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# Mathematical Modeling in Action: Design Principles for Mathematical Modeling Days

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## ABSTRACT

Mathematical modeling is fundamental for numerous scientific and technical disciplines. CAMMP has taken on the challenge of providing high school students, university students, and teachers with authentic insights into modern, computer-aided modeling practices. This article presents CAMMP's educational activities, with a particular focus on its one-day workshops, known as "CAMMP days." The underlying design principles of the teaching and learning materials for CAMMP days are presented, and the practical implementation of these principles is illustrated through a workshop on speech recognition. Experience from 450 workshops conducted with more than 7500 high school students suggests that CAMMP days successfully convey the importance and diverse applications of mathematics in the real world.

## 1 | Why Computer-Aided Mathematical Modeling?

Mathematical modeling plays a central role both within mathematics itself and in numerous applied fields such as physics, computer science, and engineering [1]. It is crucial for applications ranging from artificial intelligence and medical imaging to renewable energy technologies. Mathematical modeling serves as an indispensable tool for describing, analyzing, and predicting complex real-world phenomena. Solving real-world problems using mathematical models not only deepens our understanding of the world but also connects theoretical mathematics with practical applications. Many key concepts behind practically relevant technologies can, in fact, be explained using mathematical knowledge already taught at the school level [2–4].

Today, mathematical modeling is increasingly computer-based, driven by the availability of large data sets and rapid advances in information technology. To give students authentic insights into modern modeling practices, the use of computer-based tools should be encouraged as early as school age. This approach not only strengthens students' mathematical competencies but also prepares them for the demands of an increasingly digital workplace.

Given its fundamental importance, mathematical modeling has been incorporated into national and international school curricula from an early stage [5]. This paper presents a project focusing on workshops designed to engage high school students with computer-aided mathematical modeling. In particular, it outlines the design principles behind mathematical modeling days and discusses experiences gained from their implementation.

## 2 | The CAMMP Project and Its Formats

The project CAMMP[1] (Computational and Mathematical Modeling Program), launched in 2011, aims to introduce high school students to computer-aided mathematical modeling and to highlight the central role of mathematics in science, technology, and society. A key approach of CAMMP is to use real-world examples as a primary motivation and as an entry point to explore mathematical methods.

To support this aim, CAMMP develops innovative teaching concepts, as well as teaching and learning materials. These enable students to actively engage in solving real-world problems using mathematical modeling and computer-based tools [6]. Initially founded as an education laboratory for high school students at RWTH Aachen University, CAMMP has since expanded to include locations at Karlsruhe Institute of Technology, Paris Lodron University of Salzburg, and the University of Stuttgart.

## 2.1 | Formats

CAMMP offers various formats to reach a broad range of students and teachers. These formats vary in duration, complexity, and target group, but share the objectives mentioned above. They are primarily intended for high school students from grade 7 onward and for mathematics teachers. The duration of the formats is flexible, ranging from short introductory sessions to in-depth, multi-day workshops.

In the following, we present some CAMMP formats, followed by a more detailed description of the CAMMP day format.

- *CAMMP days*: This format includes a variety of 1-day workshops that provide insight into specific mathematical modeling problems. High school students work with digital learning materials that guide them in solving a selected real-world problem using computer-aided mathematical modeling. The program targets entire school classes and courses. The materials used are structured according to consistent design principles which are described in detail below.
- *CAMMP lessons*: Under the term CAMMP lessons, we summarize activities that are more closely aligned with the structure and timing of regular school schedules, typically spanning 1.5 h. In these sessions, high school students also engage with digital learning materials to solve a real-world problem through computer-based mathematical modeling. The design of the accompanying teaching and learning materials follows the same design principles as those used in CAMMP days. However, due to the shorter duration, the mathematical depth that can be explored is more limited.
- *CAMMP weeks*: This multi-day format offers high school students the opportunity to deeply engage with real-world modeling problems provided by partners in research and industry. As no predefined solution path is provided, participants work in small groups to explore a specific problem in depth, developing their own approaches and models to find solutions. The structure of a CAMMP week is based on the modeling week format developed in Kaiserslautern[2] and is described in detail in [7] and [8].
- *Teacher training*: CAMMP offers training courses for teachers to familiarize them with its concepts and materials. This should enable them to integrate the teaching and learning materials into their own classroom practice.

## 2.2 | Examples of CAMMP days

The topics covered during the CAMMP days range from data-driven topics in the field of artificial intelligence (e.g., speech recognition, image classification, and recommendation systems) to medical research (computed tomography) and modeling renewable energies (solar and wind power plants) [9]. This article provides a brief overview of the content of the workshops on recommendation systems, image classification, and speech recognition.

### 2.2.1 | Recommendation Systems and the Netflix Challenge

In this workshop, students develop a recommendation system for movies based on a dataset published by Netflix in 2006. Depending on the version of the workshop, the students use either the $k$-nearest neighbor method or matrix factorization [2]. The workshop is designed for students from grade 10 onwards (age 15–16).

### 2.2.2 | Image Classification

In this workshop, students from grade 11 onwards (age 16–17) develop a model for classifying traffic light images as either red or green. Students begin by developing mathematical models for simple two- and three-dimensional data sets (i.e., images with 2 or 3 pixels) and then progress to working with high-dimensional images. The underlying method, which is taught step by step, is the support vector machine [3].

### 2.2.3 | Speech Recognition

This workshop is described in detail in this paper to exemplify the application of our design principles (see Section 3.2). Students from grade 10 onwards (approx. age 16) develop a model for classifying self-recorded speech samples. They are introduced to the mathematical foundations of single word recognition through four main steps: preprocessing the speech signal and feature extraction (step 1), classification (step 2), evaluation of the classification (step 3), and model improvement through temporal adjustment of speech signals (step 4). The content of these steps is briefly explained below:

1. Step 1 focuses on preparing and visualizing the data. The speech signals are in the time domain and are converted into an amplitude spectrum using Fourier transform. Here, the Fourier transform is introduced as a "black box": its basic concept is explained and the method applied, but its detailed inner workings are not explored with the students. To filter the signal and identify typical features in the spectrum, psychoacoustic phenomena are described mathematically. At each time step, the mean values of the amplitudes of individual frequency bands between specified cut-off frequencies are computed. The mean values across all time steps are stored as a so-called speech pattern.

2. In step 2, the nearest neighbor method is used to classify the speech patterns. Here, a set of representative points are chosen. Any data point is then assigned to the closest representative point according to a selected distance measure. For speech recognition, features extracted from the speech patterns at each time step serve as data points. If an unknown speech signal is to be classified, it is compared with speech signals whose word meanings are known, called reference patterns. These reference patterns are grouped into classes with the same word meanings. At each time step, the Euclidean distance to each reference pattern is determined and used to calculate the total distance between the speech pattern and each reference pattern. These distances are then summed up over all time steps to determine the total distance to each reference pattern. Finally, the speech pattern is assigned to the class of the reference pattern with the smallest total distance.
3. Step 3 involves students evaluating the classification results. The confusion matrix, a standard evaluation instrument for machine learning methods, is introduced and linked to the $2 \times 2$ contingency tables commonly used in school. From these, quality metrics such as accuracy and precision are derived by calculating relative frequencies.
4. In step 4, the distance between the speech patterns is determined using dynamic time warping as a model improvement to take into account the temporal variability of the speech signals [10].

## 3 | Subject-Specific Design Research on Computer-Aided Modeling

### 3.1 | Fundamentals of Subject-Specific Design Research

In design research, it is common to base the design of learning opportunities on established design principles [11]. These principles are fundamental to the design of teaching and learning materials and serve as guidelines for translating theoretical considerations into practical learning opportunities [11]. They may be developed through exploratory work or by selecting principles that have already been tested for similar designs.

All CAMMP day projects have common overarching design principles that serve as guidelines for designing teaching and learning materials. These principles are presented below and their implementation is illustrated through the example of the workshop on speech recognition.

### 3.2 | Design Principles

Seven design principles (DP1–DP7) were selected and adapted for the design of the teaching and learning materials. The starting point is the central goal of the CAMMP project: to offer students the opportunity to work on real-world problems using mathematical modeling and to make the relevance of mathematics in everyday life and in social contexts tangible. Seven design principles are derived from this goal:

- **DP1**: Highlight the role of mathematics
- **DP2**: Show external and internal mathematical links
- **DP3**: Consider authenticity and relevance of the problem
- **DP4**: Use appropriate technical tools
- **DP5**: Provide opportunities for discussion and reflection in cooperative learning formats
- **DP6**: Enable internal differentiation
- **DP7**: Use a modeling cycle as a metacognitive element

The formulation and selection of the design principles were informed by existing research and theory in the didactics of mathematical modeling. In particular, Pohjolainen and Heiliö's criteria for good modeling tasks [12] were used. They are intended to support the selection of suitable problems for modeling lessons and have already been included in previous publications from the CAMMP project [6]. These background theories are especially reflected in DP1, DP2, DP3, DP4, and DP7.

Many CAMMP day problems involve data-driven modeling. Since the data itself, along with its preparation and analysis, plays an important role, insights from research on the design of statistics learning environments also informed the design of the CAMMP materials. In particular, the principles developed by Ben-Zvi et al. for designing a statistical reasoning learning environment were used, as these have been established in the context of statistics and data science education [13]. The basic ideas of Ben-Zvi et al. are incorporated in particular in DP4, DP5 and also support DP3.

The following justification and description of the design principles, along with examples of their implementation, are based on the work by Hofmann [4].

#### 3.2.1 | DP1: Highlight the Role of Mathematics

Following the recommendation of [13] to demonstrate the relevance of mathematics in solving real-world problems, it is important that students have the opportunity to identify the central mathematical content. In contrast to [13], it is not assumed here that students must independently develop all mathematical ideas. Such an expectation seems unrealistic, especially when addressing complex real-world problems within a limited time frame of a single day: the mathematical concepts required are numerous, complex, and have often not yet been covered in class. New mathematical content is therefore introduced through a problem-based approach, with particular attention to how it connects to skills that students have already developed in class and through the curriculum. Unlike a method-oriented approach, a problem-based approach introduces mathematical strategies and methods only when necessary to solve the problem at hand [6]. In this sense, students are given the opportunity to perceive and understand

> "phenomena of the world around us that concern or should concern us all, from nature, society, and culture, in a specific way [...]" [14, p. 37, author's translation].

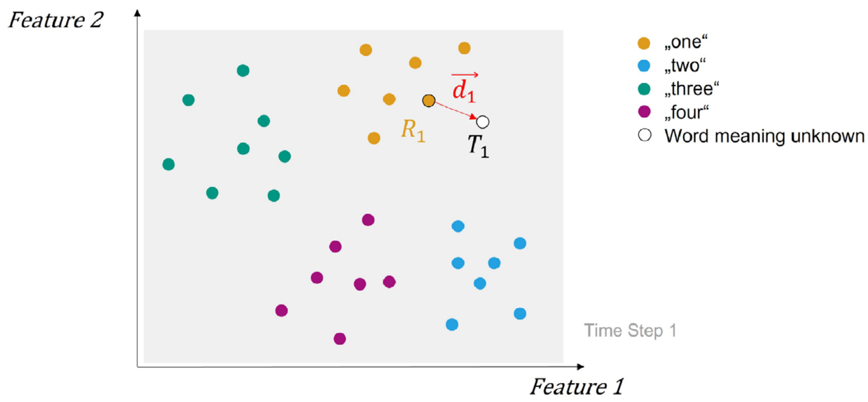


**FIGURE 1** | Distance between a reference sample and a test sample in two dimensions at the first time step.[3]

This quote reflects the first of three fundamental experiences that, according to Winter [14], should be aimed for in mathematics education.

*Implementation in the Speech Recognition Workshop*

To highlight the role of mathematics in the workshop and make the mathematical methods more accessible, the modeling process is divided into small subtasks. The students work through the mathematical formulas and procedures within the context of the problem, guided step-by-step through these subtasks.

To illustrate the procedure for developing mathematical content, step 2 of the workshop is presented here as an example: the classification of language patterns using the nearest neighbor method. The central mathematical concept to be developed is the Euclidean distance, which serves as the chosen distance measure. Rather than introducing the Euclidean distance abstractly in its multidimensional form, students work out the formula step by step by building on their prior knowledge of distances between points in 2D and in 3D. This is done using the example of the language context, with the specific goal of determining the distance between two language patterns. To reduce the dimension and therefore the complexity at the beginning and to connect with the students' existing knowledge, we initially assume that every pattern has only two features in the first time step instead of the actual number of 25 features. These two features can be interpreted as coordinates of a point in the plane. Figure 1 shows the first two features of a test pattern to be classified, along with several reference patterns in the first time step. The students are given the following example task:

*Task: Distance in the first time step for two features*

*Determine the distance of the point of one reference pattern in time step 1, called $R_1$, and from the point of the new speech pattern in time step 1, called $T_1$.*

In a subsequent subtask, three features per language pattern and per time step are considered. These features can be interpreted as points in three-dimensional space. Again, students are asked to calculate the distance and compare the resulting formula with that used for two-dimensional space. Through this analogy of the concept of distance in plane and spatial geometry, knowledge of plane geometry can be consolidated and at the same time the discovery of regularities and relationships can be trained. Analogies can be found in both geometric and algebraic approaches. Finally, the regularities in the algebraic approach to determine the distance are used in a further subtask to generalize the concept to multidimensional geometry.

### 3.2.2 | DP2: Show External and Internal Mathematical Links

Modeling real-world problems provides the opportunity to make both external and internal mathematical links visible. Highlighting external mathematical links addresses a central element of constructing meaning in mathematics instruction, namely the illustration of the relationship of mathematical concepts to the learner's world. Freudenthal refers to this idea as the principle of relationality [15], describing it as follows:

> "If you want to teach coherent mathematics, you don't have to look for the links directly in the first place; you have to understand them along the starting points where mathematics is linked to the learner's experienced reality. This - I mean reality - is the skeleton to which mathematics attaches itself." [15, p. 77, author's translation].

The principle of relationality is closely related to mathematical modeling. Initially, complex real-world problems are simplified and described using mathematical formulas and models, allowing them to be solved using mathematical methods (and computers). Through modeling tasks, students experience how and to what purpose mathematics can be applied in everyday life, technology, economy, and environmental contexts. Solving most real-world problems requires the integration of different specialist perspectives [6]. Pohjolainen and Heiliö refer to this as the "multidisciplinary nature of modeling"[12, p. 4], which, in their view, should be highlighted in teaching and learning materials on modeling. They connect this multidisciplinarity directly to the role of teamwork in solving real-world problems, which in industry and research is an automatic consequence of the interdisciplinarity of the problem (see DP5). Researchers from different disciplines typically collaborate in parallel and jointly on the development of a model. However, since the primary focus is on the complete development of a mathematical model to solve the problem (see DP1), the non-mathematical background required in addition to everyday knowledge is provided within the learning materials through detailed explanations [6].

In addition to external mathematical links, solving real-world modeling tasks usually reveals numerous internal mathematical links that can be made visible. This potential is deliberately used in the teaching and learning materials to promote integrative learning. The goal is to foster meaningful, lasting contexts that enable students to develop a deeper understanding of mathematical structures. Rather than restricting content diversity—for instance, by focusing solely on an isolated content area—this diversity should be deliberately emphasized and made didactically fruitful. According to Pohjolainen, the integration of different mathematical contents is one of the central criteria for good modeling problems [12].

*Implementation in the speech recognition workshop*

When modeling speech signals in the speech recognition workshop, physical principles from signal processing are of central importance. The representation of a sound as a sine function is based on the physical consideration of sound as a periodic pressure fluctuation in the air. Physical quantities such as frequency and amplitude also play an important role in the representation of the pitch and volume. Algorithmic work, such as determining the optimal assignment path between two speech patterns with dynamic programming, also emphasizes links to computer science. There are also links to music theory and biology.

The focus is on the mathematical content of the teaching and learning material. When developing the models, content from different areas of mathematics (e.g., analysis, geometry, stochastics) is incorporated and naturally integrated. These include, for example, trigonometric functions from analysis to describe sounds. From geometry, the calculation of distances between points in multidimensional space is introduced as the central concept of classification. Finally, stochastic concepts, such as the $2 \times 2$ contingency table and relative frequencies, are used in the evaluation of the model. Thus, internal mathematical links can be easily emphasized in the teaching and learning material.

### 3.2.3 | DP3: Consider Authenticity and Relevance of the Problem

Due to CAMMP's objective to demonstrate the importance of mathematics in solving real-world problems, various aspects of authenticity are considered when selecting problems and their method of solution. This includes the authenticity of the problem with regard to the context of the subject area. Here, Niss's definition is useful, in which a problem is embedded in real practices or subject areas outside of mathematics and addresses typical problems within those fields [16]. In addition to authenticity with respect to the factual context, the modeling problem can and should also be authentic with respect to the question posed and the use of mathematical methods [17, 18] Authenticity in the question means that the question could plausibly be asked by researchers in the corresponding domain. The mathematics used to solve the problem is authentic if it reflects methods that are actually applied in practice to solve the problem (although potentially in more complex forms) [18]. Vos emphasizes that the authenticity of the question is particularly important for the motivation and performance of students [19]. The concept of authenticity can be applied to other aspects of a task [19]. Furthermore, the authenticity of the data plays a crucial role in many data-driven CAMMP days. Ben-Zvi et al. also emphasize the importance of using real or at least realistic data sets [13]. Overall, using mathematics in authentic scenarios can help to demonstrate its practical value.

In addition to authenticity, the relevance of the modeling problem is considered. The concept of relevance can be traced back to Burkhardt, who classifies modeling problems according to the interest that students may have in the context of the problem [20]. Problems whose thematic embedding appears potentially appealing or meaningful to students are referred to as relevant in this sense. Relevant problems are particularly beneficial for promoting motivation, engagement, understanding of the real-world situation, and the ability to critically evaluate the developed model [20]. However, whether students perceive a context as relevant depends not only on the task, but also on their personal interests, experiences, and life circumstances of the individual learner and can therefore vary greatly [21]. Therefore, it is important to select contexts that are likely to resonate with a wide variety of student interests. When the benefits of mathematics are demonstrated within contexts that are meaningful to students, it is expected that the overall relevance of mathematics to their everyday lives will become more evident. This aligns with the overarching goals of CAMMP.

*Implementation in the speech recognition workshop*

Voice-based technologies—such as intelligent voice assistants, automated information systems, dictation and transcription systems, and automatic translation services—have become integral to both industry and daily life. Students experience the recognition of spoken language directly in their lives, especially through natural language assistance systems such as Siri and Alexa on smartphones, smart home devices, or voice-based search engines. Thus, speech recognition offers a concrete example from the students' everyday experience and represents a relevant modeling problem.

Furthermore, speech recognition is a dynamic and research-intensive field with roots tracing back to the 1960s [22]. Despite significant advances, the topic remains complex and continues to be the focus of active research [22]. Scientists in this field of research are still working on the central question, which is also addressed in the workshop: "How can the recognition of spoken language work (even better)?" The question can therefore be classified as authentic, as it is directly linked to a real research problem.

The speech recognition system developed in the workshop, which is based on classification with simple pattern matching and dynamic time warping, is primarily used for automatic telephone queries or pick-to-voice systems in warehouses. The reason for this is that the dynamic time warping method is only suitable for recognizing short expressions [23]. Nowadays, neural networks are primarily used as a classification method for recognizing fluent speech [22]. When choosing the method, the focus is also on making the model as simple as possible and at the same time as complex as necessary. The aim is to convey the content to students in a comprehensible way in one day and at the same time to develop a functional system (see DP1) that reliably recognizes speech signals. Although the system is limited to a small vocabulary set, the basic steps for developing a speech recognition system are the same as those presented in the workshop. In particular, the mathematical methods used for preprocessing (e.g., the Fourier transform) are also fundamental in the speech processing technologies mentioned at the beginning [23, 24], confirming the authenticity of the mathematical methods.

The data set used in the workshop consists of real audio recordings of English number words spoken by different speakers [25]. While this adds complexity to the resulting spectrograms, it enhances the realism and credibility of the speech recognition task. Therefore, the use of these audio data can be regarded as authentic and practical valuable. Overall, the modeling problem used in the workshop is both relevant and authentic in terms of its research question, the use of mathematical methods, and the data.

#### 3.2.4 | DP4: Use Appropriate Technical Tools

Digital teaching and learning materials can support the modeling process on various levels and, in many cases, enable students to solve real-world problems [26]. In particular, this includes relieving the burden of computing by outsourcing repetitive or particularly time-consuming calculations [26, 27]. This shift allows students to focus on the modeling process itself. Even more complex mathematical procedures that are not yet known at the respective grade level can be provided as black boxes. This makes it possible to address challenging questions and provides students with a realistic insight into the application process without overwhelming them with technical details. In addition, the use of digital media enables exploratory work, for example, in the form of parameter studies and systematic variations within a reasonable time frame [26]. The modeling process can also be supported by the visualization of data in different forms [26], the provision of various differentiation opportunities, and an automatic feedback system. The technical tool should therefore be designed in such a way that the aforementioned options can be implemented. To support that the teaching and learning materials are accessible to heterogeneous groups of students, they should be usable with little or no programming experience, as students' programming skills can vary significantly. In particular, the tool should also be easily accessible. If data-heavy problems are addressed within the context of CAMMP, the technical tool must meet the additional requirements of being suitable for solving these problems. This includes processing and visualization of large amounts of data in various ways. As a result, the use of technical tools has a strong influence on the way students can access and examine data [13].

*Implementation in the speech recognition workshop*

In the CAMMP days, so-called Jupyter Notebooks[4] are used, whose application for the analysis of data is also very common in industry and research [28]. Thus, their use provides an authentic picture of modeling in practice. The Jupyter Notebooks can be made accessible in a web-based format via a so-called Jupyter Hub. Therefore, no software needs to be installed for processing, just a stable internet connection is required. Jupyter Notebooks are also particularly suitable for the use as teaching and learning materials because they contain many different building blocks in a single file. Compared to pure code files, they offer the advantage that work assignments, formulas, illustrations, and code fields are directly next to each other. The ability to flexibly edit the notebooks with HTML and LaTeX, link additional notebooks with tips or additional tasks, or insert fold-out text allows the design of differentiated teaching and learning materials (see Figure 2). In the CAMMP Jupyter Notebooks, the programming languages Julia or Python are used. During CAMMP days, either a fill-in-the-gap-approach is used, as shown in Figure 2. This means that the students fill in the gaps (i.e., *None*) in the code with formulas, numbers, or equations according to the previous task. Alternatively, interactive elements such as tables that can be filled in, free text fields, sliders that can be used to change individual components of a diagram, selection boxes, or audio players are integrated. Repetitive or particularly time-consuming calculations are outsourced to the computer by implementing a function, which is executed by the students. This includes, for example, repeatedly determining the distance between language patterns during classification. This calculation is only performed once by the students for a specific example and is then taken over by the computer in the background.

In general, Jupyter Notebooks are a technical tool that can be used to outsource computationally intensive processes and easily implement different representations of data. For a more detailed description of how Jupyter Notebooks are used in CAMMP workshops, please refer to [7].

#### 3.2.5 | DP5: Provide Opportunities for Discussion and Reflection in Cooperative Learning Formats

Given the expected complexity and diversity of the mathematical content required to tackle real-world problems, it is necessary to take a more guided approach within the given time frame. To counteract the tendency for working through the materials to result in unreflective completion of tasks, opportunities for reflection and discussion about the learning content are provided. Cooperative learning methods are employed, and tasks are integrated that require exchange among students and discussions about the subject matter. Communication skills should also be promoted through plenary discussions in which partial results are consolidated, open questions are discussed, and students' ideas are debated. In general, the teaching and learning materials should encourage students to explain and justify solutions, listen attentively to the explanations and arguments of other students, and critically question alternative approaches [13].

*Implementation in the speech recognition workshop*

To allow space for discussion about the learning content, students work through the material in pairs. Additionally, reflection tasks are placed at various points in the teaching and learning

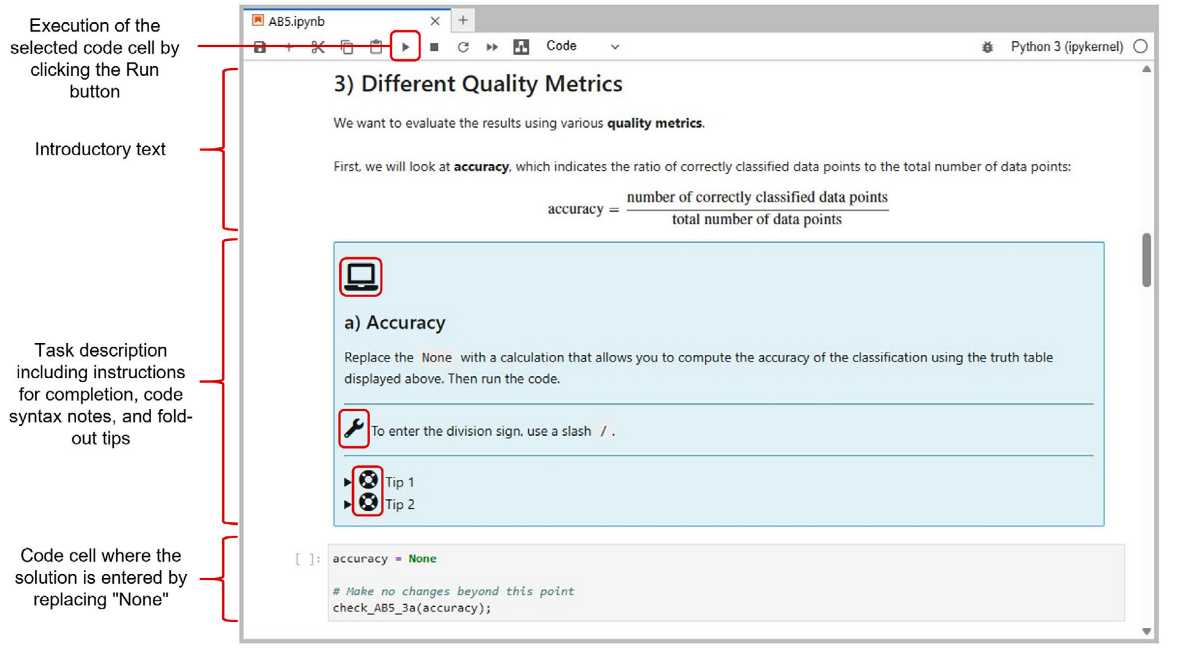


**FIGURE 2** | Screenshot of a teaching-oriented Jupyter Notebook from the teaching and learning materials on speech recognition.

material. In the case of the speech recognition workshop, a significant number of these tasks appear in step 3, where students evaluate the classification of their developed speech recognition system (see Section 2.2). The following example illustrates such a reflection task, which is presented to students after they have calculated quality metrics such as accuracy and precision:

*Task: Interpretation of performance metrics*

*Answer the following questions and enter your answers in the free text field below: What does it mean if the precision value is close to 0 or close to 1? For which words is this the case? Are the classification results satisfactory? If not, where do the results need to be improved?*

### 3.2.6 | DP6: Enable Internal Differentiation

Internal differentiation is indispensable due to the orientation of the teaching and learning material toward heterogeneous learning groups as first conceptualized by Klafki and Stöcker [29]. This means that the material should offer the possibility of individually supporting different students within the learning group without permanently dividing the entire group. Appropriate methods are taken to enable self-directed work at one's own pace during the work phases.

*Implementation in the speech recognition workshop*

To enable students to work autonomously at their own pace, the teaching and learning materials include assistance for individual tasks, which students can access as needed. This assistance is provided in the form of fold-out tips (see Figure 2) and/or links to additional digital help cards. In both cases, the tips are most of the time scaffolded hints that progressively offer increasing and, in part, more precise support, which can be revealed step by step. Figure 3 shows an example of a help card that provides scaffolded hints—initially for the two-dimensional case—for the geometric procedure to calculate the Euclidean distance between two data points. Calculating the distance is relevant for the classification procedure in step 2 of the material.

In-depth information material and additional tasks are also provided for differentiation at a higher level. To enable students to work at their own pace, the solutions are automatically checked after being entered into the digital worksheets. Since the solution cannot simply be unfolded or displayed after the first incorrect input, but instead learners receive targeted feedback—such as hints indicating where errors occurred or visualizations showing what their input would produce—they are encouraged to actively engage with the task. As a result, the solution cannot simply be copied and pasted but must actually be worked out by the students themselves. The aim is to encourage students to find their own mistakes, independently rethink their previous approach, and adapt it if necessary.

### 3.2.7 | DP7: Use a Modeling Cycle as a Metacognitive Element

Empirical research has indicated that the use of solution plans in mathematical modeling has a positive effect on organizing, elaboration, rehearsal, and planning strategies during the modeling process [30]. Based on these findings, the use of an easily accessible mathematical modeling cycle as a didactic element is proposed. In particular, the modeling cycle is used to consciously plan and delineate individual modeling steps as well as to visualize the increasing complexity of modeling approaches. The use of modeling cycles as a learning object serves as a metacognitive, strategic element that is meant to support learners in reflecting on the entire modeling process and building metaknowledge about the modeling process itself [31].

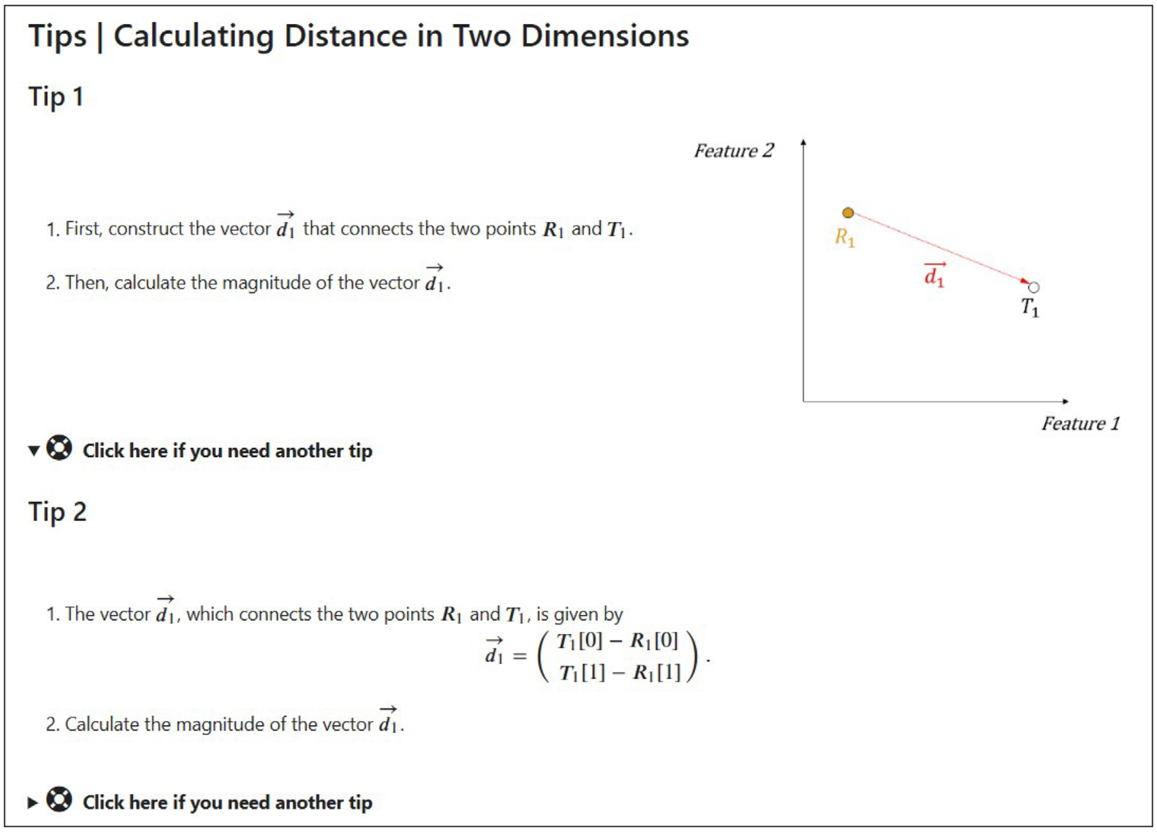


**FIGURE 3** | Screenshot of a help card from the speech recognition workshop

*Implementation in the speech recognition workshop*

In the speech recognition workshop and also in the other CAMMP days, the complete modeling cycle is fully traversed, sometimes multiple times. To avoid losing orientation in this complex process, the modeling cycle is used as a guide. For this purpose, a four-step modeling cycle, based on the cycle by Blum [32], is repeatedly addressed. Specifically, it is presented during discussion phases between work phases with digital learning materials, and the central modeling steps of the respective workshop are classified within this cycle. The cycle is intended to provide learners with orientation in joint discussions regarding both completed and upcoming modeling steps.

An example of how the completed modeling steps are categorized in the speech recognition workshop is shown in Figure 4. It illustrates the modeling cycle as it appears in the discussion phase after step 2. The real-world problem of recognizing spoken language has been simplified: instead of dealing with continuous speech, the focus is limited to the recognition of four English number words. The task of assigning a speech pattern—extracted from the audio signal—to a word meaning is mathematically described as a classification problem. The nearest neighbor method is used as classification method to predict the class labels. In the next step, the classification results will be interpreted and systematically evaluated. Thus, the modeling cycle serves both as a recap of the modeling steps completed so far and as a preview of the steps yet to come.

## 4 | Experience

CAMMP days, which typically last between 3 and 5 h, have been held regularly in Aachen since 2012 and, in subsequent years, have expanded to Karlsruhe, Salzburg, and Stuttgart. Depending on the topic, CAMMP days are aimed at different grade levels, ranging from 7 to 13 (ages 13–19). Participation in CAMMP days is generally organized by teachers, who register entire classes from grammar or comprehensive schools. As a result, the student groups are heterogeneous with respect to both their general interest in mathematics and the specific workshop topic, as well as their mathematical competence levels. Occasionally, CAMMP days are also offered to selected students who register individually; in these cases, the group of students can be assumed to be particularly interested in mathematics.

Following each workshop, students complete a questionnaire that collects sociodemographic data as well as their feedback on the workshop design, teaching materials, and perceived learning outcomes. While a comprehensive statistical analysis of these responses is not presented here, this paper highlights exemplary student comments and results of selected items that illustrate the effectiveness of our design principles.

Since 2012, more than 450 CAMMP days have taken place, involving over 7500 students. However, since not all locations used identical evaluation questionnaires and the design principles have evolved continuously over the 13-year period, this paper focuses on the results from evaluations conducted at the

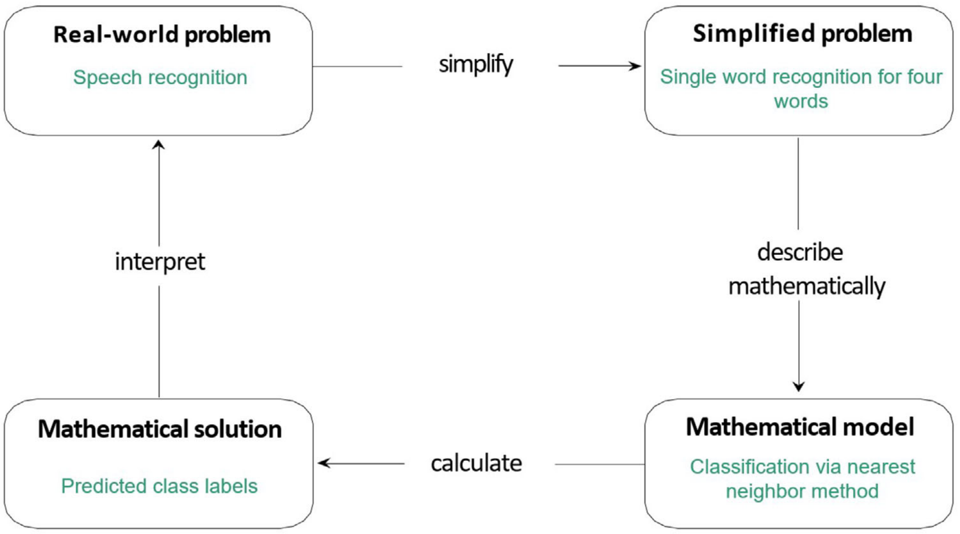


**FIGURE 4** | Modeling cycle presented in the speech recognition workshop after step 2

Karlsruhe site between September 2023 and June 2025. The questionnaire included both closed and open-ended questions. Closed questions were based on a four-point Likert scale ranging from 1 ("does not apply at all") to 4 ("applies completely"). Mean values (MW) and standard deviations (SD) for selected questions are reported, particularly those that provide insight into the extent to which our design principles and intended goals were achieved.

With regard to DP4 (appropriate technical tools) and DP6 (inner differentiation), the results for the statement "I found the automatic feedback I received after entering my solutions in the code helpful" were notably positive (Mean: 3.21, SD: 0.87, $n$ = 384), indicating that students valued the automated feedback mechanisms in the Jupyter Notebooks. Similarly, in relation to DP6, students rated the statements "I was able to work on the tasks independently" (Mean: 2.82, SD: 0.85, $n$ = 392) and "The tips/help cards were helpful" (Mean: 2.99, SD: 0.86, $n$ = 330) positively. The overall satisfaction with the workshop was also high, as reflected by the statement "All in all, I enjoyed the workshop" (Mean: 3.04, SD: 0.83, $n$ = 387).

These results suggest that students especially appreciated the automatic feedback and the design of the worksheets, with mean values consistently above 3. The ability to work independently and the usefulness of the tips were also rated positively, albeit slightly lower. Open-ended responses further underscored the impact of the workshops. When asked, "What did you learn by participating in the workshop?" students provided comments such as[5]:

- "What you can do with math" (Speech Recognition)
- "Math is not just what we do in class." (Speech Recognition)
- "Better understanding of the approach to modeling" (Speech Recognition)
- "I learned to approach big mathematical problems in a step-by-step and logical way [...]" (Netflix)
- "That mathematics is more important than I initially assumed." (Netflix)
- "That math is included in everyday things, like even Netflix." (Netflix)
- "I learned that mathematics is needed much more than I thought." (Solar Power Plant)

These statements illustrate how CAMMP days help make the role of mathematics tangible (DP1). Many students also highlighted the connection between mathematics and real-world problems addressed in the workshops. For example, in the speech recognition workshop, students frequently mentioned learning about speech recognition and AI, thereby implicitly emphasizing the relevance of mathematics to extra-mathematical, technical domains (DP2). The comments also highlight that they learned about the approach to solve real-world problems, which was one intention of the formulation of DP7.

When asked what they particularly liked about the CAMMP day, students repeatedly cited teamwork during problem-solving activities (DP5).

The positive feedback and exemplary student quotes highlighted here were selected to illustrate that it is indeed possible to introduce students to complex real-world problems and modern, applied mathematics using didactically well-prepared materials. The results thus underline the potential of the workshops to inspire students for the diverse applications and relevance of mathematics. Yet, it should be emphasized once again that the results and feedback presented here do not constitute a comprehensive scientific/empirical study on the learning effectiveness of the design principles, students' beliefs or specific workshop elements. The evaluations primarily serve the continuous further development of the CAMMP workshops. Based on feedback from both teachers and students, as well as ongoing technological advancements, the materials and structure of CAMMP days are continuously refined. For example, in the early years, Matlab was used, but as Jupyter Notebooks became more prevalent, all materials were migrated to Jupyter Notebooks. The range of topics covered by CAMMP days has also steadily expanded. From an initial three CAMMP day topics, there are now 18 workshop topics available for different age groups.

## 5 | CAMMP in University Teaching

In addition to its activities for high school students, CAMMP is embedded in various formats of university teaching at several locations. These formats enable students to experience mathematical modeling as a scientific practice and to explore connections between university-level mathematics, application domains, and in some cases also classroom implementation.

### 5.1 | Existing Activities

At several CAMMP locations, the project is already well integrated into university teaching. In the teacher education program at the University of Stuttgart, the CAMMP day materials are discussed and analyzed from a didactic viewpoint with special focus on the above design principles, and the used concepts of higher mathematics are discussed. Further, students conduct CAMMP days in course assignments. The format, therefore, supports both the deepening of mathematical content knowledge and the professional development required for future teaching practice with a particular focus on mathematical modeling.

At KIT, CAMMP day materials also serve as a basis for seminars and final theses. Students engage with mathematical content in sufficient depth to subsequently prepare it didactically and further develop it into a new CAMMP day or an additional part of an existing CAMMP day. This intensive engagement fosters a deeper conceptual understanding and provides insight into the interplay between modeling practice, didactical considerations, and the design of learning environments.

In addition, CAMMP weeks have been offered in Aachen for many years to university students in computer science, data science, mathematik, computational engineering science and simulation science. In these multi-day modeling weeks, students independently work on authentic real-world problems originating from research and industry without predetermined solution paths. The CAMMP weeks thus expand the range of university-level modeling opportunities through a research-oriented format that particularly promotes collaborative problem solving and scientific communication [33].

### 5.2 | Potential for Further Integration Into University Teaching

Building on these experiences, there is considerable potential to integrate CAMMP day materials more systematically into regular university courses, in particular to strengthen mathematical modeling competencies. One possible scenario is an inverted classroom design in which students familiarize themselves with new mathematical content by working through a real-world problem using a CAMMP day in self-study. Classroom time can then be used to discuss theoretical foundations, derive further results, and place the problem in a broader mathematical context. Given the generally stronger prior knowledge of university students, more complex content and greater levels of autonomy can be addressed.

These opportunities are particularly relevant for programs in the natural and engineering sciences. In engineering programs in particular, the explicit relevance and applicability of mathematical methods plays a central role. Here, more technical contexts, such as production processes, control problems, or optimization tasks, could be incorporated to better reflect the structure and demands of authentic engineering applications.

For such an implementation, adjustments to the existing materials would be required. This includes increasing the level of difficulty by, among other things, thematically addressing advanced mathematical concepts that are naturally required for solving the problems but had previously been treated in a more black-box manner. Furthermore, more emphasis should be placed on independent problem solving, alongside a reduction in guidance. The highly structured and finely scaffolded task design that is essential in the school context would be adapted to suit the expectations of university-level learning (concerns DP6). Similarly, the numerous short discussion phases typical of CAMMP days would likely be replaced with more extensive plenary discussions in the lecture setting (concerns DP5). In addition, a greater share of the work would shift into the self-study phase. The remaining design principles (DP1, DP2, DP3, DP4, and DP7) could be adopted unchanged.

Overall, CAMMP materials have the potential to enrich university teaching due to their problem-oriented structure and authentic application contexts. At the same time, careful adaptation is necessary to accommodate the specific conditions and learning prerequisites in higher education.

## 6 | Conclusion

This paper demonstrates how computer-aided mathematical modeling days are designed for secondary mathematics education to bridge the gap between abstract mathematical concepts and their real-world applications. Based on established design principles, CAMMP provides students with authentic opportunities to engage with complex, often data-driven problems from diverse topics such as artificial intelligence, medical imaging, and renewable energy.

The structured format of the CAMMP days allows students to experience the relevance of mathematics in solving real-world challenges with the help of computers. By emphasizing both external (real-world) and internal (mathematical) connections, the program fosters a deeper understanding of mathematical modeling and its interdisciplinary nature. Furthermore, the focus on authenticity, relevance, and differentiated access ensures that high school students with varying interests and skill levels participate and engage in productive discussions.

Student feedback confirms that CAMMP days offer valuable insights into the role of mathematics in everyday life. To further refine the design of teaching and learning materials and to better understand the impact of the format on students' learning processes, future research should complement self-assessment data with in-depth empirical studies conducted during CAMMP day implementations. In particular, the integration of CAMMP materials into regular classroom practice should be explored and supported more systematically.

Nonetheless, the theoretical considerations and insights from practical implementations presented in this paper suggest that the design principles discussed here may serve as a foundation for developing similar modeling-centered workshops for high school students and university students even beyond the CAMMP context. In particular, they can inform the creation of educational formats that aim to promote mathematical modeling competencies through authentic, interdisciplinary, and student-oriented learning environments.


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### Endnotes

[1] www.cammp.online/english/index.php

[2] For further information, see https://komms.uni-kl.de

[3] All images, including this one, have been translated for the purposes of this publication.

[4] For further information see www.jupyter.org

[5] All student quotes have been translated and any punctuation or spelling errors have been corrected.

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