Formal Process Maturity Measure

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Abstract

Measuring the maturity of a process instance is essential because we can evaluate the progress toward a mature production process; Consider we have added some sensors or actuators in the manufacturing process so we need a formal basement for measuring the maturity of the process after these changes. We used the definitions in the control theory to provide a formal measure for process maturity. We defined *Elucidability* E, *Forcability* F, and *Supervisability* S that are ostensive, interpretable, and based on quantities that can be determined or estimated. These lead to a formal definition for a measure of the process maturity M, that combines technical and economic considerations.

1 Introduction

Consider we have a manufacturing process and we do some software or hardware changes to it. 'maturity' generally can be defined as 'the state of being complete, perfect or ready'[8]. It is valuable for us to know how much we were successful to have progress in achieving our matured process[6][7]. P_i^j shows the process instance with 'hardware revision', i and 'software revision', j. The transition $P_i^j \to P_i^{j+1}$ shows software changes like control optimization and the $P_i^j \to P_{i+1}^j$ shows hardware changes and structural modifications. During

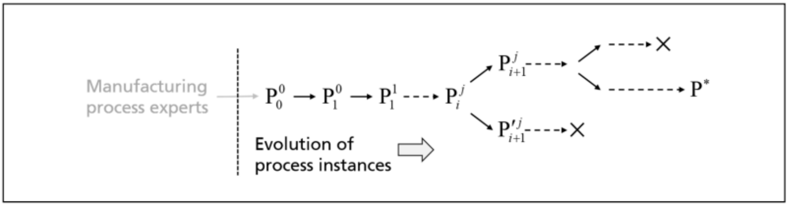


Figure 1.1: Flow diagram of the maturation of a manufacturing process. The symbol $\times$ indicates abortion, and P^* denotes the desired final mature process instance.

these changes in the project, we will have an evaluation chain of the project instance. Some changes might be done parallel in the project process and they compete with each other. In Figure 1.1 which shows the process evaluation chains, $\times$ shows abortion which means at the end of a chain we did not achieve the desired maturity, and P^* shows we achieved the desired matured process instance. Figure 1.2 shows a process instance. The input signals x and the output signals y are defined to be observable, and all non-observable dynamic quantities are in s. If the added sensor or actuator instrumentation allows components of s to be observable, those quantities are assigned to be additional components of y and x of the next process instance P_i^j. The noise n might happen in the system and the system has controller π.

The paper is organized as follows, in Section 2 we will mention the Controllability and Observability definitions. In Section 3 we explain our new definitions which are Forcability, Elucidability, and Supervisability, their formulas, and their relations with definitions which are in Section 2. In Section 4, we have a simulation example of an inverted pendulum on which we implemented our method. In the end, the paper is concluded in Section 5.

2 Controllability and Observability Definitions

Here we have the definitions of controllability and observability[4].

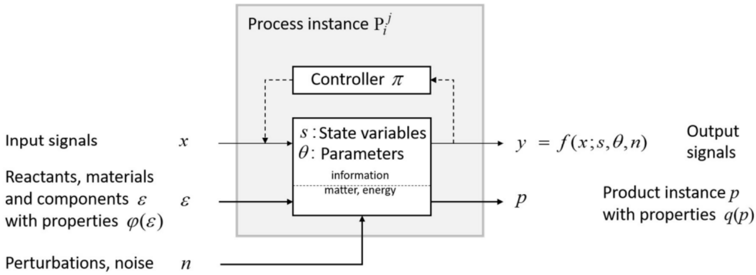


Figure 1.2: Abstract view of a process instance. This may represent the complete production process or a sub-process at any hierarchical process level. The variables x, y, n, and s are time-dependent signals.

Controllability

The system states s is controllable if there exists a control signal $\pi(t)$ over a finite time interval $0 \leq t \leq t_1$ such that a target state s^* is reached at t_1. When a system state is controllable, makes it possible to control this state to the desired point.

Observability

The system state s is observable if the state $s(0)$ at time $t = 0$ can be fully determined by the observations $y(t)$ during a finite time interval $t_0 \leq t \leq 0$.

In these definitions of controllability and observability, we can say the system is controllable/observable or not; Also it does not consider any uncertainty in the system. In this paper, we will introduce new definitions related to observability and controllability that gives us a degree of controllability and observability and considers the uncertainty in the system.

In ML, the concept of PAC learning (probably approximately correct) [9] describes the number of data samples minimally required to achieve a model error smaller than some η with a probability larger than $1 - \delta$. We apply similar notions to introduce definitions.

3 Definitions for measuring the maturity

Elucidability

For the process instance, P_i^j Elucidability which is similar to observability is approached by defining the minimum observation time τ that is necessary to achieve a certain estimation precision for the state s and parameters θ. So we find the probability of having the estimation error less than a constant precision η_θ and η_s. The internal states of the process can be estimated from the time series X_T, Y_T during the finite observation in duration T, so $X_T := \{x(0), ..., x(T)\}$, $Y_T := \{y(0), ..., y(T)\}$. The formula for Elucidability is

$$\begin{aligned} &E(T, \eta_s, \eta_\theta) := \\ &Pr(\|s(T) - \hat{s}(X_T, Y_T)\| < \eta_s \wedge \|\theta - \hat{\theta}(X_T, Y_T)\| < \eta_\theta | s(0)) \end{aligned} \tag{3.1}$$

In which the $\hat{s}(X_T, Y_T)$ is the estimation of state $s(T)$ at time T, with time series X_T and Y_T in duration T; It is the same for $\hat{\theta}(X_T, Y_T)$ which is the estimation of θ. If we do not have the time duration T we can find it with the smallest time to achieve a specific error η_s and η_θ, with a probability larger than $1 - \delta$,

$$\tau(\eta_s, \eta_\theta, \delta) := min\{T > 0 | E(T, \eta_s, \eta_\theta) > 1 - \delta\}. \tag{3.2}$$

Forcability

Forcability F which is similar to controllability is to be defined to measure the capability to steer the values of y and s with x toward target values $\breve{y}$ and $\breve{s}$. The most effective way to steer the process is to establish a closed-loop control to react instantaneously to the dynamic answers of the process. To quantify Forcability of a process instance with a given controller π, the probability of being able to force the process after a time T into the neighborhood of target values $\breve{y}$ and $\breve{s}$ is defined as follows,

$$\begin{aligned} &F_\pi(T, \eta_s, \eta_y) := \\ &Pr(\|y(T) - \breve{y}\| < \eta_y \wedge \|s(T) - \breve{s}\| < \eta_s \mid s(0), x(t) = \pi(Y_t), \breve{y}, \breve{s}) \end{aligned} \tag{3.3}$$

In close loop control, X_T is determined from the control policy π. In a condition that we do not have the $y(T)$ and $s(T)$ and we need to estimate them, we can replace the estimated $\hat{y}(T)$ and $\hat{s}(T)$ with the real value of them, so the modified formula for Forcability is

$$\begin{aligned}&F_{\pi}(T, \eta_s, \eta_y) := \\ &Pr(\|\hat{y}(T) - \breve{y}\| < \eta_y \wedge \|\hat{s}(T) - \breve{s}\| < \eta_s \mid s(0), x(t) = \pi(Y_t), \breve{y}, \breve{s})\end{aligned} \tag{3.4}$$

If we implement an optimal controller our formula will be changed to

$$F(T, \eta_s, \eta_y) := F_{\pi^*}(T, \eta_s, \eta_y) \tag{3.5}$$

If the time duration T is not fixed for a given process instance, we can use the PAC [9] in a way to find the minimum time we need to have a specific error η_s and η_y with the probability larger than $1 - \delta$,

$$\tau(\eta_s, \eta_y, \delta) = min\{T > 0 | F(T, \eta_s, \eta_y) > 1 - \delta\}. \tag{3.6}$$

Supervisability

Supervisability S is defined to measure the maturity of the quality of the process. In [1] there is information about quality management. The formula is written to find the probability of having the quality in the desired set Q in considering we have set our controller π and the input signal x, the state s, and parameter θ are in their desired set. The formula for Supervisability is defined as:

$$S(P_i^j) := Pr(q \in Q | x \in X_{adm}, s \in S_{adm}, \theta \in \Theta_{adm}). \tag{3.7}$$

In which $X_{adm}, S_{adm}, \Theta_{adm}$ are our desired set for x, s, θ. S measures the maturity of a process only from the technical point of view. To find out whether the resulting optimized process P^* is economic, the cost of the process also must be considered. Applying the economic part is beyond the scope of this project but it can be considered in the future.

4 Example

To clarify the definitions we have run an example in Matlab. The example is for an Inverted Pendulum [2] which is linearized around its stable state. The system has four states s_1, s_2, s_3 and s_4; We consider that we have added a sensor for observing the state s_3 in the matured system.

Figure 4.1 compares the probability of having a special error at different times for the state s_1.

Figure 4.2 compares the probability of having a special error at different times for the state s_2.

Figure 4.3 compares the probability of having a special error at different times for the state s_3. It is obvious that there is a comparable difference between the not Matured system and the matured system for the state s_3 for which we have added the sensor. For the not matured system the probability of having the observation error smaller than a specific constant number until some time is zero and after that growth, but for the matured system the probability started to grow at an earlier time.

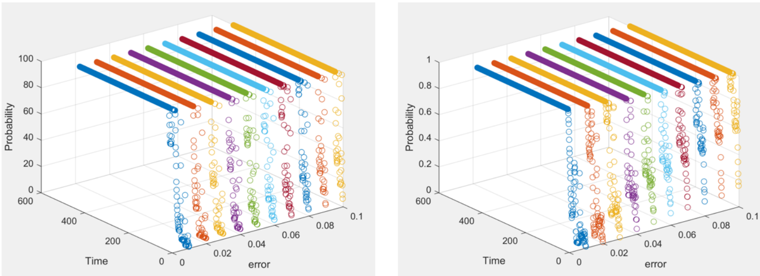
Figure 4.4 compares the probability of having a special error at different times for the state s_4. Also, this is obvious that for the state s_4 for the matured system, the probability of having an error less than a special constant number started to grow earlier.

Figure 4.5 shows the probability of having an error less than the special number for state s_1, this figure shows the probability of the smallest time that this error happens.

Figure 4.6 shows the probability of having an error less than the special number for state s_2, this figure shows the probability of the smallest time that this error happens.

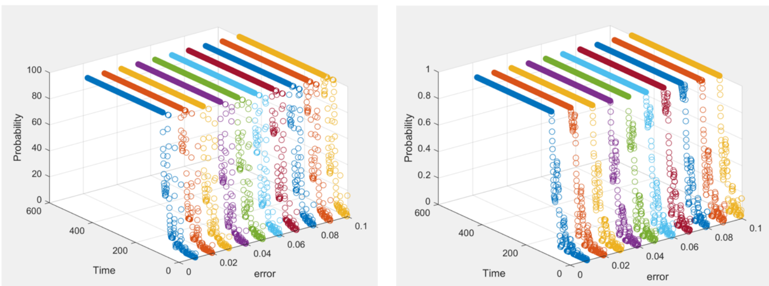
Figure 4.7 shows the probability of having an error less than the special number for state s_3, this figure shows the probability of the smallest time that the specific error happens. It is obvious that for the matured system the smallest time shifted to the earlier time.

Figure 4.8 shows the probability of having an error less than the special number for state s_4, this figure shows the probability of the smallest time that the specific error happens. It is obvious that for the matured system the smallest time shifted to the earlier time.



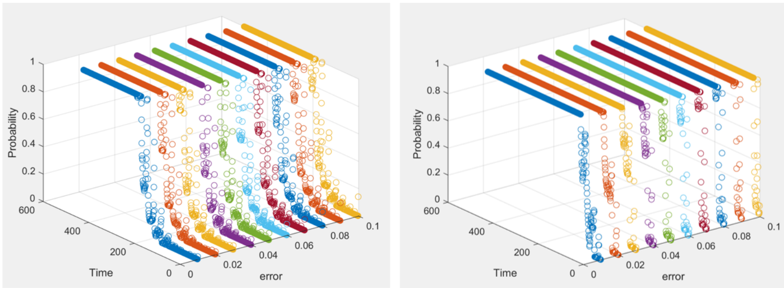
Not Matured system   Matured system

Figure 4.1: Comparing the probability of having error less than a constant number at different times, for the state s_1; Left: For the not matured system, Right: For the matured system



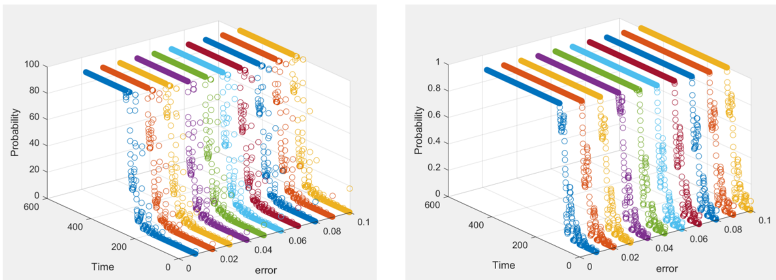
Not matured system   Matured system

Figure 4.2: Comparing the probability of having an error less than a constant number at different times for the state s_2; Left: For the not matured system, Right: For the matured system



Not matured system   Matured system

Figure 4.3: Comparing the probability of having an error less than a constant number at different times for the state s_3; Left: For the not matured system, Right: For the matured system



Not matured system   Matured system

Figure 4.4: Comparing the probability of having an error less than a constant number at different times for the state s_4; Left: For the not matured system, Right: For the matured system

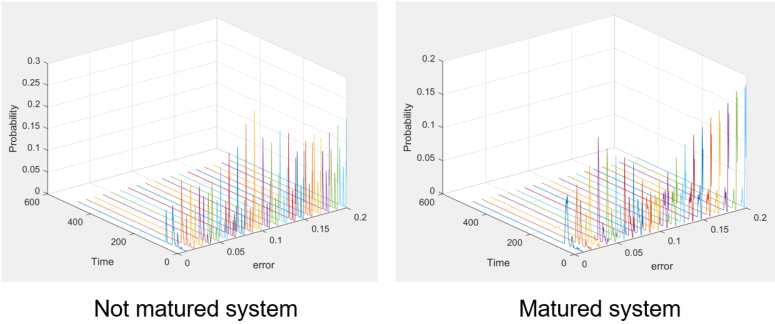


Figure 4.5: Comparing the probability of having an error less than a constant number at different times for the state s_1, this figure shows the smallest time that this error occurred; Left: for the not matured system Right: for the matured system

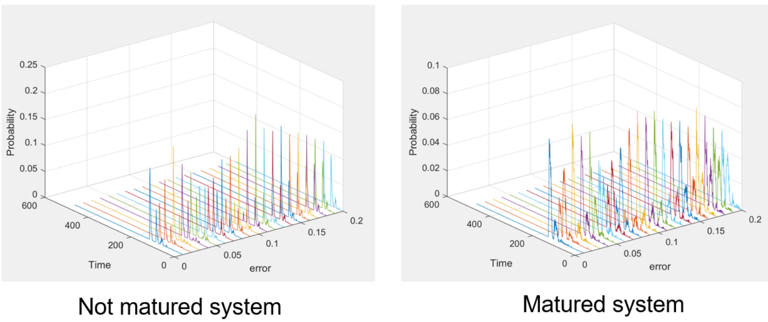


Figure 4.6: Comparing the probability of having an error less than a constant number at different times for the state s_2, this figure shows the smallest time that this error occurred; Left: for the not matured system Right: for the matured system

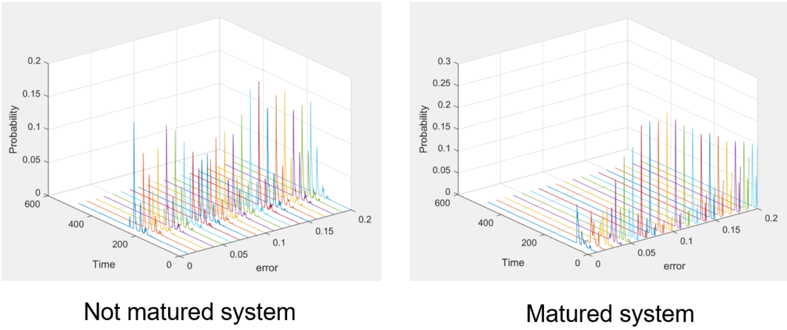


Figure 4.7: Comparing the probability of having an error less than a constant number at different times for the state s_3, this figure shows the smallest time that this error occurred; Left: for the not matured system Right: for the matured system

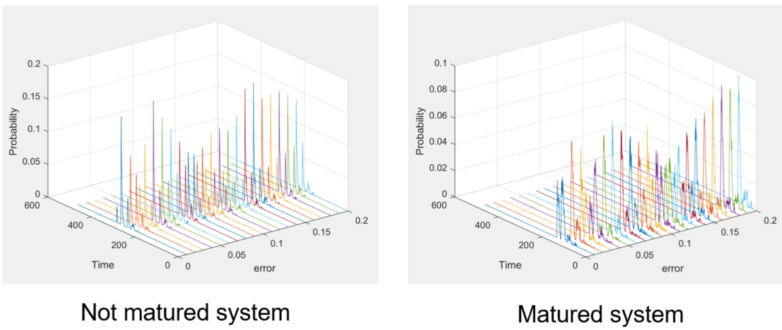


Figure 4.8: Comparing the probability of having an error less than a constant number at different times for the state s_4, this figure shows the smallest time that this error occurred; Left: for the not matured system Right: for the matured system

5 Conclusion

In this paper, we introduce definitions for measuring the maturity of a process. The Elucidability E which is similar to observability in the control theory is used for measuring the capability in estimating states and parameters of the system. Forcability F which is similar to controllability in control theory is used for measuring the capability to steer the states and outputs of the system toward their desired value. Supervisability S is defined for measuring the quality of the process. We run an example to show maturity in a process. In the example, we knew the exact physical model of the system; In practice, the exact model of the system might not be available but we can receive knowledge about the physical model of the system from experiments and have a partial model of the system and use it in machine learning[10][5][3].

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