Nonlinear Stochastic Modeling of the South Korean Power Grid Frequency Dynamics

1st Ulrich Oberhofer*
Institute for Automation and Applied Informatics
Karlsruhe Institute of Technology
Eggenstein-Leopoldshafen, Germany
ulrich.oberhofer@kit.edu

2nd Xinyi Wen*
Institute for Automation and Applied Informatics
Karlsruhe Institute of Technology
Eggenstein-Leopoldshafen, Germany
xinyi.wen@kit.edu

3rd Jaiyong Lee
Department of Energy Engineering
Korea Institute of Energy Technology
Naju, South Korea
angelclub77@kentech.ac.kr

4th Heetae Kim
Department of Energy Engineering
Korea Institute of Energy Technology
Naju, South Korea
hkim@kentech.ac.kr

5th Veit Hagenmeyer
Institute for Automation and Applied Informatics
Karlsruhe Institute of Technology
Eggenstein-Leopoldshafen, Germany
veit.hagenmeyer@kit.edu

6th Benjamin Schäfer
Institute for Automation and Applied Informatics
Karlsruhe Institute of Technology
Eggenstein-Leopoldshafen, Germany
benjamin.schaefer@kit.edu

***Abstract*—The mitigation of climate change and the associated restructuring of the energy system is one of the most relevant challenges in society at the moment. A key aspect of the transformation of energy systems is the shift from the use of conventional energy sources to renewable ones. This leads to a more economical use of fossil resources and reduces the environmental impact, but also poses challenges. The increasing amount of renewable energy sources in power grids has a major impact on the grid structure and its stability. Ensuring stability under these changing conditions is a challenge, in particular since power grids and their dynamics differ from region to region. While several studies investigated US or European power grids, Asian grids, such as the South Korean power grid, have been less of a focus. In this article, we analyze in particular the nonlinearity and the bimodality of the frequency time series, and perform a correlation analysis. Further, we use stochastic Fokker-Planck models to create synthetic frequency time series both in a linear and nonlinear manner. We examine the synthetic time series modeled in this way and compare their statistical properties with the empirical data. Thereby, we demonstrate how simple stochastic differential equations can provide a simplified model for the grid frequency, and observe that a model with nonlinear dynamics approximates the empirical data better than a purely linear one.**

***Index Terms*—power grid frequency, data analysis, stochastic modeling, Fokker-Planck equations, parameter estimation**

I. Introduction

Maintaining a balance of supply and demand at all times within the power grid is critical for its stable operation [1]. The power grid frequency mirrors this balance and is a key indicator used for monitoring and control of power systems. Therefore, developing models for the power grid frequency is important.

Data-driven models are becoming more prominent and might even be necessary to cope with the increasing complexity of modern power systems [2]. Hence, (open) data is required to fuel these models. While European Transmission System Operators (TSOs) publish data via the Transparency platform of the European TSOs [3], such open data is not always available in other regions.

In recent years, we have established an open data base for power grid frequency measurements [4], [5] and developed data-driven models [6], [7]. However, these models have been mostly limited to European power systems, while only few Asian systems have been analyzed [8]. The analysis in [8] shows that there are indicators for special dynamics in Asia, different from European grids.

Within this article, we analyze and model the South Korean power grid with its about 50 million inhabitants. The South Korean power grid has no AC lines to other grids and can therefore be considered as an islanded power grid. Further, the main sources of electricity in the grid are coal, gas and nuclear power, each with a share of around 30-35 percent. The share of renewable energy sources is currently only five percent, and it is planned to be increased to 20 percent by 2030 [9]. The impact of renewables in the Korean power grid has already been investigated by some studies. In especially, limited access to the power grid data due to regulations in [10], [11] has led to active research in South Korean power system estimation and modeling. In [12] an estimation method for the required amount of quantitative inertia is presented. [13] analyzes a power grid model of South Korea under high influence of stochastic dynamics and power electronic interfaced resources. The analysis of integrating smart grid infrastructure into the

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*: U.O. and X.W contributed equally.

power grid by stochastic modeling is shown in [12], and in [14] the relevance of energy storage systems on primary frequency control in South Korea is investigated. Recently, the synthetic test system of the Korean power grid is released mainly regarding the structural properties without functional characteristics such as frequency fluctuations [15].

The area of stochastic modeling of the power-grid frequency has attracted much attention from a broad interdisciplinary audience [16]–[22]. In particular, in order to stochastically describe frequency dynamics, Fokker–Planck equations have been used [23], [24], leading to data-based models [25], [26] and quantitative comparisons for continental synchronous areas [27].

We want to apply and generalize some of the stochastic methods presented above to power grids with different properties and, with the help of statistical analysis and data-driven stochastic modeling, understand the frequency dynamics. In particular, in the present paper we aim to apply Fokker-Planck models to represent the frequency dynamics of the South-Korean power grid. For a proper understanding of the properties of the frequency dynamics, we first apply several statistical tests and analysis methods and analyze them. Further, we adapt the models introduced in [25], [26] to the frequency dynamics of the South Korean power grid.

The remainder of the article is structured as follows. In Section II we introduce some general data analysis methods in order to describe and interpret the frequency data and its dynamics. In Section III we present the concept of stochastic models based on stochastic differential equations, Fokker-Planck equations, and purely data-based parameter estimation, and we analyze and compare the results of the models. Finally, in Section IV we provide a discussion of our results, as well as a conclusion and an outlook.

The code for reproducing the data analysis and the models which are introduced in the present paper can be found in [28].

II. Data analysis and statistical tests

To inform the construction of a synthetic model, we follow the procedure established in [8], assess the probability distributions and its bimodality, measure the linearity of the dynamics and investigate frequency increments as well as correlations.

Our analysis presented here is based on our power outlet recordings at Korea Institute of Energy Technology (KENTECH) from August 15, 2024, to December 15, 2024, with a resolution of one second. The measurements are collected by an electric data recorder (EDR). These measuring devices are being developed at our research institute [29] to analyze high-resolution current data such as current intensity and voltage, and to calculate the grid frequency from these data. We only consider complete hours of data for the analysis, that is hours with missing data are removed, leading to approximately 108 days of full-hour data available for analysis below. The acquired data align with Korea Power eXchange (KPX)'s official frequency maintenance records in [30]. See also [5], [29] and [28] for details and links to the data. For our analysis, we apply the statistical tests introduced in the following to each day of the dataset and afterwards we calculate the mean and standard deviation over the set.

A. Analyzing probability density and quantifying bimodality

We obtain a first impression of the dynamics in the South Korean power grid by inspecting the daily profile of the data and the probability density function (PDF) in Figure 1 (top and bottom left). We clearly note that the PDF does not follow a normal distribution, but instead displays two peaks. Further deviations from Gaussianity are apparent when analyzing the skewness and kurtosis, which are the third and fourth standardized moments, respectively: The skewness is -0.250 ± 0.191, indicating a slight leftward asymmetry, while the kurtosis is obtained as 2.270 ± 0.214, suggesting that the distribution is relatively flat compared to a normal distribution.

To quantify the bimodal nature, we utilize the dip statistic [31]. This statistic effectively measures the deviation from unimodality. In order to stay consistent with [8] we calculate the statistic for each hour, in contrast to the other statistics in this paper, where we calculate the values on a daily basis. Here, we obtain a mean 0.010 and a standard deviation 0.008, which is comparable to Australia (mean: 0.012, standard deviation: 0.001) and Indonesia (mean: 0.011, standard deviation: 0.006) [8]. Therefore, we have sufficient evidence to claim that the frequency dynamics in South Korea does not follow a normal distribution. This result aligns with our expectation, as the frequency distribution exhibits a bimodal pattern.

In addition to the frequency PDFs, we also analyze the frequency increments $\Delta f_\tau = f(t+\tau) - f(t)$, where $\tau = 1\,\mathrm{s}$, that is, the sampling rate of the data. We observe that the PDF of the frequency increments $\Delta f = \Delta f_{\tau=1\,\mathrm{s}}$ have a skewness of 0.108 ± 0.039 and a kurtosis of 3.602 ± 0.543. Positive skewness indicates a slight rightward asymmetry in the distribution, while kurtosis, which fluctuates on a daily horizon, indicates a distribution with significantly heavier tails compared to a normal distribution. In summary, analyzing the skewness and kurtosis of the frequency and its increments shows that the frequency is tightly controlled, whereas large power fluctuations lead to heavy-tailed increments.

B. Assessing linearity

When constructing models for dynamical systems, it is critical to assess whether a linear or nonlinear model is necessary. In our case, we examine in particular whether the time series behaves linearly on a short time scale. We follow Kantz [32] by computing a linear test (LT) quantity $LT(t)$ for the empirical and a surrogate dataset, which is obtained by taking a Fourier transform and randomizing the phases, for details see [8], [27]. Next, we consider the distance between the LT results of the original and the surrogate data, and in order to obtain a quantitative comparison, we compute the root mean squared error (RMSE). If the value of the root mean squared error (RMSE) is close to zero, it indicates that the time series exhibits linear behavior. The RMSE for Korea is of order 10^{-3}, which hints towards relevant nonlinear effects in the

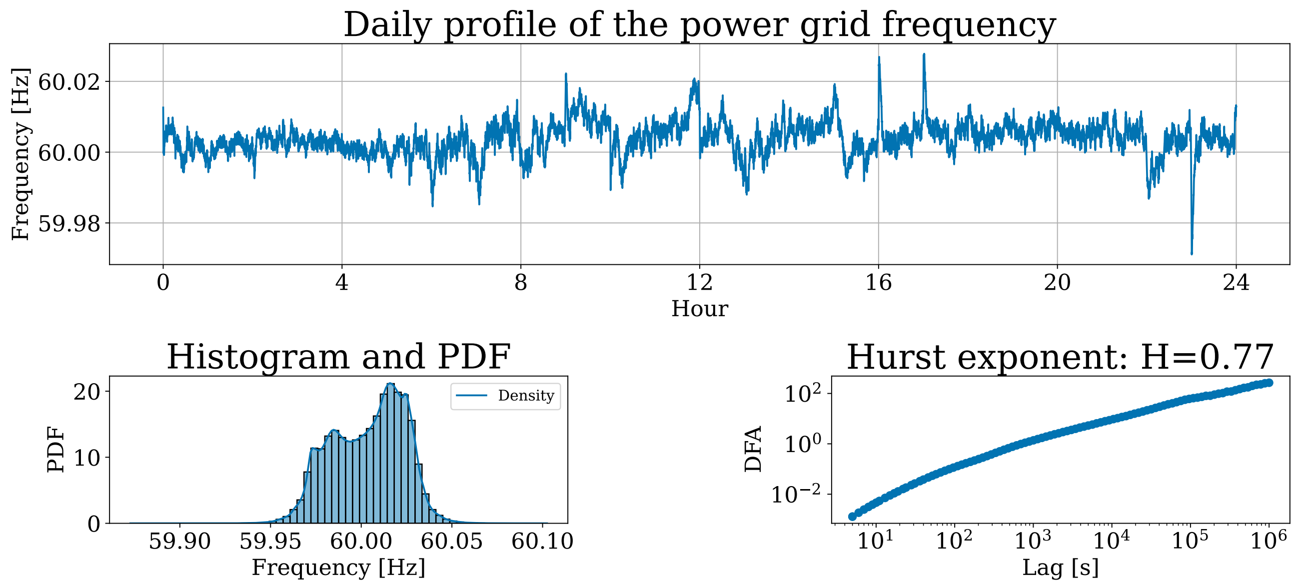

Fig. 1: **Illustration of daily profile, the probability density function (PDF) and a Detrended Fluctuation Analysis (DFA) of the South Korean frequency dynamics** Top: Daily profile of the frequency time series, showing some large deviation at the beginning of several hours. Bottom left: Histogram and density estimate of the frequency data, clearly displaying two peaks, i.e. a bimodal distribution. Bottom right: **DFA and Hurst exponent to determine correlations**. We conduct a DFA and calculate the Hurst exponent H of power-grid frequency in South Korea, revealing a positive correlation.

dynamics of the South Korean grid. A qualitative comparison of the test results that shows how the results for the original and surrogate differ in their range can also be found in [28] and [33].

C. Correlation analysis

It is critical to know whether a system is correlated or uncorrelated when modeling it. To uncover long-range correlations [34], [35] in the data, we employ Detrended Fluctuation Analysis (DFA) [36]–[38]. We generate a set of lag values that span from 5 to 10^6 seconds. The illustration at the bottom right in Fig. 1 illustrates the DFA results of the power-grid frequency in South Korea plotted against the lag values. The slope of the depicted line on the log-log scale corresponds to the Hurst exponent plus 1. For South Korea, we estimate the Hurst exponent around 0.77, which is larger than 0.5, indicating positively correlated motions.

III. Stochastic Models

In order to find explanations for the dynamics of the South-Korean power grid frequency, we aim to model synthetic time series following the dynamics of the considered power grid. Therefore, we develop stochastic models based on data-driven estimation of the Kramers-Moyal coefficients. We consider the aggregated Swing equation [23], [25], [39], [40] for the bulk angular velocity ω of a power network is given as

$$M\frac{\mathrm{d}\omega}{\mathrm{d}t} = P_{\text{control}}(\omega,t) + P_{\text{noise}}(\omega,t), \tag{1}$$

with inertia constant M. The angular velocity is connected to the power grid frequency f via $\omega = 2\pi(f - f_{\text{ref}})$, where in the South Korean grid we have $f_{\text{ref}} = 60\,\text{Hz}$. To simplify the data-driven models we divide the whole equation by the inertia constant M, and choose $P_{\text{control}}(\omega,t) = c(\omega)/M$ time-independent and $P_{\text{noise}} = \epsilon(\omega)\xi(t)/M$, with a Gaussian white noise process ξ. Therefore we obtain the stochastic differential equation

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \dot{\omega} = c(\omega) + \epsilon(\omega)\xi. \tag{2}$$

We aim to construct two synthetic models, one simple linear one and a more complex one inspired by the empirical observations from the Drift-Diffusion analysis which is presented in Section III-B.

The construction of the terms $c(\omega)$ and $\epsilon(\omega)$ is decisive for the complexity of the model. We differentiate between two cases:

Linear model: As a simple baseline model, we choose the drift $c(\omega)$ as linear of the form $c(\omega) = c^{(\text{const})}\omega$ as described in [25]. Further we estimate $\epsilon(\omega) = \epsilon^{(\text{const})}$ as a constant, which gives us the following stochastic differential equation (SDE):

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \dot{\omega} = c^{(\text{const})} + \epsilon^{(\text{const})}\xi. \tag{3}$$

Nonlinear model: To better describe the empirical data, the nonlinear model approximates $c(\omega)$ and $\epsilon(\omega)$ as polynomial functions resp. square roots of polynomial functions. By this, we can adapt the model to possibly nonlinear behavior of the data. The degree and the coefficients of the polynomials are chosen according to the drift and diffusion analysis in Section III-B.

A. Parameter estimation

Next, we have to determine the constant factors or functions for drift and diffusion. We only briefly outline our steps in the following, see [26], [41] for details. For deriving a stochastic differential equation which describes the dynamics of the power grid frequency directly from the frequency data, we transfer the aggregated Swing equation into a Fokker-Planck equation [42], [43]. A Fokker-Planck equation is a partial-differential equation which describes the dynamics of a probability distribution. We consider equation (2) and obtain the following Fokker-Planck equation for the probabililty density function $\rho(\omega)$:

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial \omega}\left(c(\omega)\rho\right) + \frac{\partial^2}{\partial \omega^2}\left(\frac{\epsilon(\omega)^2}{2}\rho\right). \tag{4}$$

To estimate the parameters for $c(\omega)$and the noise amplitude $\epsilon(\omega)$ purely from data, we turn to the Nadaraya–Watson non-parametric kernel-density Kramers–Moya-(KM) coefficients $D_n(x)$ estimation [44]–[47], which read:

$$\begin{aligned} D_n(\vec{x}) &\sim \frac{1}{n!}\frac{1}{\Delta t}\langle(\vec{x}(t+\Delta t) - \vec{x}(t))^n | \vec{x}(t) = \vec{x}\rangle \\ &\sim \frac{1}{n!}\frac{1}{\Delta t}\frac{1}{N}\sum_{i=1}^{N-1}(\vec{x}_{i+1} - \vec{x}_i)^n K(\vec{x} - \vec{x}_i), \end{aligned} \tag{5}$$

where in our case $\vec{x}$ is ω, N is the number of data points from a time series, K is a kernel with bandwidth h (c.f. [46], [47]), and Δt is the sampling rate. The Fokker-Planck equation has the form

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial \omega}\left(D_1(\omega)\rho\right) + \frac{\partial^2}{\partial \omega^2}\left(D_2(\omega)\rho\right), \tag{6}$$

where the first KM coefficient D_1 is also called the drift, and the second KM coefficient is called diffusion. Hence we receive the equations [21], [42], [43]:

$$\begin{aligned} D_1(\omega) &= c(\omega), \\ D_2(\omega) &= \epsilon(\omega)^2/2, \end{aligned} \tag{7}$$

B. Drift and Diffusion analysis

We clearly note nonlinearity in the drift and diffusion terms for the South Korean data, see Figure 2. The drift shows a strong cubic behavior, while the diffusion is showing a quadratic behavior around the origin and is also increasing for $|\mathrm{f} - 60\,\mathrm{Hz}| > 0.2\,\mathrm{Hz}$. Therefore, the two models we create based on the drift and diffusion behavior are as follows:

1) The linear model is given as described in (3), where $c^{(\mathrm{const})}$ is estimated as a linear fit of the drift, and ϵ is modeled as a constant.
2) We approximate $c(\omega)$ as a cubic polynomial and $\epsilon(\omega)$ as square root of a polynomial of degree 4 (see (7)), to represent the nonlinear drift and diffusion dynamics.

In the following we evaluate the model performance as well qualitatively in figure 3 as well as quantitatively in Table I.

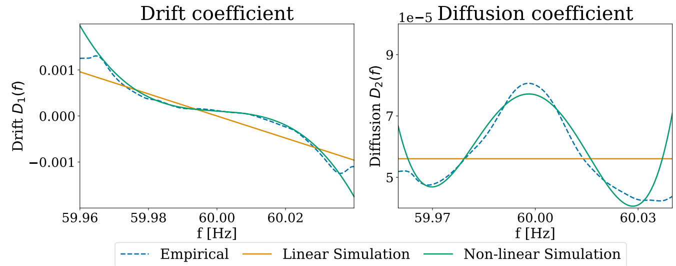

Fig. 2: **Kramers-Moyal coefficients of the Korean data with linear and nonlinear approximations** Left: Drift D_1 coefficient displaying an approximately linear or cubic behavior around the reference frequency of $60\,\mathrm{Hz}$ before saturating at low values for very large deviations. Right: Diffusion D_2 coefficient displaying a peak around the reference frequency, and slightly oscillating for larger values.

C. Model evaluation

First we present a qualitative description of the model results. In Figure 3 we notice on the left side that the linear stochastic model doesn't fit the bimodal distribution of the empirical frequency data, while the nonlinear model can approximate the bimodality quite well. On the right hand side, we note that the large tails of the empirical distribution are as well better represented by the nonlinear model. In Table I we show a quantitative comparison of the models and the empirical dataset. For this purpose, we use the statistical methods which are explained in Section II.

Regarding the first four standardized moments (mean, standard deviation, skewness and kurtosis), the distribution of the synthetic dataset created by the nonlinear model fits the distribution of the empirical data better than the one created by the linear model.

In particular the bimodal behavior of the empirical distribution is quantitatively shown by the p-value of the Dip-statistics. Here, we also observe that the tests implies a unimodal distribution for the majority of the hourly intervals of the linear model and a multimodal distribution for most of the hours of the nonlinear model.

The linearity test shows that the nonlinear model approximates the empirical frequency data better regarding the linearity of the dynamics on a short time scale, even if the linearity test implies a considerably higher appearance of nonlinear dynamics in the empirical data than for the nonlinear model.

Observing the Hurst exponent both the models exhibit a Hurst exponent of approximately 0.5, which suggests white noise. The Hurst exponent for the empirical dataset of 0.77 ± 0.03 is an indication of the occurrence of correlated noise which cannot be represented by the construction of the models. The use of fractional Gaussian noise or correlated noise remains as an extension of the models for future analyses.

The comparison of the Kullback-Leibler divergence (KL-divergence) [48] between the probability distribution of the empirical frequency data and the distribution of the linear respective the nonlinear model shows that the nonlinear model

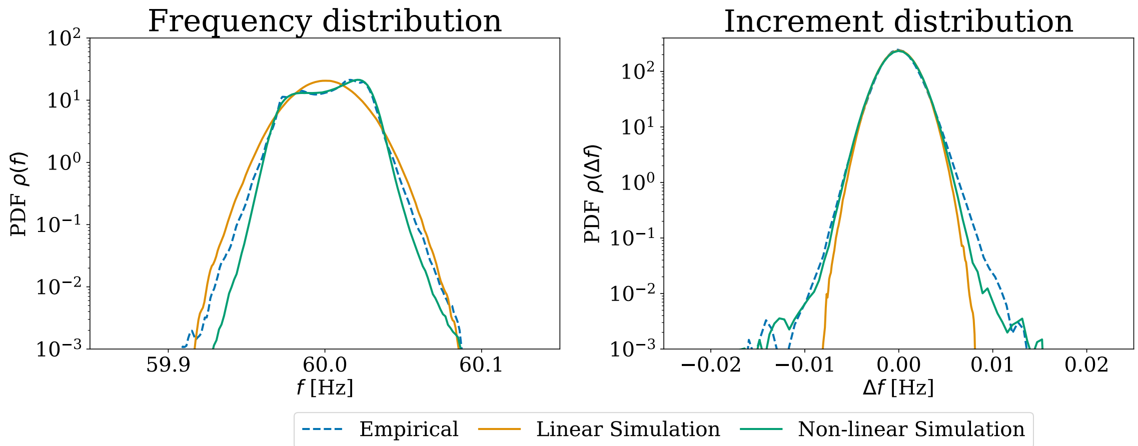

Fig. 3: **Comparison of empirical and synthetic data** We plot the PDF of both the frequency (left) and the frequency increments (right) for both the empirical data recorded in South Korea and the two synthetic models considered here.

TABLE I: Comparison of the linear and nonlinear model with the empirical frequency dataset

Statistic	Empirical dataset	Linear Model	Nonlinear model
Mean [Hz]	60.004 ± 0.003	60.000 ± 0.002	60.004 ± 0.002
Standard deviation [Hz]	0.020 ± 0.002	0.019 ± 0.001	0.019 ± 0.000
Skewness	-0.250 ± 0.191	0.006 ± 0.120	-0.252 ± 0.102
Kurtosis	2.270 ± 0.214	2.964 ± 0.186	2.145 ± 0.129
Dip statistic (modal behavior); p-value= 0.05	multimodal	unimodal	multimodal
Linearity quant. (RMSE)	0.00114 ± 0.00040	0.00016 ± 0.00011	0.00018 ± 0.00012
Hurst exponent	0.77 ± 0.03	0.49 ± 0.00	0.49 ± 0.01
KL-divergence	-	0.124 ± 0.064	0.046 ± 0.034
KL-divergence of increments	-	0.013 ± 0.014	0.011 ± 0.012

represents the distribution better than the linear one. For the KL-divergence of the increment distribution we note that the difference between the models is less pronounced but yet visible. This also confirms the observations from Figure 3.

IV. Discussion and Conclusion

In the present study, we collected and analyzed the characteristics of power grid frequency data from South Korea to gain deeper insights into its behavior and underlying patterns. Moreover, we advance previous data analyzes [8] by including stochastic modeling and Kramers-Moyal analysis.

Similarly to the observations for several other Asian countries [8], the distribution of the South Korean power grid frequency is bimodal. Furthermore, a nonlinear relationship is observed which was not seen before in literature regarding other Asian countries [8]. The power grid frequency data is also positively correlated, indicating that fluctuations in frequency tend to follow a consistent directional pattern over time.

Analyzing the stochastic models introduced in Section III reveals that the nonlinear model approximates the empirical data better than the linear model. In particular, we see that a cubic polynomial as a model of the drift, which can also be interpreted as an approximation of a deadband [24], produces a bimodal distribution. Further, modeling nonlinear dynamics and Gaussian noise can generate a non-Gaussian distribution of the increments. In order to generate correlations according to the Detrended Fluctuation Analysis, correlated noises are required in the model, or another model approach itself.

For future work, one possibility is to include techno-economic features in the models and, as described in [49], combine the stochastic models introduced in the present paper with neural networks. The relationship between control mechanisms and frequency deviations, particularly how proportional and integral controllers affect the observed multi-modal characteristics in [50]–[52], presents another promising research direction. Furthermore, parameter estimation using sparse identification of nonlinear dynamics can be helpful for the characterization of nonlinear stochastic and deterministic behavior [53], [54].

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