ROTA: Round Trip Times of Arrival for Localization with Unsynchronized Receivers

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Intelligent Sensor-Actuator-Systems

Primary Surveillance Radar (PSR)

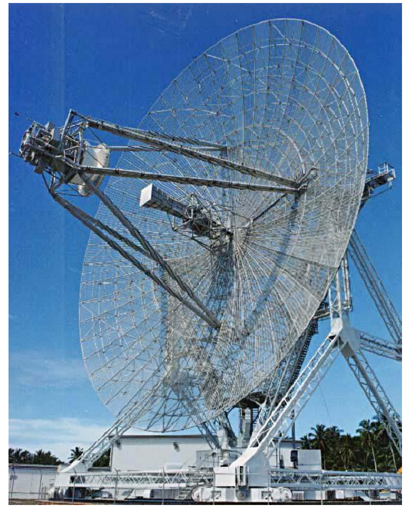
+ non-cooperative targets +

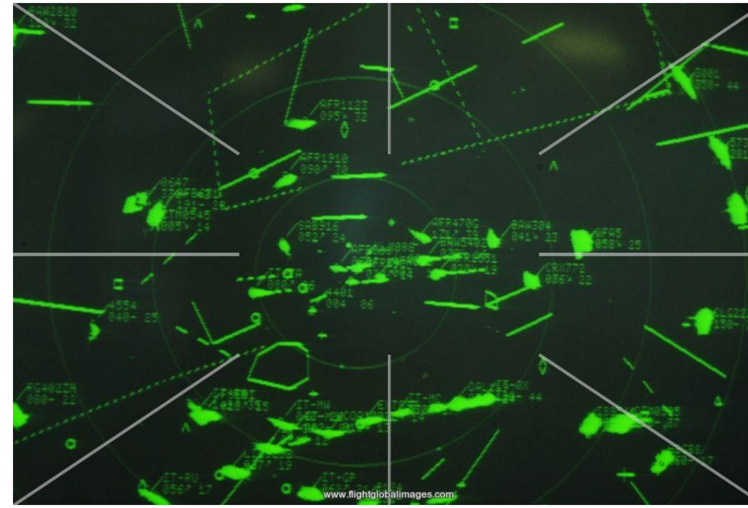
– $SNR \propto \frac{P_t}{R \cdot R \cdot R \cdot R}$ **–**

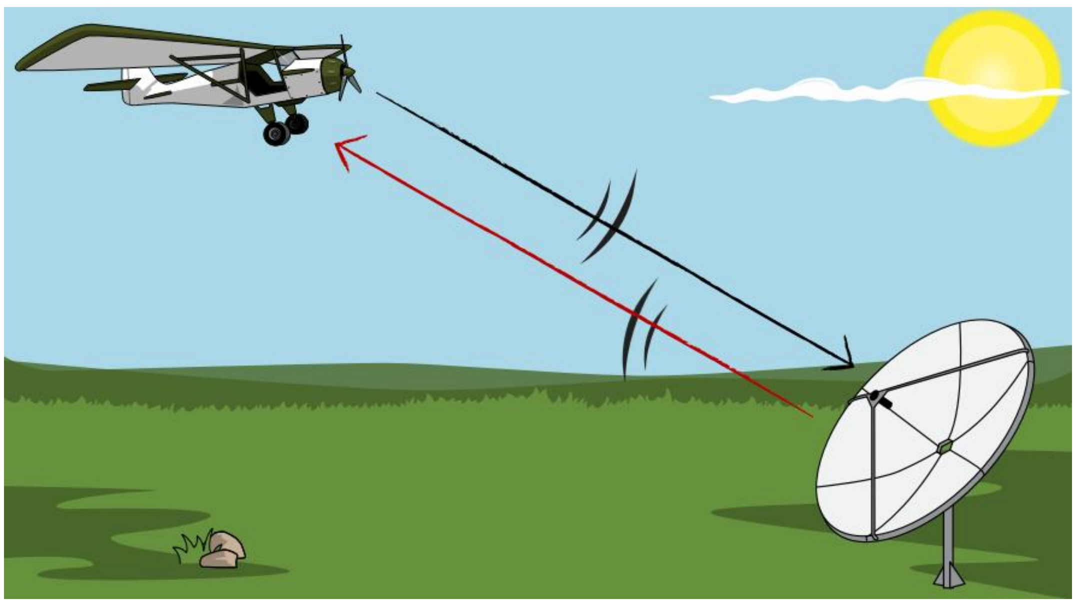
– powerful emissions –

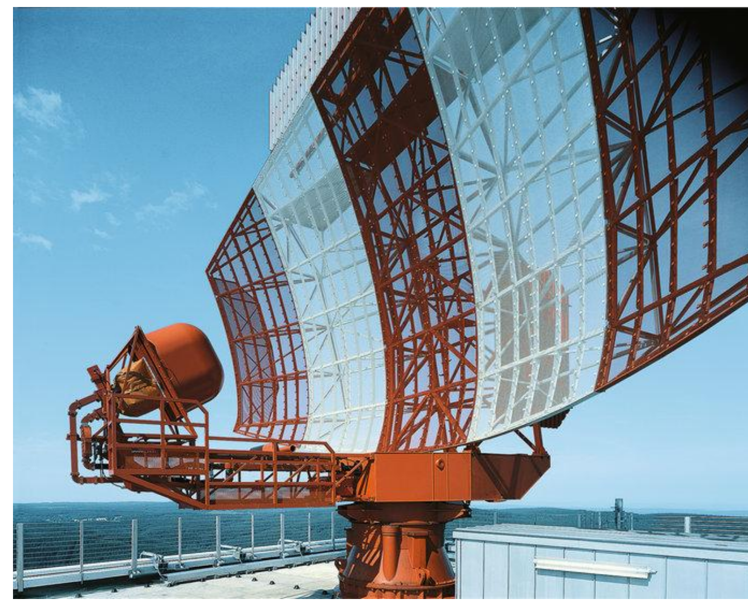
– expensive systems –

– low accuracy –









Secondary Surveillance Radar (SSR)

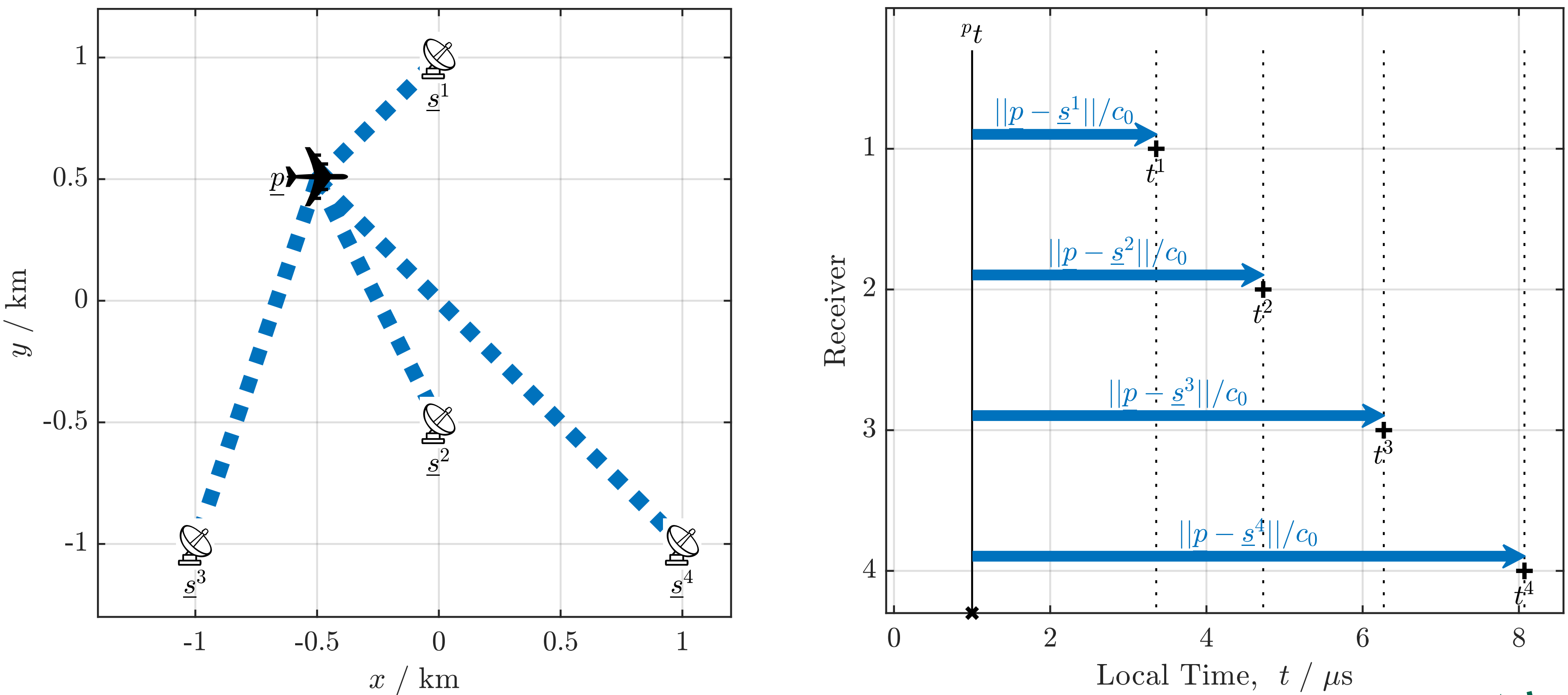
+ high localization accuracy +

+ $SNR \propto \frac{P_t}{R \cdot R}$ +

+ small transmitters & receivers +

– requires cooperative targets –

Time of Arrival (TOA)

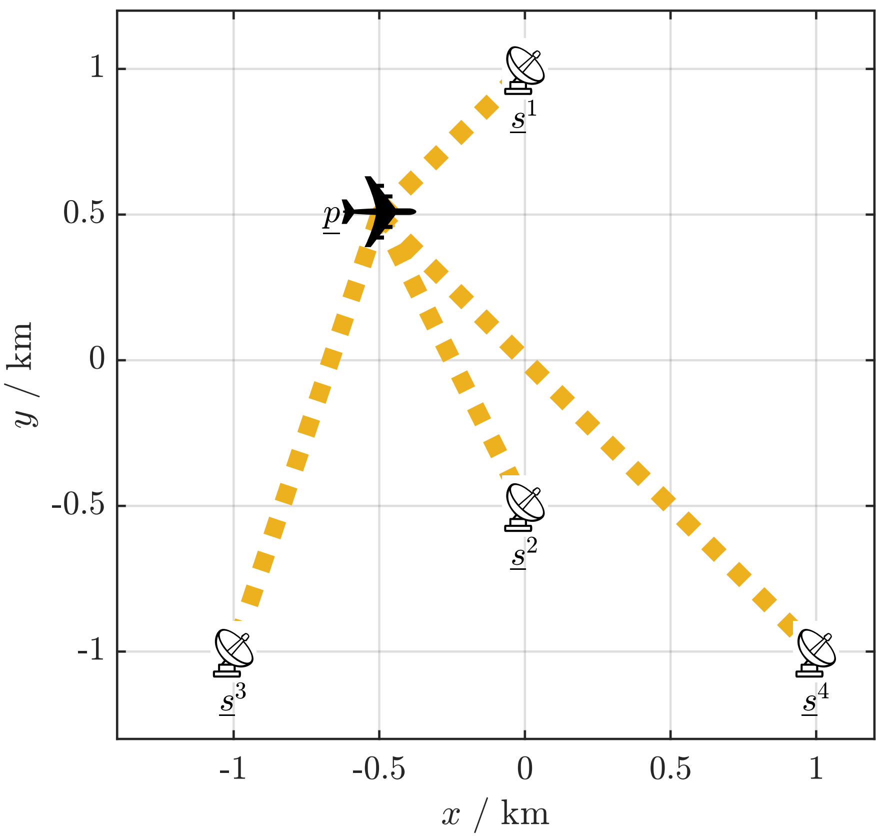


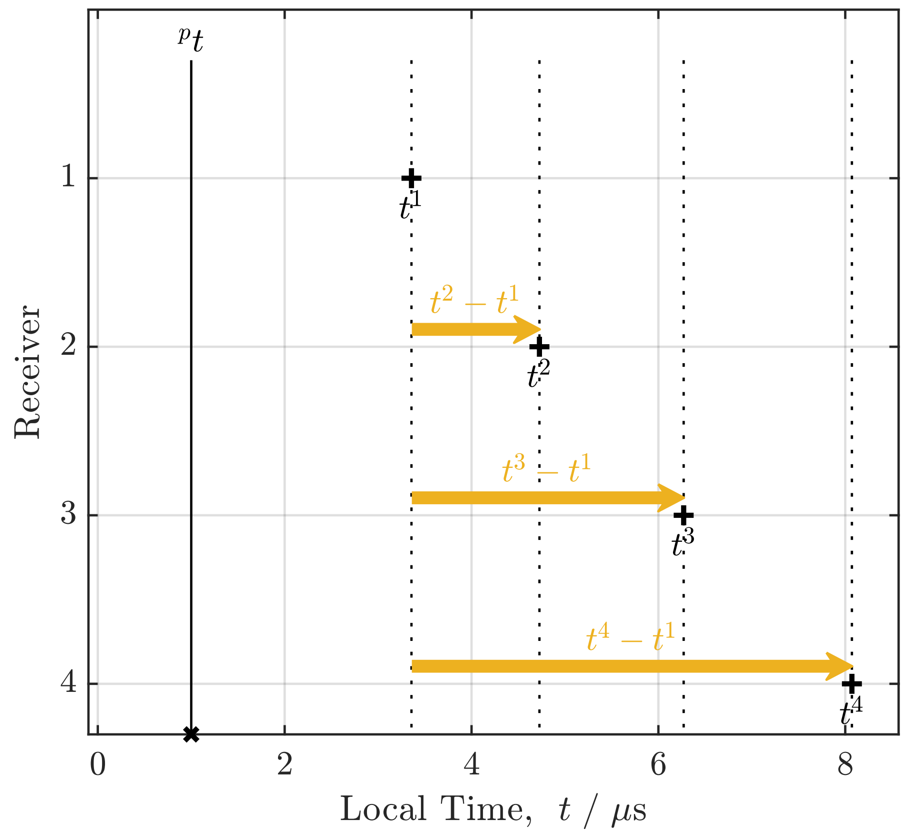
$$t^i - {}^{p}t = \left\| \underline{p} - \underline{s}^i \right\| / c_0 \ , \qquad i \in \{1, 2, 3, 4\}$$

+ Easy Measurement +
+ Accurate +
– Many Unknowns –

Literature: Dunau, Packi, Beutler, Hanebeck, "Efficient Multilateration Tracking with Concurrent Offset Estimation using Stochastic Filtering Techniques" (FUSION 2010)

Difference of Times of Arrival (DOTA) "Star"
(often called TDOA)



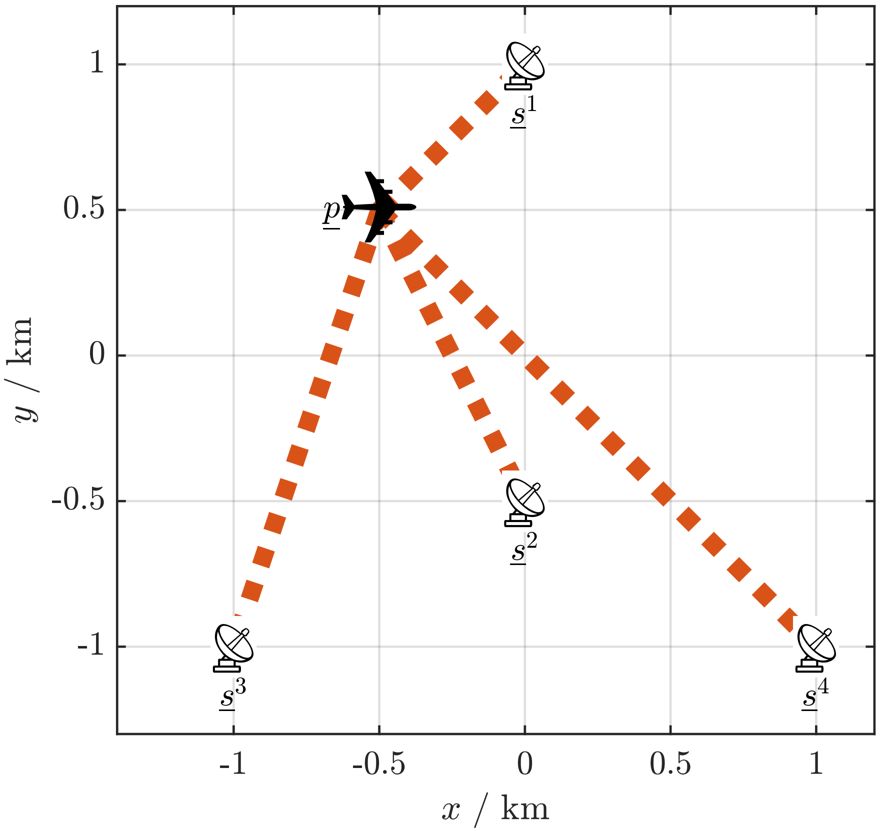


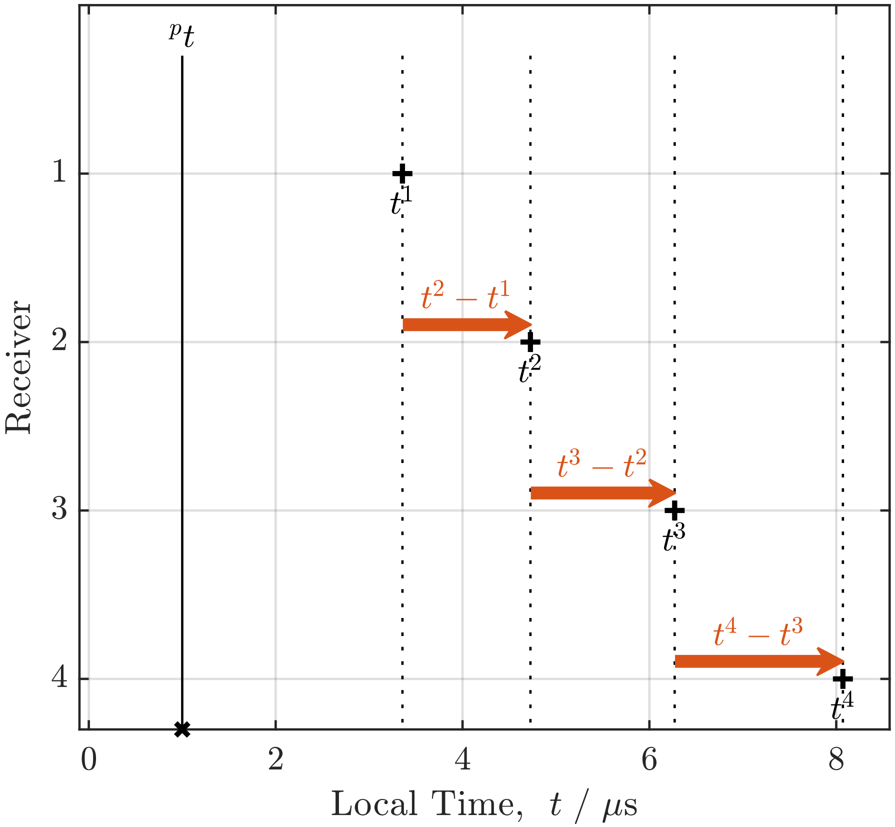
$$t^j - t^i = \frac{\left\|\underline{p} - \underline{s}^j\right\| - \left\|\underline{p} - \underline{s}^i\right\|}{c_0}, \qquad (i,j) \in \{(1,2),(1,3),(1,4)\}$$

Literature: Kaune, Musicki, Koch, “On passive emitter tracking in sensor networks” (Chapter in “Sensor Fusion and its Applications”, 2010)

Difference of Times of Arrival (DOTA) "Successive"

(often called TDOA)





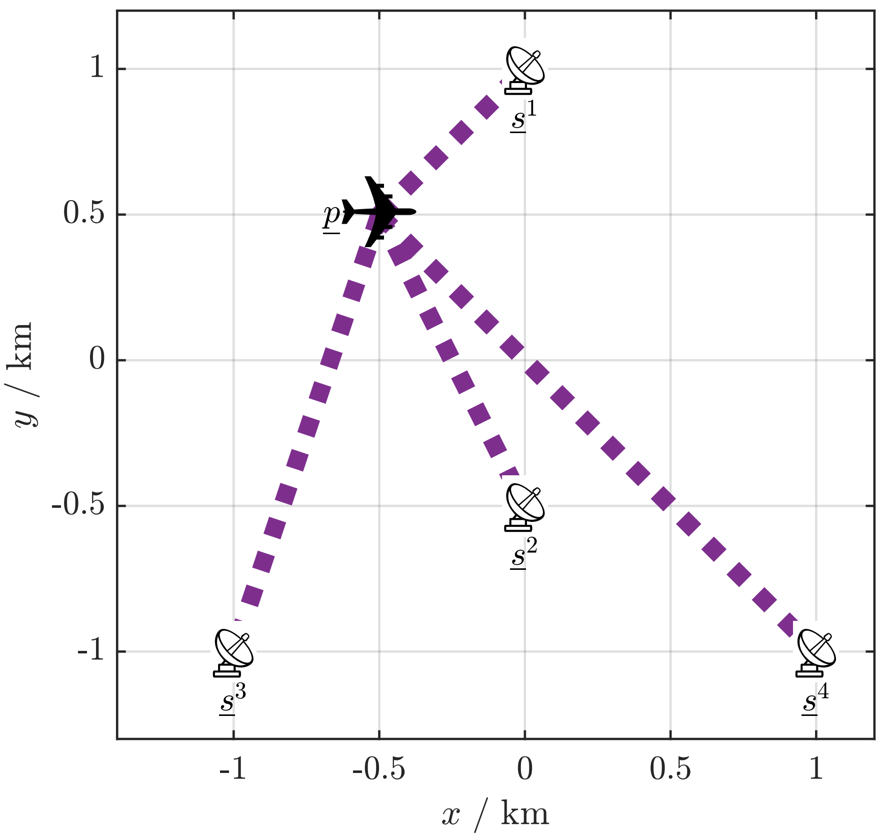
+ Simple Measurement +

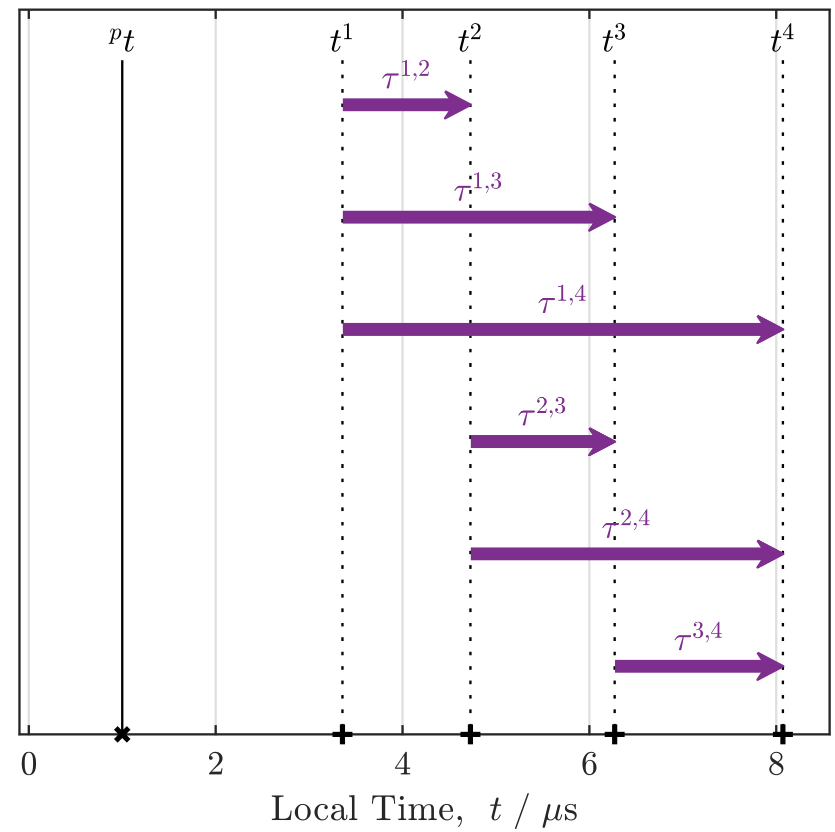
– need weighted LS –

$$t^j - t^i = \frac{\left\| \underline{p} - \underline{s}^j \right\| - \left\| \underline{p} - \underline{s}^i \right\|}{c_0}, \qquad (i, j) \in \{(1,2), (2,3), (3,4)\}$$

Literature: Kaune, Musicki, Koch, "On passive emitter tracking in sensor networks" (Chapter in "Sensor Fusion and its Applications", 2010)

Time Differences of Arrival (TDOA)





– **Costly measurement** –

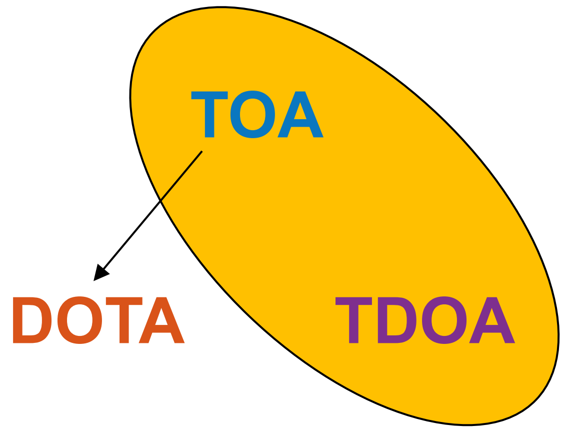
+ **High accuracy** +

$$\tau^{i,j} = \frac{\left\|\underline{p} - \underline{s}^j\right\| - \left\|\underline{p} - \underline{s}^i\right\|}{c_0}, \qquad (i,j) \in \{(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)\}$$

Literature: Kaune, Musicki, Koch, "On passive emitter tracking in sensor networks" (Chapter in "Sensor Fusion and its Applications", 2010)

Summary Synchronized SSR

- TOA or TDOA are measured
  - DOTA is **calculated** from TOA
    → Information loss

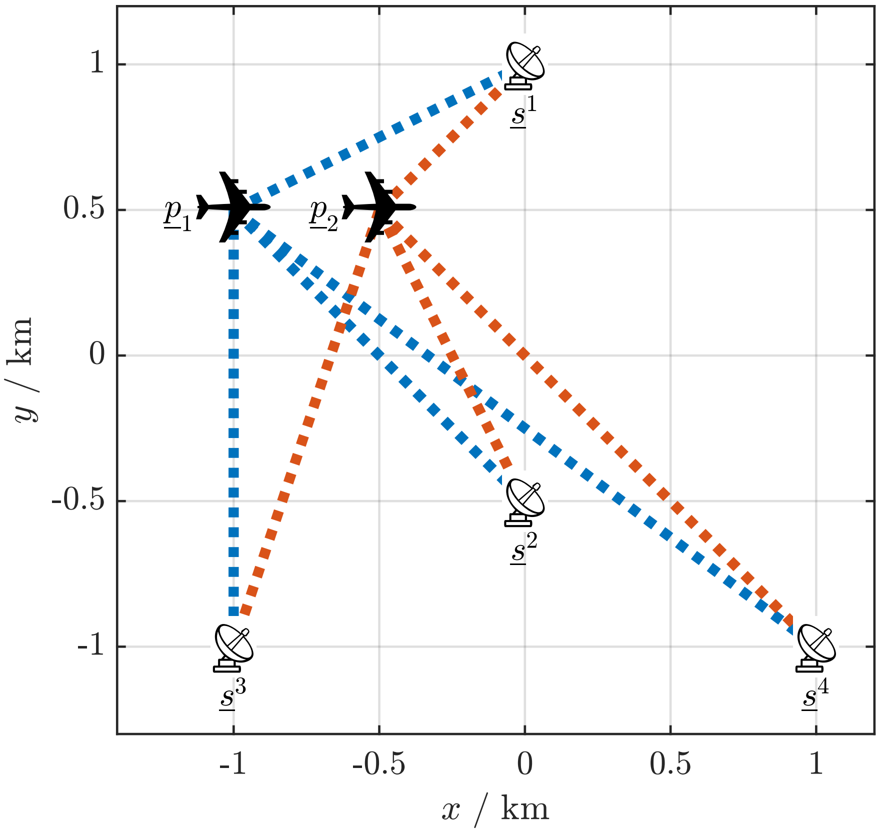


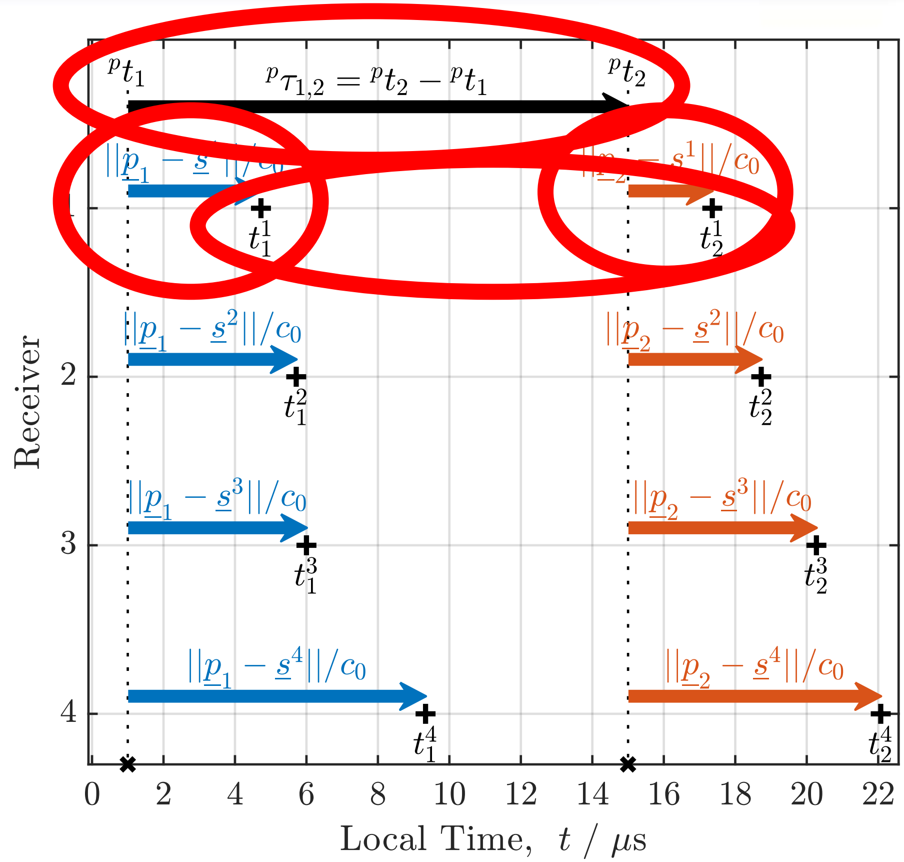
- Every DOTA algorithm
  is also TDOA algorithm
  - **different covariance**
  - TDOA easier to design,
    but more computational work

+ Static targets +
– Synchronized receivers –

- TOA, DOTA, TDOA
  need **~1ns synchronized receivers**
  - very challenging in wide area networks

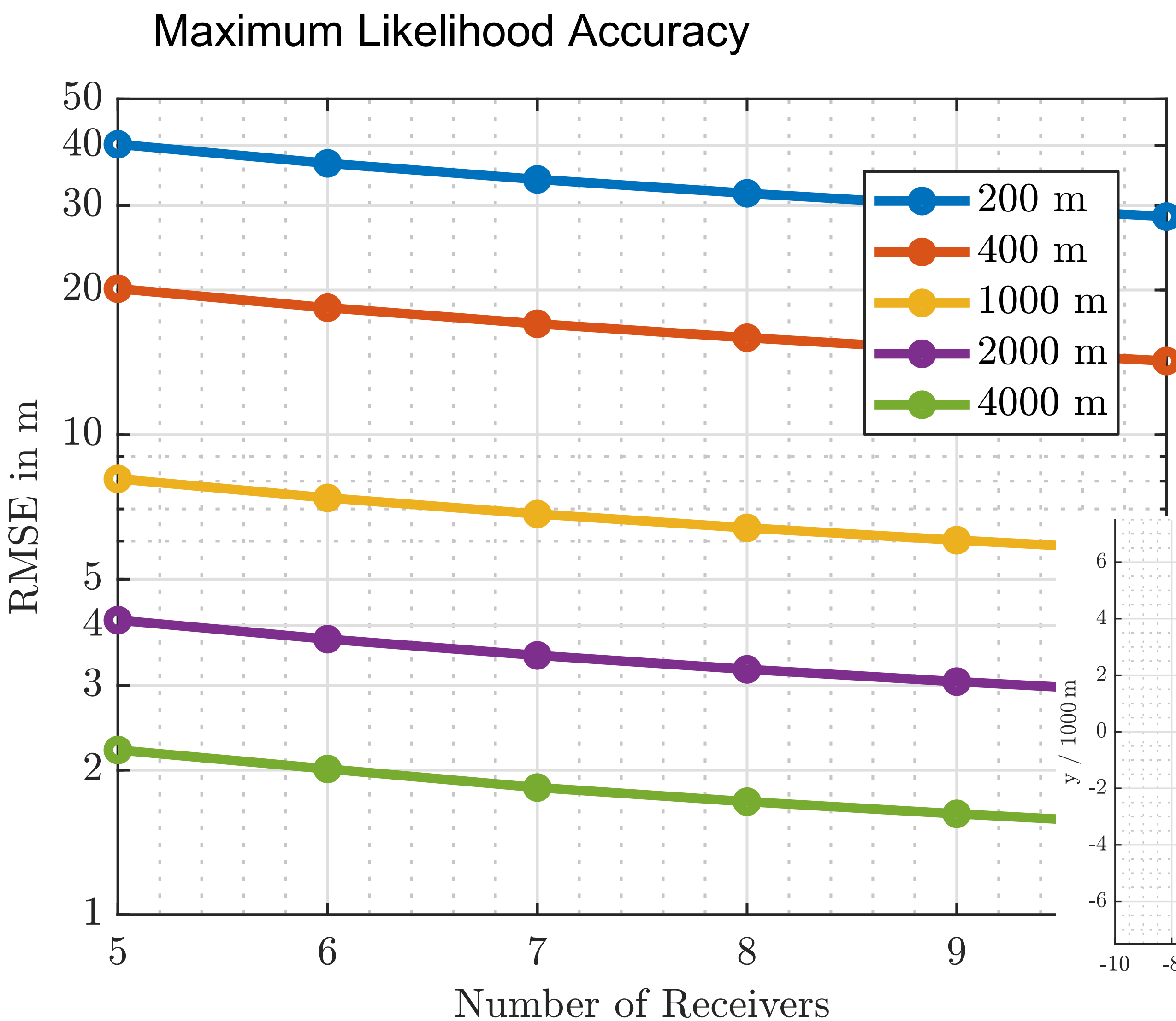
Round Trip Times of Arrival (ROTA)

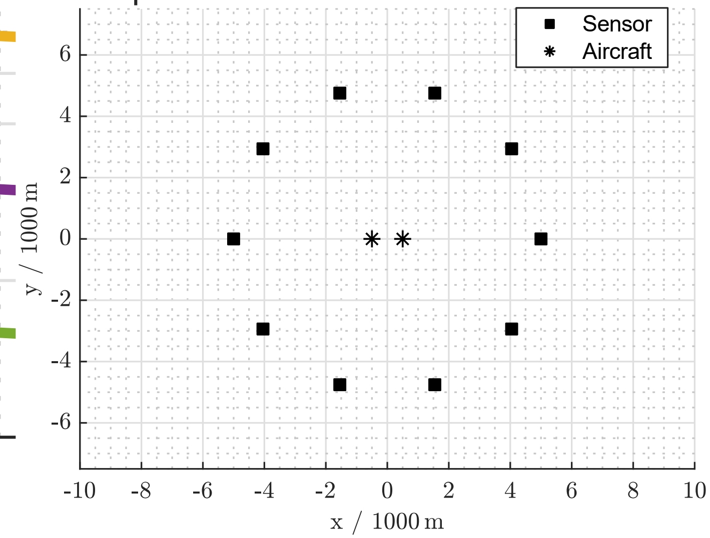




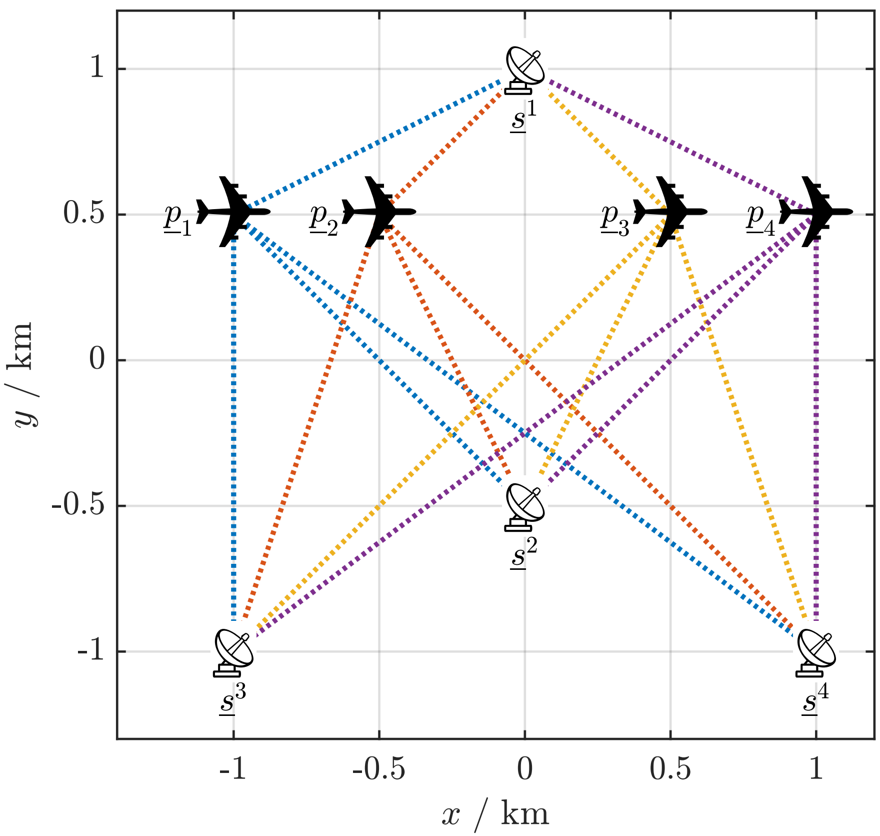
$$\frac{\left\| p_1 - s^i \right\|}{c_0} + \left(t_2^1 - t_1^1\right) = {}^p\tau_{1,2} + \frac{\left\| p_2 - s^i \right\|}{c_0}, \qquad i \in \{1, 2, 3, 4\}$$

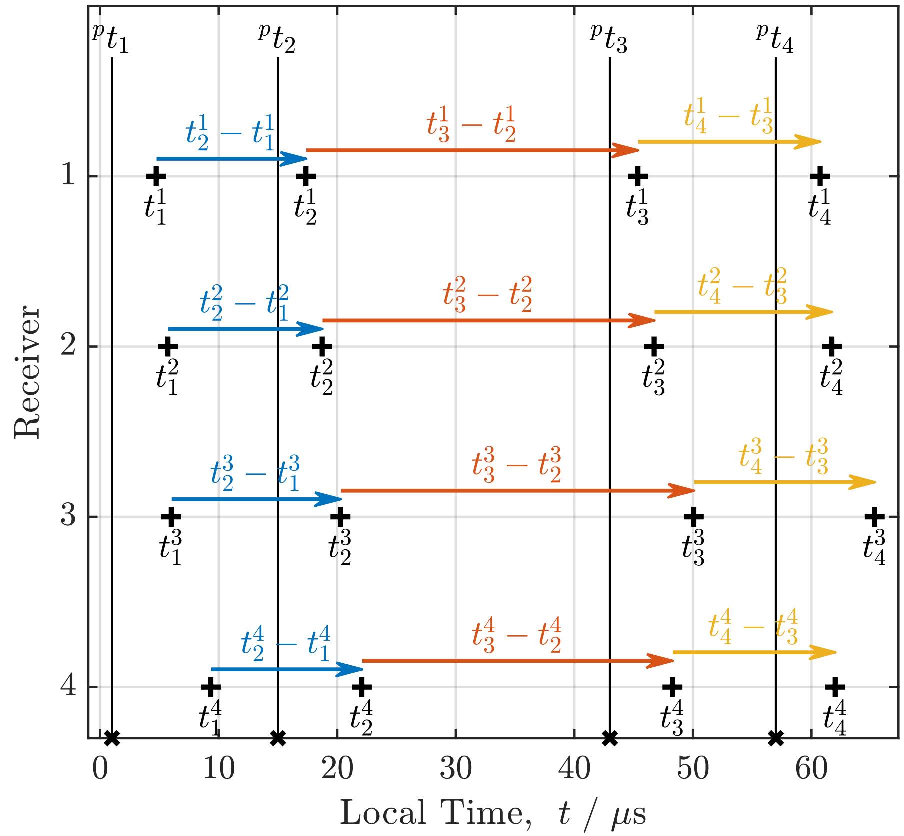
Literature: Stefanski, "Asynchronous time difference of arrival (ATDOA) method" (Pervasive Mob. Comput. 2015)


Maximum Likelihood Accuracy
RMSE in m
Number of Receivers
200 m
400 m
1000 m
2000 m
4000 m
50
40
30
20
10
5
4
3
2
1
5
6
7
8
9


Sensor
Aircraft
y / 1000 m
x / 1000 m
6
4
2
0
-2
-4
-6
-10
-8
-6
-4
-2
0
2
4
6
8
10

ROTA $\mathcal{M}$ Transmissions L-DOTA "Successive"



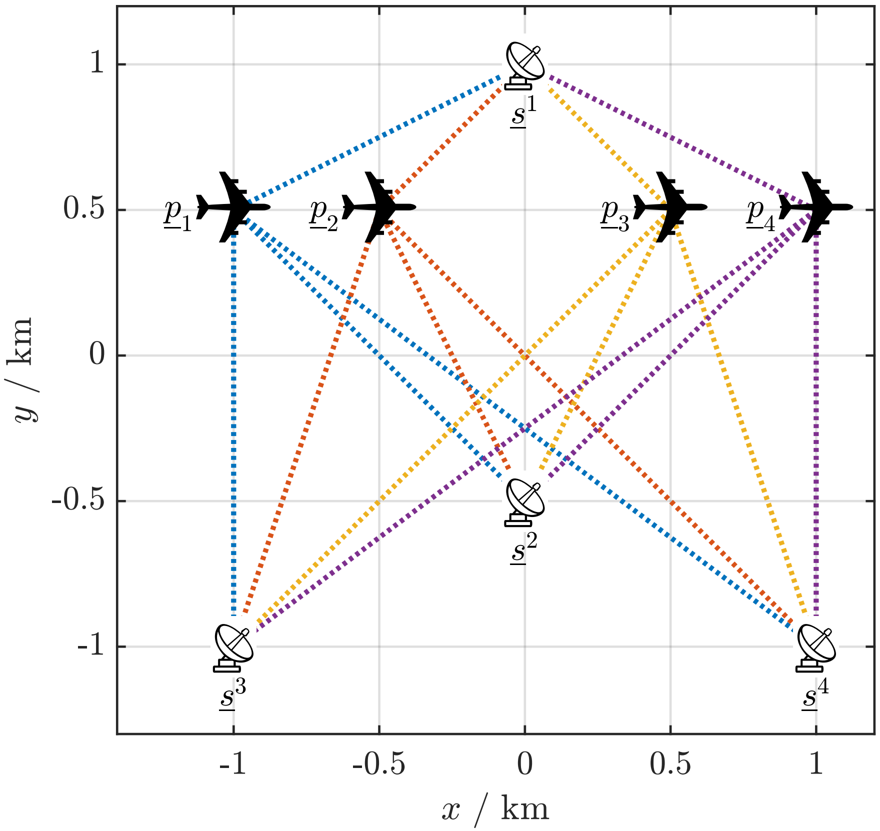


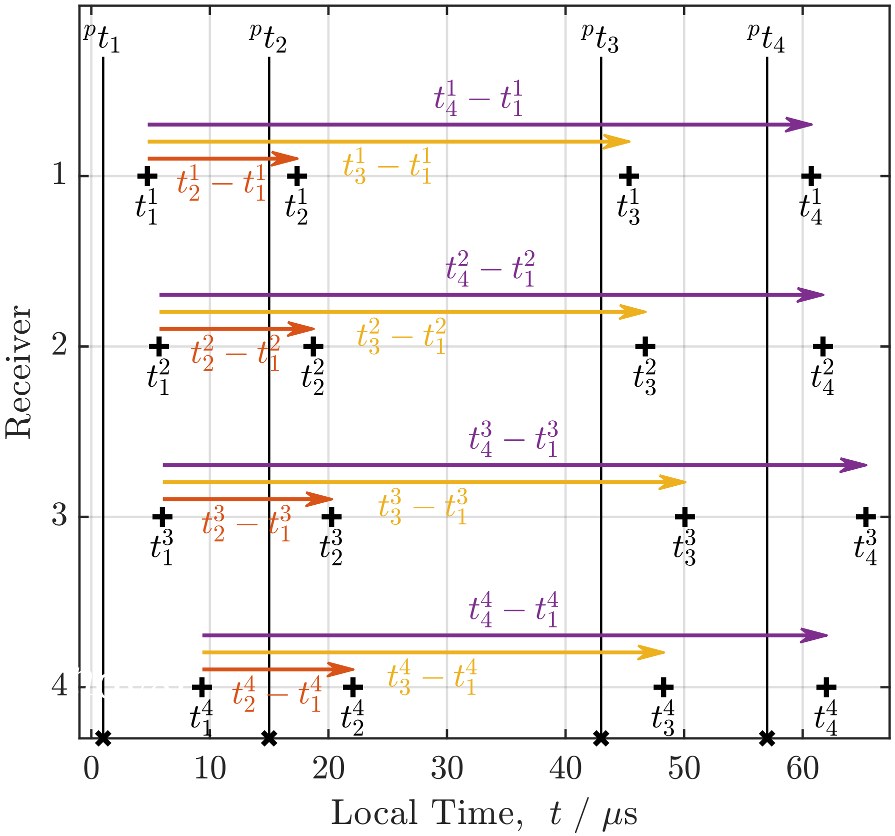
$$\frac{\left\| \underline{p}_k - \underline{s}^i \right\|}{c_0} + \left(t_l^1 - t_k^1\right) = {}^p\tau_{k,l} + \frac{\left\| \underline{p}_l - \underline{s}^i \right\|}{c_0}$$

$$i \in \{1, 2, 3, 4\}$$
$$(k, l) \in \{(1,2), (2,3), (3,4)\}$$

Literature: Stefanski, Sadowski, "TDOA versus ATDOA for wide area multilateration system" (J Wireless Com Network 2018) → but incorrect Covariance!

ROTA $\mathcal{M}$ Transmissions L-DOTA "Star"



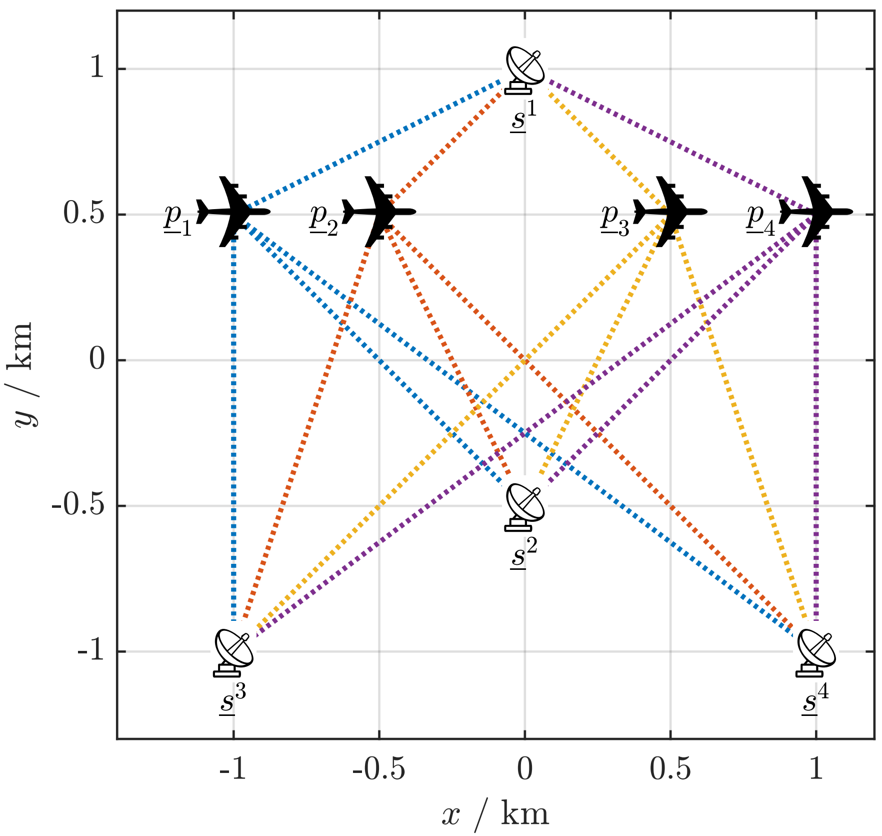


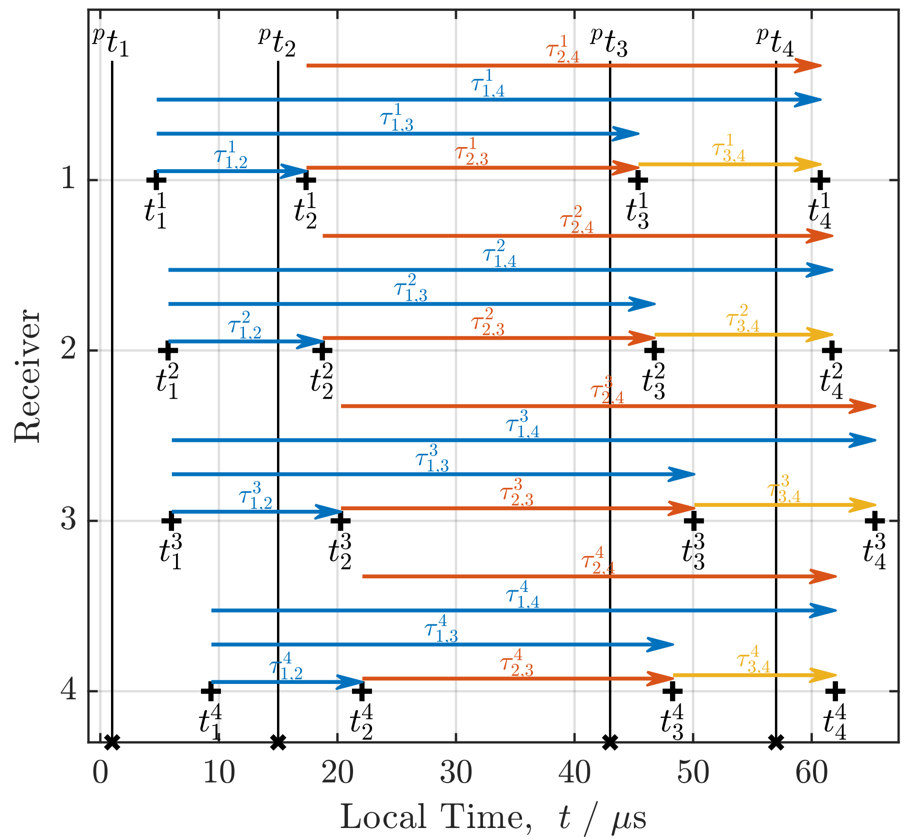
$$\frac{\left\| \underline{p}_k - \underline{s}^i \right\|}{c_0} + \left(t_l^1 - t_k^1\right) = {}^p\tau_{k,l} + \frac{\left\| \underline{p}_l - \underline{s}^i \right\|}{c_0}$$

novel

$$i \in \{1, 2, 3, 4\}$$
$$(k, l) \in \{(1,2), (1,3), (1,4)\}$$

ROTA $\mathcal{M}$ Transmissions L-TDOA





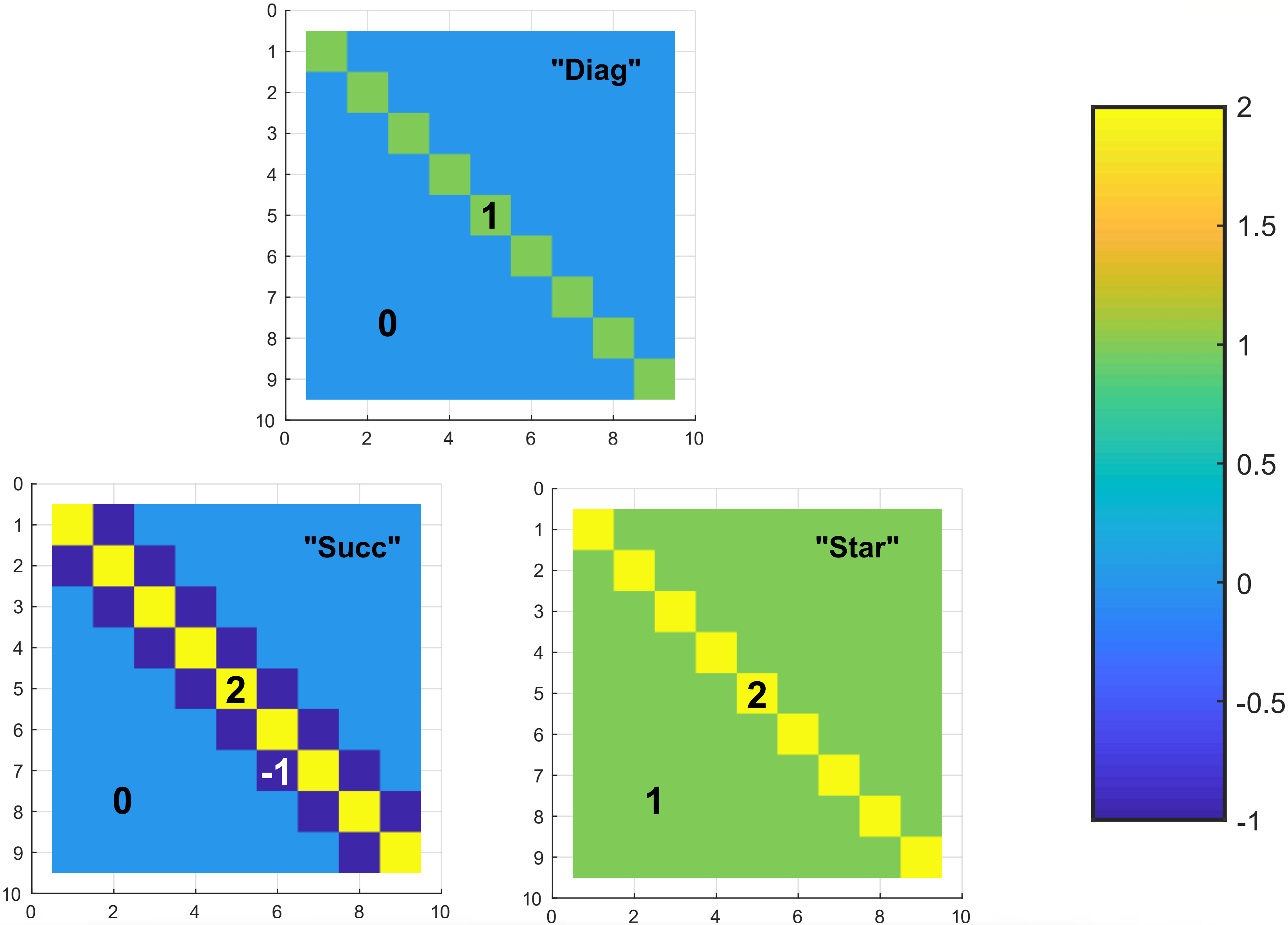
novel

$$\frac{\left\|\underline{p}_k - \underline{s}^i\right\|}{c_0} + \tau^1_{k,l} = {}^p\tau_{k,l} + \frac{\left\|\underline{p}_l - \underline{s}^i\right\|}{c_0}$$

$i \in \{1, 2, 3, 4\}$

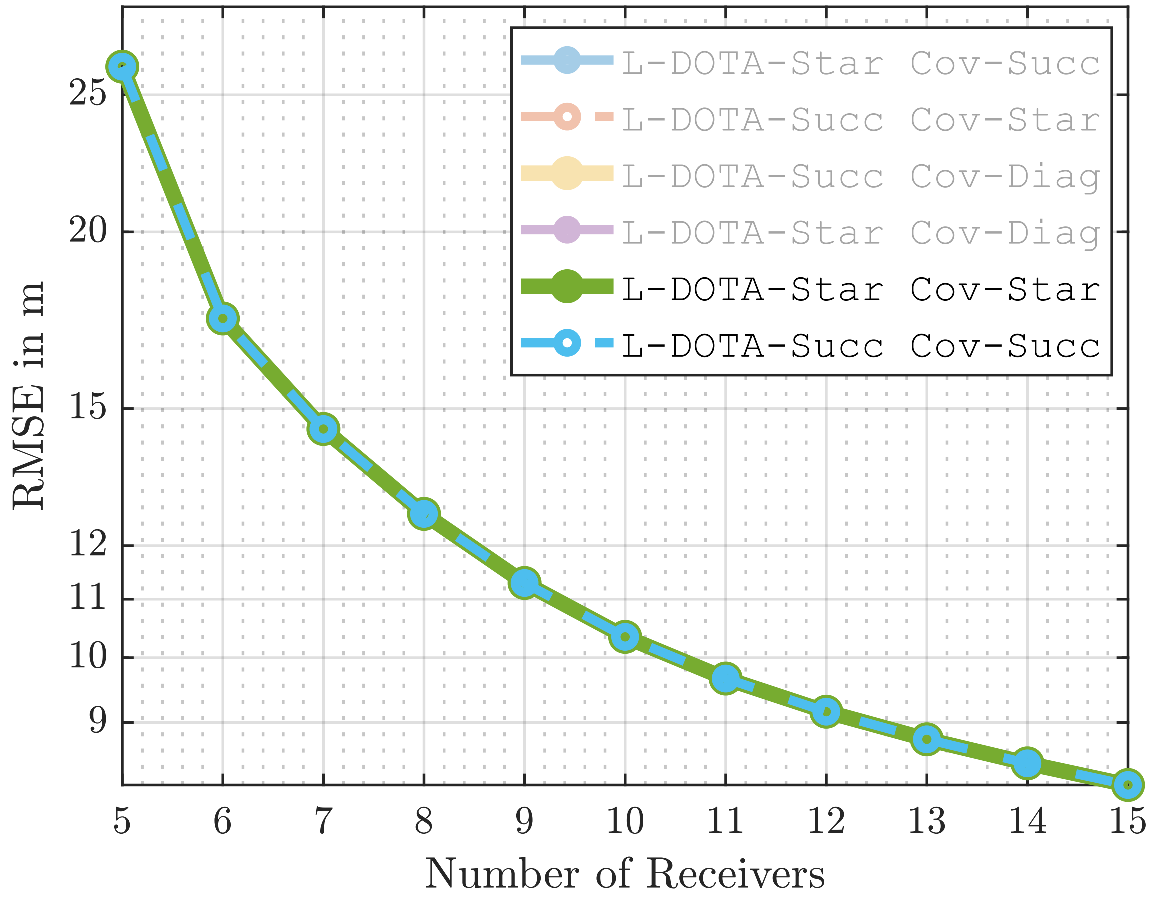
$(k, l) \in \{(1,2), (2,3), (3,4), (1,3), (1,4), (2,4)\}$

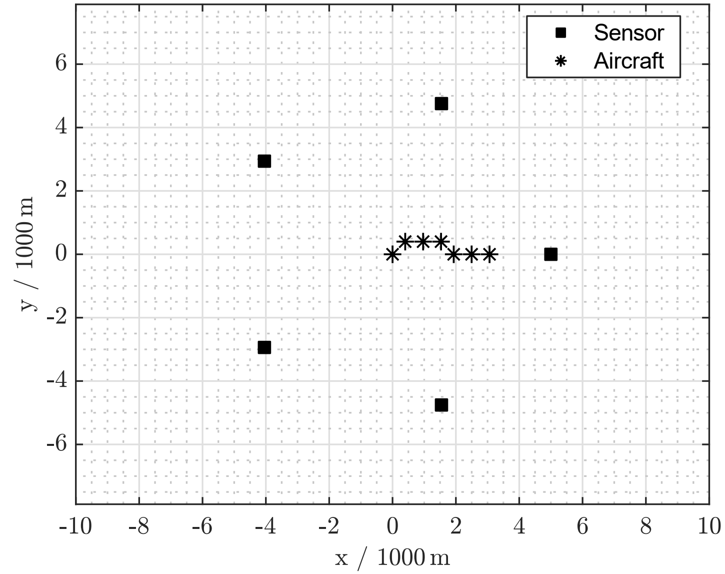
Covariance



Evaluation $\mathcal{M}$ Transmissions L-DOTA

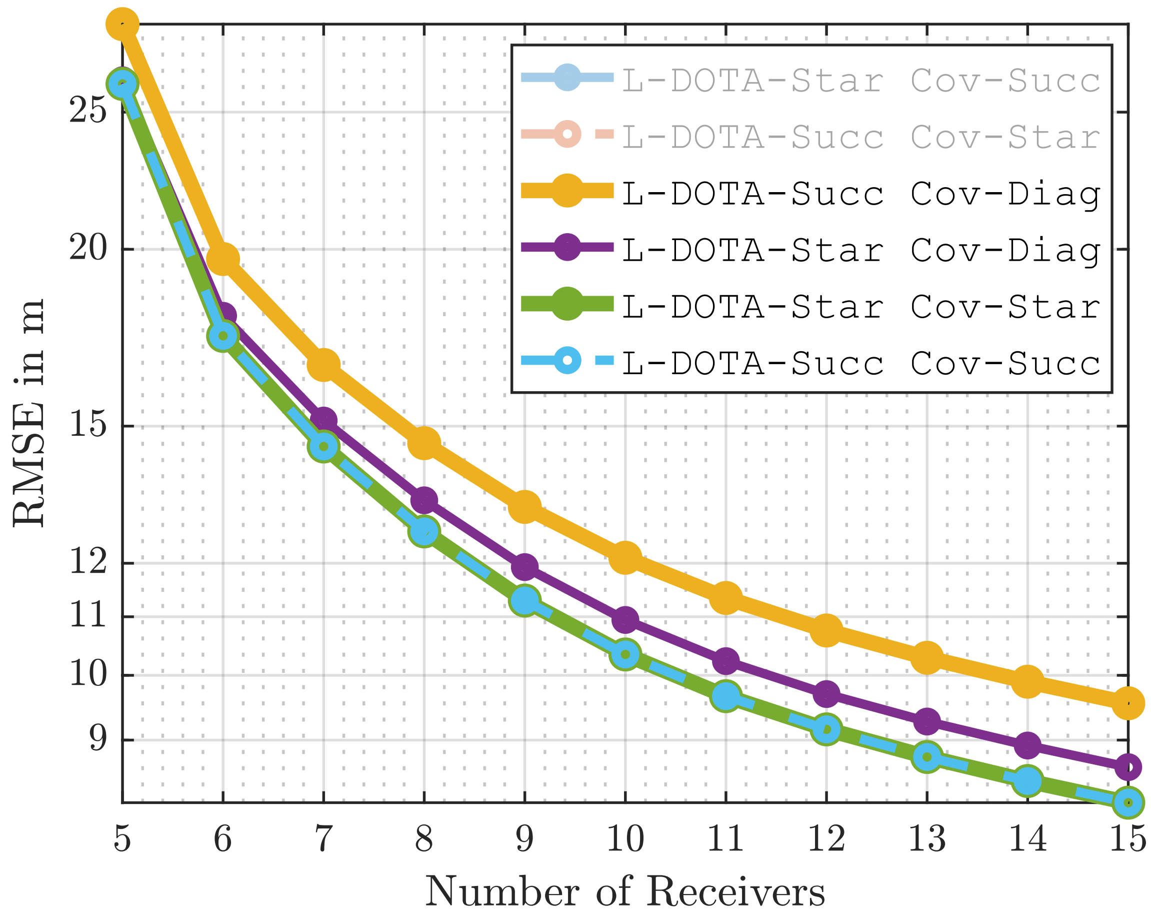
Maximum Likelihood Accuracy

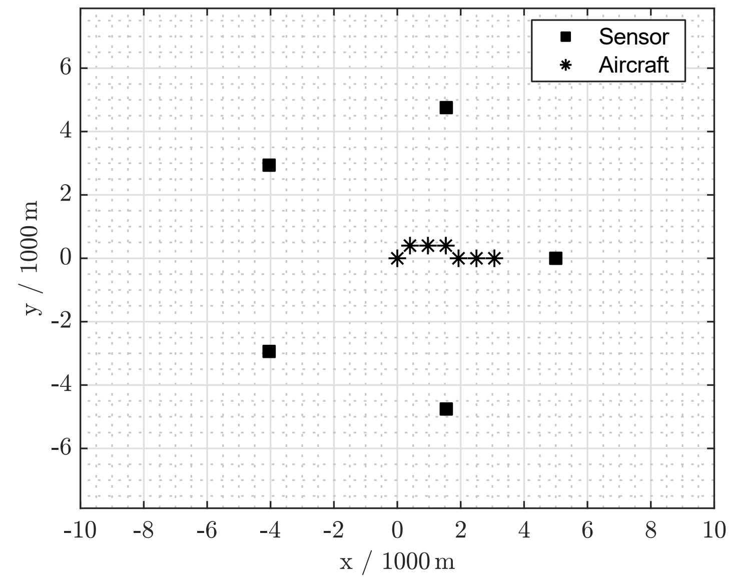




Evaluation $\mathcal{M}$ Transmissions L-DOTA

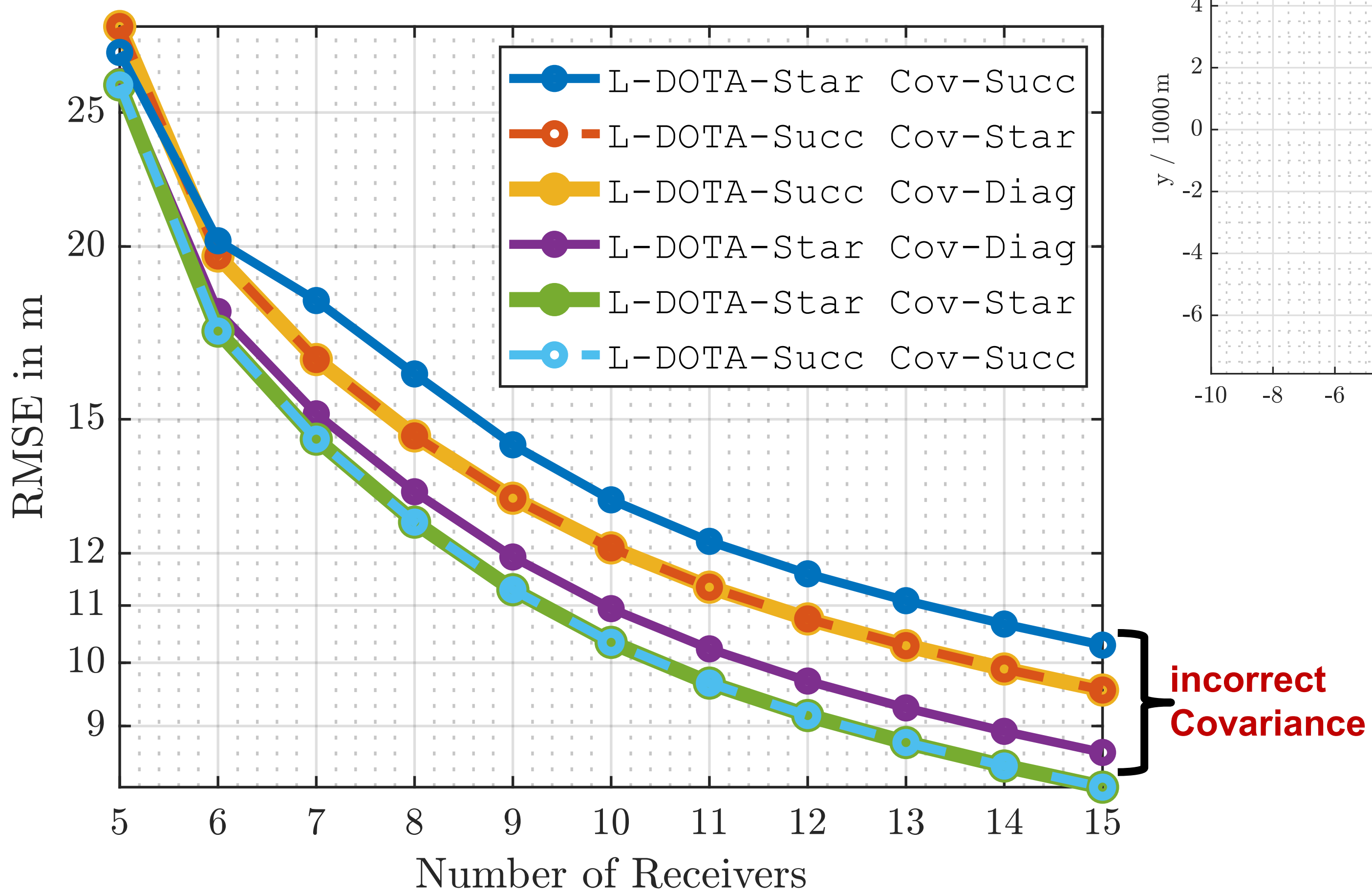
Maximum Likelihood Accuracy

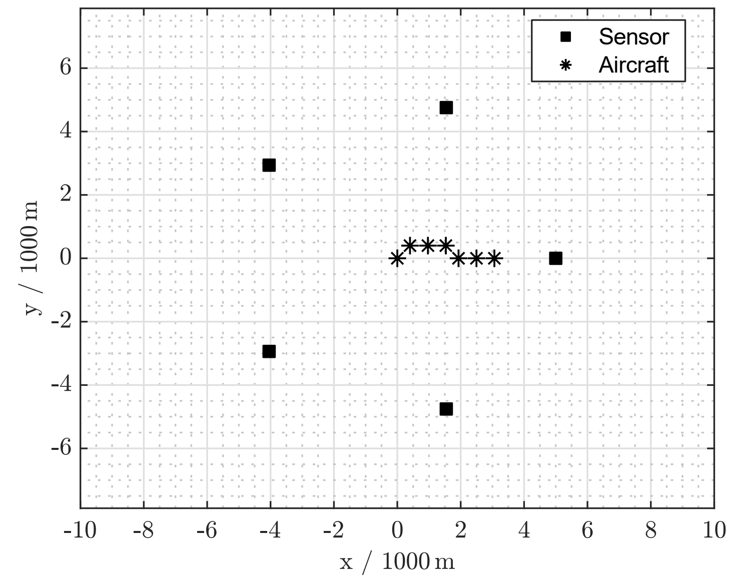




Evaluation $\mathcal{M}$ Transmissions L-DOTA

Maximum Likelihood Accuracy





Future Work

- Tracking over time, sliding window
- Multi target tracking
- Real data
- Realtime applicability
- Sensor distribution optimization

Thank you for your attention

Intelligent Sensor-Actuator-Systems

Announcement MFI 2020

Karlsruhe, Germany
September 2020

More information: www.mfi2020.org

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- Florian Pfaff, *Karlsruhe Institute of Technology*, florian.pfaff@kit.edu

Recap

- Primary Surveillance Radar (PSR)
- Secondary Surveillance Radar (SSR)
  Multilateration Algorithms
  - for Synchronized Receivers
  - for Synchronization-Free Receivers
    - Two Target Transmissions
    - Multiple Target Transmissions
      - Covariance

Likelihood Single Measurement

Deterministic Generative Model

$$\tau_{k,l}^{i} = {}^{p}\tau_{k,l} + \frac{\left\| \underline{p}_l - \underline{s}^i \right\| - \left\| \underline{p}_k - \underline{s}^i \right\|}{c_0}$$

$$\tau_{k,l}^{i} = h^i\left(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}\right)$$

AWGN

Uncertain Generative Model

$$\hat{\boldsymbol{\tau}}_{k,l}^{i} = h^i\left(\underline{\boldsymbol{p}}_k, \underline{\boldsymbol{p}}_l, {}^{p}\boldsymbol{\tau}_{k,l}\right) + \boldsymbol{v}_{k,l}^{i}$$

$$f_{k,l}^{v,i}(v) = \frac{1}{\sqrt{2\pi \cdot C_{k,l}^{v,i} \cdot \exp\left(v^2 / C_{k,l}^{v,i}\right)}}$$

Uncertain Probabilistic Model

$$f_{k,l}^{L,i}\left(\hat{\tau}_{k,l}^{i} \mid \underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}\right) = f_{k,l}^{v,i}\left(\hat{\tau}_{k,l}^{i} - h^i\left(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}\right)\right)$$

Likelihood Single Measurement

$$f_{k,l}^{L,i}\left(\hat{\tau}_{k,l}^{i} \mid \underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}\right) = f_{k,l}^{v,i}\left(\hat{\tau}_{k,l}^{i} - h^i\left(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}\right)\right)$$
$$= \mathcal{N}\left(\hat{\tau}_{k,l}^{i};\ h^i\left(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}\right);\ C_{k,l}^{v,i}\right)$$

Likelihood Two Transmissions

$$f_{k,l}^{L,1:\mathcal{R}}\left(\underline{\hat{\tau}}_{k,l}^{1:\mathcal{R}} \mid \underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}\right) = f_{k,l}^{\boldsymbol{v},1:\mathcal{R}}\left(\underline{\hat{\tau}}_{k,l}^{1:\mathcal{R}} - \underline{h}^{1:\mathcal{R}}\left(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}\right)\right)$$

$$= \mathcal{N}\left(\underline{\hat{\tau}}_{k,l}^{1:\mathcal{R}}; \underline{h}^{1:\mathcal{R}}\left(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}\right); \mathbf{C}_{k,l}^{\boldsymbol{v},1:\mathcal{R}}\right)$$

$$\underline{\hat{\tau}}_{k,l}^{1:\mathcal{R}} = \begin{bmatrix} \hat{\tau}_{k,l}^{1} \\ \hat{\tau}_{k,l}^{2} \\ \vdots \\ \hat{\tau}_{k,l}^{\mathcal{R}} \end{bmatrix} \qquad \underline{h}^{1:\mathcal{R}}\left(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}\right) = \begin{bmatrix} h^{1}\left(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}\right) \\ h^{2}\left(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}\right) \\ \vdots \\ h^{\mathcal{R}}\left(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}\right) \end{bmatrix}$$

$$\mathbf{C}_{k,l}^{\boldsymbol{v},1:\mathcal{R}} = \begin{bmatrix} C_{k,l}^{\boldsymbol{v},1} & 0 & \cdots & 0 \\ 0 & C_{k,l}^{\boldsymbol{v},2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & C_{k,l}^{\boldsymbol{v},\mathcal{R}} \end{bmatrix}$$

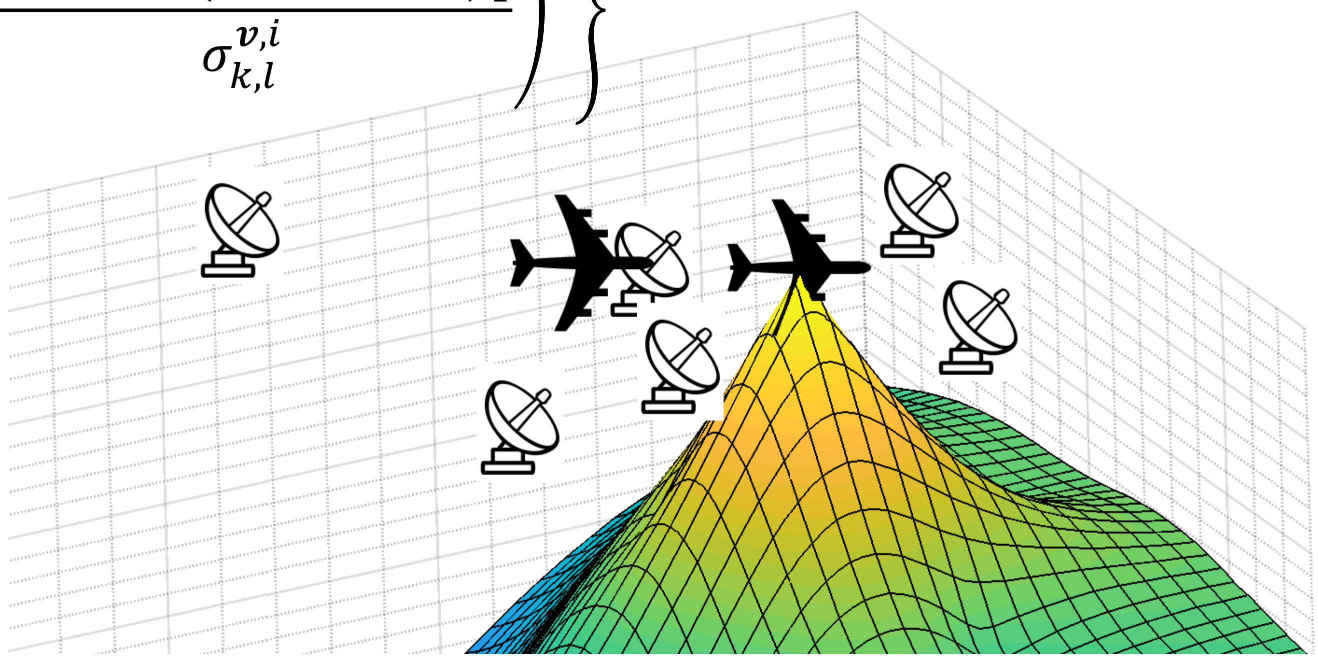
ML Estimation Two Transmissions

$$\left(\hat{\underline{p}}_k^{ML}, \hat{\underline{p}}_l^{ML}, {}^{p}\hat{\tau}_{k,l}^{ML}\right)$$

$$= \underset{\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}}{\arg\max} \left\{ \mathcal{N}\left(\underline{\hat{\tau}}_{k,l}^{1:\mathcal{R}};\ \underline{h}^{1:\mathcal{R}}\left(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}\right);\ \mathbf{C}_{k,l}^{v,1:\mathcal{R}}\right) \right\}$$

$$= \underset{\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}}{\arg\min} \left\{ \left[\underline{\hat{\tau}}_{k,l}^{1:\mathcal{R}} - \underline{h}^{1:\mathcal{R}}\left(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}\right)\right]^{\top} \left(\mathbf{C}_{k,l}^{v,1:\mathcal{R}}\right)^{-1} \left[\underline{\hat{\tau}}_{k,l}^{1:\mathcal{R}} - \underline{h}^{1:\mathcal{R}}\left(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}\right)\right] \right\}$$

$$= \underset{\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}}{\arg\min} \left\{ \sum_{i=1}^{\mathcal{R}} \left(\frac{\left[\hat{\tau}_{k,l}^{i} - h^{i}\left(\underline{p}_k, \underline{p}_l, {}^{p}\tau_{k,l}\right)\right]}{\sigma_{k,l}^{v,i}} \right)^2 \right\}$$



ML Estimation $\mathcal{M}$ Transmissions L-TDOD

$$\underline{\hat{p}}_1^{ML}, \underline{\hat{p}}_2^{ML}, \dots, \underline{\hat{p}}_{\mathcal{M}}^{ML}, (\; (\quad {}^{p}) \hat{\tau}_{1,2}^{ML}, ({}^{p} \; \hat{\tau}_{2,3}^{ML} \; , \dots, ({}^{p} \; \hat{\tau}_{\mathcal{M}-1,\mathcal{M}}^{ML}$$

$$= \underset{\dots}{\arg\min} \left\{ \sum_{i=1}^{\mathcal{R}} \left[\underline{\hat{\tau}}_{1:\mathcal{M}-1}^{i} - \underline{h}_{1:\mathcal{M}-1}^{i}(\dots) \right]^{\top} \left(\mathbf{C}_{1:\mathcal{M}-1}^{v,i} \right)^{-1} \left[\underline{\hat{\tau}}_{1:\mathcal{M}-1}^{i} - \underline{h}_{1:\mathcal{M}-1}^{i}(\dots) \right] \right\}$$

$$\underline{\hat{\tau}}_{1:\mathcal{M}-1}^{i} = \begin{bmatrix} \hat{\tau}_{k_1,l_1}^{i} \\ \hat{\tau}_{k_2,l_2}^{i} \\ \vdots \\ \hat{\tau}_{k_{\mathcal{M}-1},l_{\mathcal{M}-1}}^{\mathcal{R}} \end{bmatrix} \qquad \underline{h}_{1:\mathcal{M}-1}^{i}(\dots) = \begin{bmatrix} h^i\left(\underline{p}_{k_1}, \underline{p}_{l_1}, {}^{p}\tau_{k_1,l_1}\right) \\ h^i\left(\underline{p}_{k_2}, \underline{p}_{l_2}, {}^{p}\tau_{k_2,l_2}\right) \\ \vdots \\ h^i\left(\underline{p}_{k_{\mathcal{M}-1}}, \underline{p}_{l_{\mathcal{M}-1}}, {}^{p}\tau_{k_{\mathcal{M}-1},l_{\mathcal{M}-1}}\right) \end{bmatrix}$$

$$\mathbf{C}_{1:\mathcal{M}-1}^{v,i,\text{"Succ"}} = C^{v,i} \cdot \begin{bmatrix} 2 & -1 & & & 0 \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ 0 & & & -1 & 2 \end{bmatrix} \qquad \mathbf{C}_{1:\mathcal{M}-1}^{v,i,\text{"Star"}} = C^{v,i} \cdot \begin{bmatrix} 2 & 1 & & & 1 \\ 1 & 2 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 2 & 1 \\ 1 & & & 1 & 2 \end{bmatrix}$$

Weighted Least Squares

- Levenberg-Marquard algorithm superior performance
- Transform to sum of squares

$$\mathbf{C}^{-1} = \mathbf{R}^\top \mathbf{R}$$

$$\underline{\xi}^\top \mathbf{C}^{-1} \underline{\xi} = \underline{\xi}^\top \mathbf{R}^\top \mathbf{R}\, \underline{\xi}$$

$$= \left(\mathbf{R}\, \underline{\xi}\right)^\top \left(\mathbf{R}\, \underline{\xi}\right)$$

$$= \sum_m \left(\left[\mathbf{R}\, \underline{\xi}\right]_m\right)^2$$

Covariance

$$\hat{\boldsymbol{t}}_k = \boldsymbol{t}_k + \boldsymbol{v}_k$$

$$\hat{\boldsymbol{\tau}}_{k,l} = \underbrace{\boldsymbol{t}_l - \boldsymbol{t}_k}_{\boldsymbol{\tau}_{k,l}} + \underbrace{\boldsymbol{v}_l - \boldsymbol{v}_k}_{\boldsymbol{v}_{k,l}}$$

$$\mathrm{Cov}\{\boldsymbol{v}_{k_1,l_1}, \boldsymbol{v}_{k_2,l_2}\}$$
$$= E\{v_{l_1}v_{l_2} - v_{l_1}v_{k_2} - v_{k_1}v_{l_2} + v_{k_1}v_{k_2}\}$$