. Additionally, Sonecules moves beyond traditional earcons by incorporating spatialized noise filtering, where the positioning of noise cues indicates boundaries or critical areas within the dataset. This allows users to navigate data more fluidly and with a clearer sense of structure. By remaining backend-agnostic, Sonecules ensures that it can adapt to various sound rendering environments without being tied to a single platform. It supports multiple synthesis approaches, including oscillator bank mapping, where each variable in a dataset controls an independent oscillator, allowing for rich harmonic interactions. The ability to mix and match different sonecules within the same sonification context makes it possible to build highly detailed and nuanced auditory representations of data.

- Roy, T., & Boppana, L. (2022, July). Interactive web-based image and graph analysis using sonification for the blind. In *2022 IEEE Region 10 Symposium (TENSYMP)* (pp. 1-6). IEEE.

    - https://ieeexplore.ieee.org/document/9864411
    - The authors present Aulal (available at https://github.com/t-ROY-coder/aural) which has two main options. The first offers the possibility to upload an image and the application detects objects on the picture. The second plots and sonifies polynomials of degree at most two. The sounds are played when the user hovers the mouse over the plotted graph of the function.

- Sharif, A., Wang, O. H., Muongchan, A. T., Reinecke, K., & Wobbrock, J. O. (2022a, April). VoxLens: Making online data visualizations accessible with an interactive JavaScript plug-in. In *Proceedings of the 2022 CHI conference on human factors in computing systems* (pp. 1-19).
    - https://doi.org/10.1145/3491102.3517431
    - This article describes a JavaScript library that can be used in combination with other plotting JavaScript libraries, like d3js, chartjs or google charts. It adds an audio layer to the visual representation of data (not functions) and a text summary that can be used with a screen reader. The authors experimented with 22 users and showed that there was an improvement in the interaction and in the extraction of information. The library is available at https://github.com/athersharif/voxlens.

- Sharif, A., Wang, O. H., & Muongchan, A. T. (2022b, October). *"What makes sonification user-friendly?" Exploring usability and user-friendliness of sonified responses.* In Proceedings of the 24th International ACM SIGACCESS Conference on Computers and Accessibility (pp. 1-5).
    - https://doi.org/10.1145/3517428.3550360
    - The authors experiment with ten screen reader users different sonifications of data series, testing for clarity, confidence and pleasantness. Both continuous and discrete sonifications were tested, and several sounds were used (oscillators, synthesizers). With the output of the tests, the authors produced a JavaScript library called sonifier, and available at https://github.com/athersharif/sonifier.

- Soiffer, N., & Noble, S. (2019). Mathematics and statistics. In *Web Accessibility: A Foundation for Research* (pp. 417-443). London: Springer London.
    - https://doi.org/10.1007/978-1-4471-7440-0_23
    - Review published in 2019, summarizing the knowledge in mathematics accessibility for blind and visually impaired people. Topics covered include converting math to speech, braille output, navigation through mathematical formulas, possibilities of math input and edit in computer and accessibility to graphs via sonification, tactile tools and haptic tools. The authors describe the benefits and shortcomings of the tools presented, put them in the context of needs in teaching and learning mathematics, and cite evaluations of tools by blind testers. They identify bottlenecks in the process of making mathematics accessible to BVI people and present them as challenges for researchers and developers of new technologies. Some of the challenges have been resolved over the past 5 years, while others remain prominent today. Regarding graph sonification, the authors limit themselves only to describing a few tools. It seems that at the time of writing there was not yet enough experience for any evaluation of sonification. Finally, the authors add some personal remarks on the accessibility process and on mathematics teaching.

- Strumillo, P., Skulimowski, P., Bujacz, M., Obuchowicz, M., Borowiecka, I., Holak, M., & Radecki, A. (2018, July). A mobile application for interactive sonification of images for the visually impaired. In *2018 11th International Conference on Human System Interaction (HSI)* (pp. 391-397). IEEE.

- https://doi.org/10.1109/HSI.2018.8431362
- The mobile application for interactive sonification of images for the visually impaired aims to provide blind and visually impaired (BVI) users with an intuitive way to explore visual information through sound. A primary challenge faced by BVI individuals involves three critical activities: safe and independent mobility, spatial orientation, and information access. This application seeks to mitigate these challenges by leveraging sonification as a solution for navigating the vast amount of visual information present in everyday life. The sonification method employed in this application is inspired by the vOICe, a well-known system that maps image pixels to audio signals. Specifically, sonification is performed column-wise, where pixels at the bottom of the image correspond to lower frequencies, and brightness levels dictate the volume of individual harmonic components. However, mastering this type of sonification requires extensive training, often spanning several months. Research has demonstrated that, after prolonged training, the brain regions responsible for visual processing in blind individuals become activated when exposed to sonified representations, effectively repurposing those neural pathways for auditory-based spatial interpretation. The application allows for single-touch and multi-touch interactions, providing different levels of control over the auditory representation. It operates based on HSV (hue, saturation, value) colour mapping, which closely mirrors human visual perception, making the transformation from image to sound more intuitive. The key parameters mapped to sound include frequency, amplitude, and stereo positioning. Two main modes of sonification are supported
  - Periodic Sonification, where a region of interest (ROI) is looped to allow for in-depth auditory analysis.
  - Continuous Sonification, where different exploration methods (point, line, or area) allow users to interact with the image dynamically.

  A notable feature is discretization of the ROI, enabling users to explore specific regions of an image systematically. In the periodic sonification mode, images are divided into subregions that can be sonified individually, whereas the continuous sonification mode allows for a more fluid exploration, adapting to the user's needs. Ultimately, this application represents a significant advancement in assistive technology by making visual content more accessible through interactive sonification. It empowers visually impaired individuals to independently explore images and gain a deeper understanding of spatial and graphical information through sound.

- Thompson, J. R., Martinez, J. J., Sarikaya, A., Cutrell, E., & Lee, B. (2023, April). Chart reader: Accessible visualization experiences designed with screen reader users. In *Proceedings of the 2023 CHI Conference on Human Factors in Computing Systems* (pp. 1-18).
  - https://doi.org/10.1145/3544548.3581186
  - The authors describe a co-design approach to create Chart Reader, a web-based accessibility engine to interactively read and better understand visualizations and their underlying data by screen reader users. They worked together with 10 blind and low vision individuals over five months to explore visualization prototypes with them. In their application, they incorporate point-wise sonification, discrete sequentially played sounds and used earcons for boundaries. Moreover, they offer spatial comparison which allows users to sonify the value of each series at the currently focused x-value through a set of

discrete, sequentially played tones that are spatialized using stereo audio. The software can display single and multiple line charts as well as bar charts. The authors found that their design partners developed their own personal styles of engaging with charts throughout their study, reflecting their autonomy in deciding how to consume visualizations with their tool. They find evidence that autonomous chart exploration is particularly important. Another finding was that a change was necessary from using the users‘ screen readers to the screen reader present in the web application because of difficult interactions.

- Tomlinson, B. J., Winters, R. M., Latina, C., Bhat, S., Rane, M., & Walker, B. N. (2017, June). Solar system sonification: Exploring earth and its neighbors through sound. In *The 23rd international conference on auditory display (ICAD 2017)* (pp. 128-134).
  - https://doi.org/10.21785/icad2017.027
  - The article presents a study on the use of sonification in planetarium shows to enhance the understanding of astronomy. Researchers developed an auditory model of the solar system, using spatial audio to represent planetary characteristics such as mass, orbital periods, temperature, and gravity. The sonifications were integrated into a planetarium show, accompanied by visual anchors and narration. The study involved 40 planetarium visitors and employed two auditory perspectives: the solar system view and the planetary view. In the solar system view, mass was represented inversely to pitch, the length of a day influenced the sonification volume, and the length of a year was conveyed through the direction of sounds. In the planetary view, characteristics such as moons, rings, temperature range, and gravitational strength contributed to creating a unique timbre for each planet. However, the study did not include user interaction. The results were positive, with participants rating the experience highly in terms of interest, pleasantness, usefulness, understandability, and relatability of concepts. Sonification proved helpful in understanding complex astronomical ideas, particularly the scale and relationships between planets, though specific planetary details were harder to remember compared to broader system-wide characteristics. The study suggests that auditory displays can be effective in informal learning environments and may enhance accessibility, especially for individuals with visual impairments. Future developments could involve refining sonification mappings, introducing interactive exhibits, and expanding educational applications beyond planetariums.

- Trayford, J. W., & Harrison, C. M. (2023). *Introducing Strauss: A flexible sonification python package*. In Proceedings of the 28th International Conference on Auditory Display (ICAD) 2023 (pp. 249-256).
  - https://doi.org/10.21785/icad2023.1978
  - STRAUSS (Sonification Tools and Resources for Analysis Using Sound Synthesis) is an open-source Python library designed to integrate sonification seamlessly into existing code-based data analysis workflows without disrupting their structure. Unlike other tools that are either GUI-based (e.g., StarSound), code-based (e.g., Astronify), or online platforms (e.g., Sonification Studio), STRAUSS is meant to be embedded directly into an analyst’s workflow, making sonification accessible for beginners while offering deep customization for advanced users. It is specifically optimized for astronomy, a field characterized by multivariate, high-dimensional datasets that often contain transient events and hidden patterns that are difficult to visualize. STRAUSS is object-oriented and designed to provide a modular, flexible approach to sonification. It supports a wide range of applications, from scientific data analysis to outreach and

accessibility. The tool allows researchers to explore data through sound, leveraging the human ability to detect auditory patterns. Accessibility has been a key focus in its development, with plans for future enhancements to improve screen reader compatibility and usability for visually impaired users. The package offers several innovative sonification choices. To enhance the perception of pitch without adding harmonic complexity, it employs power chords (dyads of root and fifth) instead of single notes. A low-pass filter (LPF) is used to control timbre, with higher values corresponding to higher cutoff frequencies, making the sound brighter—an intuitive mapping where "higher" corresponds to "brighter." This approach ensures that pitch remains unchanged while the sonic quality of the data shifts dynamically. In some cases, this method has even been applied to white noise to create a "wind" effect. Additionally, STRAUSS has experimented with the simultaneous sonification of an entire spectral dataset in a single auditory event using an inverse Fourier transform, allowing all spectral details to be heard at once. Multivariate sonification is another key feature of STRAUSS, enabling the simultaneous representation of multiple variables, which is often necessary for understanding complex relationships in astrophysical data. Typically analysed using differential equations, these relationships can now be explored through sound. For example, STRAUSS maps stellar formation rates, which indicate speed or frequency, to LFO modulation, where a higher formation rate results in a faster oscillation or flickering effect—potentially a more distinguishable cue than pitch variation. Similarly, stellar metallicity is mapped to a low-pass filter's cutoff frequency, where a higher metal content produces a brighter, more metallic sound, aligning with intuitive physical properties. The package also features full spatial audio capabilities, including ambisonic representation. This allows for precise placement of sound sources in a 3D space based on their altitude and azimuth relative to an observer. STRAUSS supports up to 16 channels, while standard VR applications currently use only 8, making it a powerful tool for immersive experiences. Using the ambiX format, it enables the positioning and mixing of an arbitrary number of sound sources in three-dimensional space. This feature was showcased in the "Stars Appearing" VR experience, where stars gradually emerge in the night sky, their brightness and colour mapped to distinct sonic characteristics, creating an accurate and engaging spatialized auditory experience.

- United Nations Office for Outer Space Affairs. (2023). *Sonification: A tool for research, outreach and inclusion in space sciences (Special Report)*. United Nations.
  - https://www.unoosa.org/documents/pdf/Space4PersonswithDisabilites/UNOOSA_Special_Report_on_Sonification_2023.pdf
  - Sonification is increasingly recognized as a powerful tool in space sciences, transforming complex data into sound to facilitate analysis, communication, and engagement. This technique is particularly valuable for accessibility, offering a way for blind and visually impaired (BVI) individuals to participate in scientific research. With approximately 15% of the global population experiencing some form of disability, sonification aligns with the principles of Universal Design, as outlined in the Convention on the Rights of Persons with Disabilities (CRPD), promoting inclusive scientific practices. However, despite its potential, the field lacks standardization, requiring a balance between flexibility for diverse user needs and the establishment of best practices. NASA and other institutions have implemented various sonification mapping techniques in X-ray astronomy, such as pitch variations to represent energy levels, volume intensity linked to brightness, discrete notes for distinct celestial

objects, and evolving sounds for extended structures like gas clouds. Projects like StarSound and VoxMagellan explore 3D sonication using ambisonic technologies, integrating virtual and augmented reality for an immersive auditory experience. The United Nations Office for Outer Space Affairs (UNOOSA) report highlights sonification's role in achieving Sustainable Development Goals (SDGs), particularly in reducing inequalities. Sonification can enhance accessibility for BVI researchers, engage the public through multisensory learning, and complement traditional visual analysis in scientific discoveries. However, challenges persist, including a lack of standardized methodologies, scepticism in the scientific community, and limited funding for research and development. In education and outreach, sonification has proven beneficial in making space sciences more engaging and accessible. Initiatives like Astronify, which converts light curve data into sound, enhance understanding for both scientists and students. Public-facing projects like A Universe of Sound, developed by NASA's Chandra X-ray Observatory, allow audiences to experience space through sound, making astronomical data more immersive and inclusive. Despite these advancements, continued efforts are needed to integrate sonification as a mainstream scientific tool and to ensure broader acceptance within the research community.

- Vines, K., Hughes, C., Alexander, L., Calvert, C., Colwell, C., Holmes, H., Kotecki, C., Parks, K., & Pearson, V. (2019). Sonification of numerical data for education. *Open Learning: The Journal of Open, Distance and e-Learning, 34*(1), 19-39.
    - https://doi.org/10.1080/02680513.2018.1553707
    - Sonification is becoming an increasingly relevant tool in education, accessibility, and data exploration, particularly in STEM fields. Traditional graphs are an essential way to visualize numerical data, but they create barriers for blind or visually impaired (BVI) learners. This research investigates whether sonification—transforming data into sound—can serve as a meaningful alternative or complement to traditional graphical representations, allowing students to understand data trends through auditory perception. The study highlights the importance of giving users direct access to raw data rather than relying on mediated textual descriptions that could unintentionally influence their interpretations. An effective sonification should allow users to grasp the same patterns and trends as those who view a visual graph, and it should do so within a comparable time frame. Ideally, sonification should integrate seamlessly with other learning tools, such as tactile graphics and text, rather than acting as a standalone replacement. To test its effectiveness, researchers conducted a two-phase study. The first phase involved controlled tests in which students engaged with sonified data and were asked to interpret it. The second phase implemented sonifications in an online statistics course, allowing students to engage with the material in a real-world educational setting. Results revealed several interaction design challenges. There was no clear preference for sonification over text-based descriptions, suggesting that its effectiveness is highly dependent on individual learning styles, prior exposure to auditory-based learning, and personal preference. Some key challenges emerged. Many users expected sonifications to present data as discrete sounds rather than continuous tones, which sometimes made it difficult to identify specific points in the dataset. Additionally, while scatterplots typically make outliers visually distinct, these anomalies were not as easily perceived in auditory form. A critical usability finding was that tactile exploration provided greater user control—allowing self-paced, non-linear navigation—whereas sonification followed a predefined, linear temporal path that constrained

exploration strategies. Despite these challenges, the study reaffirmed that sonification can be a valuable tool in education, especially for BVI students and learners with specific needs. However, it should not be seen as a direct replacement for visual graphs but rather as an additional modality that enhances comprehension when combined with other accessible tools. The best approach is a multimodal one, where students can switch between visual, tactile, and auditory formats depending on their needs and preferences. While sonification does not hinder learning, its effectiveness varies between individuals, making it a useful but context-dependent tool in STEM education.

- Wang, R., Jung, C., & Kim, Y. (2022, June). Seeing through sounds: Mapping auditory dimensions to data and charts for people with visual impairments. In *Computer Graphics Forum* (Vol. 41, No. 3, pp. 71-83).
    - https://doi.org/10.1111/cgf.14523
    - The focus of the research article is to assess how different auditory channels (e.g. pitch, volume, etc.) and conditions (continuity, tempo, polarity) impact the data and visualization perception among people with visual impairments. The authors designed an exploratory experiment and recruited 20 blind or partially-sighted people to participate in a 1-hour online session consisting of 3 parts. In the 1st part they evaluated the intuitiveness and accuracy of mapping auditory channels to data. The goal of the 2nd part was to consider intuitiveness among the type of the data and its auditory representation in order to communicate the data effectively for the visually impaired. The last part was determined to examine the prior experience of the participants with audio charts and to receive their feedback on the experiment. The authors used mainly the Sonification Sandbox app. to create all the auditory inputs for the participants. They found evidence that the data type can impact the intuitiveness and effectiveness of auditory channels to represent such data. The experiment's participants rated pitch to be the most intuitive, while the number of tappings and the length of sounds yielded the most accurate perception in decoding data.

- Wörtwein, T., Schauerte, B., Müller, K., & Stiefelhagen, R. (2016, July). Mobile interactive image sonification for the blind. In the *International Conference on Computers Helping People with Special Needs* (pp. 212-219). Cham: Springer International Publishing.
    - https://doi.org/10.1007/978-3-319-41264-1_28
    - The authors describe a web-based, mobile sonification to give blind users access to graphical information and to solve various everyday as well as job-related tasks. The combination of a touch screen, image processing, and sonification allows the user to hear the content of every image region that he or she indicates with his/her finger position on a tablet or mobile phone. They mapped all distances and function values not directly to output frequencies, because a twice as high frequency is not perceived as being twice as high by the human auditory perception. Therefore, they use the Mel transformation to have a meaningful auditory representation. However, since too low frequencies are hardly heard, they add a constant offset just before the Mel transformation to have perceivable frequencies. The following functions are sonified: a linear, hyperbolic, parabola, sine, square root, and an exponential function. They included tactile graphics as a baseline in their evaluation. All introduced sonifications reduced the time that the participants require to interpret mathematical graphs, bar charts, and floor maps.

- Zhang, Z., Thompson, J. R., Shah, A., Agrawal, M., Sarikaya, A., Wobbrock, J. O., Cutrell, E., & Lee, B. (2024, October). ChartA11y: Designing accessible touch experiences of visualizations with blind smartphone users. In *Proceedings of the 26th International ACM SIGACCESS Conference on Computers and Accessibility* (pp. 1-15).
  - https://doi.org/10.1145/3663548.3675611
  - The ChartA11y project introduces an iOS app (not yet publicly available) designed to improve accessibility of 2D data visualizations for blind users through a co-design process involving 13 sessions with two blind participants. The system leverages multimodal and multisensory interactions, combining text-to-speech (TTS), sonification, and vibrotactile feedback to provide a more intuitive and comprehensive understanding of complex visualizations. It integrates two primary interaction modes: the Semantic Navigation Framework (SNF), which enables structured exploration using VoiceOver gestures, and Direct Touch Mapping (DTM), which allows users to explore data freely through touch. One of the main challenges in touch-based visualization is that blind users often struggle to maintain directional movement and trace data patterns due to the lack of hand-eye coordination. To address this, different sonification techniques were tested, including pitch variations, duration encoding, and tonal cues, allowing trends and data distributions to be represented across line charts, bar charts, and scatter plots. Through iterative refinement, users showed a strong preference for adaptive speech output, contextual sonification, and seamless mode-switching, which helped minimize cognitive load. As a result, participants who had previously relied on sighted assistance were able to independently interpret complex multi-series scatter plots, demonstrating a level of understanding comparable to sighted users. ChartA11y highlights the importance of participatory design in sonification and introduces a customizable and flexible tool that significantly enhances non-visual access to data visualizations. The integration with VoiceOver gestures, along with the smooth transition between SNF and DTM modes, ensures an intuitive and efficient user experience.

- Zhao, Y., Lu, J., & Nacenta, M. A. (2024, October). Speech-based mark for data sonification. In *Proceedings of the 26th International ACM SIGACCESS Conference on Computers and Accessibility* (pp. 1-5).
  - https://doi.org/10.1145/3663548.3688514
  - This paper introduces SpeechTone, a novel speech-based sonification method that extends the Erie declarative grammar by encoding data into text-to-speech (TTS) attributes such as pitch, speed, timbre, and text content. By leveraging these vocal characteristics, SpeechTone provides a structured and efficient approach to auditory data representation, with a particular focus on improving data accessibility for visually impaired users. The approach is demonstrated through three examples: mapping car model counts per origin to speech pitch, encoding production trends over time into speech speed, and using voice timbre to differentiate car origins while ordering models by fuel efficiency. These demonstrations highlight the rich interplay between speech attributes, enabling multi-dimensional auditory encoding. The benefits of SpeechTone include efficient information transmission and redundant encoding, enhancing accessibility and comprehension. However, challenges remain in perceptual ranking of attributes and empirical validation, as the study currently consists only of demonstrations without formal user testing. SpeechTone is available on Chromium-based browsers such as Edge and Chrome, making it widely accessible for web-based applications. As a new method for data mapping, it

opens opportunities for further research, particularly in user studies, interactivity enhancements, and integration with other sensory modalities to refine and expand its practical applications.

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