Editors' Suggestion

Superconductivity of Incoherent Electrons near the Relativistic Mott Transition in Twisted Dirac Materials

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We demonstrate that superconductivity driven by strong quantum-critical fluctuations can emerge near relativistic Mott transitions in twisted two-dimensional materials. In twisted double-bilayer WSe_2, all time-reversal-even, gap-opening collective modes promote pairing, whereas time-reversal-odd modes do not. In twisted bilayer graphene, all transitions into intervalley-coherent insulators give rise to superconductivity. Hence, the two separate superconducting domes of insulating or semimetallic undoped systems are expected to merge near the Gross-Neveu transition angle. A crucial ingredient of the theory is that critical fluctuations render the electronic states strongly incoherent, allowing attractive pairing channels to overcome the bare Dirac semimetal behavior. The richer the Dirac structure, the more readily pairs can form. Finally, we demonstrate a direct relation between boson-mediated pairing and the formation of charge-carrying skyrmions in the proximate insulating state.

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Twisted two-dimensional materials have emerged as a versatile setting to study the tunable interplay of electron correlations, unconventional order, and topology [1–13]. Through the control of twist angle, carrier density, and displacement fields, these systems can be driven into a remarkable variety of ordered and topological states, ranging from correlated insulators to superconductors. At the single-particle level many twisted materials possess symmetry-protected massless Dirac points. Prominent examples, where the Dirac point is right at the Fermi level, include twisted bilayer graphene (TBG) at charge neutrality [8,10] or stacked twisted double-bilayer WSe_2 at filling $\nu = 2$ [13–17]. Changing the twist angle increases correlation effects and opens a gap at the Dirac points [6,7,13,18–20]. The description of this semimetal-insulator transition in terms of the Gross-Neveu (GN) mechanism [21,22] was shown to yield quantitative agreement for, e.g., the value of the twist angle of the transition in models that include states in the entire Brillouin zone [23–25]. For such a relativistic Mott transition, the critical point represents the onset of spontaneous symmetry breaking that gaps the Dirac spectrum and is governed by strong, quantum-critical fluctuations. It suggests that at neutrality these fluctuations are governed by the Dirac point only.

In a parallel manuscript [26], we analyzed the conditions under which quantum-critical fluctuations near GN criticality can induce superconductivity at the neutrality point of a generic two-dimensional Dirac system. As illustrated in Fig. 1, strong-coupling superconductivity can emerge even though only a small number of carriers are thermally or quantum-mechanically excited. The nature of the pairing state depends on the underlying critical bosonic mode. Remarkably, superconductivity occurs only when the fermions are sufficiently incoherent: well-defined quasiparticles remain nonsuperconducting, whereas strongly renormalized, ill-defined fermions are able to condense. An important open question is how the relativistic Mott transition in concrete twisted systems intertwines with this mechanism of superconductivity. In this Letter, we investigate superconductivity in the vicinity of relativistic Mott transitions in twisted two-dimensional materials. To this end, we generalize and apply the theoretical framework of Ref. [26] to the concrete cases of twisted double-bilayer WSe_2 and TBG. For twisted WSe_2, we demonstrate that all time-reversal-even fluctuations promote pairing, whereas time-reversal-odd fluctuations do not induce superconductivity. In TBG, focusing on intervalley-coherent insulators [27–29], we uncover a rich spectrum of degenerate and nearly degenerate superconducting states. Thus, close to the twist angle where the fermions become gapped, we expect superconductivity at neutrality. Furthermore, we establish a direct connection between our framework and

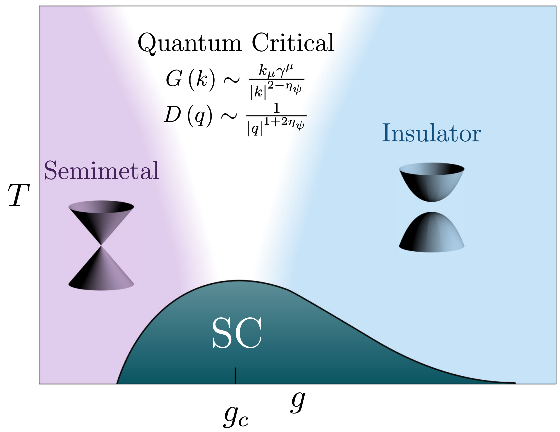

FIG. 1. Schematic phase diagram for the behavior near the Gross-Neveu mass-generating transition (at g_c) of two-dimensional Dirac systems within the approach of Ref. [26]. For systems with a sufficiently large anomalous fermion dimension, η_ψ, in the electron Green's function $G(k)$, critical boson fluctuations [with propagator $D(q)$] give rise to superconductivity (SC) even in the absence of a sharp Fermi surface or any well-defined charge carriers.

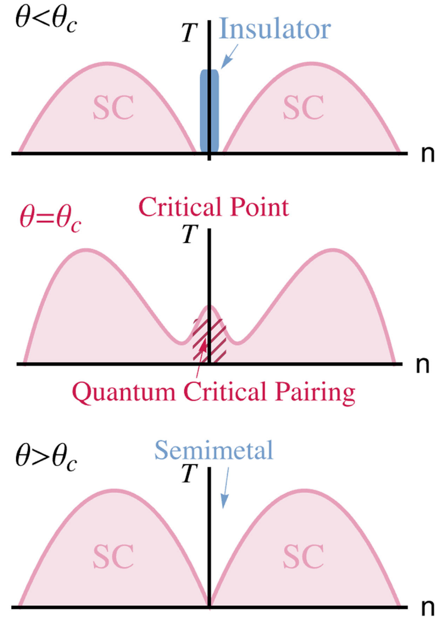

FIG. 2. Schematic temperature-doping phase diagram of a twisted Dirac system near the critical Gross-Neveu transition for different twist angles, where θ_c corresponds to the angle at the transition, i.e., where $g(\theta_c) = g_c$, with g_c of Fig. 1. For θ smaller and larger than θ_c, two separate superconducting domes (SC) occur, either separated by an insulating state or a nonsuperconducting semimetal in the weak coupling limit. At the critical point the two domes merge, giving rise to the quantum-critical pairing described in this Letter.

the intriguing possibility of charged skyrmions as a topological mechanism for pairing [30–32], as has been intensely discussed in the context of TBG [33–35]. In Fig. 2 we illustrate how our findings fit into the overall phase diagram of twisted materials near neutrality. While two separated domes as function of doping exist when the undoped system is either insulating or a semimetal, we expect them to merge close to the angle of the Gross-Neveu transition. If confirmed, this phase diagram implies that superconductivity in the two domes shares a common origin in fluctuations of the nearby insulating state.

We are interested in the solution of the two-dimensional coupled Dirac-fermion boson problem with the following Hamiltonian:

$$H = v_{\mathrm{F}} \int d^2 x \psi^\dagger(\boldsymbol{x})(-i\nabla)\cdot \boldsymbol{\alpha}\psi(\boldsymbol{x}) + \frac{1}{2}\int d^2 x \sum_a \left(\pi_a^2(\boldsymbol{x}) + \omega_0^2 \phi_a^2(\boldsymbol{x}) + v_{\mathrm{B}}^2[\nabla\phi_a(\boldsymbol{x})]^2\right) + g\sum_a \int d^2 x (\psi^\dagger(\boldsymbol{x})\Upsilon_a \psi(\boldsymbol{x})\phi_a(\boldsymbol{x}) + \mathrm{H.c.}). \quad (1)$$

Here, ψ is an n_γ-component Dirac spinor with $\{\alpha_i, \alpha_j\} = 2\delta_{ij}$ and ϕ_a is an m_ϕ-component critical bosonic field $(a = 1\cdots m_\phi)$. π_a is its conjugate momentum. ω_0 is the bare mass of the boson, $v_{\mathrm{F,B}}$ are the fermion and boson velocities, respectively, and g is the coupling constant, where the symmetry of the boson determines the $n_\gamma \times n_\gamma$ matrices Υ_a that act in spinor space and that have a common parity $\tau_\phi = \pm$ under time reversal.

To address the strong coupling behavior of this model at the GN critical point, we utilize a generalized Sachdev-Ye-Kitaev approach [36–39]. Following Refs. [40–42] we introduce additional N fermion and M boson flavors, randomize the coupling constant g between these additional flavors, and perform a controlled large N and M expansion at fixed M/N, a limit where so-called melonic diagrams dominate the physical behavior. For Dirac systems this theory was formulated for the normal state in Ref. [42] and extended to the superconducting state in Ref. [26]. In the normal state and at the GN-critical point one obtains universal power-law behavior for the critical fermions and bosons (see Fig. 1), with a comparatively large anomalous fermion dimension $0 < \eta_\psi < \frac{1}{2}$ within a controlled theory.

To determine possible superconducting states we analyze the anomalous self energy $\Phi(k)$ in the Nambu-Gorkov description of superconductors, which is also an $n_\gamma \times n_\gamma$ matrix in spinor space [26]. The linearized equation for $\Phi(k)$ is

$$\Phi(k) = \frac{\lambda_p \tau_\phi}{m_\phi}\sum_{a=1}^{m_\phi}\int \frac{d^3 p}{4\pi}\frac{p_\mu p_\nu \Upsilon_a \gamma^0 \gamma^\mu \Phi(p)\gamma^\nu \gamma^0 \Upsilon_a}{|p|^{4-2\eta_\psi}|k-p|^{1+2\eta_\psi}}. \quad (2)$$

At the GN-critical point, the power-law structure of the kernel and the dimensionless pairing strength,

$$\lambda_p = \frac{2^{1+2\eta_\psi}(3-\eta_\psi)\Gamma(2-\eta_\psi)\Gamma(\frac{1}{2}+\eta_\psi)\sin(\pi\eta_\psi)}{2\pi^{3/2}(1+\eta_\psi)}, \quad (3)$$

are universally determined by the anomalous exponent η_ψ, i.e., exclusively set by the universality class of the GN transition and independent on the initial coupling constant g. λ_p plays a role similar to the 't Hooft coupling [43] in strongly coupled gauge theories. It vanishes if $\eta_\psi \to 0$ and is of order unity if η_ψ is no longer small (it reaches $(5/3\pi)$ for $\eta_\psi \to \frac{1}{2}$).

The anomalous self-energy determines the usual pairing function $\Delta(k)_{ab} \sim \langle \psi_a \psi_b \rangle$ via $\Delta(k) = \Phi(k) u_T$, where u_T is the unitary component of the time-reversal operator $\mathcal{T} = \mathcal{K} u_T^\dagger$ with complex conjugation $\mathcal{K}$. Fermi statistics further implies $\Delta(k) = -\Delta(-k)^T$. To determine the pairing state we expand

$$\Phi(k) = \sum_{J=1}^{n_\gamma^2} \chi_J(k) \Gamma_J \quad (4)$$

in a complete set of Hermitian $n_\gamma \times n_\gamma$ matrices Γ_J. Only solutions that are isotropic in momentum-frequency space and hence only a function of $|k| = \sqrt{\omega^2/v_\mathrm{F}^2 + \boldsymbol{k}^2}$ occur. The detailed $|k|$ dependence of $\chi_J(k)$ was analyzed in Ref. [26], but will not be important for our considerations, except that $\Phi(k \to 0) \neq 0$ without fine-tuning. We focus on the structure of superconductivity in spinor space. The superconducting instability only occurs if the normal-state anomalous dimension exceeds a critical value, i.e., $\eta_\psi \geq \eta_\psi^c$ [44]. Thus, within our theory the *incoherent* nature of electrons at criticality is essential for, and drives, superconductivity.

We now turn to the question of the Γ_J that contribute to Φ, which establish the types of pairing order that can occur. Indeed, for the dominant pairing instability of Eq. (2) this is fully determined by rather easy-to-analyze algebraic conditions [26]: it must hold that

$$[\Gamma_J, \alpha_i] = 0, \qquad \Gamma_J \Upsilon_a = \tau_\phi \Upsilon_a \Gamma_J, \quad (5)$$

for $i = 1, 2$ and $a = 1 \cdots m_\phi$. In addition, the pairing state must obey Fermi statistics, i.e., $(\Gamma_J u_T)^T = -\Gamma_J u_T$. If these conditions are fulfilled, superconductivity at the GN critical point will be possible. To understand the implications of these conditions, we assume, for the moment, constant-in-k mean-field order parameters for superconductivity and the critical boson. Then, the Bogoliubov–de Gennes Hamiltonian of the fermions becomes an effective Dirac theory in Nambu space with $m_\phi + 2$ mass terms that are either due to the condensation of the bosonic mode or the superconducting order parameter: $\mu_r = (\mu_1, \ldots, \mu_{m_\phi}, \mathrm{Re}\chi, -\mathrm{Im}\chi)$. The algebraic conditions of Eq. (5) can easily be shown to yield the Bogoliubov energies

$$E_{\boldsymbol{k}} = \pm\sqrt{(v_\mathrm{F}\boldsymbol{k})^2 + \sum_r \mu_r^2}, \quad (6)$$

where the gaps due to particle-hole and particle-particle condensation add up in squares (see [45]), revealing a remarkably simple coexistence of GN critical superconductivity with particle-hole correlations.

We will next scrutinize the possible pairing states and couplings compatible with Eq. (5) for twisted double-bilayer WSe_2 and TBG within their respective Dirac theories.

Twisted double-bilayer WSe_2—As shown in Ref. [14], an emergent D_6 symmetry implies that low-energy Γ-valley moiré holes realize a single-orbital model on a honeycomb lattice, i.e., artificial graphene but with a tunable interaction strength. Furthermore, recent experiments [13] identified a semimetal-insulator transition as a function of the twist angle in AB-BA stacked twisted double-bilayer WSe_2, which is considered a strongly correlated version of single-layer graphene [13–17] (and therefore a likely candidate for the present scenario). To apply Eq. (5) we need to identify the Dirac matrices α_i and the coupling matrices Υ_a. The quantum numbers are spin, valley, and sublattice, which we denote by the Pauli matrices σ_{j_1}, τ_{j_2}, and ρ_{j_3}, respectively. The 8×8 matrices α_i are then given by

$$(\alpha_1, \alpha_2) = \sigma_0(\tau_3\rho_1, \tau_0\rho_2). \quad (7)$$

Time reversal is determined by $u_T = i\sigma_2\tau_1\rho_0$. The complete set of matrices needed for the determination of the pairing state is most conveniently written as $\Gamma_J = \sigma_{j_1}\tau_{j_2}\rho_{j_3}$, labeled by $J = (j_1, j_2, j_3) = 1 \ldots 64$.

Let us first discuss the possible collective bosons that induce upon condensation a gap in the Dirac spectrum and whose fluctuations may serve as pairing glue. The corresponding ordered states have all been discussed in great detail; see, e.g., Refs. [46–52]. Spin-orbit interaction in this system is believed to be very small [14,15], so it is natural to consider Heisenberg couplings of the type

$$\vec{\Upsilon}_{j_2,j_3} = (\Upsilon_{1,j_2,j_3}, \Upsilon_{2,j_2,j_3}, \Upsilon_{3,j_2,j_3}) = \vec{\sigma}\tau_{j_2}\rho_{j_3}, \quad (8)$$

or coupling in the charge sector,

$$\Upsilon_{0,j_2,j_3} = \sigma_0\tau_{j_2}\rho_{j_3}. \quad (9)$$

For the combined valley and sublattice index (j_2, j_3) there are then in total four options that open an isotropic gap, i.e., where the coupling matrices anticommute with α_1 and α_2 simultaneously. This is the case for $(j_2, j_3) = (0, 3)$, (1, 1), (2, 1), and (3, 3). For the point group D_6 the spatial parts of these interactions transform as B_2, A_1, B_1, and A_2, respectively. The first option $\Upsilon_{\mu,03} = \sigma_\mu \tau_0 \rho_3$ breaks the sublattice symmetry (either in the spin or in the charge sector). $(\Upsilon_{\mu,11}, \Upsilon_{\mu,21}) = \sigma_\mu(\tau_1, \tau_2)\rho_1$ are valley-mixing bond-order states akin to Kekulé order. They are closely related to the intervalley-coherent (IVC) insulators that will be discussed in the context of TBG below. Finally, $\Upsilon_{\mu,33} = \sigma_\mu \tau_3 \rho_3$ corresponds for $\mu = 0$ to the Haldane state [53], a loop-current ordered state that breaks time-reversal symmetry, while $\vec{\Upsilon}_{33} = \vec{\sigma}\tau_3\rho_3$ is a time-reversal-even ordered state with opposite currents for the two spin flavors (parallel and antiparallel to $\vec{\Upsilon}_{33}$), leading to an anomalous spin-Hall effect. Notice $\sigma_3\tau_3\rho_3$ is the celebrated Kane-Mele spin-orbit term [54] that would be spontaneously generated in the gapped phase.

We have analyzed all possible pairing states due to fluctuations of these collective bosons by analyzing the algebraic rules of Eq. (5) and find no stable superconducting solution due to time-reversal-odd fluctuations $\vec{\Upsilon}_{03}$, $\vec{\Upsilon}_{11}$, and $\vec{\Upsilon}_{21}$. Equally, no superconductivity emerges due to fluctuations of the Haldane state $\Upsilon_{0,33}$, also time-reversal-odd. All other time-reversal-even states do, however, induce pairing: charge-sublattice fluctuations with $\Upsilon_{0,03}$ induce $\vec{\Phi} = \vec{\sigma}\tau_3\rho_0$, which yields for the superconducting order parameter,

$$\vec{\Delta}_{B_1} = i\sigma_2\vec{\sigma}\tau_2\rho_0, \tag{10}$$

i.e., a spin triplet, valley singlet. B_1 gives the transformation of the spatial part with regard to D_6. The same coupling also induces a singlet s-wave $\Phi = \sigma_0\tau_0\rho_0$, which becomes

$$\Delta_{A_1} = i\sigma_2\tau_1\rho_0. \tag{11}$$

$\vec{\Delta}_{B_1}$ and Δ_{A_1} are, at the level of the continuum theory, degenerate; a degeneracy that will be lifted by lattice corrections to the Dirac theory.

Next, we consider the valley-mixing Kekulé-type bond-order fluctuations $\Upsilon_{0,11}$ and $\Upsilon_{0,21}$. If we consider them individually, as single-component order parameters, they induce either $\vec{\Phi} = \vec{\sigma}\tau_2\rho_2$ or $\vec{\Phi} = \vec{\sigma}\tau_1\rho_2$, which correspond to

$$\vec{\Delta}_{A_2} = i\sigma_2\vec{\sigma}\tau_3\rho_2 \quad \text{and} \quad \vec{\Delta}_{B_2} = i\sigma_2\vec{\sigma}\tau_0\rho_2. \tag{12}$$

These are spin triplets and sublattice singlets with out-of-phase or in-phase order parameter in the valleys. They are again degenerate with the s-wave of Eq. (11). Alternatively, we can consider $(\Upsilon_{0,11}, \Upsilon_{0,21})$ as a degenerate two-component order parameter, a degeneracy caused by the $U_{\text{valley}}(1)$ symmetry that, within the Dirac theory, conserves charge in each valley separately. Then both spin triplets are frustrated by the other boson component, which acts as a pair breaker. As a consequence, only Δ_{A_1} survives. Finally, the anomalous spin-Hall state $\vec{\Upsilon}_{33} = \vec{\sigma}\tau_3\rho_3$ gives rise to the s-wave state Δ_{A_1} of Eq. (11). The pairing states of WSe_2 are illustrated in Fig. 3 (top row).

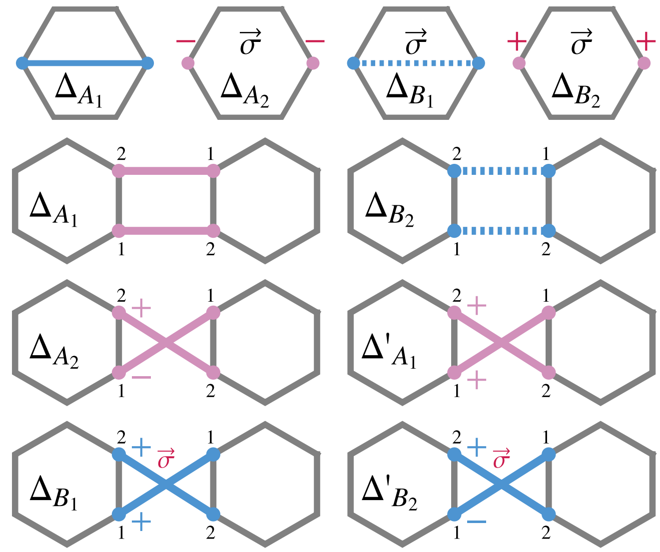

FIG. 3. Top row: the four possible SC instabilities in double-bilayer WSe_2 given in Eqs. (10)–(12) labeled by the D_6 irreducible representation. Pairing correlations that are intrasublattice ($\propto \rho_0$) or intersublattice ($\propto \rho_2$) are labeled blue and pink, respectively, with solid (dashed) lines connecting intervalley pairing of type τ_1 (τ_2). Panels with no lines indicate intravalley pairing that is either in phase ($\propto \tau_0$, indicated with $+$) or out of phase ($\propto \tau_3$, indicated with $-$). Bottom rows: the six possible SC instabilities in TBG depending on the nature of the insulating state (KIVC or TIVC), as listed in Eqs. (16) and (17) and labeled by the D_6 irreducible representation. In each panel the left and right hexagons are the mini-Brillouin zones associated with opposite valleys, with numbers 1, 2 labeling minivalleys. The valley pairing is labeled by color (pink for τ_1 and blue for τ_2). The pairing can also mix minivalleys, with μ_1 (or μ_2) pairing given by solid (or dashed) lines, or be in the same minivalley, with $+-$ (or $++$) labeling μ_z (or μ_0) giving the phase. $\vec{\sigma}$ indicates triplet pairing, with the other cases singlet.

To summarize, superconductivity at the critical twist angle for twisted double-bilayer WSe_2 occurs for bosons that do not break time-reversal symmetry. The most frequent pairing state is the s-wave of Eq. (11).

Twisted bilayer graphene—In this case we need to include, in addition to spin, valley, and sublattice, the minivalley quantum number (the two points $\boldsymbol{K}$ and $\boldsymbol{K}'$ of the mini-Brillouin zone) [27,28,33,55,56], which we denote by Pauli matrices μ_{j_4}. Hence, we now have $n_\gamma = 16$-component Dirac spinors and will expand our pairing states in the 256 matrices $\Gamma_J = \sigma_{j_1}\tau_{j_2}\rho_{j_3}\mu_{j_4}$ and analyze the conditions in Eq. (5) to determine which of them is an allowed superconducting state. The Dirac dispersion is described by the two Dirac matrices,

$$(\alpha_1, \alpha_2) = \sigma_0\,(\tau_3\rho_1, \tau_0\rho_2)\,\mu_0. \tag{13}$$

Time reversal is determined by $u_T = i\sigma_2\tau_1\rho_0\mu_0$. For simplicity we focus on bosons that yield IVC insulators, i.e., that break the translation invariance of the original graphene lattice by forming bond- or loop-current order [28,55] and that have been observed, albeit at different fillings [57]. The effects of all other gap-inducing bosons for TBG are listed in Supplemental Material [45]. The coupling of Dirac fermions to an IVC state depends on whether it does or does not break time-reversal symmetry [25,58]. In the former case we have the Kramers-IVC (KIVC) with coupling matrices

$$(\Upsilon_1, \Upsilon_2)_{\mathrm{KIVC}} = \sigma_0(\tau_1, \tau_2)\rho_1\mu_2, \tag{14}$$

or, alternatively, the time-reversal symmetric IVC (TIVC) with

$$(\Upsilon_1, \Upsilon_2)_{\mathrm{TIVC}} = \sigma_0(\tau_1, \tau_2)\rho_1\mu_1. \tag{15}$$

In Ref. [45] we discuss similar results for other TIVC states, where we also consider a two-component, XY-order parameter.

For both interactions we find from the algebraic conditions, Eq. (5), four degenerate pairing solutions. In the case of KIVC they are

$$\Delta_{\mathrm{A}_1} \propto i\sigma_2\tau_1\rho_0\mu_1, \qquad \Delta_{\mathrm{A}_2} \propto i\sigma_2\tau_1\rho_0\mu_3,$$
$$\vec{\Delta}_{\mathrm{B}_1} \propto i\sigma_2\vec{\sigma}\tau_2\rho_0\mu_0, \qquad \Delta_{\mathrm{B}_2} \propto i\sigma_2\tau_2\rho_0\mu_2. \tag{16}$$

Except for the spin-triplet B_1 state, all of these superconductors are spin singlets. While the A_1 order parameter transforms trivially under all point group symmetries, it corresponds to a finite-momentum superconductor with nontrivial phase shift under moiré translation; this is also true for B_2 in Eq. (16), while the remaining two states have Cooper pairs with zero center-of-mass momentum.

Considering TIVC fluctuations, Δ_{A_1} and Δ_{B_2} are unchanged while instead of the other two, we now find

$$\Delta'_{A_1} \propto i\sigma_2\tau_1\rho_0\mu_0, \qquad \vec{\Delta}'_{B_2} \propto i\sigma_2\vec{\sigma}\tau_2\rho_0\mu_3. \tag{17}$$

These four states are again degenerate at the level of the Dirac theory. The six distinct pairing states of TBG, discussed above, are illustrated in Fig. 3 (bottom three rows). Most importantly, in TBG we find superconductivity emerging at the onset of intervalley coherent insulators, irrespective of whether time-reversal symmetry is broken or preserved. The additional minivalley flavor structure substantially enhances the range of possible pairing states, in comparison to WSe_2.

To estimate the superconducting transition temperature T_c in these twisted Dirac materials, we note that the maximal T_c obtained in Ref. [26] is approximately $2\times 10^{-3}\Lambda$, where Λ denotes the cutoff energy of the Dirac model, i.e., a sizable portion of the electronic bandwidth. Interestingly, T_c does not depend on the coupling constant g in Eq. (1), which decreases as one increases the twist angle starting from the insulating state, while Λ increases. For both WSe_2 at a twist angle of $\theta \approx 2.7°$ and for TBG at $\theta \approx 1.2°$ one finds $\Lambda \sim 20$ meV [13,20], which yields an optimal transition temperature of $T_c \approx 0.4$ K, i.e., smaller than but comparable to the maximum T_c in TBG; see Fig. 2.

For both systems, our results for pairing at the semimetal to insulator transition were obtained in the limit where we only consider the Dirac point and ignore the behavior in the rest of the mini-Brillouin zone or due to higher bands. Hence, we should ask whether there are implications beyond the Dirac physics near the Gross-Neveu critical point. In this context, it is interesting that the second condition in Eq. (5) was recently obtained in the study of pairing instabilities of TBG in the extreme flat-band limit [59], i.e., not in the Dirac regime. This strongly suggests that it may in fact be of more general relevance than either limit. Of course, the first condition in Eq. (5) is specific to the Dirac theory.

Of the many approaches to link superconductivity to nearby ordered states, a particularly beautiful and powerful one is the observation that the seeds of pairing are planted in the topological excitations of the insulator and vice versa [30–35]. This is the result of a Wess-Zumino-Witten term,

$$S_{\mathrm{WZW}} = i\frac{3\mathcal{N}}{4\pi}\int \epsilon_{\alpha\beta\gamma\delta\nu} n_\alpha \partial_u n_\beta \partial_\tau n_\gamma \partial_x n_\delta \partial_y n_\nu, \tag{18}$$

of the coupled superconducting and mass-generating boson problem. Similar to the multiple mass terms in the Bogoliubov spectrum of Eq. (6), the five-component order parameter n_α comprises the real and imaginary part of the superconducting order parameter together with three suitably chosen collective bosonic modes, denoted by $\vec{\Upsilon}$. The integration $\int\cdots = \int d^3x \int_0^1 du$ extends over space-time and the auxiliary variable u [30–35]. Reference [33] formulated three conditions for $\mathcal{N}\neq 0$ in a particularly convenient way for our purposes. Remarkably, the pairing conditions of Eq. (5), derived from the analysis of the critical pairing problem, coincide with two of these three criteria. The remaining condition imposes an additional constraint on the three components of $\vec{\Upsilon}$, namely

$$\mathrm{tr}(\alpha_{i_1}\alpha_{i_2}\Upsilon_{a_1}\Upsilon_{a_2}\Upsilon_{a_3}) = -8\mathcal{N}\epsilon_{i_1 i_2 a_1 a_2 a_3} \neq 0, \tag{19}$$

which also determines the coefficient $\mathcal{N}$ in Eq. (18).

Analyzing options for such topologically enabled pairing for WSe_2, we find the partner states $\vec{\Upsilon} = (\Upsilon_{011}, \Upsilon_{021}, \Upsilon_{003})$ and $\vec{\Upsilon}_{33}$, both with Δ_{A_1} of Eq. (11). Hence, either a combination of bond with charge order or the anomalous spin-Hall state, already discussed in Ref. [32], form natural partner orders with s-wave superconductivity to yield a topological origin of pairing. Here, $\mathcal{N} = 1$ and skyrmion defects in the insulator carry charge $2e$. For the TBG problem, the behavior is again much richer and we find in total 56 such partner states, all combined with a wide array of superconducting states. They are listed in Supplemental Material [45]; see also Ref. [33]. Examples are the two states for WSe_2 multiplied by the unit matrix in minivalley space: $\vec{\Upsilon}\mu_0$ and $\vec{\Upsilon}_{33}\mu_0$. As a result of the additional minivalley quantum number, we have $\mathcal{N} = 2$, so that skyrmions carry charge $4e$. For both systems, we find the common theme that only bosons that are even under time reversal yield nonzero S_{WZW}.

These results are rather unexpected. *A priori*, there is no reason to expect a connection between the conditions arising from a pairing instability due to critical boson exchange and those required for a topological term in the action. Yet our analysis implies that all insulators with charged skyrmions, together with their superconducting partners, form a subset of the states we obtain from Eq. (5). The common physical feature of both phenomena is that the masses generated by particle-hole and particle-particle condensation add in quadrature, both for skyrmion condensation and for quantum-critical pairing; see Eq. (6). Our theory shows not only that these particle-hole and particle-particle partners are natural candidates for topological pairing, but also that the superconducting partners are in fact induced by fluctuations of particle-hole modes, i.e., we provide an energetic mechanism that leads to topological pairing. Consequently, these pairing states may be relevant beyond neutrality or the specific critical point studied.

In summary, we investigate superconductivity emerging near relativistic Mott transitions in twisted two-dimensional Dirac materials. Using a controlled strong-coupling framework, based on a generalized Sachdev-Ye-Kitaev approach, it was shown in Ref. [26] that superconductivity may appear once the fermionic anomalous dimension induced by Gross-Neveu criticality exceeds a universal threshold, enabling attractive pairing to overcome the bare Dirac semimetal behavior. Applying this theory to twisted double-bilayer WSe_2, we find that all time-reversal-even bosons cause superconductivity. In contrast, for TBG we find a manifold of degenerate and nearly degenerate superconducting states that can be either time-reversal-even or odd. Broadly speaking, the more intricate the Dirac structure (i.e., the larger the Dirac representation of the twisted material), the more readily pairs can form. Finally, we established a direct link between pairing due to critical bosons and the formation of charged topological solitons, i.e., skyrmions, in the associated insulator. The latter are a subset of the former. Hence, twisted Dirac materials provide a platform that links relativistic Mott criticality, its insulating phases, and unconventional superconductivity.

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Data availability—No data were created or analyzed in this study.

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[45] See Supplemental Material at http://link.aps.org/supplemental/10.1103/sgnp-ywsh for a discussion of the pairing condition, tables of all pairing states for WSe_2 and TBG, and a list of all partner states for TBG that give rise to a Wess-Zumino-Witten term in the action.

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