

Erratum: Nonleptonic B -meson decays to next-to-next-to-leading order

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Upon reexamining our published work, we identified an error in the derivation of the dependence on the renormalization scale μ_b at NLO for the $b \rightarrow c\bar{c}s$ channel. Specifically, the ancillary file for $G_{ij}^{(1)}$, as defined in eq. (4.1) of the original paper, contained a mistake. This error affects figures 9 and 11, as well as the associated discussion. However it does not affect the expressions given at $\mu_b = m_b$, in particular eqs. (4.20)–(4.27), which remain unchanged. We corrected and updated the repositories [1, 2], in particular the file `b2ccs/b2ccs_NLO_exact.m`.

The corrected version of figure 9 is given in figure 1 and the paragraph describing the plot must be substituted with the following one:

In figure 1 we show the dependence on the renormalization scale μ_b . As observed already in [3–5], the $O(\alpha_s)$ corrections are rather large, about 25% of the LO prediction at $\mu_b = m_b$, and similarly the $O(\alpha_s^2)$ corrections are 16% of the NLO prediction. At NLO we observe a scale variation at the level of 9.2%, we do not observe a significant reduction at NNLO while using the pole scheme for the quark masses. We performed a preliminary study of the decay adopting short-distance scheme for the masses (see also ref. [6]). Using the kinetic scheme for the bottom and the $\overline{\text{MS}}$ scheme for the charm, we observe a scale uncertainty of about 13% at NLO after varying $m_b^{\text{kin}}/2 \leq \mu_b \leq 2m_b^{\text{kin}}$. The uncertainty is reduced to 1.5% after inclusion of the NNLO corrections, thus showing the improvement in the prediction also for the channel $b \rightarrow c\bar{c}s$.

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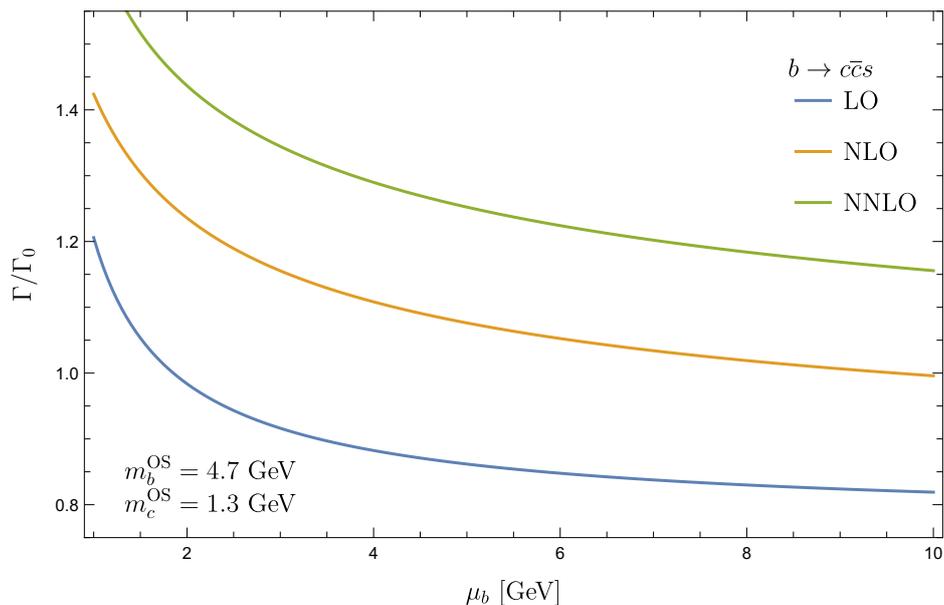


Figure 1. The dependence of the rate for $b \rightarrow c\bar{c}s$ on the renormalization scale $\mu_b \sim m_b$ at LO, NLO and NNLO in the on-shell scheme.

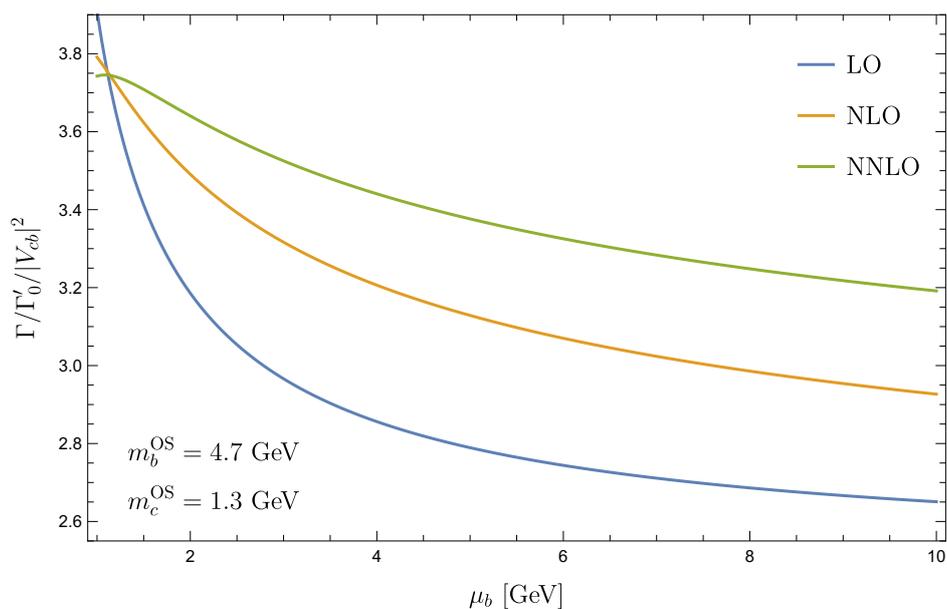


Figure 2. All contributing decay channels combined.

The corrected version of figure 11 is given in figure 2 and the paragraph describing the plot must be substituted with the following one:

We observe that the NNLO corrections come with the same sign as the NLO corrections in the relevant region of the renormalization scale μ_b . Using `RunDec`, we obtain $\alpha_s(\mu_b = 4.7 \text{ GeV}) = 0.2166$ and find that the term of $O(\alpha_s^2)$ is roughly 50–60% of the one at $O(\alpha_s)$ for $\mu_b = m_b^{\text{OS}}$. The scale uncertainty reduces from a relative 27% at NLO to 13% at NNLO.

Moreover, the second paragraph of the conclusions should be updated as follows:

We perform a preliminary numerical study of the impact of the NNLO corrections in the various channels considered using pole scheme for the quark masses. Overall we find that the theoretical uncertainties stemming from the scale variation is reduced by more than a factor of three for $b \rightarrow c\bar{u}d$. The reduction for the channel $b \rightarrow c\bar{c}s$ is much milder as we do not observe a significant improvement in the pole scheme after inclusion of the NNLO corrections to this channel. However the convergence is improved when adopting a short-distance mass scheme for the bottom and charm masses, see ref. [6].

Data Availability Statement. This article has no associated data or the data will not be deposited.

Code Availability Statement. This article has no associated code or the code will not be deposited.

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