Efficient Deterministic Conditional Sampling of Multivariate Gaussian Densities

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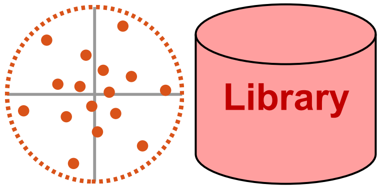
Intelligent Sensor-Actuator-Systems

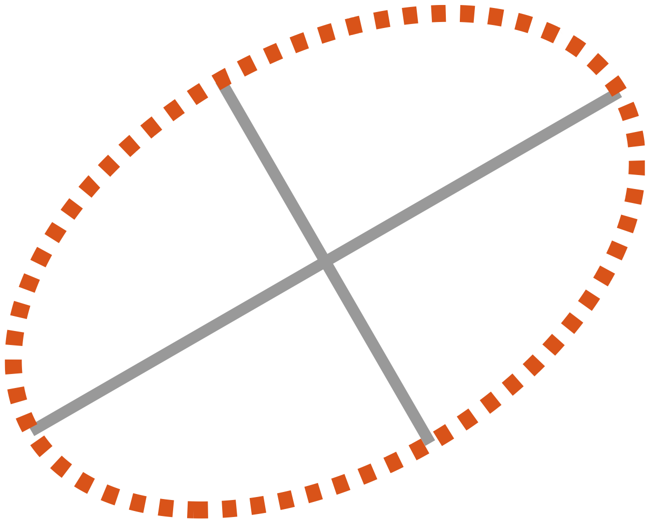
Overview

Inputs
- #Samples
- Covariance
- Mean

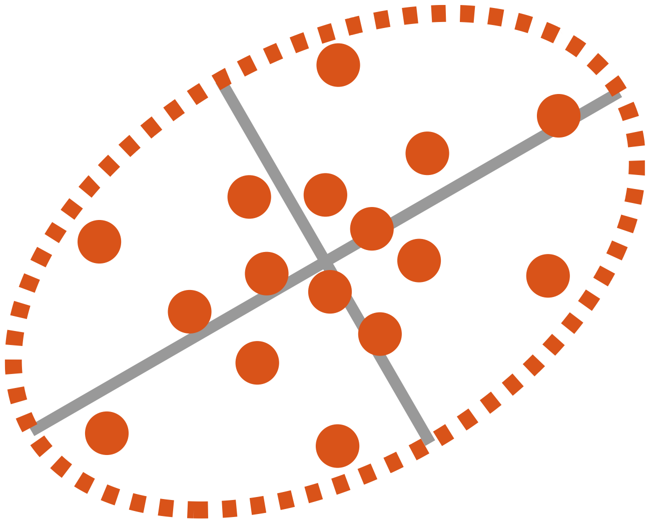
→ **Conditional Sampling** →



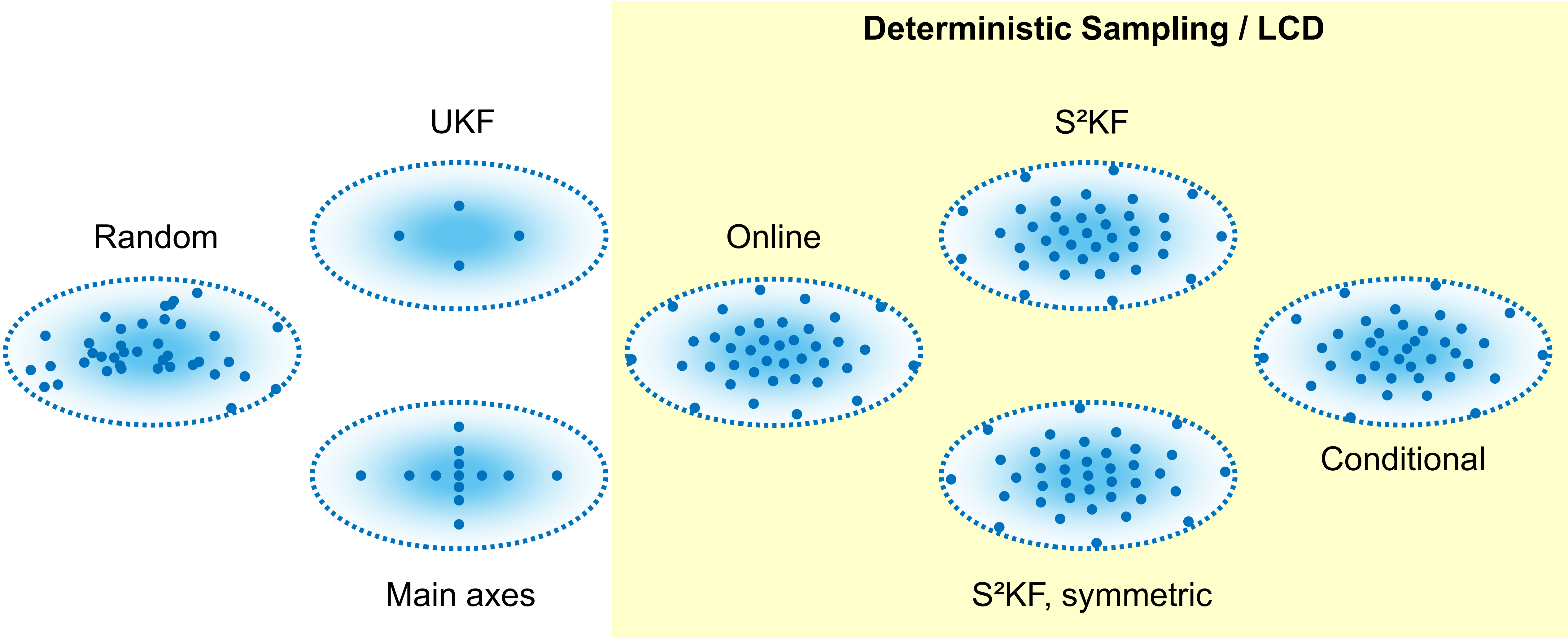
Output
Samples



Methods
- Conditional Sampling
- Distance Measure
  - Localized Cumulative Distribution (LCD)
  - Modified Cramér-von Mises Distance
- Covariance Adaptation



Gaussian Sampling – History



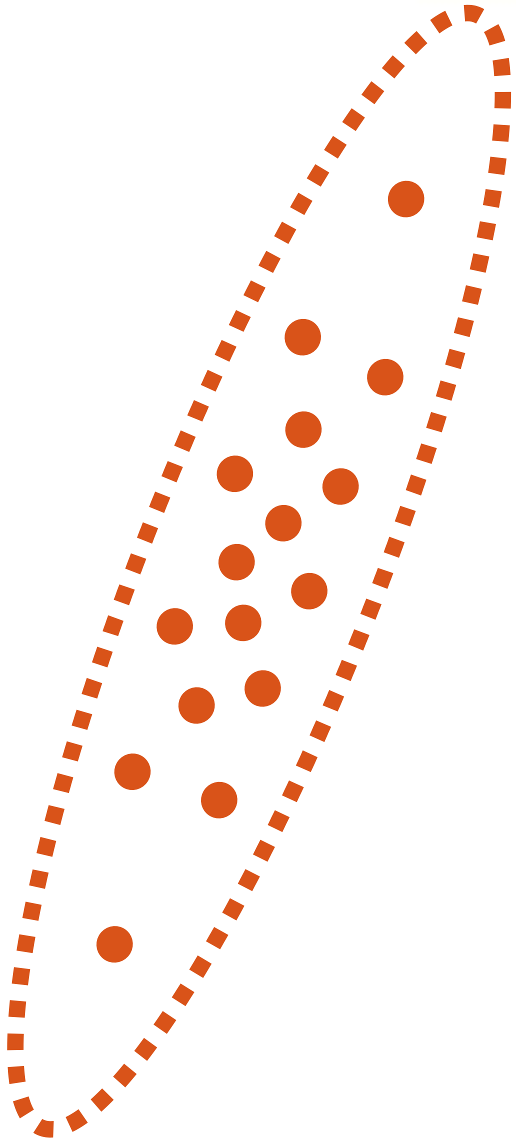
$$F(\underline{m}, b) = \int_{\mathbb{R}^n} f(\underline{x})\, K(\underline{x}, \underline{m}, b)\, \mathrm{d}\underline{x}$$

LCD

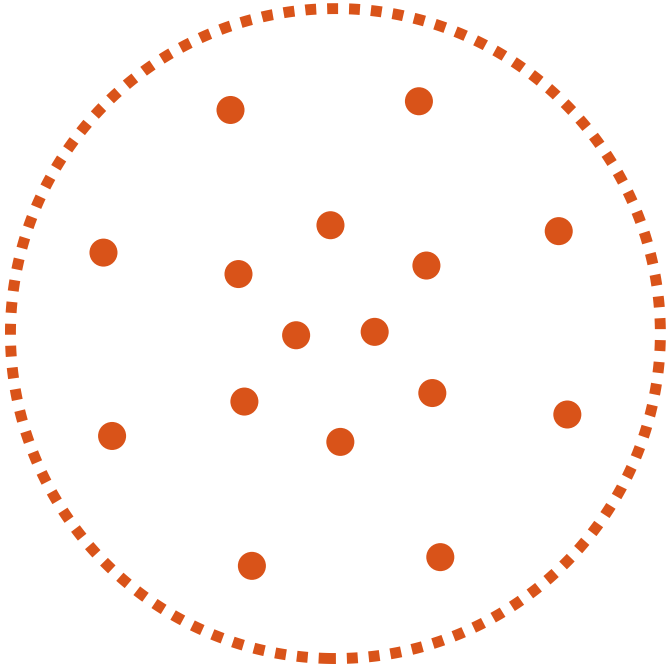
$$D_n^2 = \int_{\mathbb{R}^+} w(b) \int_{\mathbb{R}^n} \left(\tilde{F}(\underline{m}, b) - F(\underline{m}, b)\right)^2 \mathrm{d}\underline{m}\, \mathrm{d}b$$

Distance measure for comparing all types of densities:

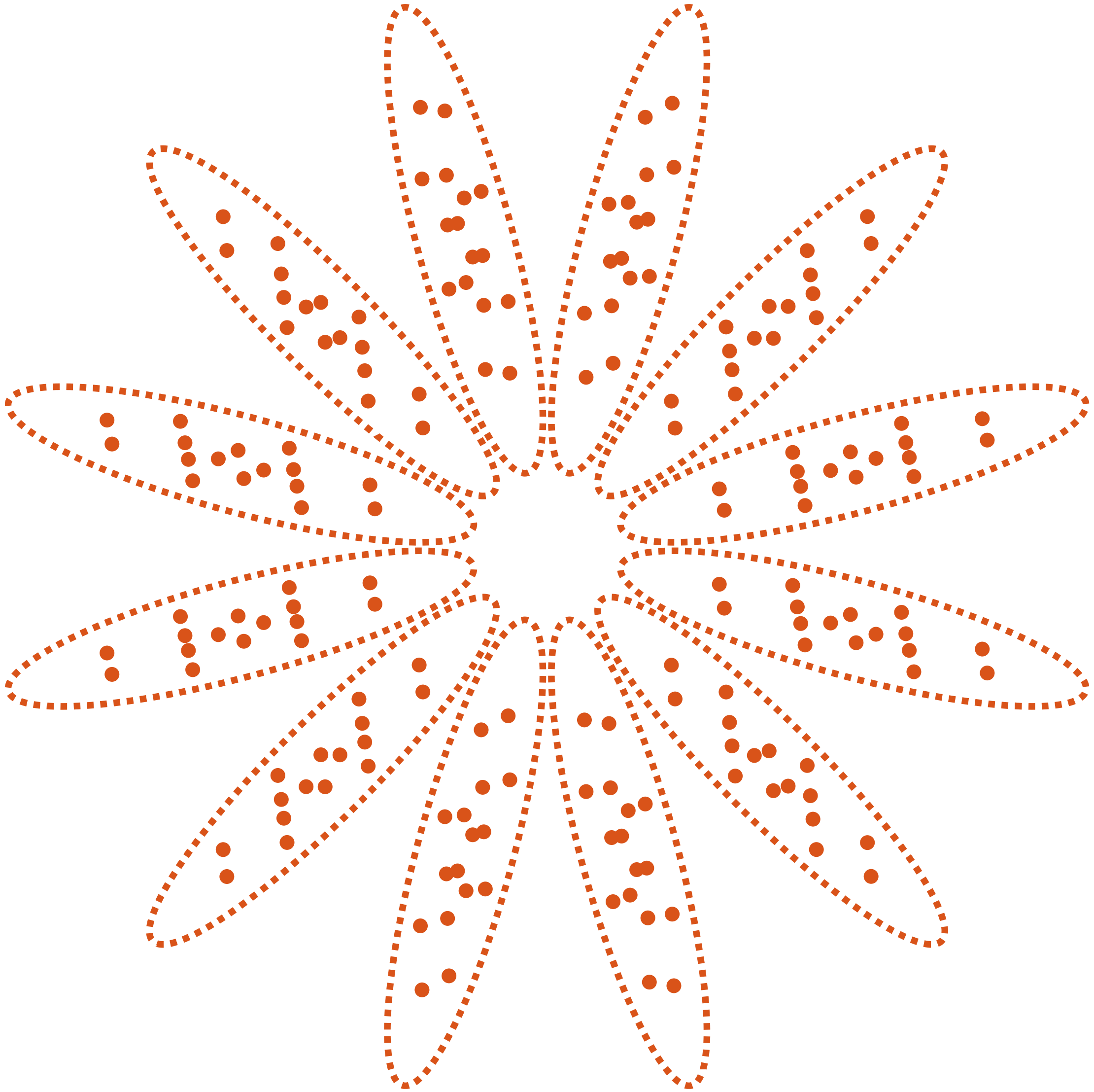
- discrete vs discrete
- **discrete vs continuous**
- continuous vs continuous



State of Art



$$\mathrm{chol}(\mathbf{C})\,\underline{x} - \underline{\mu}$$

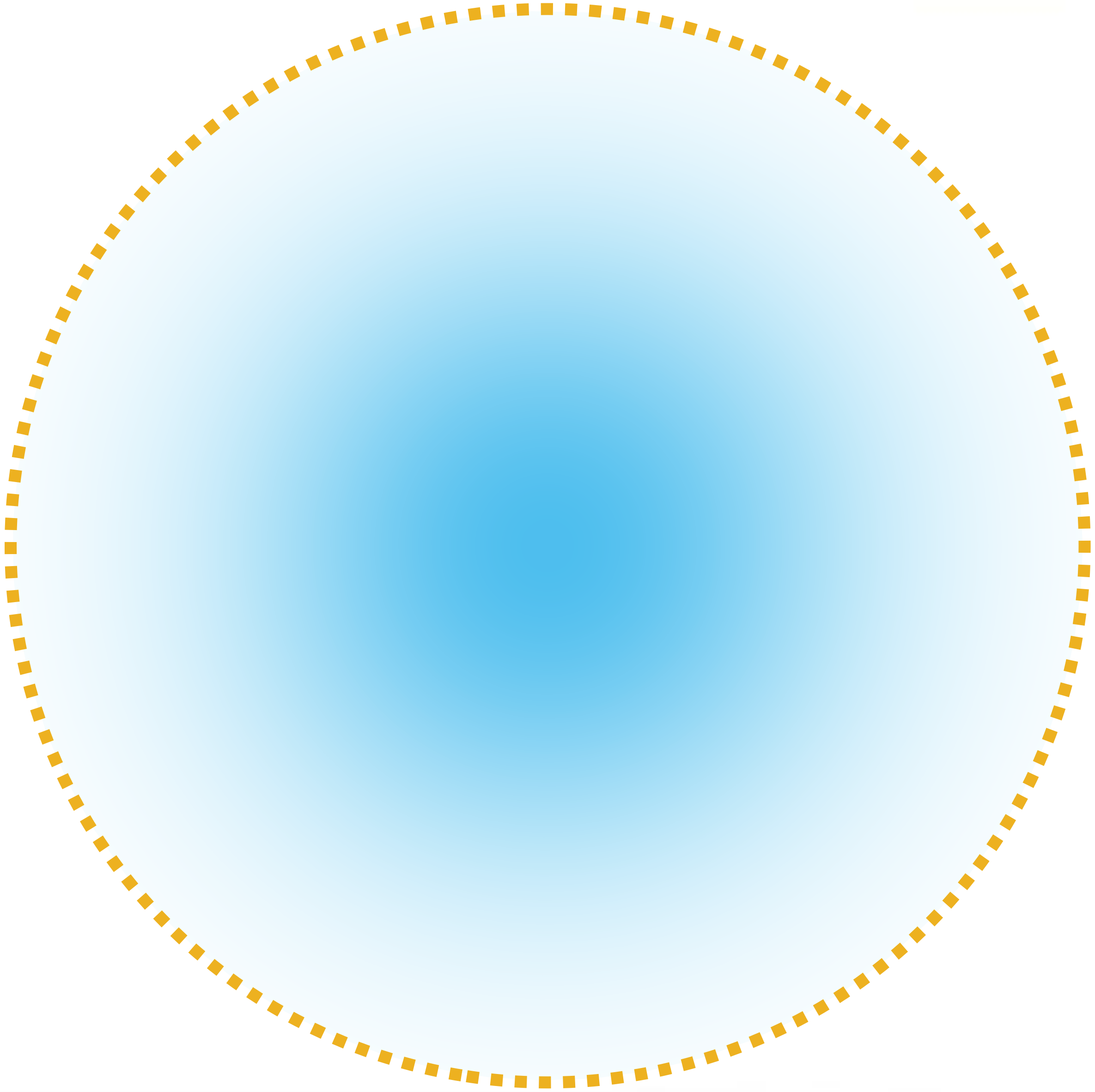


Fast realtime application

- Offline library
- Online linear adaption
- No preferential direction for scaling

Conditional Sampling (proposed) – Offline

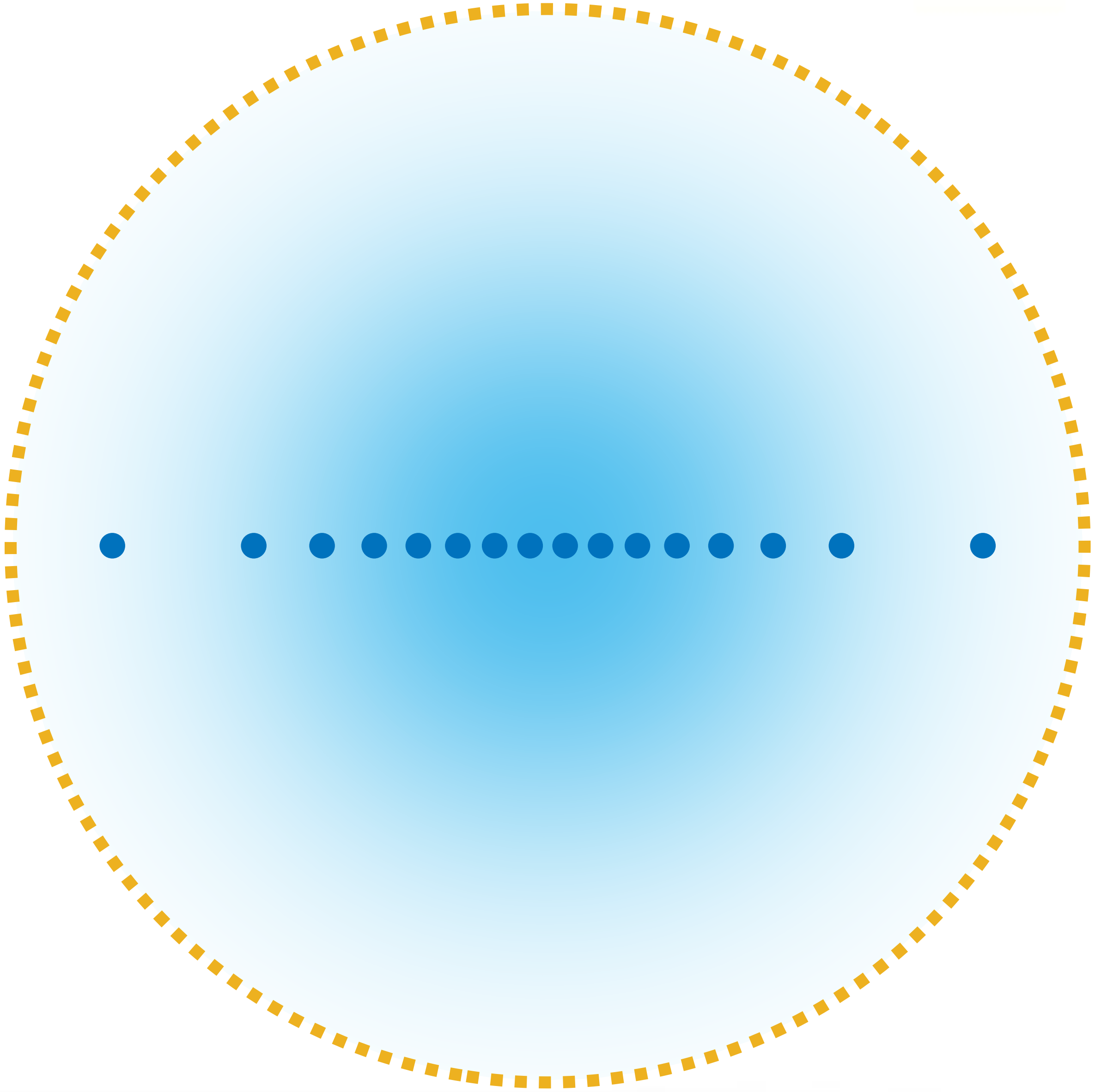
Standard normal samples (offline)



Conditional Sampling (proposed) – Offline 1D

Standard normal samples (offline)

- Start with 1D marginal
  - x coordinates
    $$x^{\mathrm{s}}_{1:L} = \underset{x_{1:L}}{\arg\min} \Big\{ D_1\big({}^{\mathrm{G}}f^{\mathrm{s}}_1(x), {}^{\mathrm{DM}}f^{\mathrm{s}}_1(x)\big) \Big\}$$

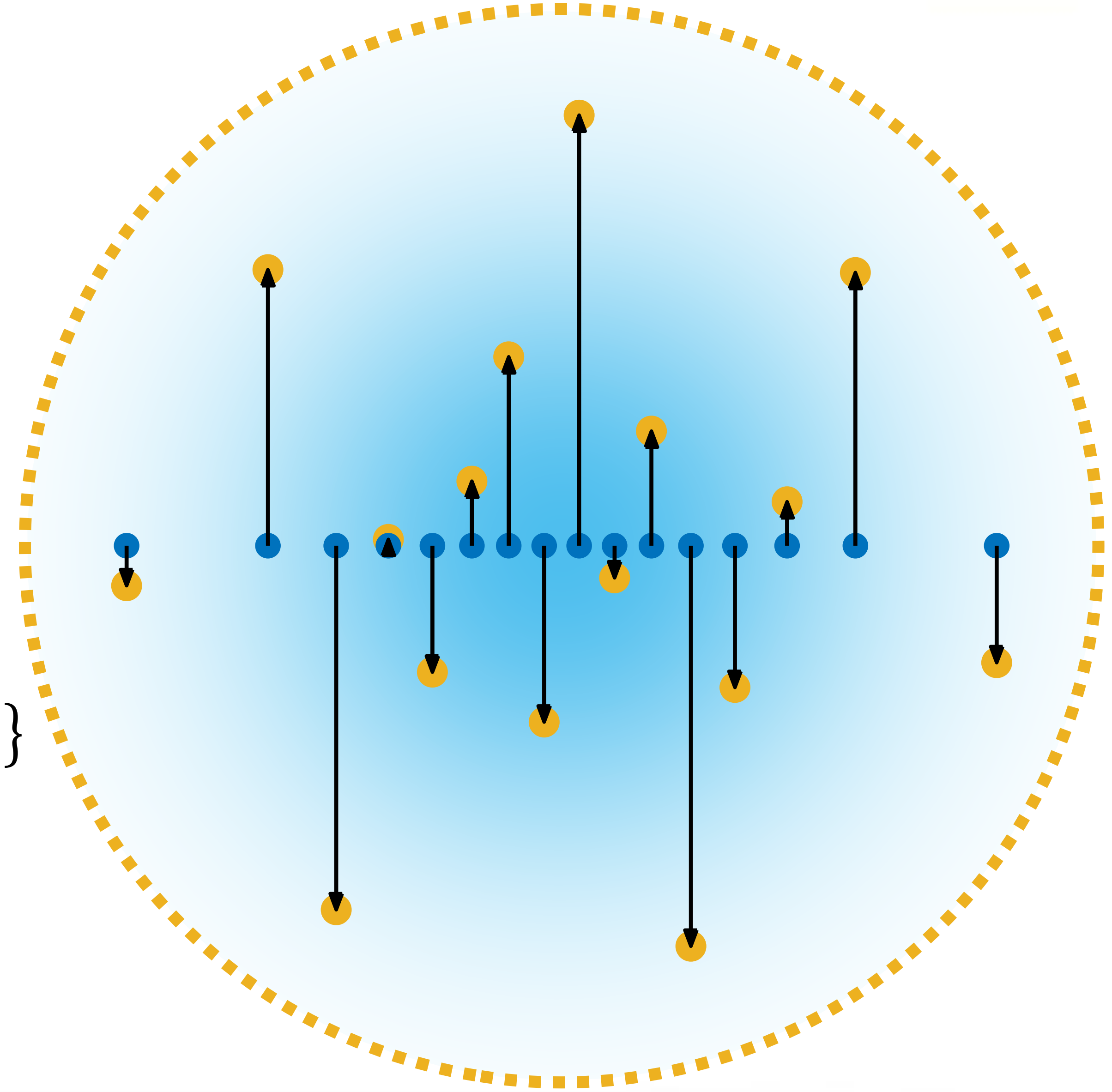


Standard normal samples (offline)

- Start with 1D marginal
  - x coordinates

    $$x_{1:L}^{s} = \underset{x_{1:L}}{\arg\min} \left\{ D_1\left({}^{\mathrm{G}}f_1^{s}(x), {}^{\mathrm{DM}}f_1^{s}(x) \right) \right\}$$
- Augment 2nd dimension
  - y coordinates

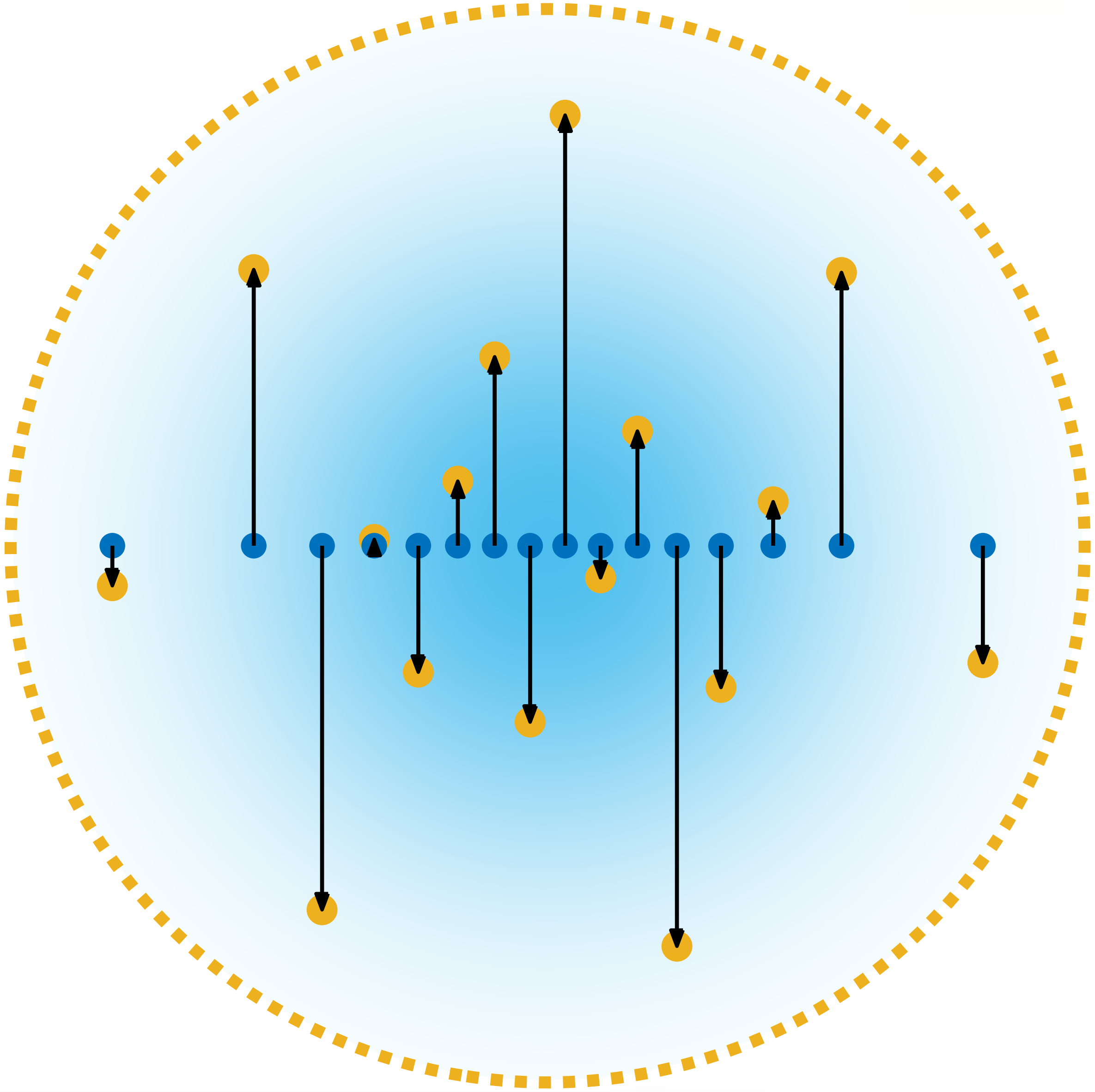
    $$y_{1:L}^{s} = \underset{y_{1:L}}{\arg\min} \left\{ D_2\left({}^{\mathrm{G}}f_2^{s}(x,y), {}^{\mathrm{DM}}f_2^{s}(x,y) \right) \right\}$$
- Augment 3rd dimension…



$$\mathbf{C} = \mathbf{R}\boldsymbol{\Sigma}\mathbf{R}^\top \qquad \sqrt{\boldsymbol{\Sigma}}\,\underline{x}_i$$

Linear adaption (online)

- Anisotropic scaling
  - Eigenvalues of Cov
  - Along preferential axis
  - Avoiding "lumps"



$\mathbf{C} = \mathbf{R}\boldsymbol{\Sigma}\mathbf{R}^\top$ $\sqrt{\boldsymbol{\Sigma}}\,\underline{x}_i$

Linear adaption (online)

- Anisotropic scaling
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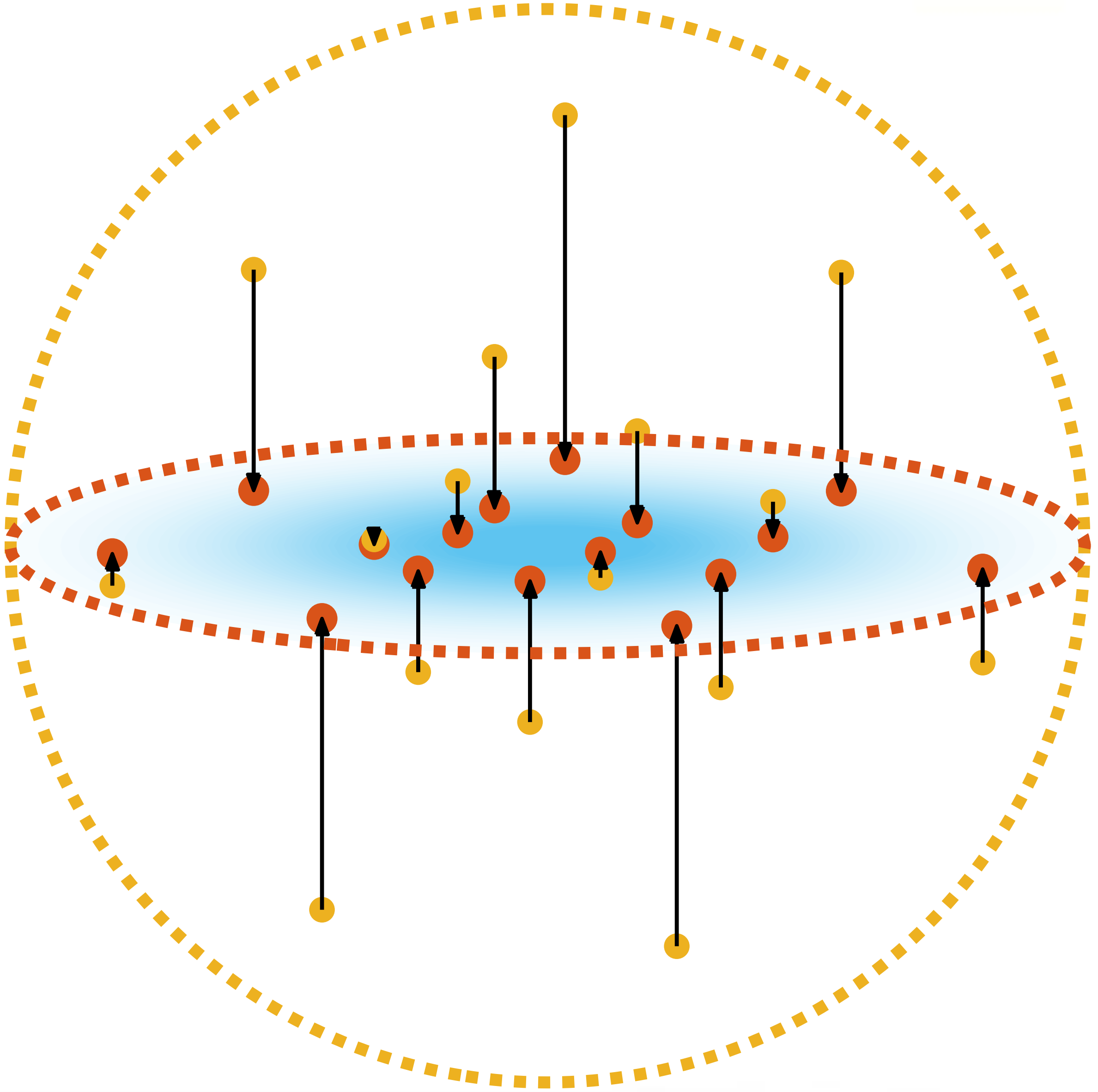
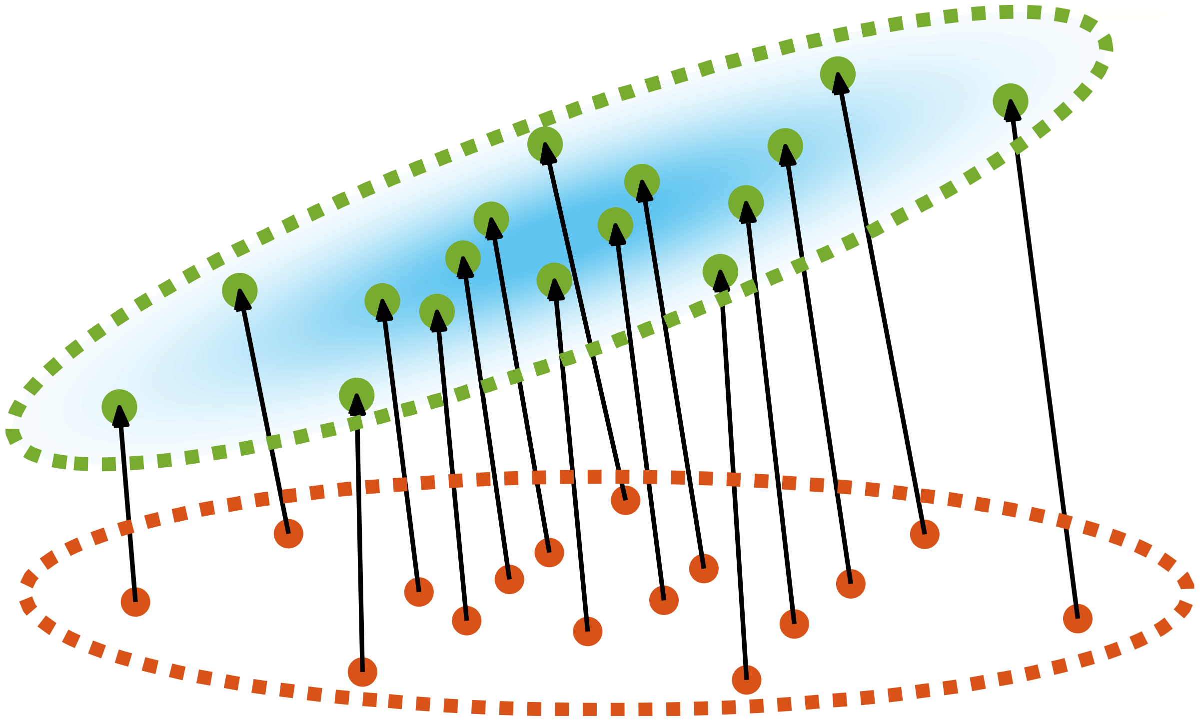


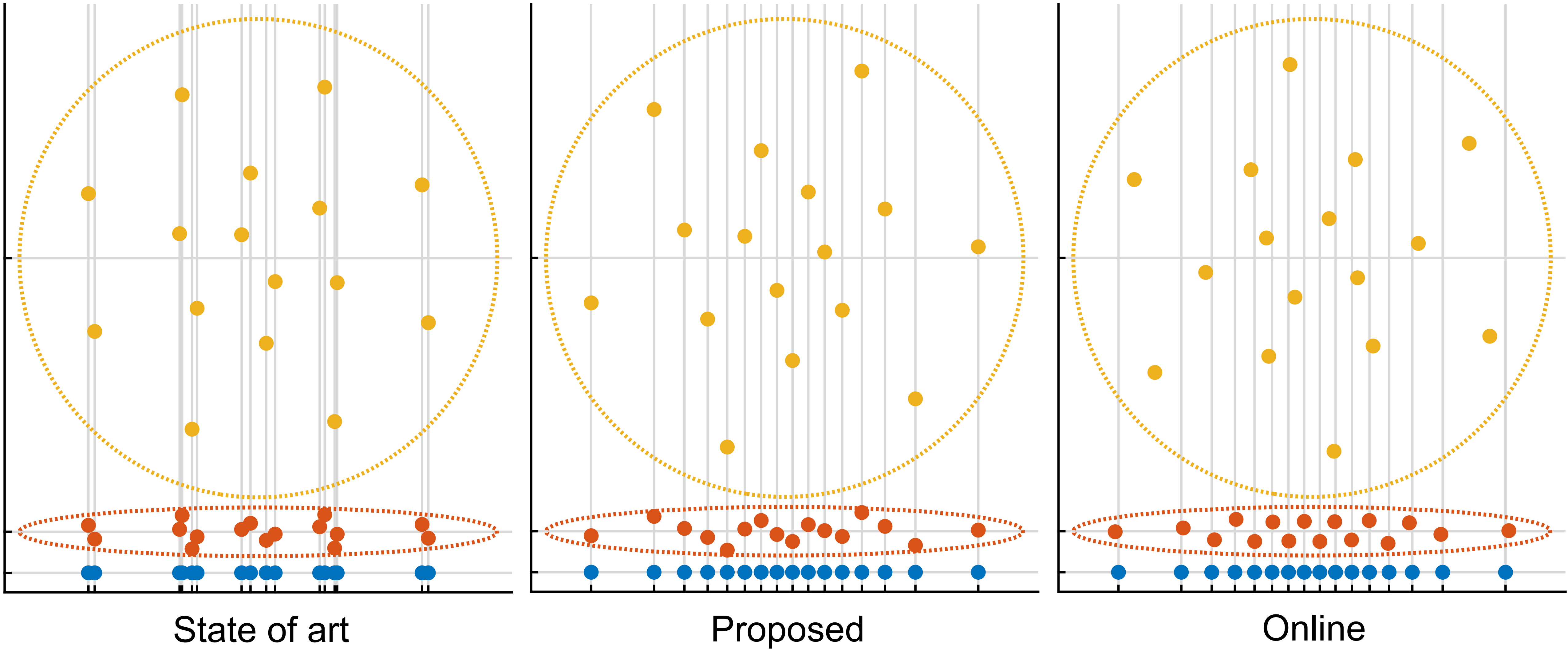
$$\mathbf{C} = \mathbf{R}\mathbf{\Sigma}\mathbf{R}^\top \qquad \mathbf{R}\sqrt{\mathbf{\Sigma}}\,\underline{x}_i + \underline{\mu}$$



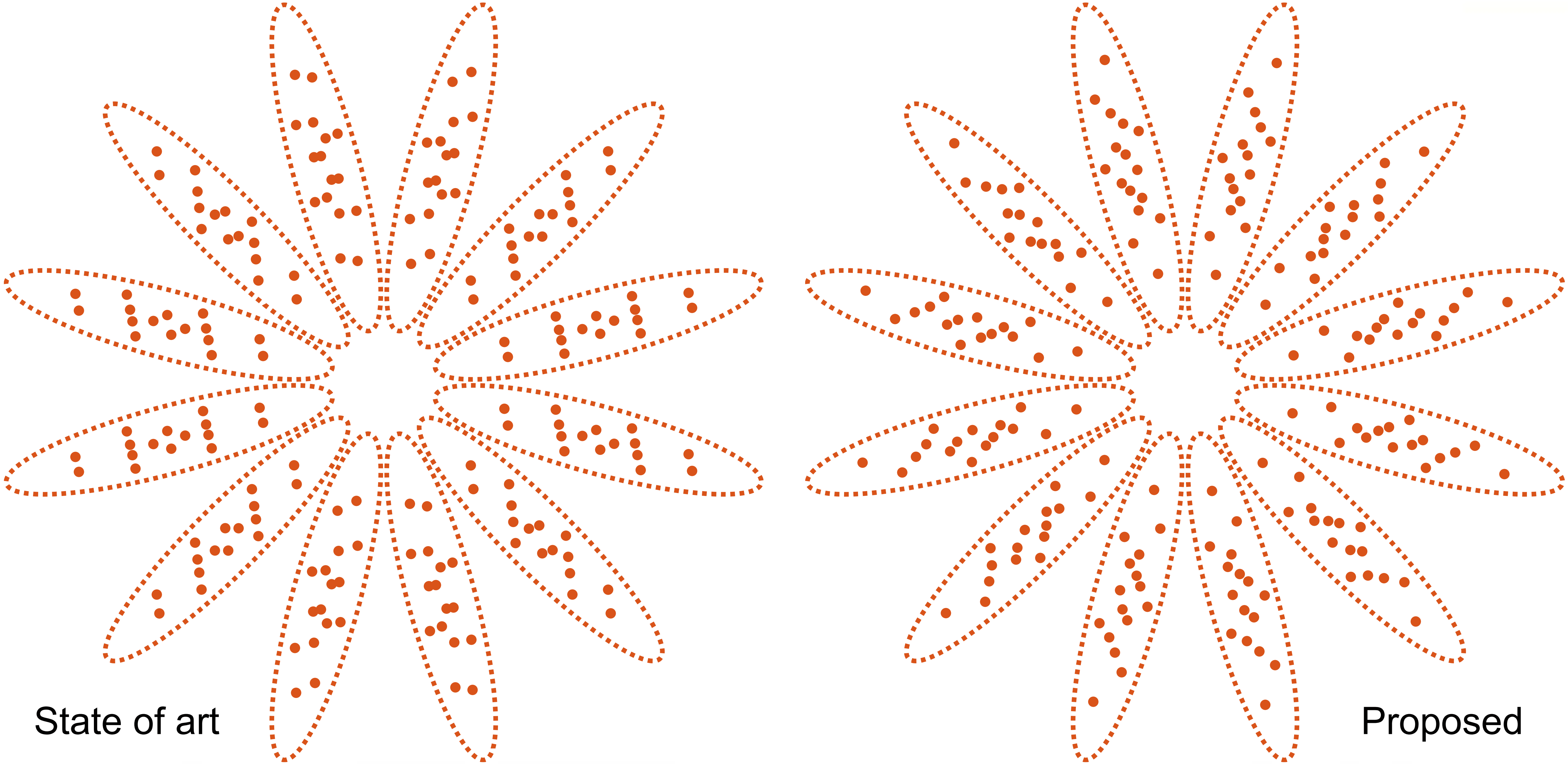
Linear adaption (online)

- Anisotropic scaling
  - Along preferential axis
  - Avoiding “lumps”
  - Eigenvalues of Cov
- Rigid transformation
  - Rotation: eigenvectors of Cov
  - Translation: mean

Evaluation 1



Evaluation 2



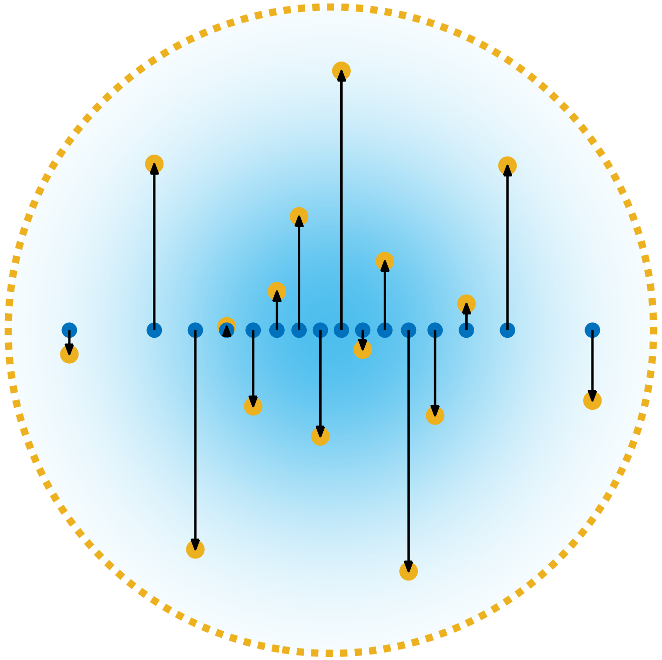
Conclusion

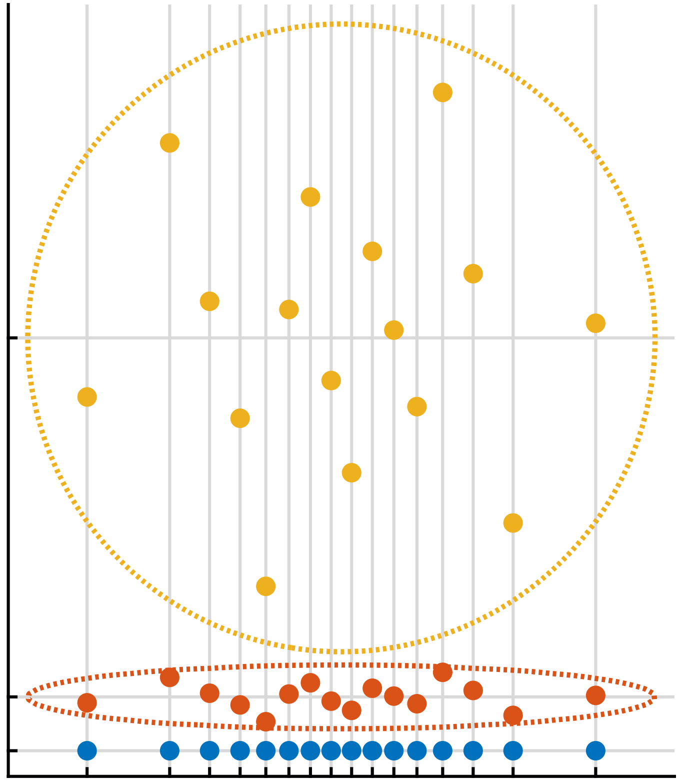
Pros

- Fast
- Better for „narrow“ Cov
- Library smaller

Cons

- Need eigenvalues and -vectors of Cov
- Inferior for standard normal Cov





Outlook

- Select from both variants
- Explore higher dimensions
- Use in S²KF, PGF, etc.

Thank you for your attention

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