Optimal Sensor Placement for Multilateration Using Alternating Greedy Removal and Placement

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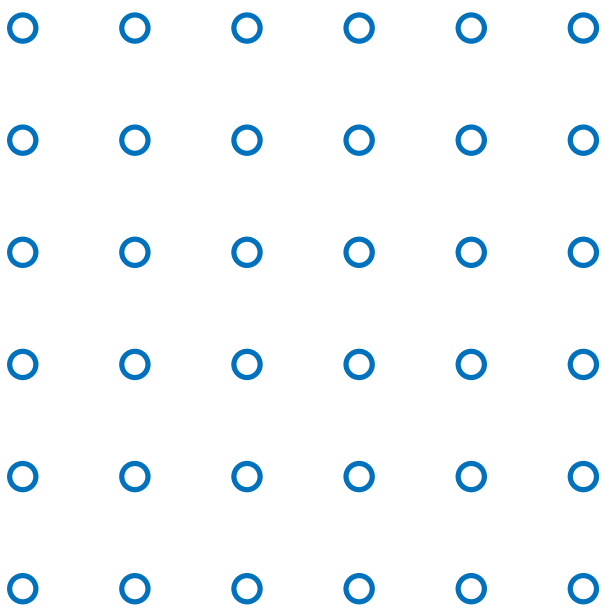
Intelligent Sensor-Actuator-Systems

Overview

Input

Multilateration Setup

- All satellite (sensor) locations



Overview

Input

Multilateration Setup

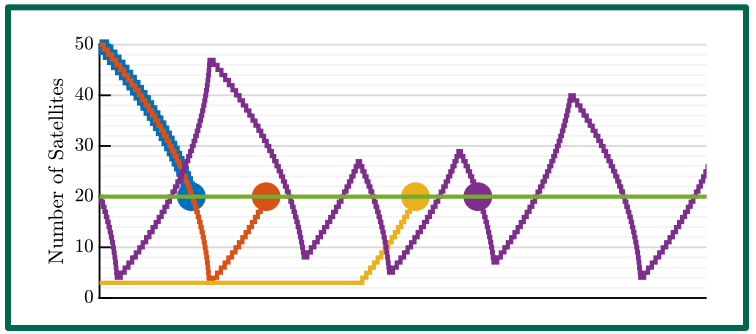
- All satellite (sensor) locations
- Coverage area

AGREP

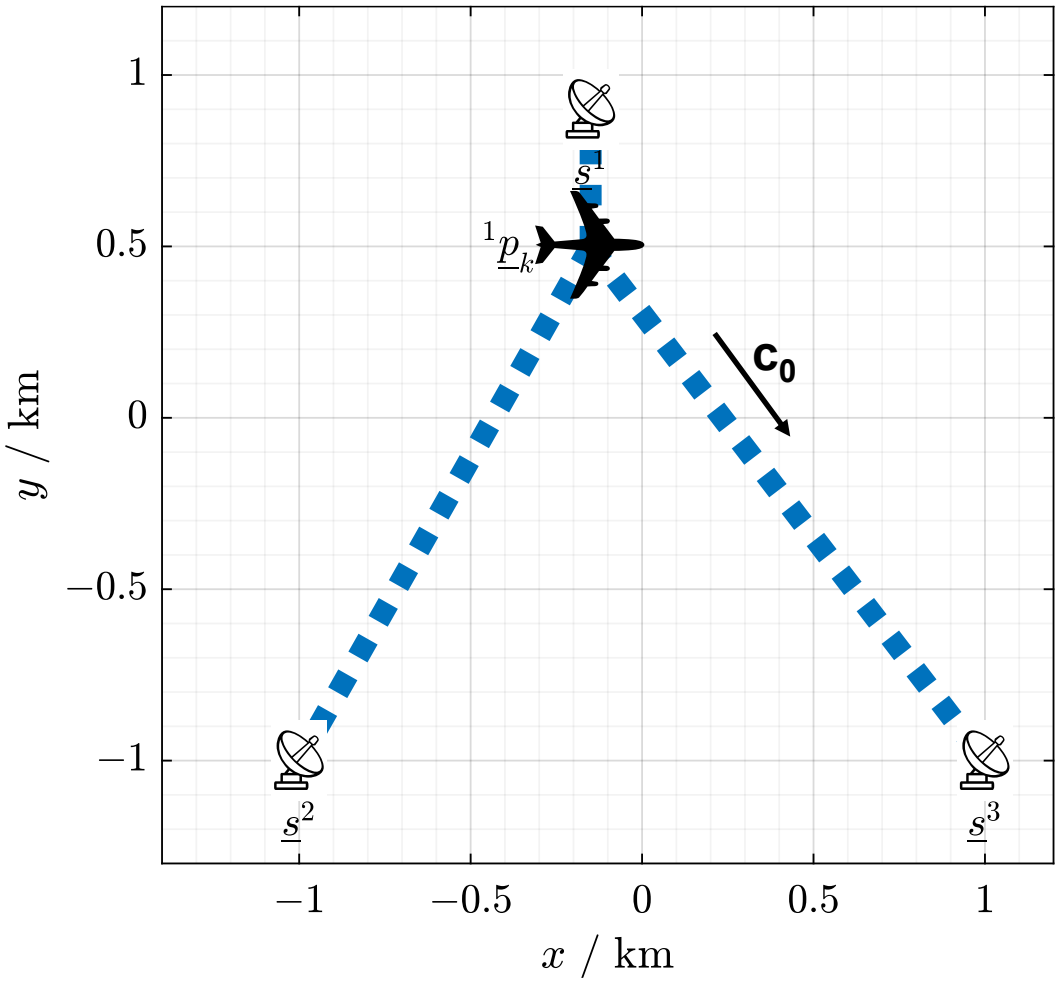
Output

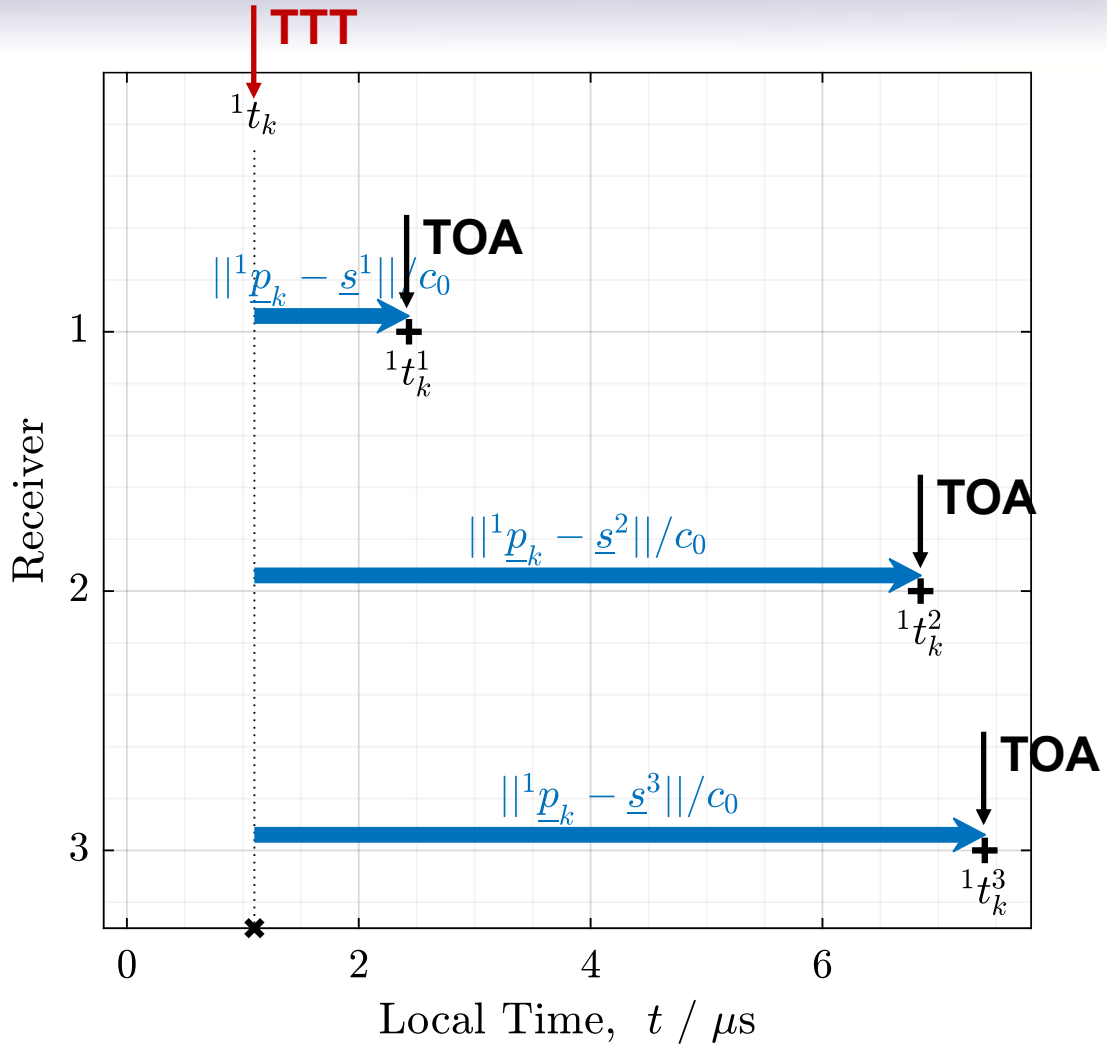
Best Sensor Locations

- Selected satellite locations
- Coverage requirements fulfilled



Multilateration

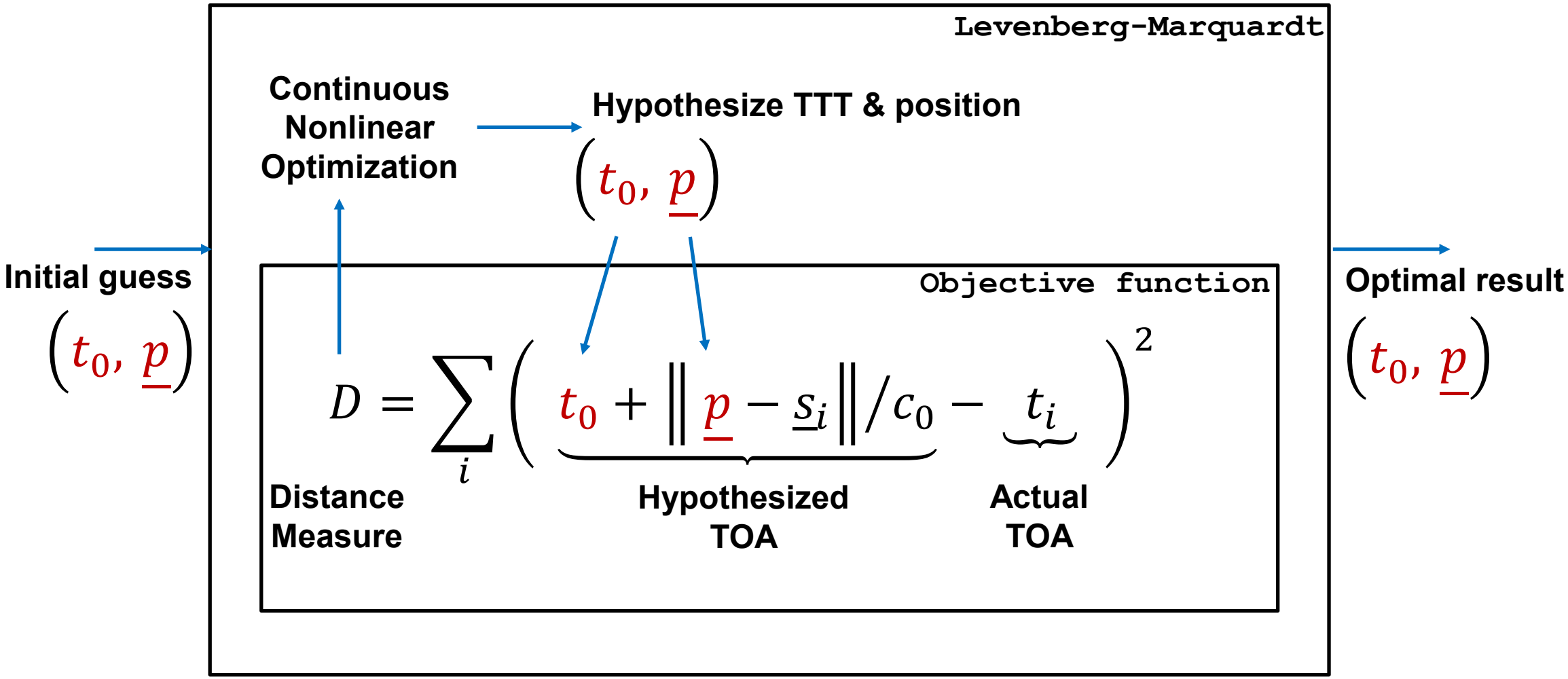




$$t_i = t_0 + \left\| \underline{p} - \underline{s}_i \right\| / c_0 \ , \qquad i \in \{1, 2, 3\}$$

+ Nonlinear Equation System +

Solve NLE


Levenberg-Marquardt
Continuous Nonlinear Optimization
Hypothesize TTT & position
$(t_0, \underline{p})$
Initial guess
$(t_0, \underline{p})$
Objective function
$D = \sum_i \left(t_0 + \| \underline{p} - \underline{s}_i \| / c_0 - t_i \right)^2$
Distance Measure
Hypothesized TOA
Actual TOA
Optimal result
$(t_0, \underline{p})$

MLAT and Accuracy

$$\underline{x} = \begin{bmatrix} \underline{p} \\ t_0 \end{bmatrix}$$

$$h_i(\underline{x}) = t_0 + c_0^{-1} \left\| \underline{p} - \underline{s}_i \right\|$$

$$t_i = h_i(\underline{x}) + v_i$$

$$\hat{\underline{x}} = \arg\min_{\underline{x}} \sum_{i=1}^{N} \left(h_i(\underline{x}) - t_i \right)^2$$

Differentials:

$$\nabla h_i(\underline{x}) = \begin{bmatrix} \dfrac{p^{(x)} - s_i^x}{\left\| \underline{p} - \underline{s}_i \right\|} & \dfrac{p^{(y)} - s_i^y}{\left\| \underline{p} - \underline{s}_i \right\|} & \dfrac{p^{(z)} - s_i^z}{\left\| \underline{p} - \underline{s}_i \right\|} & 1 \end{bmatrix}^\top$$

$$\mathbf{H} = \begin{bmatrix} \nabla h_1(\underline{x}) \\ \vdots \\ \nabla h_N(\underline{x}) \end{bmatrix}$$

$$\Delta \underline{t} = \mathbf{H}\, \Delta \underline{x} \qquad \mathbf{C}^v = \mathbf{H} \mathbf{C}^x \mathbf{H}^\top$$

$$\mathbf{C}^x = (\mathbf{H}^\top (\mathbf{C}^v)^{-1} \mathbf{H})^{-1}$$

$$\mathbf{J}^x = \mathbf{H}^\top \mathbf{J}^v \mathbf{H}$$

MLAT Fisher Information

$$\mathbf{J}^x = \mathbf{H}^\top \mathbf{J}^v \mathbf{H}$$

$$\mathbf{C}^v = \mathbf{I}$$

$$\mathbf{J}^x = \mathbf{H}^\top \mathbf{H}$$

$$= \sum_{i=1}^{N} \Delta h_i [\Delta h_i]^\top$$

$$\mathbf{J}_i^x = \Delta h_i [\Delta h_i]^\top$$

Total FIM

$$\mathbf{J}^x = \sum_{i=1}^{N} \mathbf{J}_i^x$$

Sensor-specific FIM

Information Metrics / Matrix Means

- T – optimality $\Theta = \mathrm{trace}(\mathbf{J}^x)$
  Trace
  - Sensor superposition
  - Constant

- A – optimality $\Theta = \mathrm{trace}(\mathbf{C}^x)$
  Average Variance
  - No sensor superposition
  - Depends on sensor locations

GDOP
PDOP
HDOP

$$\mathbf{C}^x = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} & \sigma_{xt} \\ \sigma_{yx} & \sigma_y^2 & \sigma_{yz} & \sigma_{yt} \\ \sigma_{zx} & \sigma_{zy} & \sigma_z^2 & \sigma_{zt} \\ \sigma_{tx} & \sigma_{ty} & \sigma_{tz} & \sigma_t^2 \end{bmatrix}$$

- D – optimality $\Theta = \det(\mathbf{J}^x)$
  Determinant

- E – optimality $\Theta = \Lambda_{\min}(\mathbf{J}^x)$
  Smallest Eigenvalue

Rank-1 Update

$$\mathbf{J}_i^x = \Delta h_i [\Delta h_i]^\top$$

Rank-1 Matrix

$$\mathbf{C}^x = \left(\sum_{i=1}^{N} \mathbf{J}_i^x \right)^{-1}$$

Add 1 sensor:
Rank-1 update

$$\mathbf{C}^x \mathrel{+}= -\frac{\mathbf{C}^x \Delta h_i [\mathbf{C}^x \Delta h_i]^\top}{1 + [\Delta h_i]^\top \mathbf{C}^x \Delta h_i}$$

$$\Theta = \mathrm{trace}(\mathbf{C}^x)$$

$$\Theta \mathrel{+}= -\frac{\|\mathbf{C}^x \Delta h_i\|_2^2}{1 + [\Delta h_i]^\top \mathbf{C}^x \Delta h_i}$$

Minimize Θ

Search Routines

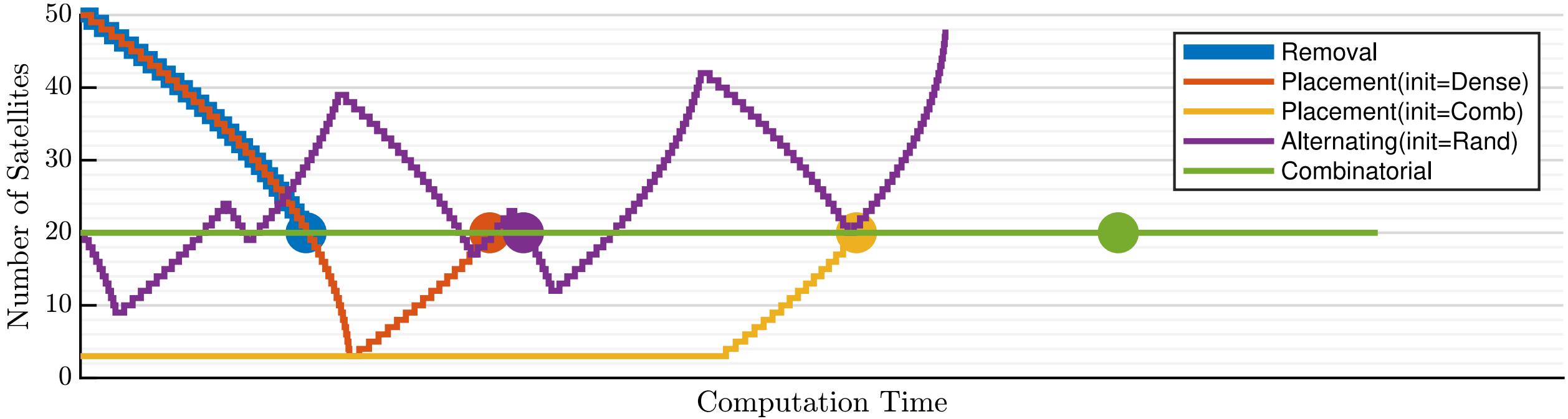
- Combinatorial Search
  - Complexity: $\mathcal{O}\left(\binom{M}{N}\right)$
  - Combinatorial Gray Code

- Greedy Removal
  - Init: Dense
  - Remove least important

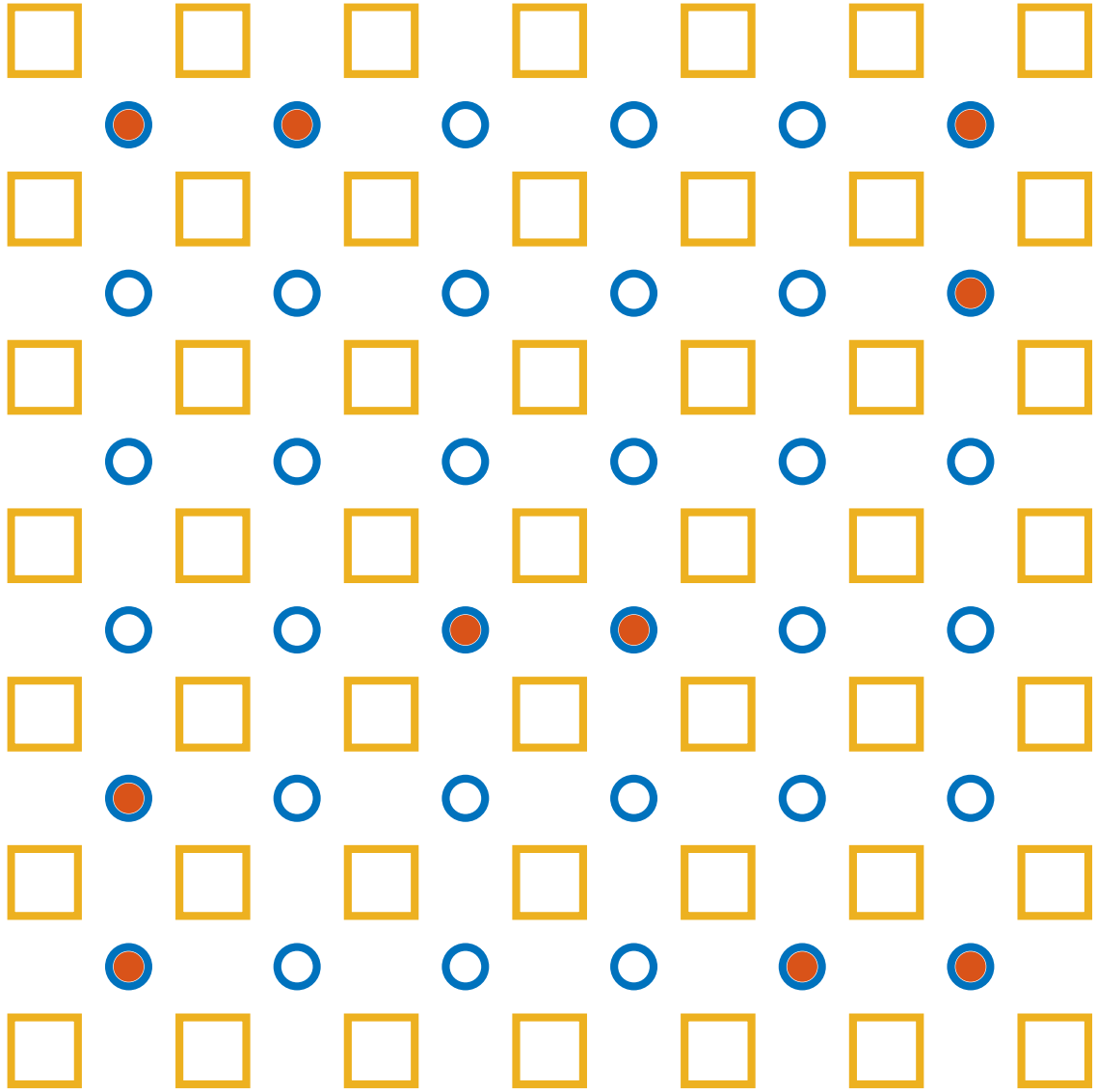
- Greedy Placement
  - Init: Few sensors
    - Combinatorial
    - OR Greedy Removal
  - Add most beneficial

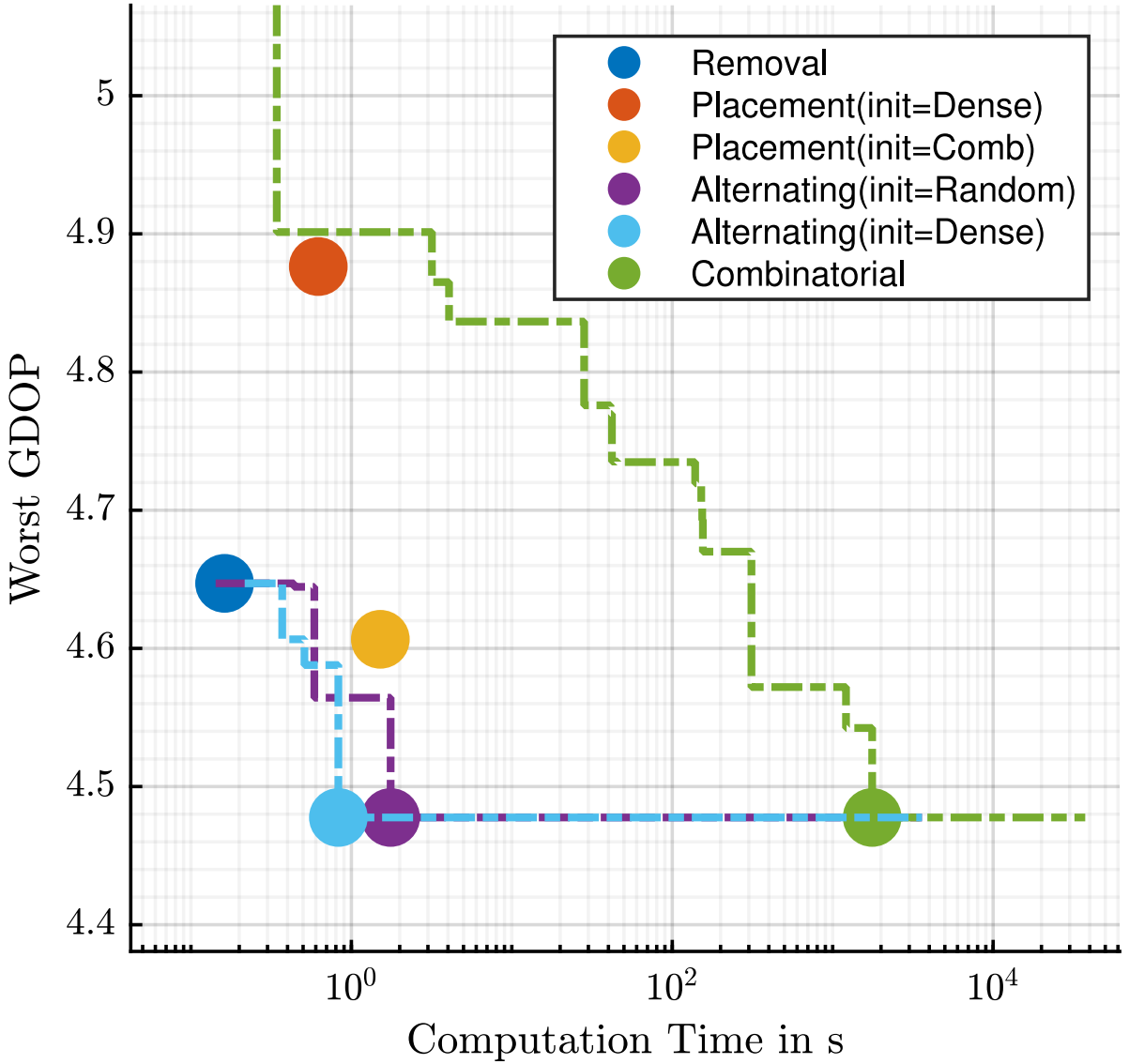
- Alternating Greedy Removal and Placement (AGREP)
  - Init:
    - Dense
    - OR Random
  - Greedy Removal
  - Greedy Placement
  - Greedy Removal
  - …

Search Routines



Evaluation




Removal
Placement(init=Dense)
Placement(init=Comb)
Alternating(init=Random)
Alternating(init=Dense)
Combinatorial
Worst GDOP
5
4.9
4.8
4.7
4.6
4.5
4.4
10^0
10^2
10^4
Computation Time in s

Conclusion

Achieved

- Sensor Selection Algorithm AGREP
- Tailored for MLAT
- Finds global optimum fast (2D LOS)

Future Work

- More complex geometries
  - Non-square floor plans
  - NLOS
- Realistic measurement model
- Improve AGREP
  - More random decisions
  - Include GP aspects

Thank you for your attention