

$B \rightarrow K +$ axionlike particles: Effective versus UV-complete models and enhanced two-loop contributions

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An axionlike particle a (ALP) can explain the excess of $B \rightarrow K +$ invisible events at Belle II. However, many analyses of ALP scenarios are oversimplified. We revisit the $B \rightarrow Ka$ transition rate in a popular minimal and UV complete model with two Higgs doublets (2HDM) and a complex singlet [Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) model]. To this end we compare our results with previous studies which derived the $\bar{b}sa$ vertex from the $\bar{b}SA$ vertex, where A is the heavy pseudoscalar of the 2HDM, in terms of an $a - A$ mixing angle. We find this approach to work only at the leading one-loop order, while it fails at the two-loop level. Furthermore, while an approximate Z_2 symmetry suppresses the leading-order amplitude by a factor of $1/\tan\beta$, which is the ratio of the two vacuum expectation values of the Higgs doublets, we find the two-loop contribution unsuppressed and phenomenologically relevant for $\tan\beta \gtrsim 5$. We determine the allowed parameter space and underline the importance of better searches for $\Upsilon \rightarrow \gamma +$ invisible and for a possible excess in $B \rightarrow K\mu^+\mu^-$. We further study the low-energy axion effective theory which leads to a divergent and basis-dependent amplitude. As a conceptual result, we clarify the ambiguities and identify which low-energy framework is consistent with the DFSZ model.

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I. INTRODUCTION

Recently Belle II has reported an excess in $B^+ \rightarrow K^+\nu\bar{\nu}$ over the Standard Model (SM) prediction with a significance of 2.8σ [1]. Since this excess is rather localized in the visible kaon energy, a fit under the assumption of a two-body decay $B^+ \rightarrow K^+a$ with invisible a also gives an excellent fit to the data and has been performed in Ref. [2]. Using Belle II data the authors obtain a significance of 3.6σ for $m_a \approx 2$ GeV and $\text{BR}(B^+ \rightarrow K^+a) = (8.8 \pm 2.5) \times 10^{-6}$. In a global analysis of Belle II and *BABAR* data the significance is reduced to about 2.4σ with $\text{BR}(B^+ \rightarrow K^+a) = (5.1 \pm 2.1) \times 10^{-6}$.

However, the lightness of a remains puzzling. A common interpretation is that a is the pseudo-Goldstone boson of a spontaneously broken global symmetry. When the symmetry is the Peccei-Quinn $U(1)_{\text{PQ}}$ [3,4] (with a soft breaking term), a is called an axionlike particle (ALP). Since a renormalizable theory with only a and SM particles is inconsistent, many related discussions are thus based on effective theories, see for example Refs. [5–13]. Although

this is sufficient to explain the data, it does not allow an unambiguous connection between $b \rightarrow sa$ and other processes, such as a decaying to SM particles. This is because, in the most general axion effective theory (axion-EFT) [14], the $b - s - a$ coupling strength is a free parameter. Correlations to the other interactions are, thus, not predicted [15]. Therefore, a complete renormalizable model is needed.

The minimal benchmark, the Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) model [18,19], was discussed in Ref. [20], where the authors assumed $b \rightarrow sa$ is induced by the mixing angle θ in $a - A$ mass term:

$$\mathcal{A}(b \rightarrow sa) = -\sin\theta \times \mathcal{A}(b \rightarrow sA). \quad (1)$$

Here A is a heavy CP -odd scalar of the Type-II two-Higgs-doublet model (2HDM) [21,22], where $\mathcal{A}(b \rightarrow sA)$ is a finite one-loop effect [23–25]. But we notice Eq. (1) requires clarification: when A is heavier than the b quark, the $b \rightarrow sA$ amplitude is off-shell and therefore unphysical and gauge-dependent. In fact, gauge-dependence challenges the entire mixing picture due to the subtlety of the aG^+H^- vertex, as in the Higgs portal model discussed in Ref. [26]. Another problem is that $\mathcal{A}(b \rightarrow sa)$ is suppressed when $\tan\beta$, the ratio of the two vacuum expectation values (VEVs) of 2HDM, is sizable. We find that $\mathcal{A}(b \rightarrow sa)$ can avoid the $\tan\beta$ suppression through higher-loop effects. So clearly, one needs to revisit $\mathcal{A}(b \rightarrow sa)$

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including all physical and auxiliary states, and compare the results from the $a - A$ mixing picture provided in Ref. [20].

After clarifying the DFSZ model prediction, the next question is whether $B \rightarrow Ka$ can be described without specifying UV physics. It is known that the W^\pm, G^\pm bosons contribute to $b \rightarrow sa$ with a divergent amplitude. So for a theory without tree-level a flavor violating couplings, the effect of UV physics, whatever it is, must cancel the divergence and yield a model independent leading-log term $\log(\Lambda_{\text{UV}}/m_W)$. Interestingly, Ref. [27] noted that the leading-log result differs by a factor of 4 in two reasonable operator bases: (i) $a\bar{q}\gamma_5 q$ and (ii) $\partial_\mu \bar{q}\gamma^\mu \gamma_5 q$. We will show that only (ii) gives a leading-log result consistent with DFSZ model, and explore the reason in more detail. It reflects the fact that the decoupling of heavy physics requires gauge invariance in the light sector, as discussed in [28] nearly 50 years ago. This behavior is subtle but not unique, and some very similar results were found for $\mu \rightarrow e\gamma$ decays [29–31].

II. $b \rightarrow sa$ IN THE DFSZ MODEL

We start with the DFSZ model, a minimal renormalizable theory for invisible ALPs. The scalar fields and their related interactions read [18,19,32]:

$$\Phi_\alpha = \begin{pmatrix} \phi_\alpha^- \\ v_\alpha + (\rho_\alpha + i\eta_\alpha)/\sqrt{2} \end{pmatrix}, \quad \alpha = u, d, \quad (2)$$

$$\Phi_s = f + \frac{r_0 + ia_0}{\sqrt{2}}.$$

$$V_\Phi = \tilde{V}_{\text{moduli}}(|\Phi_u|, |\Phi_d|, |\Phi_u \Phi_d|, |\Phi_s|) + \lambda \Phi_s^2 \Phi_u \Phi_d^\dagger + \text{H.c.},$$

$$\mathcal{L}_Y = Y_u \bar{Q}_L u_R \Phi_u + Y_d \bar{Q}_L d_R \Phi_d + Y_e \bar{L}_L e_R \Phi_s + \text{H.c.} \quad (3)$$

Here, $\tilde{V}_{\text{moduli}}$ contains all gauge invariant combination of Φ_u, Φ_d, Φ_s without phase dependence, and $\tilde{\Phi}_d = i\sigma_2 \Phi_d^\dagger$. Y_u, Y_d, Y_e are 3×3 matrices, and the generation indices are implicit.

To get insights, let us first focus on the limit $\lambda = 0$, which is the Peccei-Quinn-Wilczek (PQWW) model [3,4,33,34] plus a non-interacting complex singlet scalar. In this case, the interaction of Eq. (3) admits three $U(1)$ symmetries, corresponding to independent phase rotations of Φ_u, Φ_d, Φ_s . All $U(1)$ symmetries are spontaneously broken when the scalar fields take their VEVs, giving three massless Goldstone modes. One of them is gauged by the hypercharge $U(1)_Y$, and the other two are PQWW visible axion A_0 and the massless radial mode of a_0 from Φ_s . The PQWW $b \rightarrow sA_0$ amplitude was analyzed by [23–25], and $b \rightarrow sa_0$ is clearly zero due to the $a_0 \rightarrow -a_0$ (or $\Phi_s \rightarrow \Phi_s^*$) symmetry. Then, by introducing a tiny $a_0 - A_0$ mass matrix that only softly breaks the $U(1)$ symmetries but generates a physical mixing angle θ , the amplitude for the physical state a reads:

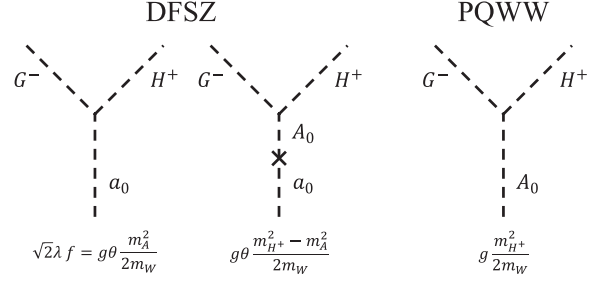


FIG. 1. Feynman rules for $G^- H^+ - \text{ALP}$ coupling in DFSZ (left) and PQWW (right) models.

$$\mathcal{A}(b \rightarrow sa)_{\text{DFSZ}} = -\sin \theta \times \mathcal{A}(b \rightarrow sA_0)_{\text{PQWW}}. \quad (4)$$

Equation (4) clarifies the meaning of Eq. (1) analyzed in Ref. [20] in the limit $\lambda = 0$.

What if $\lambda = 0$? It breaks global $U(1)_{\text{PQWW}} \times U(1)_s$ to $U(1)_{\text{PQ}}$. One of the $a_0 - A_0$ mixing states, A , acquires large mass, and the $b \rightarrow sA_0$ amplitude could be off shell. In addition, some Feynman rules change: The $G^- H^+ A_0$ vertex rule is modified by a m_A^2 term, and a new $G^- H^+ a_0$ vertex appears, as shown in Fig. 1. Therefore, we conclude:

$$\mathcal{A}(b \rightarrow sa)_{\text{DFSZ}} \neq -\sin \theta \times \mathcal{A}(b \rightarrow sA_0)_{\text{DFSZ}},$$

$$\mathcal{A}(b \rightarrow sA_0)_{\text{DFSZ}} \neq \mathcal{A}(b \rightarrow sA_0)_{\text{PQWW}}. \quad (5)$$

And strictly speaking, the off-shell amplitude $\mathcal{A}(b \rightarrow sA_0)_{\text{DFSZ}}$ is not physical since its gauge dependent. However, we find the Feynman rule of the effective $G^- H^+ a_0$ vertex remains the same as the $G^- H^+ A_0$ one of PQWW multiplied by θ . As illustrated in Fig. 1, the m_A^2 term cancels and Eq. (4), the explicit expression of Ref. [20], is correct. We think the change is not accidental. After all, the $\lambda \Phi_s^2 \Phi_u \Phi_d^\dagger$ term does not contain the physical state a , when parameterized exponentially. In the nonlinear basis, the $G^- H^+ a$ vertex becomes $G^- \overset{\leftrightarrow}{\partial}^\mu H^+ \partial_\mu a$, and is manifestly independent of m_A .

The $\lambda \Phi_s^2 \Phi_u \Phi_d$ term implies that a can directly interact with the scalar fields and the mixing picture breaks down unless $\lambda = 0$. However, λ does not appear in the explicit expression. This can be understood by analyzing the symmetry in the broken phase. In the limit $v_d \ll v_u \ll f$, all mixing angles are suppressed, aligning the mass and interaction basis for the scalar fields:

$$\Phi_u \simeq H_u = \begin{pmatrix} G^- \\ v + \frac{h+iG^0}{\sqrt{2}} \end{pmatrix}, \quad \Phi_d \simeq H_d = \begin{pmatrix} H^- \\ \frac{H+iA}{\sqrt{2}} \end{pmatrix},$$

$$\Phi_s \simeq f + \frac{r+ia}{\sqrt{2}}. \quad (6)$$

If in addition $\lambda \ll 1$, the following Z_2 symmetry emerges in V_Φ and \mathcal{L}_Y :

$$d_R \rightarrow -d_R, \quad H_d \rightarrow -H_d, \quad \text{others unchanged.} \quad (7)$$

Let us define

$$\theta \equiv \frac{2v_d}{f}, \quad \frac{1}{\tan \beta} \equiv \frac{v_d}{v_u}, \quad \lambda \quad (8)$$

as Z_2 spurions, and the effective Hamiltonian for $b \rightarrow sa$ is then fixed up to the loop factors X_1 , X_2 and X_3 :

$$\mathcal{H}_{\text{eff}} = \theta \frac{g^3 V_{ts}^* V_{tb}}{128\pi^2} \frac{m_t^2}{m_W^3} \bar{s} \gamma^\mu P_L b \partial_\mu a \times \left(X_1 \frac{1}{\tan \beta} + X_2 \frac{1}{\tan^3 \beta} + X_3 \frac{\lambda}{16\pi^2} \right). \quad (9)$$

X_1 and X_2 are given in Ref. [20], while λ enters the expression only with an additional loop factor $1/16\pi^2$. This is because λ carries Planck units \hbar . As a coupling constant, λ arises only at the next order in the perturbative expansion. This is why its contribution is absent in the one-loop expression of Ref. [20]. We notice the one-loop result is suppressed when $\tan \beta$ is sizable, so the λ contribution can be important.

Let us define $m_H \equiv m_{H^+}$ and take the limit that $m_H^2 \simeq m_A^2 = \lambda f^2 \tan \beta \gg m_W^2$, the loop factors in Eq. (9) read:

$$\begin{aligned} X_1 &= -\log \frac{m_H^2}{m_t^2} + \frac{3m_W^4}{(m_t^2 - m_W^2)^2} \log \frac{m_t^2}{m_W^2} + \frac{3(m_t^2 - 2m_W^2)}{m_t^2 - m_W^2}, \\ X_2 &= 0, \\ X_3 &= \log \frac{m_H^2}{m_t^2} + \frac{6m_W^2}{m_t^2 - m_W^2} \log \frac{m_t^2}{m_W^2} + \frac{1}{2}. \end{aligned} \quad (10)$$

Our results for X_1 and X_2 agree with Ref. [20] in the large m_H limit. X_3 is new and could in principle contain transcendental-weight-two functions, but only the logarithm functions appear after expanding in $1/m_H$.

To calculate X_3 , we work in the general R_ξ gauge, to check our results by showing ξ independence. We use the packages FeynRules [35,36] and FeynArts [37] to generate the new interacting vertexes and new diagrams. In total thousands of diagrams contribute to $b \rightarrow sa$ at the two-loop level, but only very few contribute to X_3 , as shown in Fig. 2. For simplicity, the diagrams from exchanging $b \leftrightarrow s$, and the ones replacing the internal a , H with r , A [38], are not shown explicitly. The others come with additional factor of $\tan \beta$ and θ are not relevant.

To evaluate the diagrams, we Taylor expand the Feynman amplitudes in the small external momentum $p_i^\mu \sim m_b$ [39]:

$$\mathcal{A} = \mathcal{A}|_{p_i^\mu=0} + p_i^\mu \frac{\partial \mathcal{A}}{\partial p_i^\mu} \Big|_{p_i^\mu=0} + \mathcal{O}(p_i^2). \quad (11)$$

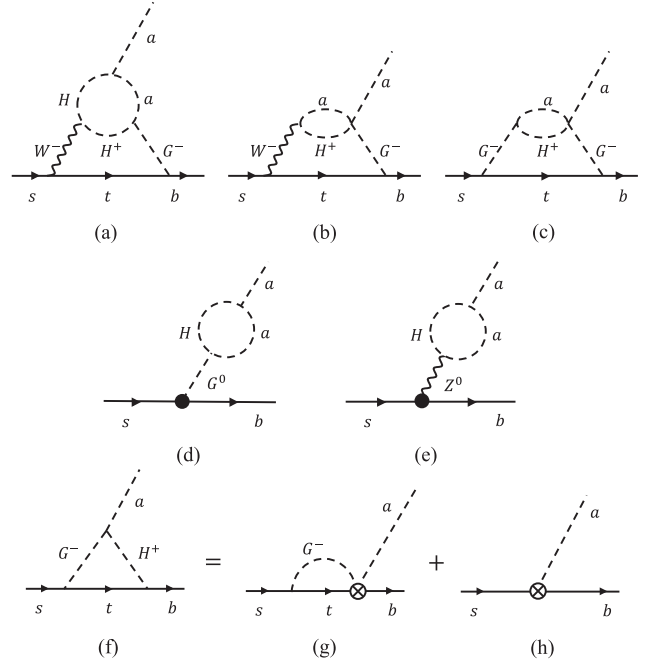


FIG. 2. Some Feynman diagrams related to the discussion, where the black dots in (d) and (e) indicate one-loop vertices and the crossed dots in (g) and (h) indicate the effective vertices after integrating out H^+ .

In the large $\tan \beta$ limit, we find the $\mathcal{A}|_{p_i^\mu=0}$ do not contribute to X_3 . Its effect is canceled after renormalizing the quark-mixing matrix, which arise from the same diagrams by replacing the external a with the vacuum tadpole f . So only $\frac{\partial \mathcal{A}}{\partial p_i^\mu} \Big|_{p_i^\mu=0}$ is relevant, and we evaluate it using FeynCalc [40–42], FIRE [43], and FeynHelpers [44]. The new functions for multiloop tensor reduction and topological identification in FeynCalc 10 [45] are applied. The master integrals are reduced to the vacuum bubble ones, whose analytical expressions are given in [46,47].

The cancellation of the gauge parameter ξ_W is similar to the case of $b \rightarrow s\mu^+\mu^-$ in SM [48]. Intuitively, the ξ_W dependent contribution from the boxlike diagrams (a–c) cancel those from the penguin-like diagrams (d, e). Thus, the loop-induced kinetic-mixing term $\partial_\mu G^0 \partial^\mu a$ plays an important role. To perform $a - G^0$ wave-function renormalization, we apply the standard method of subtracting the Goldstone boson self-energies at zero momentum [49,50]. This eliminates the tadpole contributions and simplifies the diagrams.

III. $b \rightarrow sa$ WITHOUT SPECIFYING UV PHYSICS

Is it possible to find the properties of the $b \rightarrow sa$ amplitude of the DFSZ model from an effective theory, without specifying any UV physics? Naively, one can start with the renormalizable Lagrangian of Eq. (3) but drop all heavy particles:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + ia \sum_{q=t,b} c_q \bar{q} \gamma_5 q. \quad (12)$$

This looks quite reasonable, since the light ALP a only changes the IR structure of the SM. The low-energy effective field theory [51,52] respects the QED \times QCD symmetry, under which the quarks and leptons are vector-like. Therefore, the a -fermion couplings can be renormalizable. With c_q matched from the DFSZ model, one can calculate the $b \rightarrow sa$ amplitude, with UV divergence. Applying the RG equations, one finds the leading-log term appears:

$$\mathcal{A}(b \rightarrow sa) \sim \log(\Lambda_{\text{UV}}^2/m_t^2). \quad (13)$$

In the DFSZ model Λ_{UV} is equal to m_H [53]; however, we find the coefficient of Eq. (13) in disagreement with the X_1 term of Eq. (10). Thus, something is wrong.

We carefully checked the DFSZ calculation and found the missing term comes from Fig. 2(f). The low-energy theory cannot capture its contribution because the Feynman rules of the $G^- H^+ a$ vertex is proportional to m_H^2 . Figure 2(f) is not suppressed by $1/m_H^2$ although it contains a heavy particle. Although H^+ is much heavier than W^+ , they are equally important in $b \rightarrow sa$. This challenges the naive understanding about IR-UV mixing/decoupling.

In our opinion, the reason is that without H^+ , the renormalizable theory of Eq. (12) is not invariant under $SU(2)_L \times U(1)_Y$. The necessary condition for decoupling, that the light theory must be gauge invariant [28], is not satisfied. Decoupling is hidden in the dimensionless coupling:

$$c_q \sim \theta \sim \frac{1}{f} \sim \frac{1}{m_H}. \quad (14)$$

Here, m_H can not be arbitrarily heavy given finite c_q . This behavior is somehow uncommon, but not unique. For instance, the $\mu \rightarrow e\gamma$ decay amplitude in a 2HDM is not directly suppressed by the heavy mass either [29], but by the misalignment parameter $c_{\alpha\beta} \sim 1/m_H^2$ [55]. With finite c_q or $c_{\alpha\beta}$, one can not recover $SU(2)_L \times U(1)_Y$.

To reveal the decoupling picture, the $SU(2)_L \times U(1)_Y$ gauge symmetry must be respected by the low energy theory. It is chiral, unlike QED \times QCD. The $SU(2)$ doublet Higgs must join the low-energy-theory of Eq. (12) so renormalizability cannot hold anymore. The gauge invariant Lagrangian reads:

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{\text{SM}} + i \frac{a}{v} (c_b \bar{Q}_L b_R \tilde{H}_u + c_t \bar{Q}_L t_R H_u + \text{H.c.}) \\ &= \mathcal{L}_{\text{SM}} + ia \sum_{q=t,b} c_q \bar{q} \gamma_5 q + i \frac{a}{v} [c_b V_{tb} \bar{l}_L b_R G^+ \\ &\quad + c_t (V_{tb}^* \bar{b}_L t_R G^- + V_{ts}^* \bar{s}_L t_R G^-) + \text{H.c.}] + \dots \end{aligned} \quad (15)$$

The key difference is the appearance of nonrenormalizable operators with unphysical Goldstone Mode G^+ . They have a clear UV origin, as illustrated in Fig 2(g). Splitting the propagator of H^- into two pieces [56],

$$\frac{1}{k^2 - m_H^2} = -\frac{1}{m_H^2} + \frac{1}{m_H^2} \frac{k^2}{k^2 - m_H^2}, \quad (16)$$

the $-1/m_H^2$ term leads to the nonrenormalizable operator, while the m_H dependence is canceled since the $G^- H^+ a$ vertex is proportional m_H^2 . This effective operator, as shown Fig. 2(g), leads to a divergent amplitude, and we checked that it exactly reproduces the leading-log term missing in Eq. (13). As previously discussed, the light theory of the 2HDM is also not gauge invariant. Very similar operators with Goldstone bosons contribute to $\mu \rightarrow e\gamma$ with a leading-log term. We refer the reader to Ref. [31], for details about this closely related example.

If one picks the unitary gauge, Eqs. (15) and (12) are the same, since the gauge fixing condition sets $G^+(x^\mu) \equiv 0$. We have checked that the missing leading-log term of Eq. (13) now originates from the longitudinal part of W propagator $k_\mu k_\nu / m_W^2$. Decoupling works, because the gauge symmetry is strictly speaking still preserved, just hidden by gauge fixing. And again, the cost is losing renormalizability, known as a consequence of the unitary (non-renormalizable) gauge.

By applying the equations of motion, Eq. (15) becomes the general axion EFT where the $U(1)_{\text{PQ}}$ symmetry is manifest [14,16,17]:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_{\psi_L = Q_L, l_R^c, b_R^c} \frac{c_\psi}{f} \bar{\psi}_L \gamma^\mu \psi_L \partial_\mu a + \dots \quad (17)$$

Anomalous terms such as $a \tilde{W}_{\mu\nu} W^{\mu\nu}$ [17] are higher order for flavor violating processes [6], so we don't show them explicitly here. Clearly, this derivative basis produces the same $b \rightarrow sa$ amplitude as the one of Eq. (15). However, the Yukawa basis of Eq. (12) gives a different result. The authors of Ref. [27] have commented on this discrepancy in a footnote and correctly connected it to the dimension-5 operators. Here, we emphasize that Eq. (12) is inconsistent without gauge fixing.

Before finishing the bottom-up discussions, we want to emphasize that $\log(\Lambda_{\text{UV}}^2/m_t^2)$ is large and the terms without this leading log are not available without specifying UV physics. UV physics is hidden in the counter term of Fig. 2(h) [from the third term of Eq. (16)]. The general ALP effective theory allows tree level flavor violating couplings. Strictly speaking, $b \rightarrow sa$ itself is a *definition* of the renormalization scheme, about how Fig. 2(h) cancels the divergence of Fig. 2(g), not a *prediction* of the EFT.

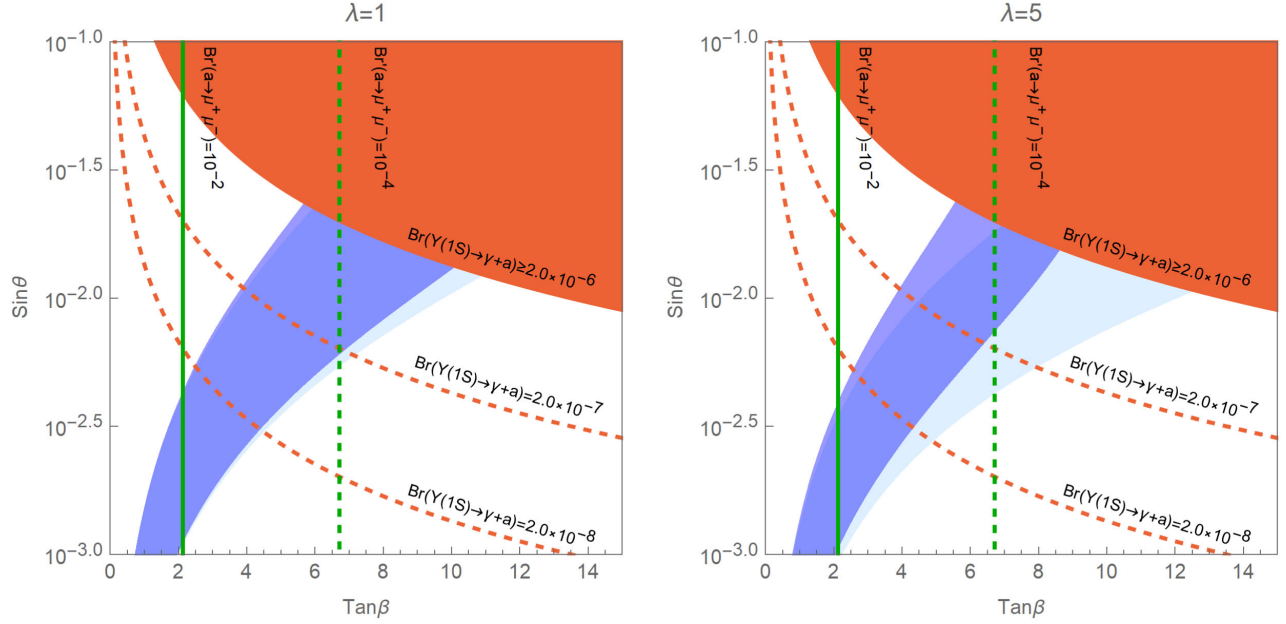


FIG. 3. Parameter space explaining the Belle II excess in the DFSZ model, for $\lambda = 1$ (left) and $\lambda = 5$ (right). The dark (light) blue region gives $\text{Br}(B \rightarrow Ka) = (1 \sim 9) \times 10^{-6}$, with complete (one loop only) calculation. The red region is excluded by the search for $\Upsilon(nS) \rightarrow \gamma a$. The green contours indicate the visible branching ratio $\text{Br}'(a \rightarrow \mu^+\mu^-) = \text{Br}(a \rightarrow \mu^+\mu^-)/\text{Br}(a \rightarrow \text{visible})$.

IV. PHENOMENOLOGY

The $b \rightarrow sa$ decay amplitude alone is not sufficient to explain the Belle II excess in a self-consistent way. The invisible signal requires a to escape detection. However, the DFSZ model implies that a is short-lived and decays inside the detector. Explaining the Belle II excess needs invisible a decay channels, which is beyond the model prediction. The DFSZ model has to be extended with a dark sector, for example, a light sterile particle χ with $a\bar{\chi}\chi$ coupling only. Here, we do not try to build dark matter models, but assume $\text{Br}^{\text{inv}} \equiv \text{Br}(a \rightarrow \text{invisible}) \approx 1$ for simplicity. If $\text{Br}^{\text{inv}} \ll 1$, the required value for the mixing angle θ must be enhanced by a factor of $1/\sqrt{\text{Br}^{\text{inv}}}$.

We also checked the consistency with other search limits. The various visible decay rates of a general axion-EFT are shown in Ref. [57]. Considering the detecting limits, the only relevant visible channel is the charged lepton one, $a \rightarrow \mu^+\mu^-$. It contributes an excess in $B \rightarrow K\mu^+\mu^-$ at low $q^2 \sim 2$ GeV. If the total visible decay rate is non-negligible, escaping the current limits requires sizable $\tan\beta$ so that H_d weakly couples to leptons. On the other hand, very large $\tan\beta$ is excluded by $\Upsilon(1S) \rightarrow \gamma + \text{invisible}$ searches. The current limit disfavors the $\tan\beta \gtrsim 10$ region.

We illustrate the three phenomenologically relevant processes in Fig. 3, in the benchmark scenario $\lambda = 1$ and $\lambda = 5$. The dark blue regions represent the values of $\sin\theta$ and $\tan\beta$ that can explain the Belle II excess. The light blue regions are based on one-loop calculations alone and mostly overlap with the dark ones. However, they differ when both $\tan\beta$ and λ are sizable enough. In this case, the

two-loop calculation we newly computed in this work becomes important. Notably, the two-loop and one-loop amplitudes have opposite sign and partly cancel, so favoring a larger value of $\sin\theta$. Assuming a sizable visible branching ratio, the viable $\sin\theta - \tan\beta$ parameter space to explain the Belle II excess becomes fully constrained. Consequently, either a $B \rightarrow K\mu^+\mu^-$ excess or $\Upsilon \rightarrow \gamma + \text{invisible}$ should be observed in future experiments. Detection can only be avoided if the total visible decay rate is negligible.

V. CONCLUSION AND DISCUSSION

We revisited the $B \rightarrow Ka$ transition rate in the DFSZ model, which is a minimal UV-complete benchmark for an ALP a . Studying approximate symmetries suppressing the known one-loop amplitudes, we determine new unsuppressed two-loop contributions. When $\tan\beta$ is sizable, our result becomes essential. In addition, while it is possible to capture the key features of DFSZ model with a bottom-up approach, the choice of the low energy theory is subtle. Only the gauge invariant EFT yields the correct leading-log term, with the cost of losing renormalizability.

From a practical side of view, we agree that the operator $a\bar{s}\gamma_5 b$ alone is sufficient to explain the Belle II excess. However, the DFSZ model should be taken more seriously as a minimal benchmark for light new particles with minimal flavor violation [58,59]. Here, the rare $B \rightarrow Ka$ decay rate is suppressed by a loop factor $1/(16\pi^2)$, small flavor mixing angles, and a possible hierarchy between two VEVs. So the new physics for UV completion needs not be

superheavy, but could be in the TeV range. In other words, some other beyond-SM processes should not be far away from detection. Therefore, it is reasonable to expect detecting $\Upsilon \rightarrow \gamma + a$ and $B \rightarrow Ka \rightarrow K\mu^+\mu^-$ signals in future experiments, which will support the model.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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