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Non-perturbative effects in Higgs boson decays to electroweak vector bosons and photons

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Abstract: We estimate the magnitude of the leading *non-perturbative* QCD corrections to the decays of the Higgs boson to the γZ and $\gamma\gamma$ final states. These corrections originate from the light-quark contributions to such decays. We show that the non-perturbative effects are suppressed by the small Yukawa couplings of light quarks, but that there is no further quark-mass suppression. This is at variance with what is found in the standard perturbative calculations of the light-quark contributions. We demonstrate that the non-perturbative corrections modify the $H \to \gamma Z$ and $H \to \gamma\gamma$ decay rates by $\mathcal{O}(10^{-5})$, well below the expected precision with which such decays can be studied both at the high-luminosity LHC and at future colliders.

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1 Introduction

Measurements of the Higgs boson couplings to electroweak gauge bosons are expected to reach the percent-level precision at the high-luminosity LHC [1, 2]. This precision can be further improved at the future e^+e^- colliders [1, 2]. Since perturbative computations in the Standard Model, commensurate with this level of precision, are feasible [3–14], an unambiguous and instructive comparison of the results of the experimental measurements and the theoretical predictions is expected to be possible.

This point of view, widely shared among particle physicists, tacitly assumes that all contributions to these decays, originating from the *non-perturbative* domain of QCD, are small, compared to the percent or per mille accuracy. This assumption is very natural, as there is a huge disparity between the mass of the Higgs boson m_H and the energy scale of non-perturbative QCD $\Lambda_{\rm QCD}$. It is certain that the decays of Higgs bosons to electroweak vector bosons and photons are predominantly determined by the short-distance physics. In contrast, the non-perturbative QCD effects involve large distances. For this reason they have always been thought to be negligible.

Recently, this story received an interesting twist [15, 16]. It has been known for a long time that leading contributions of light quarks to the $H \to \gamma\gamma$ decay rate, which originate from the interference of light-quark-mediated amplitudes with amplitudes mediated by top quarks and W bosons, are proportional to the *second* power of light-quark masses $\mathcal{O}(m_q^2/m_H^2)$. One power of m_q comes from the Yukawa coupling, and the other one from the helicity flip on the light-quark line as required for the decay of a scalar particle to two spin-one particles through a fermion loop.

While the appearance of the relevant Yukawa coupling in any amplitude where the Higgs boson couples to a light quark is indisputable, it is less obvious that the second power of the light-quark mass is actually present when the *non-perturbative* contribution to the Higgs decay amplitude is considered [15]. Indeed, it was argued in ref. [15] that in the non-perturbative contribution this additional power of the quark mass is replaced by the quantity related to the so-called quark condensate [17]. Since the quark condensate density is the order parameter of the spontaneous chiral symmetry breaking in QCD [17], it remains non-vanishing in the massless quark limit, eliminating the second power of the light quark mass in the Higgs decay amplitudes to electroweak vector bosons and photons.

It was also argued in ref. [15] that the non-perturbative effects in the decays $H \to \gamma\gamma$ and $H \to \gamma Z$ are likely to be enhanced and, in the case of $H \to \gamma\gamma$, could be as large as a few percent. If these results are confirmed, they will significantly affect the program of Higgs precision studies at the high-luminosity LHC and at the future colliders, since this would mark the very first time that $\mathcal{O}(1\%)$ contribution to the $H \to \gamma\gamma$ decay is identified, that cannot be fully controlled.

More recently, the non-perturbative QCD corrections to the $H \to \gamma\gamma$ decay were studied in ref. [16]. There, the dispersion relation in the invariant mass of the two photons was considered for the corresponding form factor. The spectral density in the dispersion relation receives contributions from low-energy hadronic states, including $\pi^+\pi^-$, K^+K^- etc. Estimating such *hadronic* contributions to the $H \to \gamma\gamma$ form factor, the non-perturbative effects were found to be much smaller than the result in ref. [15] and, in fact, in line with widely shared expectations that such effects can only appear well below the percent-level precision target. However, no statement about the light-quark mass dependence was provided in ref. [16], probably because the computational method used there is not conducive to addressing such a question.

Our goal in this paper is to discuss the non-perturbative corrections to Higgs boson decays to electroweak vector bosons and photons one more time. In doing that, we would like to *i*) elaborate on the issue of the degree of suppression of these decays by the light-quark masses, and *ii*) provide alternative estimates of the magnitude of the non-perturbative effects.

We find it convenient to discuss the Higgs decay processes in a particular order, starting with the $H \to Z^*Z^*$ transition, continuing with $H \to \gamma Z$ and, finally, arriving at the $H \to \gamma\gamma$ decay. This order is motivated by a degree of theoretical sophistication required to perform the analysis of the non-perturbative effects in a particular Higgs decay. Indeed, while the non-perturbative phenomena in the $H \to Z^*Z^*$ transition can be analyzed within the framework of the short-distance operator product expansion [18], description of the $H \to \gamma Z$ and $H \to \gamma\gamma$ decays requires an operator product expansion on the light cone [19–23] which introduces non-perturbative quantities beyond the familiar vacuum condensates of quark and gluon fields.

In general, methods that we employ in this analysis are closer to the approach described in ref. [15], and we will confirm the absence of the second power of the light-quark mass in the non-perturbative corrections to the relevant Higgs decay amplitudes. However, our numerical estimate of the non-perturbative contributions in $H \to \gamma\gamma$ are a factor 10^{-4} *smaller* than the results in ref. [15] and, thus, are fully in line with those reported in ref. [16].

2 Higgs boson transition to two Z bosons

There is a direct coupling of the Higgs boson to two Z bosons in the Standard Model, $m_Z^2/v\, H Z_\mu Z^\mu$, where m_Z is the Z-boson mass and v is the Higgs-field vacuum expectation value. This coupling does not lead to a decay to two on-shell Z-bosons because $m_H < 2m_Z$. The decay $H \to ZZ^*$, where one of the Z bosons is off-shell, has a $\mathcal{O}(3\%)$ branching ratio in the Standard Model. There are several ways in which the $H \to ZZ^*$ decay can be affected by the non-perturbative QCD effects, and we will not discuss all of them. In fact, we are particularly interested in the non-perturbative effects related to the light-quark contributions to the $H \to ZZ^*$ transition, because the same effects will provide the *leading* non-perturbative corrections to the $H \to \gamma Z$ and $H \to \gamma\gamma$ decays.

To understand why these light-quark contributions are peculiar, we write the amplitude of the $H(q_{12}) \to Z^*(q_1) + Z^*(q_2)$ transition, moderated by a light quark q, in the following

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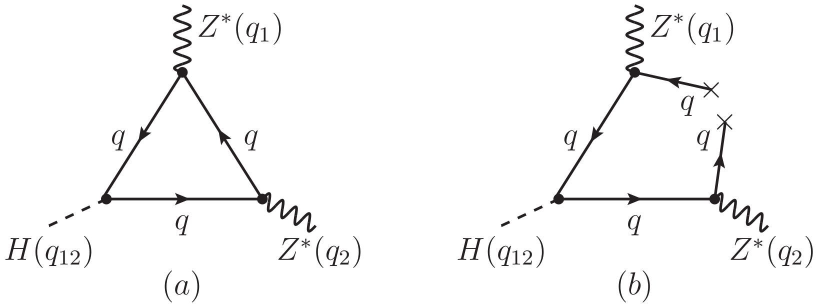

Figure 1. Contribution of light quarks to the $H \to Z^*Z^*$ transition: (a) an example of a short-distance perturbative diagram, (b) an example of the non-perturbative contribution. Terminated quark lines imply the quark condensate.

way (see figure 1)

$$\mathcal{M}^q_{H\to ZZ} = y_q\, g^2_{Zq}\, T^{\mu\nu}\, \epsilon^{(1)}_\mu \epsilon^{(2)}_\nu. \tag{2.1}$$

In eq. (2.1) $y_q = m_q/v$ is the light-quark Yukawa coupling, g_{Zq} is the vector coupling constant between the Z boson and the quark q, and $\epsilon^{(i)}$, $i = 1, 2$, are the polarization vectors of the two Z bosons that satisfy the standard transversality conditions, $\epsilon^{(i)} \cdot q_i = 0$. Furthermore,

$$T^{\mu\nu} = \langle 0|\, \hat{T}^{\mu\nu}\, |0\rangle\,, \tag{2.2}$$

where the operator $\hat{T}^{\mu\nu}$ is the time-ordered product of three currents,

$$\hat{T}^{\mu\nu} = \int d^4x\, d^4y\, e^{iq_1\cdot x + iq_2\cdot y}\, T\left\{\bar{\psi}_q(0)\psi_q(0),\, \bar{\psi}_q(x)\gamma^\mu\psi_q(x),\, \bar{\psi}_q(y)\gamma^\nu\psi_q(y)\right\}\,. \tag{2.3}$$

In eq. (2.3) ψ_q is the field operator of the quark q, and for each current the summation over quark colors is assumed. We have dropped the axial-vector coupling of the Z-bosons to light quarks for simplicity, and because keeping it does not affect our reasoning and conclusions. For the on-shell Z bosons, $q_1^2 = q_2^2 = m_Z^2$; however, it is more convenient to keep these invariant masses as free parameters in the computation.

In perturbation theory

$$T^{\mu\nu} = -iN_c \left\langle \gamma^\mu \frac{1}{\not{k} - m_q}\, \gamma^\nu \frac{1}{\not{k} + \not{q}_2 - m_q}\, \frac{1}{\not{k} - \not{q}_1 - m_q} \right\rangle + (\mu \leftrightarrow \nu,\, q_1 \leftrightarrow q_2), \tag{2.4}$$

where

$$\langle \cdots \rangle = \int \frac{\mathrm{d}^4k}{(2\pi)^4} \mathrm{Tr}\left[\cdots\right]. \tag{2.5}$$

It is easy to see that the amplitude $T^{\mu\nu}$ in eq. (2.4) vanishes if m_q is set to zero, i.e. in the chiral limit. Hence, we conclude that perturbatively $T^{\mu\nu} \sim m_q$.[1] Since the Yukawa coupling y_q in eq. (2.1) is also proportional to m_q, we find $\mathcal{M}^q \sim m_q^2$ which is the familiar

[1]For clarity, we note that we do not discuss here the so-called singlet contribution to Higgs decays, that proceeds through an intermediate gg state, i.e. $H \to gg \to ZZ$. Singlet corrections receive contributions from the light-quark loops, but they are not proportional to the light-quark masses. However, they only appear in higher perturbative orders.

quadratic dependence of the $H \to ZZ^*$ decay amplitude facilitated by the light quark q. We note, that the integral over the loop momentum k in the perturbative amplitude is saturated at $k \sim q_1 \sim q_2 \gg m_q$.

However, in the integral over the loop momentum k, there are regions where the perturbative expansion breaks down. This happens whenever the momenta comparable to the hadronic scale $\Lambda_{\rm QCD} \sim 1$ GeV flow through quark propagators. To find the non-perturbative contributions to the amplitude that originate from these regions, we apply the operator product expansion (OPE) [18, 24] to products of currents in the operator $\hat{T}^{\mu\nu}$ in eq. (2.3),

$$\hat{T}^{\mu\nu} = \sum_i C_i^{\mu\nu}(q_1, q_2)\, \mathcal{O}_i \,. \tag{2.6}$$

In the above equation, $C_i^{\mu\nu}$ and $\mathcal{O}_i$ are the Wilson coefficients and local operators, respectively. The leading operator associated with the non-perturbative regions is $\mathcal{O}_q = \bar{\psi}_q \psi_q$ where the summation over colors is implied. The vacuum average of this operator is the quark condensate [17].

It is straightforward to compute the OPE coefficient $C_q^{\mu\nu}$ for the operator $\mathcal{O}_q$. To this end, it is sufficient to calculate the matrix element

$$\langle q(p_2)|\, \hat{T}^{\mu\nu}\, |q(p_1)\rangle \tag{2.7}$$

in the limit of vanishing momenta $p_{1,2}$ of the initial and final quarks which we take to be *massless*. Diagrammatically, this calculation involves cutting various propagators in the perturbative triangle diagram (see figure 1b) and, assuming that vanishingly-small momentum flows through the cut propagator, computing the tree amplitude composed of the product of the two remaining quark propagators through which the large momenta $\sim q_1, q_2$ flow.

For simplicity, we write the resulting OPE coefficient of the $q\bar{q}$ operator contracted with the polarization vectors of the two Z-bosons,

$$C_q = C_q^{\mu\nu} \epsilon_\mu^{(1)} \epsilon_\nu^{(2)} = -\frac{q_{12}^2}{q_1^2\, q_2^2} \left\{ \left[1 - (r_1 - r_2)^2\right] \epsilon^{(1)} \cdot \epsilon^{(2)} - 2\left[1 + r_1 + r_2\right] \frac{q_2 \cdot \epsilon^{(1)}\; q_1 \cdot \epsilon^{(2)}}{q_{12}^2} \right\}. \tag{2.8}$$

We have defined $r_i = q_i^2/q_{12}^2$, $i = 1, 2$, and have used the transversality conditions $\epsilon^{(i)} \cdot q_i = 0$, $i = 1, 2$, to simplify the above equation.

Using this result to find the non-perturbative contribution of one nearly massless quark to the Higgs decay amplitude in eq. (2.1), we obtain

$$\mathcal{M}_{H \to ZZ}^{q,\,\rm np} = y_q g_{Zq}^2 \,\langle 0|\bar{\psi}_q \psi_q|0\rangle\, C_q \,. \tag{2.9}$$

To estimate the numerical impact of this contribution, we note that the non-perturbative amplitude in eq. (2.9) interferes with the leading order amplitude of the $H \to Z^* Z^*$ transition, $\mathcal{M}_{H\to ZZ} = 2(m_Z^2/v)\, \epsilon^{(1)} \cdot \epsilon^{(2)}$. Thus, the correction is proportional to the ratio of non-perturbative and perturbative amplitudes, which evaluates to

$$\frac{\mathcal{M}_{H\to ZZ}^{q,\,\rm np}}{\mathcal{M}_{H\to ZZ}} \approx -(2\pi\alpha)\, \frac{m_q \langle 0|\bar{\psi}_q \psi_q|0\rangle}{m_Z^4}. \tag{2.10}$$

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To obtain the above result, we used $y_q = m_q/v$, $g_{Zq}^2 \sim 4\pi\alpha$, $\mathcal{M}_{H\to ZZ} \sim 2m_Z^2/v$ and $C_q \sim -m_Z^{-2}$, since for the purpose of this order-of-magnitude estimate, we take $m_H^2 = q_{12}^2 \sim q_1^2 \sim q_2^2 \sim m_Z^2$.

To obtain the full result, one must sum over all light-quark flavors. This sum is strongly dominated by the strange-quark contribution. Hence, focusing on the strange quarks and using the Gell-Mann-Oakes-Renner relation [17], we write[2]

$$\frac{\mathcal{M}_{H\to ZZ}^{\rm np}}{\mathcal{M}_{H\to ZZ}} \approx \frac{(\pi\alpha) f_K^2 m_K^2}{m_Z^4} \sim \text{few} \times 10^{-12}, \tag{2.11}$$

where we used $f_K = 155.7$ MeV [25] and $m_K = 498$ MeV.

We note that non-perturbative corrections to the $H \to W^+W^-$ transition can be estimated along the same lines. Because in this case flavor-changing quark currents are involved, the details of the analysis will be different, but the numerical suppression will be similar to the result in eq. (2.11).

Hence, in spite of being proportional to the first power of the light-quark mass only, the non-perturbative correction to $H \to ZZ^*$ appears to be tiny. There are two reasons for this very strong suppression. The first one is the fine-structure constant α, which appears because we compute the loop-induced non-perturbative correction to the decay amplitude $H \to ZZ^*$ which by itself is not loop-induced.

The second reason for the suppression is the fourth power of the hard scale $q_{1,2} \sim m_Z$ in the denominator in eq. (2.10). This high power appears because the mass-dimension of the non-perturbative matrix element $\langle 0|m_q \bar{\psi}_q \psi_q|0\rangle$ is equal to four. Since there are other non-perturbative quantities of the same mass-dimension, for example, the gluon condensate $\langle 0|\alpha_s/\pi \, G_{\mu\nu}G^{\mu\nu}|0\rangle$, there are other non-perturbative corrections to the $H \to ZZ^*$ transition that are of the same order as the light-quark contribution shown in eqs. (2.10), (2.11). Hence, the light-quark contribution to $\mathcal{M}_{H\to ZZ}^{\rm np}$ is certainly peculiar but not unique in any way.

We will discuss the non-perturbative contributions to $H \to \gamma Z$ and to $H \to \gamma\gamma$ in the following sections, and it is interesting that in those cases *both of the above points become invalid*. Indeed, because the $H \to \gamma Z$ and $H \to \gamma\gamma$ decays are both loop-induced, the fine-structure constant or any other electroweak coupling constant will not be present in the ratios of non-perturbative and perturbative amplitudes in these cases.

We will also see that the non-perturbative contributions of light quarks to $H \to \gamma Z$ and $H \to \gamma\gamma$ are suppressed by the *second* power of the hard scale and, therefore, are much larger. One can anticipate this because the Wilson coefficient in eq. (2.8) reads $C_q \sim q_{12}^2/(q_1^2 q_2^2)$. Hence, a naive extrapolation of the above result to regions where $q_{1,2}^2 = 0$, which is exactly what is needed to describe the Higgs boson decays $H \to \gamma Z$ and $H \to \gamma\gamma$, indicates that a significant enhancement of the non-perturbative effects in such decays can be expected [15].

As we explain in the next section, this expectation is partially correct. Technically, to arrive at this result, we need to appreciate that the operator product expansion for processes with a massless final-state particle becomes different. In fact, the proper theoretical framework

[2]In eq. (2.10), m_q and $\langle 0|\bar{\psi}_q \psi_q|0\rangle$ are defined at the high renormalization scale $\mu \sim m_Z$. However, since the product of the quark mass m_q and the quark condensate $\langle 0|\bar{\psi}_q \psi_q|0\rangle$ does not depend on the renormalization scale, we can use the Gell-Mann-Oakes-Renner relation, naturally associated with low hadronic scales, to estimate their product.

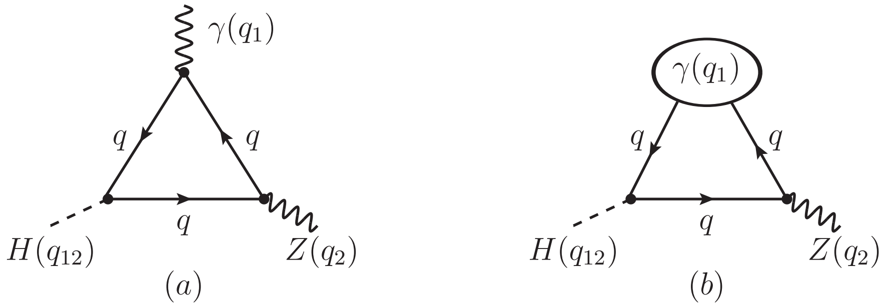

Figure 2. (a) The perturbative short-distance contribution to $H \to Z\gamma$. (b) The non-perturbative fragmentation of a $q\bar{q}$ pair to the photon distribution amplitude. Diagrams with the opposite fermion-flow direction are not shown.

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to analyze the $H \to \gamma Z$ decay is an operator product expansion near the light cone, familiar from studies of hard exclusive processes [19–23, 26].

3 Higgs boson decay to photon and Z boson

Consider the Higgs boson decay to a photon with the momentum q_1 and a Z-boson with the momentum q_2, $H(q_{12}) \to \gamma(q_1) + Z(q_2)$. The amplitude for this process reads

$$\mathcal{M}_{H\to\gamma Z} = y_q\, e_q\, g_{Zq}\, T^{\mu\nu} \epsilon^{(1)}_\mu \epsilon^{(2)}_\nu, \tag{3.1}$$

where $T_{\mu\nu}$ can be taken from eqs. (2.2), (2.3) and we should set $q_1^2 = 0$ there. Furthermore, e_q is the electric charge of the quark q. Leading contributions to the $H \to \gamma Z$ decay amplitude are shown in figure 2. The first, shown in figure 2a, is entirely short-distance one and can be computed in perturbation theory. Similarly to the case $H \to ZZ^*$, it vanishes in the chiral limit, and we do not discuss it further. The second contribution shown in figure 2(b) is more complex as it involves both the short- and the long-distance parts. The short-distance part is the transition of the Higgs boson to the Z-boson and a collinear $q\bar{q}$ pair. The long-distance part corresponds to the fragmentation of a nearly collinear $q\bar{q}$ pair to a photon.

The $q\bar{q} \to \gamma$ fragmentation is described by the so-called photon distribution amplitude which was first introduced in ref. [27], (see also ref. [22]) in the context of QCD sum rules based on the light-cone operator product expansion. In ref. [26] a comprehensive discussion of these distribution amplitudes, including the separation of short- and long-distance effects in the photon-quark interactions can be found.

In principle, without further ado, this standard methodology[3] can be straightforwardly applied to analyze the non-perturbative contributions to the $H \to \gamma Z$ decay. However, we find it useful to discuss the main ideas behind this approach. To this end, we note that the long-distance fragmentation phenomenon that we need to describe makes it inconvenient to work with the correlator $\hat{T}_{\mu\nu}$. Instead, we write $e_q T^{\mu\nu} \epsilon^{(1)}_\mu$ as the matrix element of the

[3]For example, in refs. [28, 29] light-cone sum rules with the photon distribution amplitude have been used to describe the $B \to \gamma \ell \nu_\ell$ decay and a correlation function similar to eq. (3.2), albeit with completely different quark currents, was computed.

T-product of the scalar and vector currents between the vacuum and the single-photon state

$$\begin{aligned} \mathcal{M}_{H\to\gamma Z} &= y_q\, g_{Zq}\, \langle\gamma(q_1)|\hat{\Pi}^\nu|0\rangle\, \epsilon_\nu^{(2)}, \\ \hat{\Pi}^\nu &= i\int d^4x\, e^{-iq_{12}\cdot x}\, T\{\bar{\psi}_q(x)\psi_q(x), \bar{\psi}_q(0)\gamma^\nu\psi_q(0)\}\,. \end{aligned} \tag{3.2}$$

Equation (3.2) contains both, perturbative and non-perturbative, contributions shown in figure 2. However, for the massless quarks the perturbative contribution vanishes. Then, eq. (3.2) is very convenient since the non-perturbative long-distance physics is isolated into the matrix element, and the short-distance physics is described by the product of two current operators that appear explicitly in $\hat{\Pi}^\nu$.

To construct the OPE of the product of currents in $\hat{\Pi}^\nu$ at small x, we compute its matrix element between the on-shell massless quark states with momenta p and $p+q_1$. Since $q_1^2=0$, the requirement that $(p+q_1)^2=0$ and $p^2=0$ implies that $p\cdot q_1=0$, which means that p is either aligned with q_1 or is transversal to it. At tree level, we find

$$\langle q(p+q_1)|\,\hat{\Pi}^\nu\,|q(p)\rangle = -\bar{u}_i(p+q_1)\left[\frac{1}{\not{p}-\not{q}_2}\gamma^\nu + \gamma^\nu\frac{1}{\not{p}+\not{q}_{12}}\right]u^i(p)\,, \tag{3.3}$$

where $u^i(p)$ is the spinor wave function for the quark with the color i and momentum p. Rewriting products of Dirac matrices through anti-commutators and commutators, we obtain

$$\begin{aligned} \langle q(p+q_1)|\,\hat{\Pi}^\nu\,|q(p)\rangle = &-\left[\frac{(p-q_2)^\nu}{q_2^2-2pq_2} + \frac{(p+q_{12})^\nu}{q_{12}^2+2pq_{12}}\right]\bar{u}_i(p+q_1)u^i(p) \\ &-\left[\frac{(p-q_2)_\alpha}{q_2^2-2pq_2} - \frac{(p+q_{12})_\alpha}{q_{12}^2+2pq_{12}}\right]\bar{u}_i(p+q_1)\sigma^{\alpha\nu}u^i(p)\,, \end{aligned} \tag{3.4}$$

where $\sigma^{\alpha\nu} = [\gamma^\alpha,\gamma^\nu]/2$.

It is convenient to start with the case of the small photon and quark momenta, p, $q_1 \ll q_2$ which corresponds to the limit when the Higgs and the Z boson masses are very close. In this limit, the first term in eq. (3.4) drops and the second survives. Rewriting $\bar{u}\sigma^{\alpha\nu}u$ in terms of the quark-field operators, we conclude that the OPE of $\hat{\Pi}^\nu$ takes the form

$$\hat{\Pi}^\nu = \frac{2q_{2\alpha}}{q_2^2}\;\bar{\psi}_q(0)\,\sigma^{\alpha\nu}\psi_q(0)\,. \tag{3.5}$$

It remains to compute the matrix element of $\hat{\Pi}^\nu$ in eq. (3.5) between the photon and vacuum states. This matrix element is known [30]; it is parametrized by a particular quantity χ called the magnetic susceptibility of the quark condensate,

$$\langle\gamma(q_1)|\bar{\psi}_q(0)\sigma_{\alpha\nu}\psi_q(0)|0\rangle = e_q\,\chi\,\langle 0|\bar{\psi}_q\psi_q|0\rangle f_{\alpha\nu}^{(1)}\,. \tag{3.6}$$

In this equation, $\langle 0|\bar{\psi}_q\psi_q|0\rangle$ is the quark condensate, and $f_{\alpha\nu}^{(1)} = q_{1\,\alpha}\epsilon_\nu^{(1)} - q_{1\,\nu}\epsilon_\alpha^{(1)}$ is the field-strength tensor of the photon. Thus, we conclude that the non-perturbative part of $H\to\gamma Z$ amplitude in the limit of small photon momentum q_1, i.e. for $m_H^2 - m_Z^2 \ll m_H^2$, reads

$$\mathcal{M}_{H\to\gamma Z}^{q,\,\mathrm{np}} = e_q\, g_{Zq}\, y_q\, \frac{\langle 0|\bar{\psi}_q\psi_q|0\rangle\;\chi}{m_H^2}\; f_{\mu\nu}^{(2)} f^{(1)\mu\nu}\,, \tag{3.7}$$

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where $f^{(2)}_{\mu\nu} = q_{2\,\mu}\epsilon^{(2)}_\nu - q_{2\,\nu}\epsilon^{(2)}_\mu$. An interesting property of this result is that the non-perturbative amplitude is only suppressed by the *second* power of the hard scale. This feature is related to the appearance of the magnetic susceptibility χ which has the mass-dimension -2 and, parametrically, is determined by the soft QCD scale, $\chi \sim \mathcal{O}(\Lambda^{-2}_{\rm QCD})$. Thus, it provides an enhancement of the non-perturbative effects in $H \to \gamma Z$ decay, which was advertised at the end of the previous section, albeit so far derived only for the unphysical case $m_H \approx m_Z$.

We continue with the discussion of the realistic case, where the photon momentum is of the same order as m_H and m_Z. To this end, we expand eq. (3.4) to higher powers in the quark momentum p to derive the OPE coefficients of operators of higher mass-dimensions.[4] The leading operator is $\bar{\psi}_q\,\sigma^{\alpha\nu}\psi_q$, that was already introduced; it has mass-dimension 3 and spin 1, so its twist[5] is 2. Powers of momentum p in the expansion would lead to the appearance of higher spin operators with derivatives of the quarks fields. For example, a term that is linear in p introduces the following operator

$$\mathcal{O}^{\alpha\nu}_\mu = \bar{\psi}_q\,\sigma^{\alpha\nu} i D_\mu \psi_q\,, \tag{3.8}$$

where D_μ is the covariant derivative. It provides the following addition to the leading contribution to $\hat{\Pi}^\nu$ in eq. (3.5)

$$\delta\,\hat{\Pi}^\nu \sim \frac{q_2^\mu q_{2\,\alpha}}{(q_2^2)^2}\,\mathcal{O}^{\alpha\nu}_\mu . \tag{3.9}$$

Normally, $\delta\hat{\Pi}^\nu$ is a small correction to $\hat{\Pi}^\nu$, but since we are interested in the matrix element of $\hat{\Pi}^\nu$ between the photon with a large momentum q_1 and the vacuum, this is not true anymore. Indeed, the relevant matrix element is proportional to the photon momentum

$$\langle\gamma(q_1)|\,\mathcal{O}^{\alpha\nu}_\mu\,|0\rangle \sim q_1^\mu\,\langle\gamma(q_1)|\bar{\psi}_q\sigma^{\alpha\nu}\psi_q\,|0\rangle, \tag{3.10}$$

which implies that for $q_1 \cdot q_2 \sim q_2^2$,

$$\langle\gamma(q_1)\,|\,\delta\hat{\Pi}^\nu\,|0\rangle \sim \langle\gamma(q_1)\,|\,\hat{\Pi}^\nu\,|0\rangle, \tag{3.11}$$

and there is no suppression. Hence, all terms with additional derivatives acting along the light-cone direction, defined by the photon momentum q_1, cannot be discarded. The summation of all such contributions provides a non-perturbative object that is known as the twist-two photon distribution amplitude [22].

The twist-two photon distribution amplitude depends on the ratio of the hard scale of the process we are interested in, and the non-perturbative QCD scale $\Lambda_{\rm QCD}$. In our case, this ratio is very large $m_H/\Lambda_{\rm QCD} \sim 10^3$. Because of this, we are interested in the so-called asymptotic form of this amplitude [21] which is obtained by taking the hard scale to infinity. To introduce it, we note that the operator $\mathcal{O}^{\alpha\nu}_\mu$ can be re-written in the following way

$$\mathcal{O}^{\alpha\nu}_\mu = \frac{1}{2}\,i\partial_\mu(\bar{\psi}_q\,\sigma^{\alpha\nu}\psi_q) + \frac{1}{2}\,\bar{\psi}_q\,\sigma^{\alpha\nu} i\overset{\leftrightarrow}{D}_\mu\psi_q\,. \tag{3.12}$$

[4] Since we are interested in the matrix of the operator $\hat{\Pi}^\mu$ between the photon and the vacuum, we can discard the first term on the right-hand side of eq. (3.4).

[5] Twist of an operator is the difference between its mass-dimension and spin.

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The first term in the above equation is the total derivative of the leading operator whose matrix element in eq. (3.6) defines the magnetic susceptibility. The matrix element of the second term in eq. (3.12) has a similar form but it is a different operator nonetheless. In principle, one should define its matrix element by introducing another susceptibility-like quantity that will differ from the susceptibility χ in eq. (3.6).

In principle, the contribution of *both* operators in eq. (3.12), as well as all other multi-derivative operators, must be taken into account. However, it is known [21] that all operators which in addition to total derivatives, contain other quantities, are suppressed by the logarithm $\log(m_H/\Lambda_{\rm QCD})$ which, as we already mentioned, is large. Hence, in the limit $m_H \gg \Lambda_{\rm QCD}$ only operators that are total derivatives of $\bar{\psi}_q \sigma^{\alpha\nu}\psi_q$ should be retained. These operators provide the asymptotic form of the photon distribution amplitude which is therefore completely determined by the single non-perturbative parameter χ.

The photon distribution amplitude $\phi_\gamma(\xi)$ describes how the photon momentum q_1 is shared between a quark and an antiquark fragmenting into the photon. Our convention is that the antiquark carries momentum ξq_1 and the quark carries the rest. To account for this, we write $p^\mu = -\xi q_1^\mu$ in eq. (3.4), extending the operator $\hat{\Pi}^\nu$ to non-vanishing quark momenta

$$\hat{\Pi}^\nu = \int_0^1 {\rm d}\xi\, \phi_\gamma(\xi) \left[\frac{1}{(1-\xi)q_{12}^2 + \xi q_2^2} + \frac{1}{\xi q_{12}^2 + (1-\xi)q_2^2}\right] q_{2\,\alpha} \bar{\psi}_q\, \sigma^{\alpha\nu}\psi_q\,. \tag{3.13}$$

Finally, taking the matrix element between the photon and the vacuum state, making use of eq. (3.6) and substituting $q_{12}^2 = m_H^2$, $q_2^2 = m_Z^2$, we find

$$\mathcal{M}^{q,{\rm np}}_{H\to\gamma Z} = e_q g_{Zq} y_q \frac{\langle\bar{\psi}_q\psi_q\rangle\chi}{2m_H^2}\, f^{(2)}_{\mu\nu} f^{(1)\,\mu\nu} \int_0^1 {\rm d}\xi \left[\frac{1}{1-(1-r)\xi} + \frac{1}{r+\xi(1-r)}\right]\phi_\gamma(\xi), \tag{3.14}$$

where $r = m_Z^2/m_H^2$. To compute the remaining integral, we employ the asymptotic form of the photon distribution amplitude[6]

$$\phi_\gamma = 6\,\xi(1-\xi), \tag{3.15}$$

and obtain

$$\mathcal{M}^{q,{\rm np}}_{H\to\gamma Z} = e_q\, g_{Zq}\, y_q \frac{\langle\bar{\psi}_q\psi_q\rangle\chi}{m_H^2}\, f^{(2)}_{\mu\nu} f^{(1)\,\mu\nu}\, 3F\left(\frac{m_Z^2}{m_H^2}\right), \tag{3.16}$$

where

$$F(r) = \frac{1+r}{(1-r)^2} + \frac{2r}{(1-r)^3}\ln r\,. \tag{3.17}$$

The function $F(r)$ is finite at $r=0$ and at $r=1$, and for $r\in[0,1]$ assumes numerical values between 1 and 1/3.

To estimate the numerical impact of the non-perturbative corrections to the $H\to\gamma Z$ decay amplitude on the decay rate, we note that the main effect comes from the interference

[6] The asymptotic form of the *pion* distribution amplitude is derived in ref. [21], but it can be equally well applied to the photon case.

of $\mathcal{M}^{\rm np}_{H\to\gamma Z}$ with the leading perturbative amplitude that contains loops of heavy quarks and vector bosons. This amplitude was computed in ref. [31] for the first time.

Given the fact that the non-perturbative effects that we discuss in this paper are quite small, it is sufficient to provide a rough estimate of the perturbative amplitude. To this end, we write

$$\mathcal{M}_{H\to\gamma Z} \sim \frac{e\, g_Z}{4\pi v}\, f^{(2)}_{\mu\nu} f^{(1)\mu\nu}, \tag{3.18}$$

where g_Z is the electroweak coupling constant. The ratio of the non-perturbative and perturbative amplitudes evaluates to

$$\frac{\mathcal{M}^{q,\rm np}_{H\to\gamma Z}}{\mathcal{M}_{H\to\gamma Z}} \sim \frac{6\pi\, m_q\, Q_q \chi \langle\bar{\psi}_q\psi_q\rangle}{m_H^2} \to -\frac{3\pi\, Q_s \chi f_K^2 m_K^2}{m_H^2}\,, \tag{3.19}$$

where we have used $F(m_Z^2/m_H^2) \approx 1/2$ for the physical masses of the Z and Higgs bosons. Furthermore, in the last step we took into account that strange quarks provide the largest contribution and again used the Gell-Mann-Oakes-Renner relation [17].

Similarly to the case $m_H \sim m_Z$ discussed earlier, the most striking feature of eq. (3.19) in comparison with the $H \to ZZ^*$ case, is that the degree of suppression is reduced from $1/m_Z^4$, to $1/m_H^2 \sim 1/m_Z^2$. This (dimension-full) difference is accounted for by the magnetic susceptibility of the vacuum χ, whose mass-dimension is minus two. Writing $\chi = M_\chi^{-2}$, we express eq. (3.19) as follows

$$\frac{\mathcal{M}^{\rm np}_{H\to\gamma Z}}{\mathcal{M}_{H\to\gamma Z}} \sim -\frac{3\pi\, Q_s f_K^2 m_K^2}{m_H^2 M_\chi^2}. \tag{3.20}$$

For numerical estimates, we require magnetic susceptibility χ at the high scale. However, for simple numerical estimates, we will neglect its running. We use $\chi(\mu = 1~\text{GeV}) = 2.85 \pm 0.5~\text{GeV}^{-2}$ [32],[7] so that M_χ evaluates to $\mathcal{O}(0.6~\text{GeV})$. This implies

$$\frac{\mathcal{M}^{\rm np}_{H\to\gamma Z}}{\mathcal{M}_{H\to\gamma Z}} \sim \text{few} \times 10^{-5}. \tag{3.21}$$

Therefore, we find that, in the $H \to \gamma Z$ case, the non-perturbative effects are suppressed by only *two* powers of the hard scale, whereas in the $H \to ZZ^*$ case, they are suppressed by *four* powers. This is in line with the proposal in ref. [15], which effectively advocates the replacement of both factors $1/q_i^2$, $i = 1, 2$, in eq. (2.8) with the factor $1/M_V^2$, where M_V is a mass of a typical light vector meson M_ρ, M_ω, M_ϕ, as a way to describe the non-perturbative contributions to $H \to \gamma Z$ and, eventually, to $H \to \gamma\gamma$ decays. In the next section, we will discuss whether the extension of our analysis to the $H \to \gamma\gamma$ case supports this approach.

4 Higgs boson decay to two photons

It remains to discuss the $H \to \gamma\gamma$ case. We can derive the non-perturbative corrections to the $H \to \gamma\gamma$ amplitude by using the results for the $H \to \gamma Z$ amplitude discussed in the previous

[7] Similar estimates of the magnetic susceptibility were obtained in refs. [26, 30, 33].

section, and extrapolating them to $q_2^2 = 0$. This extrapolation is straightforward because the function $F(r)$ possesses smooth $r = q_2^2/m_H^2 \to 0$ limit, $F(0) = 1$. Thus,

$$\mathcal{M}^{q,\,\mathrm{np}}_{H\to\gamma\gamma} = 6\, e_q^2\, y_q\, \frac{\langle\bar{\psi}_q\psi_q\rangle\,\chi}{m_H^2}\, f^{(2)}_{\mu\nu} f^{(1)\,\mu\nu}\,. \tag{4.1}$$

In comparison to eq. (3.16), we replaced g_Z with e_q, set $F(0) \to 1$, and multiplied by 2 because each of the two photons can be produced in the fragmentation of the collinear $q\bar{q}$ pair. For the perturbative short-distance amplitude of the $H \to \gamma\gamma$ decay, first computed in refs. [34, 35], we use eq. (3.19), where we make a replacement $g_Z \to e_q$ for obvious reasons. The result in eq. (4.1) implies that the ratio of the non-perturbative amplitude to the perturbative one in the $H \to \gamma\gamma$ case is nearly identical to eq. (3.20).

The physical picture of the non-perturbative corrections to the $H \to \gamma\gamma$ amplitude consistent with this result can be formulated as follows. The long-distance fragmentation of the collinear $q\bar{q}$ pair to a photon is the main source of the leading non-perturbative correction to the $H \to \gamma\gamma$ decay. However, only *one* of the two photons in the decay is produced by this mechanism, whereas the second photon is produced at short distances. Hence, the production of the second photon is not subject to an additional power enhancement by the ratio of the square of the short-distance scale to the hadronic scale, represented by the magnetic susceptibility of the QCD vacuum.

It is exactly this point that distinguishes our analysis from the enhancement mechanism discussed in ref. [15], since in that reference the long-distance enhancement is postulated for *both* photons. Although we believe that our analysis of the non-perturbative effects in the $H \to \gamma Z$ amplitude is better motivated than the discussion in ref. [15], there is no doubt that the result in ref. [15] can be used to provide an order-of-magnitude estimate of the non-perturbative effects in the $H \to \gamma Z$ case and, numerically, our results are similar. However, for the $H \to \gamma\gamma$ decay, our result is *smaller* than the result in ref. [15] by a factor $(\Lambda_{\mathrm{QCD}})^2/m_H^2 \sim 10^{-4}$ and, therefore, it is more in line with the findings in ref. [16].

5 Conclusions

We have discussed the leading non-perturbative corrections to the $H \to \gamma Z$ and $H \to \gamma\gamma$ decays. These corrections originate from the light-quark loop contributions, see figure 2. It is peculiar that, in contrast to the regular perturbative light-quark short-distance contributions to these decays, which are suppressed by two powers of the light-quark mass, the non-perturbative effects are only suppressed by one power of m_q. This was pointed out earlier in ref. [15], and our analysis supports these findings.

We have shown that one can use the well-established method of the operator product expansion on the light cone, to estimate the non-perturbative corrections to the $H \to \gamma Z$ and $H \to \gamma\gamma$ decays. The leading non-perturbative contributions are determined by the twist-two photon distribution amplitude, the quark condensate and the magnetic susceptibility of the QCD vacuum. Our analysis suggests that the *leading* non-perturbative correction to the $H \to \gamma\gamma$ decay amplitude, originates from kinematic configurations where one photon is produced by a long-distance fragmentation of the $q\bar{q}$ pair, and the second one is produced at short distances. While one *can* identify non-perturbative contributions to $H \to \gamma\gamma$ decay where

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both photons are produced at long distances, our analysis shows that they will be suppressed by $\Lambda_{\rm QCD}^2/m_H^2 \sim 10^{-4}$ relative to the leading non-perturbative mechanism established above.

Numerically, the non-perturbative effects are tiny. They modify the $H \to \gamma Z$ and $H \to \gamma\gamma$ decay rates by about 10^{-5}. Similar level of suppression was observed in ref. [16] which utilized the dispersion relation for the $H \to \gamma\gamma$ form factor to estimate the non-perturbative corrections. We therefore conclude that non-perturbative corrections to the $H \to \gamma Z$ and $H \to \gamma\gamma$ decays are *not an obstacle* for the exploration of these processes at the high-luminosity LHC and at future colliders with a percent-level precision.

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