Benchmarking convective heat transfer in laminar pipe flow

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Numerical models are increasingly employed to predict the behavior of complex physical systems. To ensure that decisions based on these models are reliable, their results must rest on a rigorous foundation. Simplified benchmark problems with known analytical solutions are therefore widely used to validate model performance, enabling quantitative assessment of accuracy, stability, and convergence.
In this study, we investigate coupled fluid flow and heat transfer by comparing numerical predictions with the analytical solution of the Graetz problem [1]. Specifically, we consider laminar pipe flow with conjugate heat transfer in the fully developed regime. Within a diffuse-interface framework, the no-slip condition at the fluid–solid boundary is enforced through an additional dissipative term [2] in the momentum equations. Because the chosen approximation for the diffuse model strongly influences accuracy, we evaluate several formulations for the heat flux [3] through direct comparison with the analytical solution. This systematic assessment highlights the relative performance of each formulation across a range of flow and thermal conditions.
The findings provide practical guidance for selecting diffuse-interface heat-flux formulations in simulations of conjugate heat transfer.

References

[1] J. Neuhauser, J.-H. Metsch, D. Gatti , B. Frohnapfel, Solution of the extended Graetz problem for nonuniform heat flux, *International Journal of Heat and Mass Transfer* **249**, 127198, 2025

[2] C. Beckermann, H.-J. Diepers, I. Steinbach, A. Karma, X. Tong, Modeling Melt Convection in Phase-Field Simulations of Solidification, *Journal of Computational Physics* **154**, pp. 468–496, 1999

[3] M. Nicoli, M. Plapp, H. Henry, Tensorial mobilities for accurate solution of transport problems in models with diffuse interfaces, *Physical Review E* **84**, 046707, 2011

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