Pre-Filtered Numerical Integration for Machine Tool Vibration Measurements

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***Abstract*—Accurate knowledge of spindle displacement is essential for assessing the dynamic behavior and machining performance of machine tools, as relative tool–workpiece motion directly affects surface quality and dimensional accuracy. In practice, however, spindle vibrations are commonly measured using accelerometers. Therefore, the displacement has to be calculated from accelerometer data. A major challenge in deriving displacement from acceleration measurements is the accumulation of low-frequency noise and offsets during numerical integration, which leads to unrealistic drift and unphysical displacement signals. As a result, direct double integration of raw acceleration data is not suitable for reliable machine tool vibration analysis with respect to displacements. This paper addresses this problem by applying pre-filtered numerical integration of acceleration signals, following the methodology proposed by Hofmann (2013). High-pass Butterworth filtering with varying cutoff frequencies is applied prior to double integration to suppress low-frequency disturbances while preserving relevant dynamic content. The proposed approach is experimentally validated on a milling machine spindle using broadband impulse hammer excitation. Reconstructed displacement signals are directly compared with reference measurements obtained from a laser interferometer in both the time and frequency domains. The results show that a high-pass cutoff frequency of approximately 10 Hz provides the best agreement with the reference data, enabling physically meaningful spindle displacement reconstruction from accelerometer measurements.**

***Index Terms*—machine tool, measurement, vibration**

I. Introduction

Machine tool accuracy depends directly on the relative displacements between the tool and the workpiece. As Altintas [1] noted, the positioning accuracy and structural deformations of the machine determine whether specified tolerances can be met. These structural deformations manifest as dynamic relative motions, i.e. vibrations, between tool and workpiece. Thus, for assessing machining performance, displacement is the most relevant vibration quantity. In practice, accelerometers are widely used for vibration measurements due to their robustness and bandwidth. However, they inherently measure acceleration, while displacement must be derived by numerical integration. Direct double integration of acceleration is not feasible since measurement noise and low-frequency disturbances accumulate, resulting in unrealistic drift.

To address these issues, Hofmann [2] proposed a method that combines filtering with subsequent numerical integration of acceleration signals. The filtering step removes offsets and low-frequency components that would otherwise dominate the integrated response, thereby enabling physically meaningful reconstruction of vibration displacements.

This technical note examines the applicability of such preprocessing for milling machine vibrations. In particular, the focus is on the spindle, where displacement information is required to evaluate machine dynamics.

II. Background

Accelerometers provide measurements of dynamic accelerations with high bandwidth and minimal influence on the structure under test. Through integration, acceleration can be transformed into velocity and displacement. In principle, the relations are straightforward:

$$\ddot{s}(t) = \dot{v}(t) = a(t) \tag{1}$$

Where $s(t)$ is the displacement, $v(t)$ is the velocity and $a(t)$ is the acceleration. In practice, however, even small offsets or noise components in the acceleration signal are amplified by integration. As Hofmann [2] showed, the physically correct vibration behavior is typically not obtained directly from numerical integration. Instead, signal preprocessing, in particular high-pass (HP) filtering, is necessary to suppress low-frequency drift and restore realistic vibration amplitudes.

Previous recommendations indicate that filtering with a cutoff frequency of about 5 Hz is sufficient for velocity estimation, while 10 Hz is required for displacement reconstruction [2]. The question remains whether such procedures can be reliably applied to milling spindles, where mode frequencies and excitation conditions differ from generic vibration tests.

III. Methodology

An experimental setup was designed to evaluate spindle displacement reconstruction from accelerometer data. The machine tool (Deckel Maho DMC 60 H) was excited at the spindle nose using an instrumented impulse hammer (Kistler 9726A20000). This provides broadband frequency content, allowing the dynamic behavior of the spindle-machine system to be characterized over a wide frequency range. Consequently, multiple structural modes can be excited in a single measurement. The experimental setup is shown in Figure 1.

For validation, a Renishaw ML10 laser interferometer was employed to provide direct displacement measurements at a sample rate of 5000 Hz. The accelerometer was mounted on

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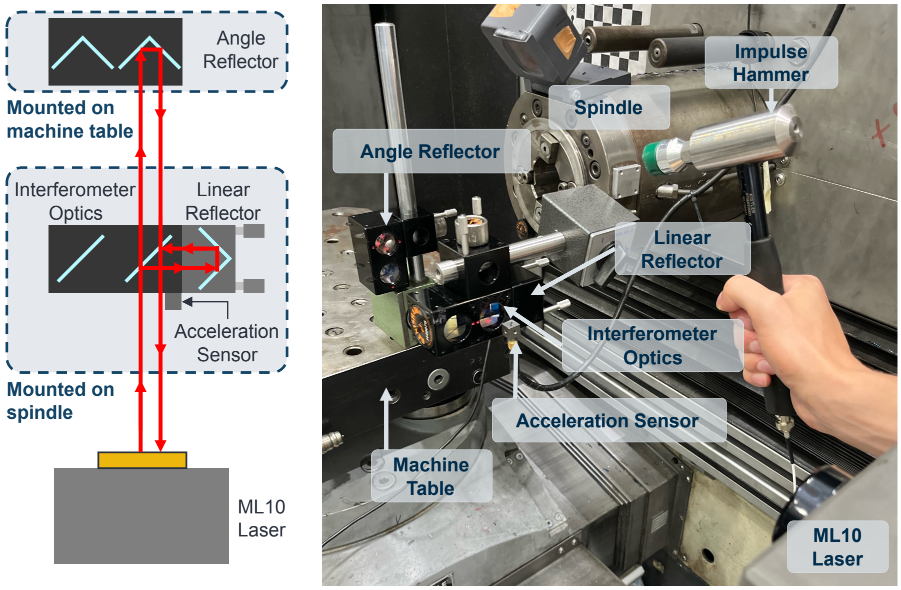

Fig. 1. Experimental setup: Schematic on the left and realization on the right. The machine tool is excited at the spindle nose by an instrumented impulse hammer. The resulting spindle nose vibration is measured in two ways: as the relative displacement between the machine table and the spindle nose using a laser interferometer setup, and as the acceleration recorded by an accelerometer mounted on the angle interferometer optic of the displacement measurement system.

the moving part of the interferometer optics, with a linear reflector attached to the spindle nose using a magnetic mount. The angle reflector was fixed to the machine table, thus capturing the relative spindle–table displacement. This configuration allows direct comparison between interferometer-based displacement and accelerometer-based reconstructed displacement.

Acceleration signals were acquired using a PCB-356A33 triaxial accelerometer at the same sample rate as the displacement measurements (5000 Hz). The signal processing related workflow for calculating the displacement based on acceleration measurement data is shown in Figure 2. The acceleration signals are preprocessed using sixth-order Butterworth filters with varying cutoff frequencies $f_{c1} = f_{c2}$ before numerical integration. The trapezoidal method is employed for numerical integration. The influence of filter selection on displacement accuracy was systematically investigated.

IV. Results and Discussion

The experimental results demonstrate the strong influence of signal preprocessing on displacement reconstruction from accelerometer data. Figure 3 illustrates the effect of numerical integration with and without pre-filtering in the time domain.

The upper plot shows the measured acceleration response of the spindle following excitation by the impulse hammer. The middle plot depicts the displacement obtained by direct double integration of the raw acceleration signal. As expected, this approach leads to a pronounced low-frequency drift, resulting in an unphysical displacement signal that does not resemble the measured spindle motion.

The lower plot shows the displacement obtained after pre-filtering the acceleration signal according to the method proposed by Hofmann [2]. HP filtering effectively suppresses low-frequency offsets and drift components, yielding a physically

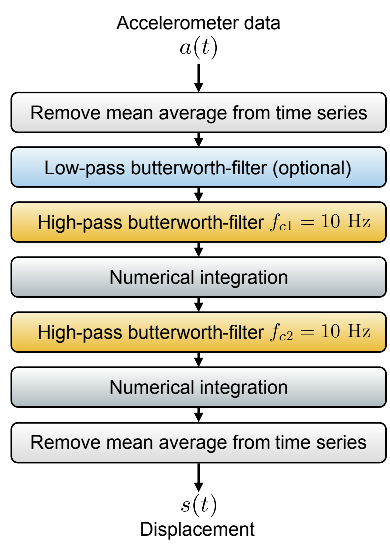

Fig. 2. Signal Processing: Obtaining the displacement vibration signal from measured accelerometer data using Butterworth filtering with cutoff frequencies f_{c1}, f_{c2} and numerical integration.

meaningful displacement signal. The reconstructed displacement is compared directly with the reference measurement obtained from the laser interferometer. The influence of the selected cutoff frequency is clearly visible.

A HP filter with a cutoff frequency of 10 Hz and no additional low-pass (LP) filtering (HP 10 Hz, LP 0 Hz) provides the best agreement with the interferometer signal in the time domain. During the transient response immediately following the hammer impact, up to approximately 0.18 s, the reconstructed displacement closely matches the reference measurement. During subsequent free decay, small deviations

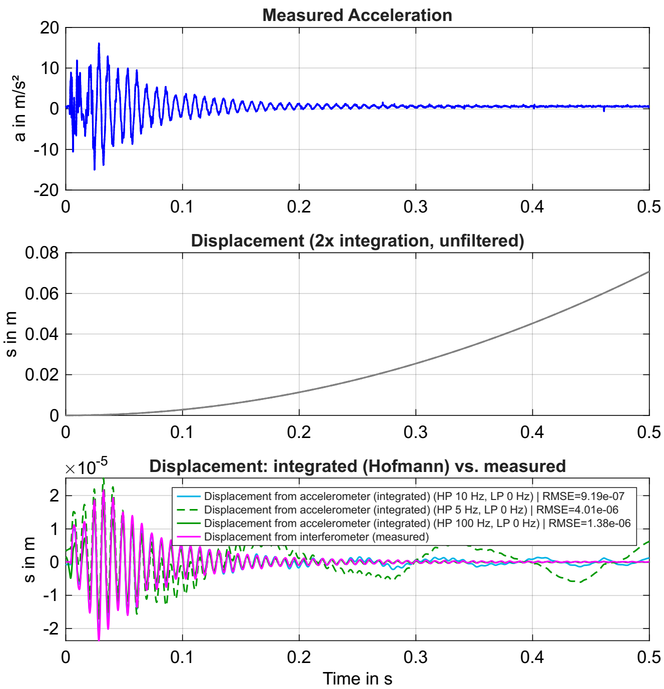

Fig. 3. Time-domain signals: (a) measured acceleration response following impulse hammer excitation, (b) displacement obtained by direct double integration of the raw acceleration signal, exhibiting strong low-frequency drift, and (c) displacement reconstructed from pre-filtered acceleration data for different HP cutoff frequencies, compared with the reference displacement measured by the laser interferometer.

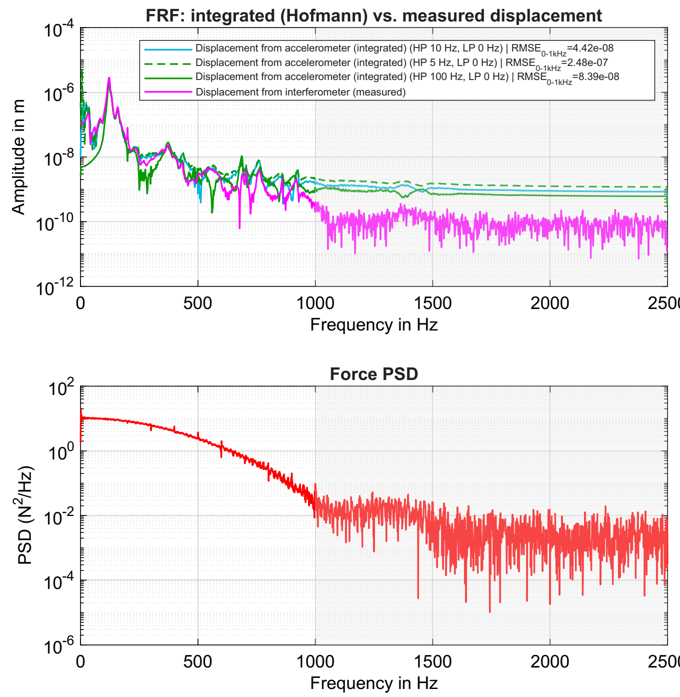

Fig. 4. Frequency-domain analysis: upper plot shows FRFs derived from accelerometer-based displacement reconstruction for different HP cutoff frequencies in comparison with the interferometer-based reference; lower plot shows the PSD of the excitation force applied by the impulse hammer. Evaluation is limited to frequencies below 1000 Hz (shown with gray background for unused frequencies), where sufficient excitation energy is present.

become apparent, attributed to accumulated integration errors and residual low-frequency components. The root-mean-square error (RMSE) between reconstructed and measured displacement (laser interferometer) amounts to 9.17×10^{-7}.

Reducing the HP cutoff frequency to 5 Hz results in noticeably larger deviations, particularly at low frequencies. Residual drift components remain present, leading to a significantly increased RMSE of 4.01×10^{-6}. Conversely, increasing the cutoff frequency to 100 Hz suppresses low-frequency drift effectively and yields good agreement during the free-decay phase. However, in the time interval immediately following excitation (approximately 0 s to 0.05 s), significant discrepancies with respect to the interferometer signal occur. This indicates that relevant low-frequency content excited by the impulse hammer is attenuated excessively.

To further evaluate the influence of the cutoff frequency, the reconstructed displacement signals were analyzed in the frequency domain. Figure 4 shows the frequency response functions (FRFs) derived from the accelerometer-based displacement signals in comparison with the interferometer-based reference. The lower plot depicts the power spectral density (PSD) of the excitation force applied by the impulse hammer.

The force PSD reveals that the excitation energy decreases rapidly above approximately 1000 Hz and transitions into a noise-dominated regime. A corresponding reduction in vibration amplitude is observed in the interferometer measurement. Consequently, we limit the evaluation of displacement reconstruction quality to the frequency range from 0 Hz to 1000 Hz, where sufficient excitation energy is present. All RMSE values calculated for the frequency-domain analysis were therefore computed within this range.

Among the investigated filter configurations, the HP 10 Hz setting yields (similar to the results in Figure 3) the best overall agreement with the reference FRF, with an RMSE of 4.42×10^{-8}. A qualitatively good match is observed up to approximately 400 Hz. Beyond this frequency, increasing deviations occur; however, dominant resonance peaks present in the reference signal remain clearly identifiable in the reconstructed displacement up to approximately 950 Hz, albeit with reduced amplitude accuracy.

The HP 5 Hz configuration exhibits larger deviations at low frequencies, particularly below approximately 20 Hz, where a noticeable shift relative to the reference signal occurs. Above this range, the agreement improves, but remains inferior to the HP 10 Hz case. The corresponding RMSE is 2.48×10^{-7}. The HP 100 Hz filter leads to the strongest attenuation of low-frequency content and performs worst overall, despite showing good agreement in isolated frequency bands. Its RMSE amounts to 8.39×10^{-8}.

Above approximately 900 Hz, all filter configurations show strong deviations from the reference signal. However, this frequency range coincides with a significant reduction in excitation energy and coherence due to the limited bandwidth of the impulse hammer. As a result, discrepancies in this region are not considered representative for assessing displacement reconstruction quality.

V. Conclusion

This study demonstrates that physically meaningful spindle displacement signals can be reconstructed from accelerometer measurements if appropriate signal preprocessing is applied prior to numerical integration. Direct double integration of raw acceleration data leads to severe low-frequency drift and unphysical results, rendering such an approach unsuitable for machine tool vibration analysis.

By applying HP filtering according to the methodology proposed by Hofmann [2], low-frequency offsets and drift components can be effectively suppressed. Experimental validation using a laser interferometer reference measurement shows that a HP cutoff frequency of approximately 10 Hz provides the best compromise between drift suppression and preservation of relevant vibration content for milling spindle applications.

Both time- and frequency-domain analyses confirm that this filter setting yields the closest agreement with interferometer-based displacement measurements across the frequency range excited by the impulse hammer. Lower cutoff frequencies result in residual drift, while higher cutoff frequencies excessively attenuate low-frequency dynamics that are essential for accurately capturing the spindle response.

It should be noted that the applied preprocessing inherently removes static and quasi-static displacement components. However, since accelerometers are insensitive to static motion, such information cannot be recovered in any case. The proposed approach, therefore, offers a practical and reliable method for dynamic spindle displacement estimation, extending Hofmann's general framework [2] to the specific requirements of machine tool vibration analysis and providing a valuable basis for dynamic characterization and chatter avoidance.

References

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Acronyms

FRF frequency response function 3

HP high-pass 1–4

LP low-pass 2

PSD power spectral density 3

RMSE root-mean-square error 3