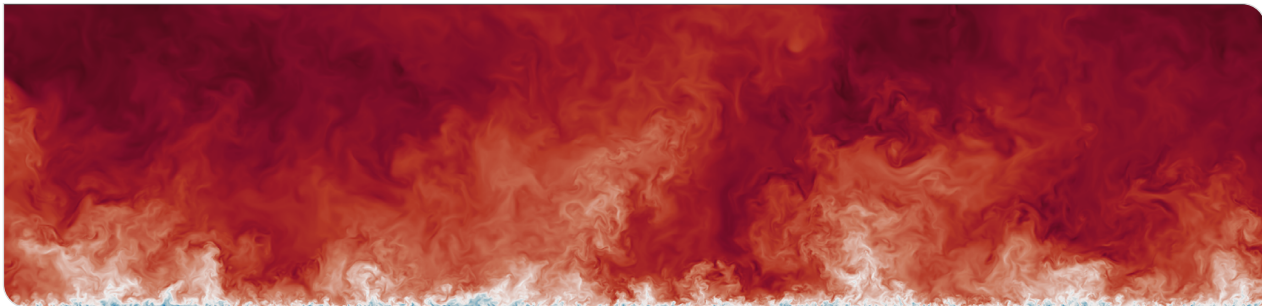
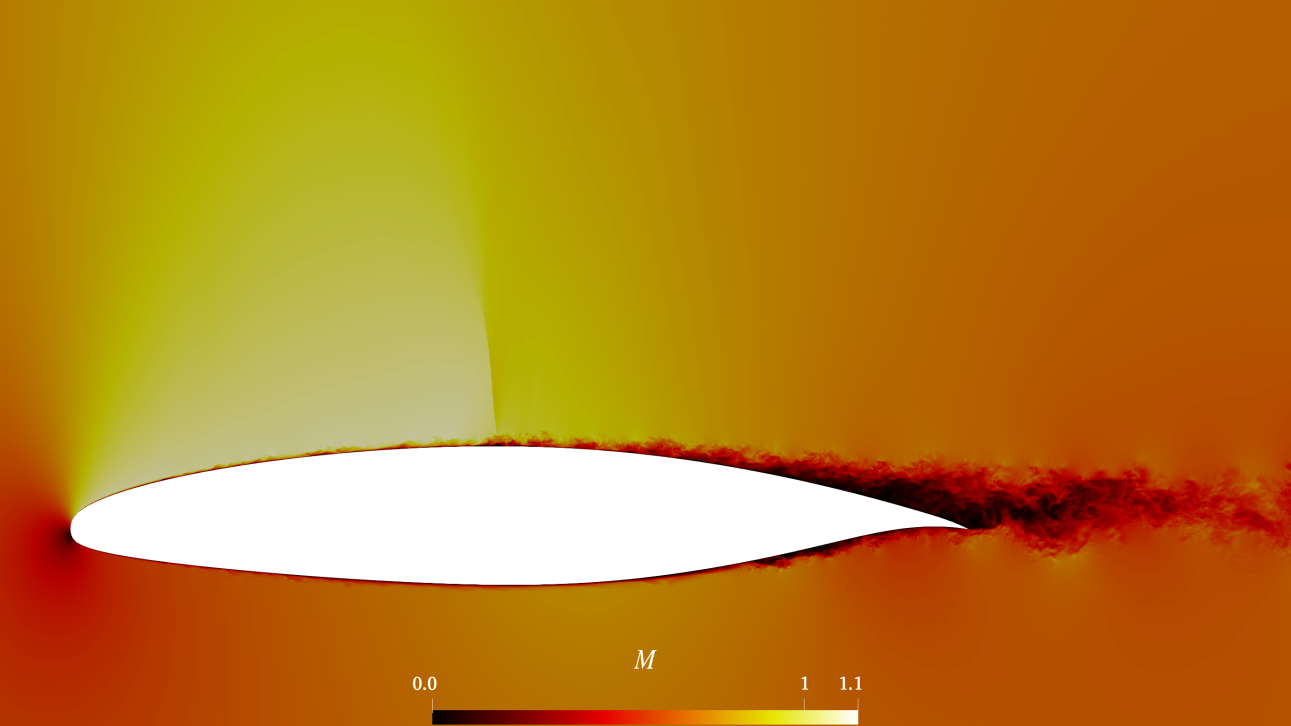


Turbulent drag reduction: what we do know, and what we don't

GAMM 95th annual meeting, Poznań, Poland

Daive Gatti | 10th April 2025



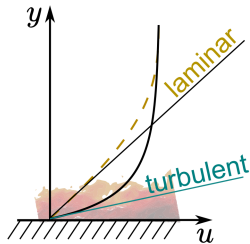


0.0

M

1

1.1



wall shear stress

$$\tau_w = \mu \left. \frac{\partial u}{\partial y} \right|_w$$

M

0.0

1

1.1



Turbulent skin-friction drag reduction

some example applications

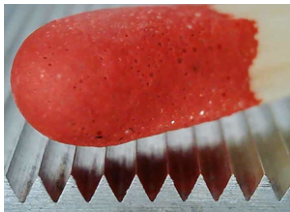


©Singletrack

drag reducing additives

in pipelines

~50% less drag



von Deyn, Gatti, Frohnapfel, JFM (2025)



©Lufthansa Group & BASF

riblets

on airplanes and other vehicles

~8% less drag

fuel consumption reduction: ~200kg/h

for a B777-300ER, Lufthansa Group, 2024

Main challenges

on the way from research to applications

- **Reynolds number:** Research at **lower** values of Re , applications at **larger** ones!
 - Does low- Re evidence transfer to high- Re flows?

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- **Flow complexity:** Research in **simple** flows, applications in **complex** ones!
 - How does drag reduction affect lift and other sources of drag?
- **Physics: not fully understood!** More on this in the next talk
 - What is the most realisable and efficient way of achieving drag reduction?

Main challenges

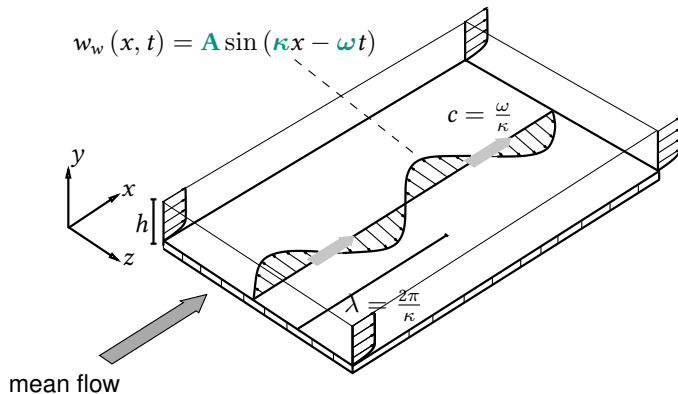
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The **Reynolds-number** effect
on turbulent skin-friction drag reduction

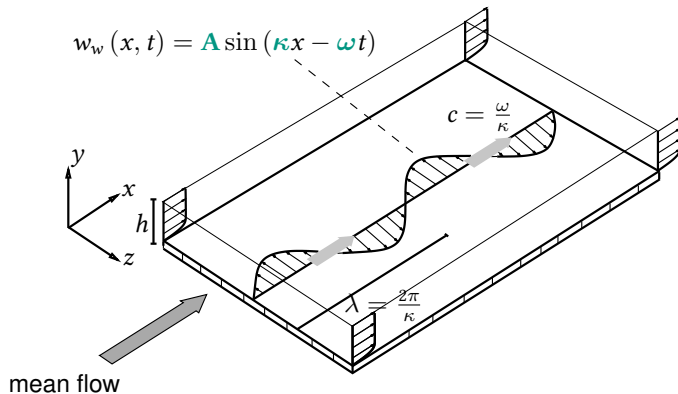
Streamwise-travelling waves of spanwise wall velocity

Quadrio, Ricco & Viotti, JFM (2009)



Streamwise-travelling waves of spanwise wall velocity

Quadrio, Ricco & Viotti, JFM (2009)



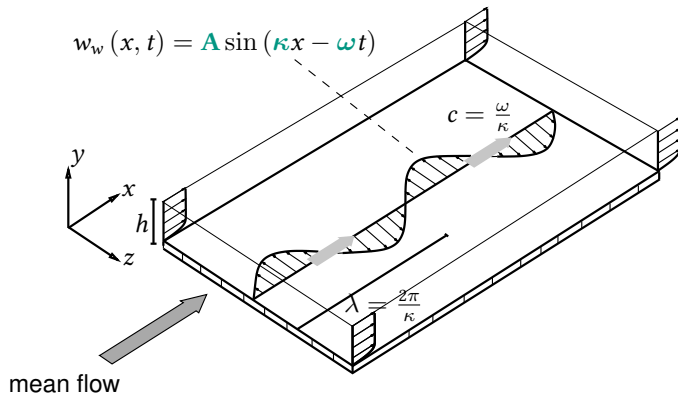
Pumping power: P_p

Drag reduction rate:

$$\mathcal{R} = 1 - \frac{P_p}{P_{p0}} = 1 - \frac{c_f}{c_{f0}}$$

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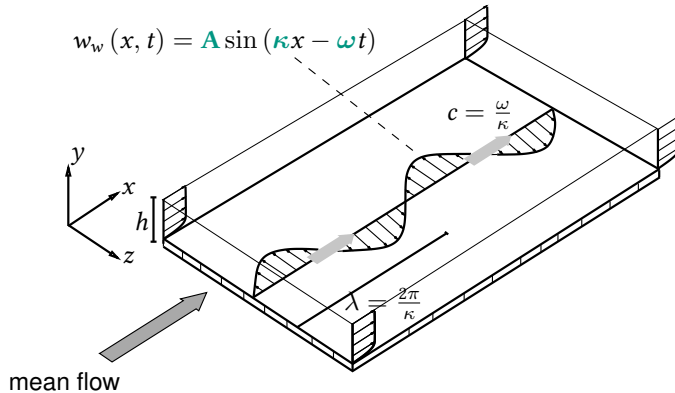
Control input power: P_{in}

Net power saving rate:

$$\mathcal{S} = \mathcal{R} - \frac{P_{in}}{P_{p0}}$$

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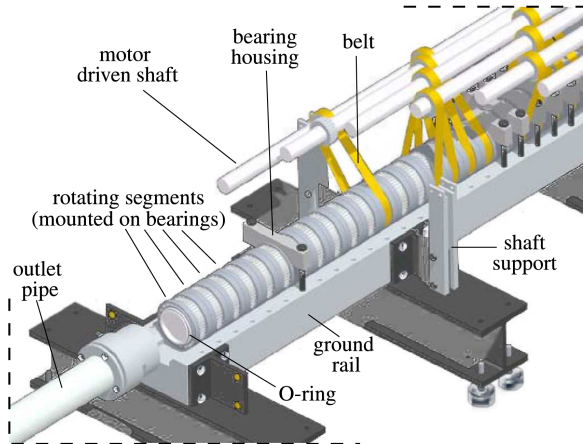
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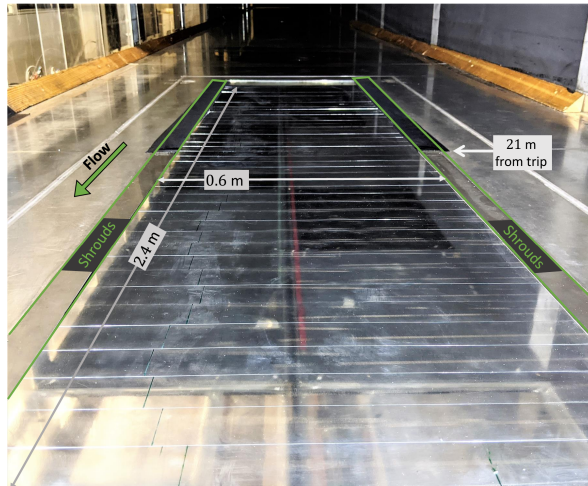
\mathcal{R} and \mathcal{S} depend on A , κ , ω
and Reynolds number $Re!$

Experimentally verified!



Aueri *et al.*, PoF (2010)

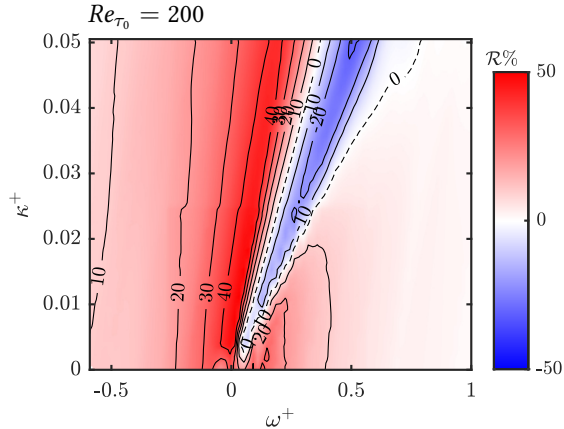
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Marusic *et al.*, Nat.Comm. (2021)

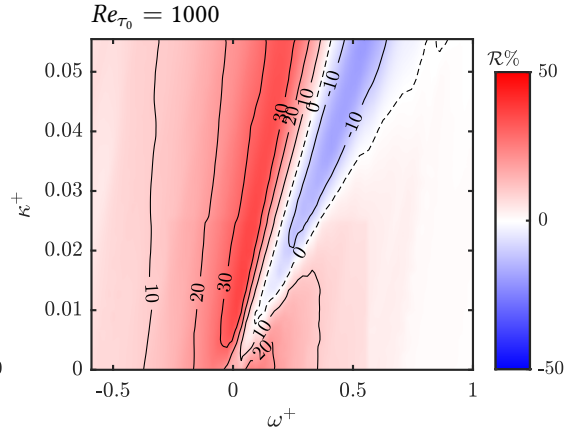
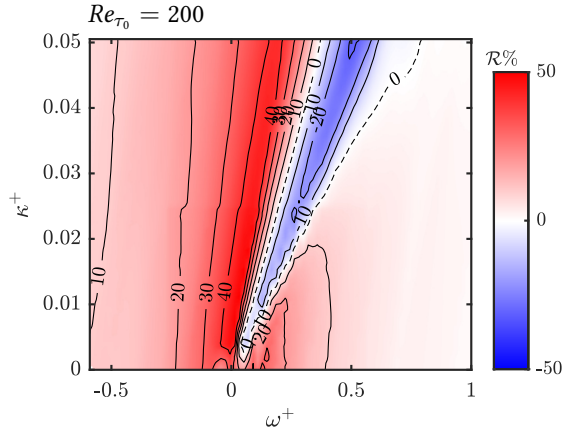
The drag reduction map at $A^+ = 12$

Data by Gatti & Quadrio, JFM (2016)



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The *Re*-dependency of drag reduction

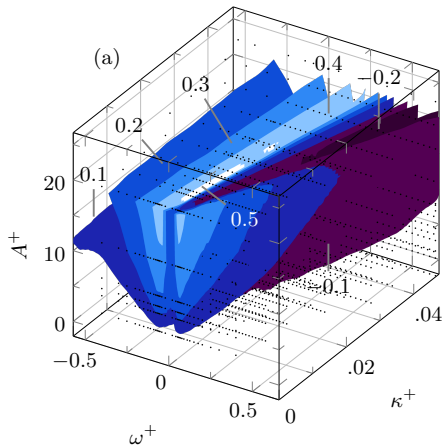
Twenty years of research, and counting...

- Difficult to study: many parameters, difficulty to reach large Re
- \mathcal{R} decreases with Re
- Early studies on wall oscillations ($\kappa = 0$):
 - \mathcal{R} decreases rapidly with Re as $\mathcal{R} \sim Re_{\tau_0}^\gamma$
Choi, Xu & Sung, AIAA (2002); Touber & Leschziner, JFM (2012)
- Later studies on travelling waves:
 - \mathcal{R} can decrease slowly with Re
 - $\gamma = \gamma(A, \kappa, \omega)$
Gatti & Quadrio, PoF (2013); Hurst, Yang & Chung, JFM (2014)
- First predictive model by Gatti & Quadrio, JFM (2016)

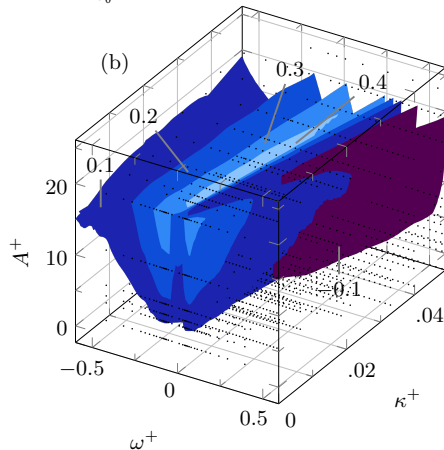
The GQ database

4020 Direct Numerical Simulation (DNS) in small domains, 20 DNS in large domains

$Re_{\tau_0} = 200$

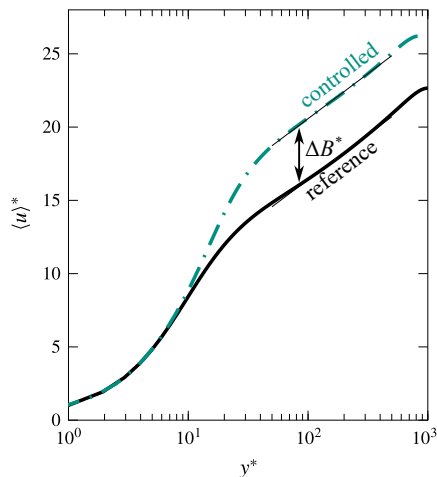


$Re_{\tau_0} = 1000$



The GQ model

A simple predictive model for the Re -dependency of drag reduction



- Near-wall manipulation induces a “roughness function” ΔB^*

$$U^*(y^*) = \frac{1}{k} \ln y^* + B_0^* + \Delta B^*$$

- $\Delta B^* = \Delta B^*(A^*, \kappa^*, \omega^*)$ is Re -independent

- A modified P-vK friction relation describes the Re -dependency of \mathcal{R} :

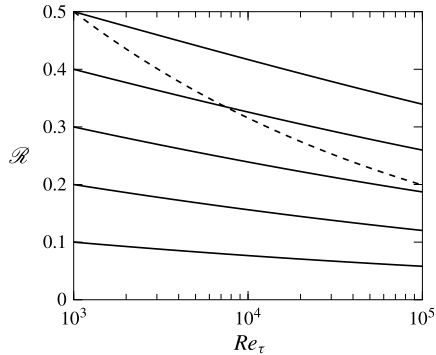
$$\Delta B^* = \sqrt{\frac{2}{C_{f_0}}} \left[(1 - \mathcal{R})^{-1/2} - 1 \right] - \frac{1}{2k} \ln(1 - \mathcal{R})$$

- \mathcal{R} decreases with Re due to Re -dependency of C_{f_0}

- Rouhi *et al.*, JFM (2023) confirm the GQ model up to $Re_{\tau_0} = 4000$ using LES in small domains

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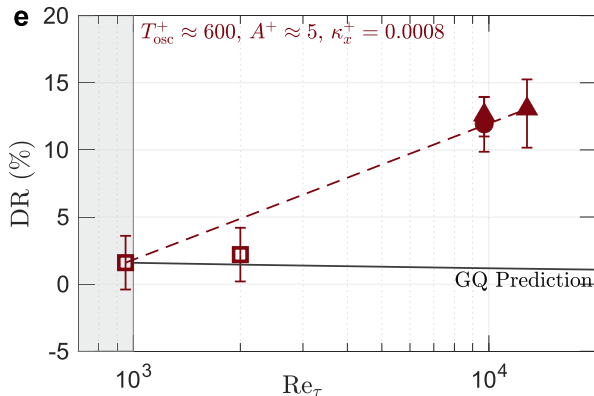
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A contrasting evidence: can \mathcal{R} increase with Re ?

A recent study by Marusic et al., Nature comm. (2021)

$T_{osc}^+ > 350$: Large-eddy actuation



$Re_{\tau_0} \leq 2000$:

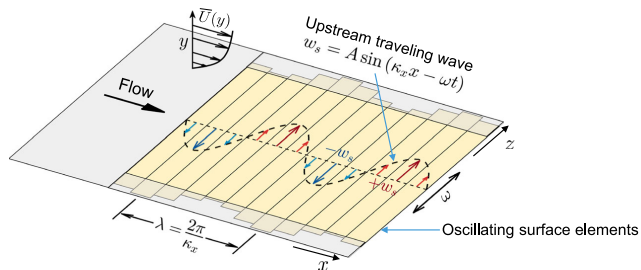
- Large Eddy Simulation
- Turbulent open channel flows in small domains
- Ideal forcing

$6000 \leq Re_{\tau_0} \leq 12800$:

- Laboratory experiment
- Turbulent boundary layers
- Discrete forcing

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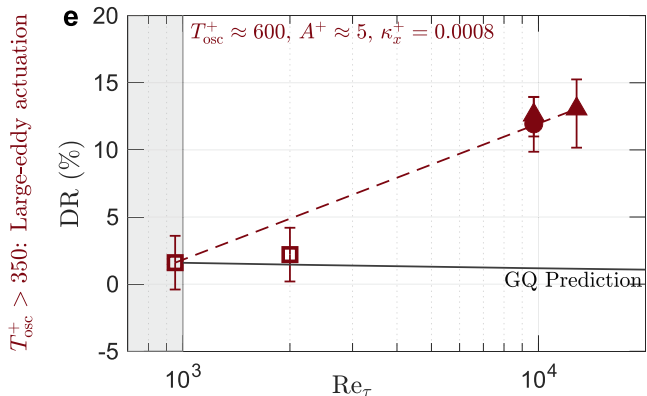
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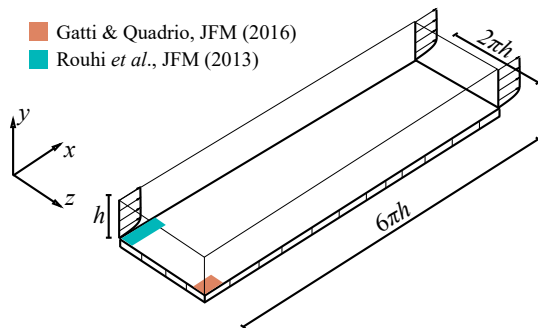
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The new numerical database

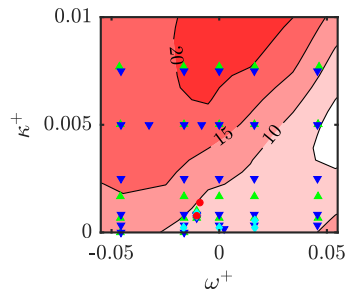
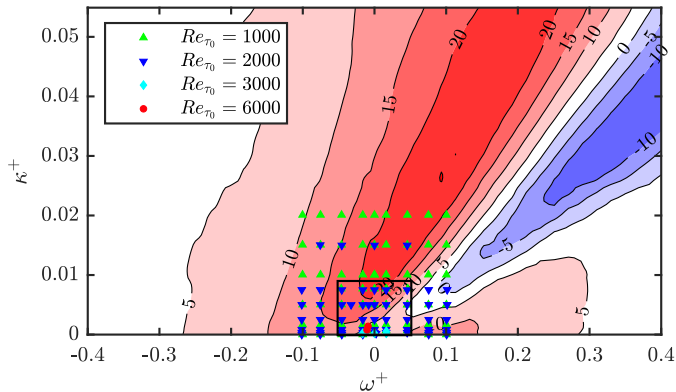
Setup

- Direct numerical simulation of turbulent open channel flows
- $Re_{\tau_0} = \{1000, 2000, 3000, 6000\}$
- Domain size: $L_x = 6\pi h$, $L_z = 2\pi h$
- Resolution: $\Delta x^+ \sim 8.5$, $\Delta z^+ = 4$
- Natural grid stretching by Pirozzoli & Orlandi, JCP (2021) in wall-normal direction
- GPU-accelerated second-order finite difference solver
- Uncertainty on C_f always below 1% with method by Russo & Luchini, JCP (2017)

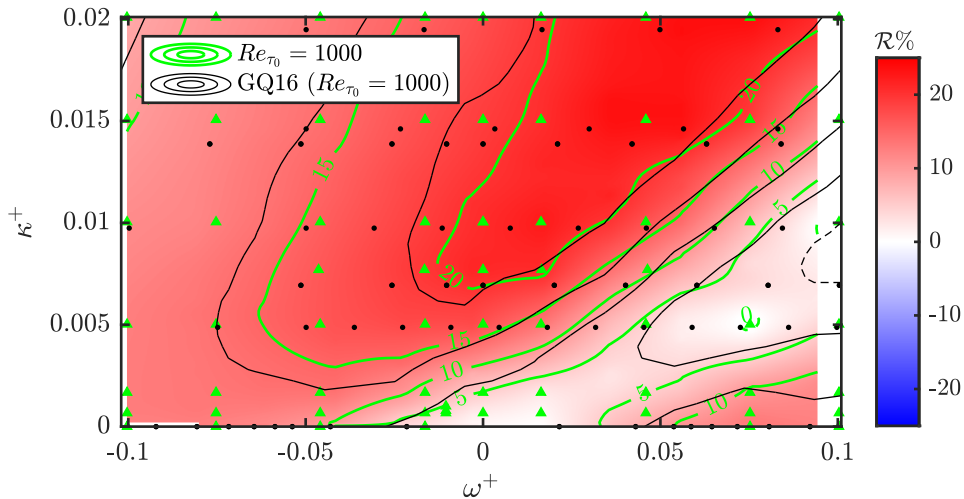


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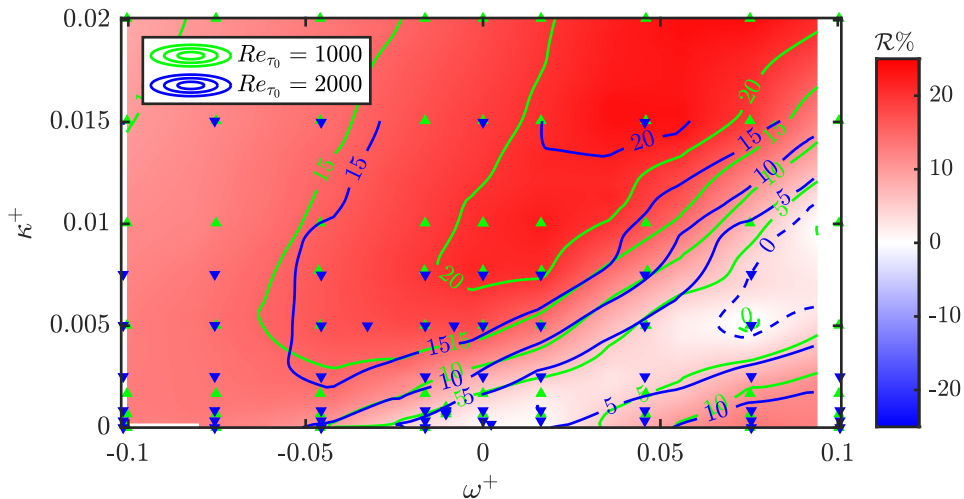
147 simulations at $A^+ = 5$ up to $Re_{\tau_0} = 6000$



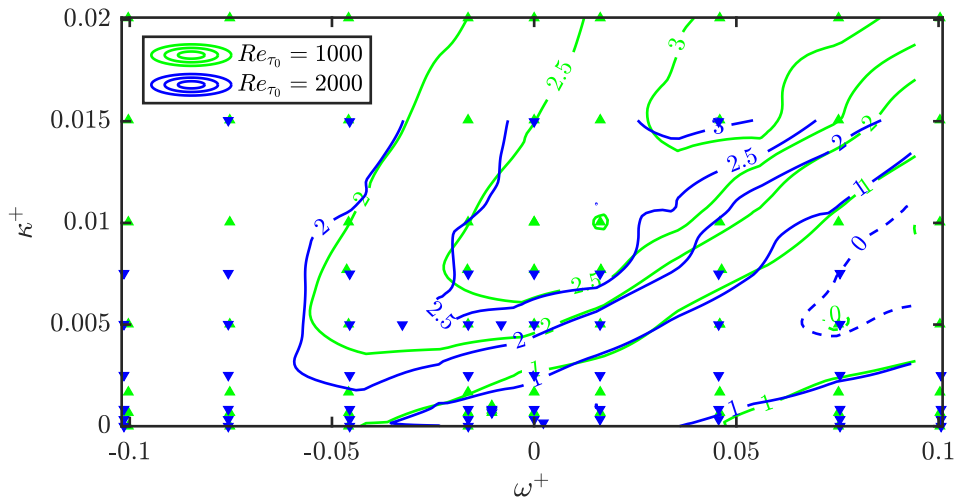
Drag reduction rate \mathcal{R}



Drag reduction rate \mathcal{R}

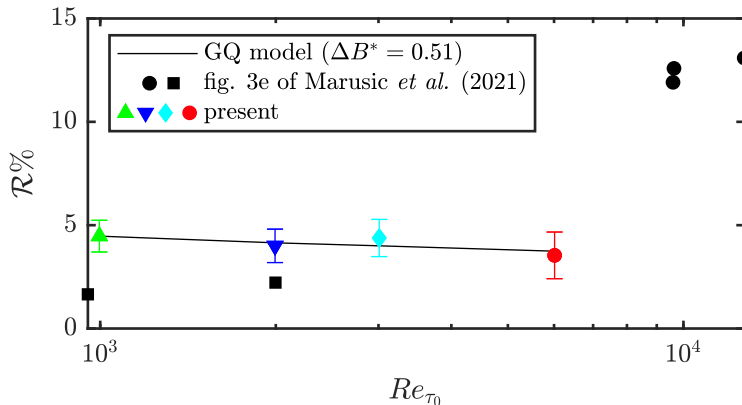


Roughness function ΔB^*



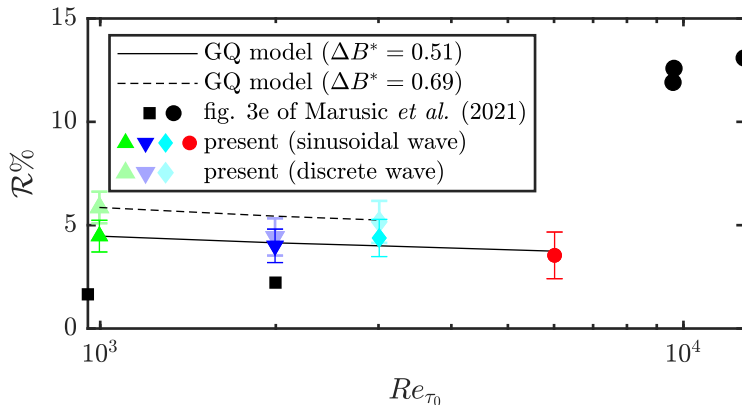
Does \mathcal{R} increase with Re ?

Marusic et al. (2021) parameters: $A^+ = 5$, $\kappa^+ = 0.00078$, $\omega^+ = -0.0105$



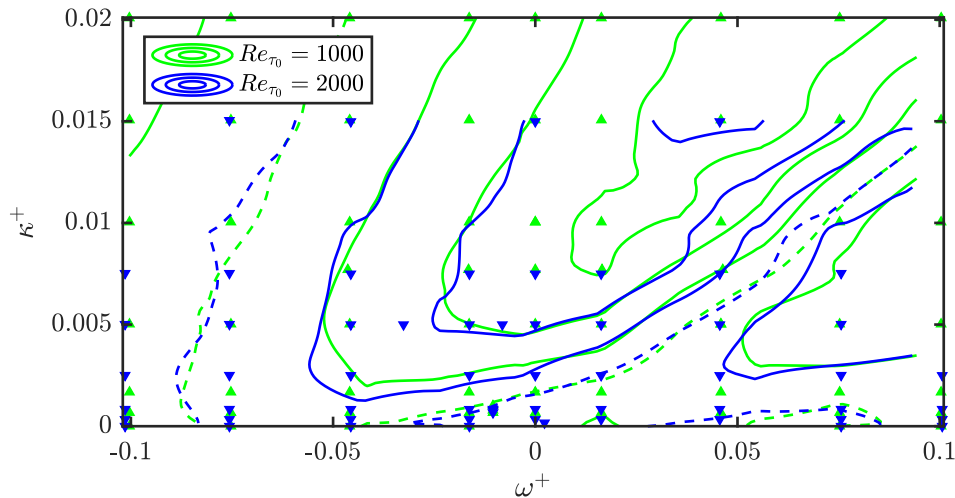
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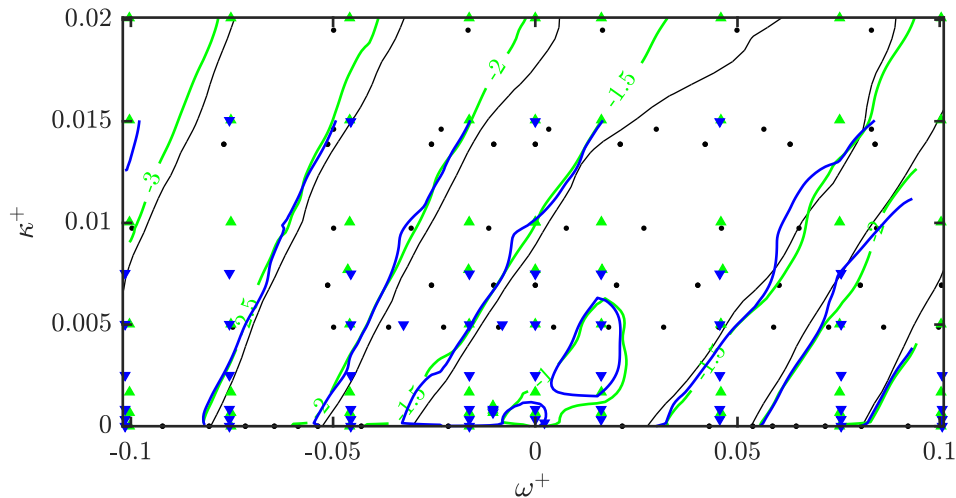
Net power saving \mathcal{S}

$$\mathcal{S} = \mathcal{R} - P_{in}/P_{p0} = \mathcal{R} - P_{in}^+/U_{b0}^+$$



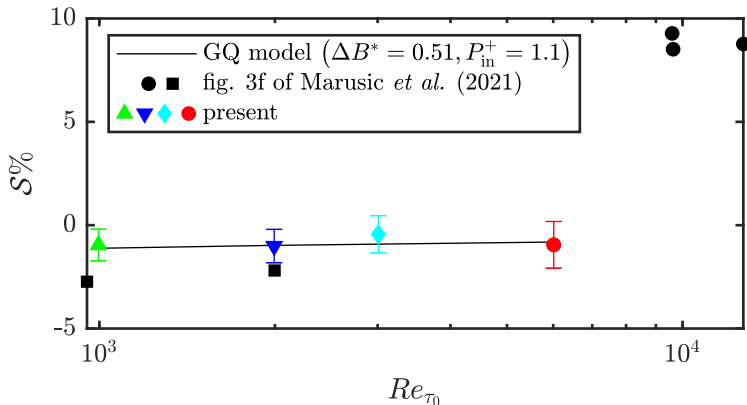
Control input power P_{in}^+

Constant with Re in viscous units



Reynolds-number dependence of \mathcal{S}

Marusic et al. (2021) parameters: $A^+ = 5$, $\kappa^+ = 0.00078$, $\omega^+ = -0.0105$



The *Re*-dependency of drag reduction

What do we know now?

- 😞 GQ model confirmed: \mathcal{R} decreases with Re

The Re -dependency of drag reduction

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- Increase of \mathcal{R} with Re observed by Marusic *et al.*, Nat. Comm. (2021) not confirmed

The *Re*-dependency of drag reduction

What do we know now?

- 😞 GQ model confirmed: \mathcal{R} decreases with Re
- S also predictable, decay is slower than \mathcal{R}
- Increase of \mathcal{R} with Re observed by Marusic *et al.*, Nat. Comm. (2021) not confirmed
- 😊 GQ model predicts only mild decay of \mathcal{R} with Re

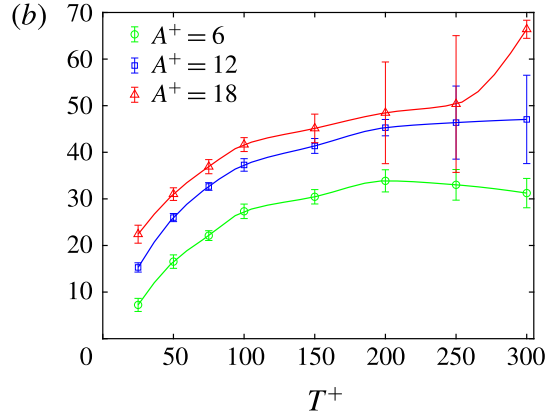
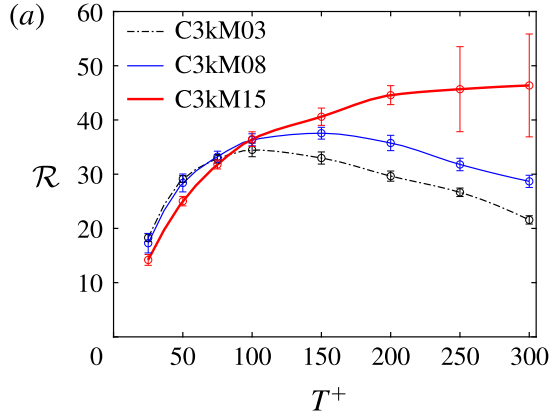
Some still open question:

- Would other flows behave similarly?
- What is the reason for the discrepancy?
 - e.g. \mathcal{R} at $A^+ = 2.5$, $\kappa^+ = 0.0014$, $\omega^+ = -0.009$, $Re_{\tau_0} = 6000$
 - 6% Chandran *et al.*, JFM (2023)
 - $2.3\% \pm 1.1$ present

The Mach-number effect
on turbulent skin-friction drag reduction

The Ma -dependency of drag reduction

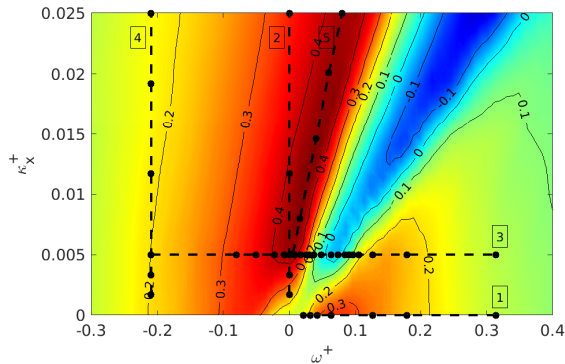
Yao & Hussain, JFM (2019): \mathcal{R} increases with Ma



The compressible numerical database

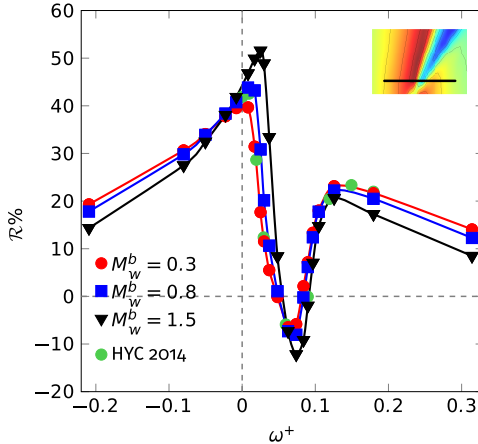
Setup

- Direct numerical simulation of turbulent **channel flows**
- STREAMS solver by Bernardini *et al.*, CPC (2021)
- Domain size: $L_x = 6\pi h$, $L_z = 2\pi h$
- Resolution: $\Delta x^+ \sim 8.5$, $\Delta z^+ = 4$
- Constant flow rate (CFR) : $U_b = \text{const}$
- $Re_{\tau_0} = 400$
- $M_w^b = U_b/c_w = U_b/\sqrt{\gamma RT_w} = 0.3, 0.8, 1.5$ (as Yao & Hussain, JFM 2019)
- Control: $A^+ = 12$, 42 combinations of (ω^+, κ_x^+)



Drag reduction \mathcal{R}

$$\kappa_x = 0.005$$



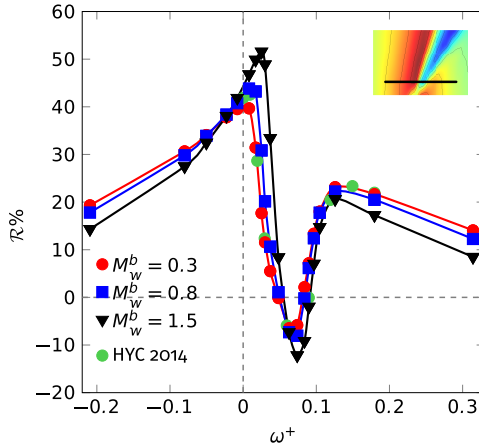
Compressibility effect ($M_w^b \uparrow$):

- Large positive effect at small ω
- Small negative effect at large $|\omega|$

$$\mathcal{R}_{M=0.3} \approx 40 \% \quad \mathcal{R}_{M=1.5} \approx 52 \%$$

Drag reduction \mathcal{R}

$$\kappa_x = 0.005$$



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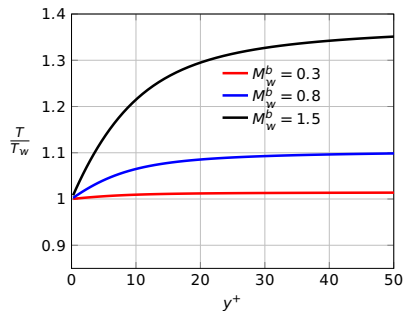
- Large positive effect at small ω
- Small negative effect at large $|\omega|$

Is the increase of \mathcal{R}
really due to compressibility?

The effect of the temperature

- $T_b \uparrow$ with Ma and \mathcal{R} depending on (ω, κ_x)

$$T_b = \frac{1}{2h\rho_b U_b} \int_{-h}^h \langle \rho u T \rangle dy$$

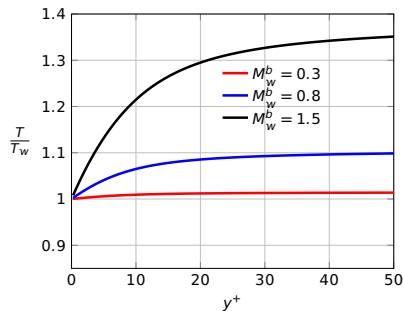


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Compressibility or indirect effect of $T_b \uparrow$?



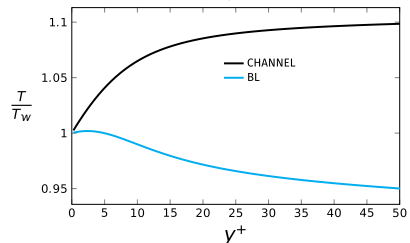
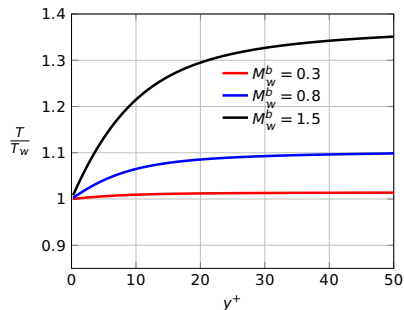
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$$T_b = \frac{1}{2h\rho_b U_b} \int_{-h}^h \langle \rho u T \rangle dy$$

Compressibility or indirect effect of $T_b \uparrow$?

- T profile not representative of external flows



New comparison strategy

$$\frac{\partial \rho e}{\partial t} + \frac{\partial \rho (e + p/\rho) u_j}{\partial x_j} = \frac{\partial \sigma_{ij} u_i}{\partial x_j} - \frac{\partial q_j}{\partial x_j} + f u_1$$

New comparison strategy

$$\frac{\partial \rho e}{\partial t} + \frac{\partial \rho (e + p/\rho) u_j}{\partial x_j} = \frac{\partial \sigma_{ij} u_i}{\partial x_j} - \frac{\partial q_j}{\partial x_j} + f u_1$$

Zero Bulk Cooling (ZBC)

$$\Phi = 0$$

T_b/T_w varies with Ma and \mathcal{R}

$$\frac{\partial \rho e}{\partial t} + \frac{\partial \rho(e + p/\rho)u_j}{\partial x_j} = \frac{\partial \sigma_{ij}u_i}{\partial x_j} - \frac{\partial q_j}{\partial x_j} + fu_1 - \Phi$$

Zero Bulk Cooling (ZBC)

$$\Phi = 0$$

T_b/T_w varies with Ma and \mathcal{R}

Constrained Bulk Cooling (CBC)

$$\Phi = f(\Theta = \text{const} = 0.75)$$

Diabatic Parameter (Θ)

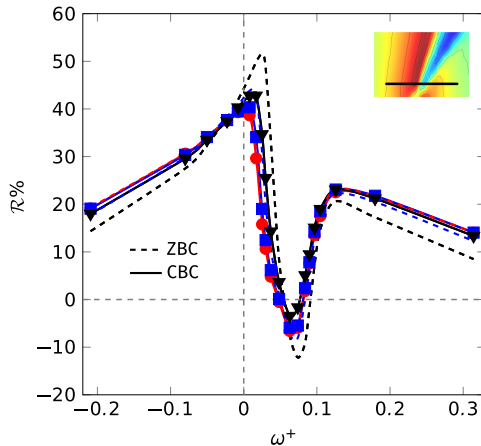
Θ = Fraction of the available kinetic energy transformed into thermal energy at the wall

$$\Theta = \frac{T_w - T_b}{T_r - T_b}, \quad \frac{T_r}{T_b} = \left(1 + \frac{\gamma - 1}{2} r (M_w^b)^2\right)$$

T_b/T_w constant

Drag reduction \mathcal{R}

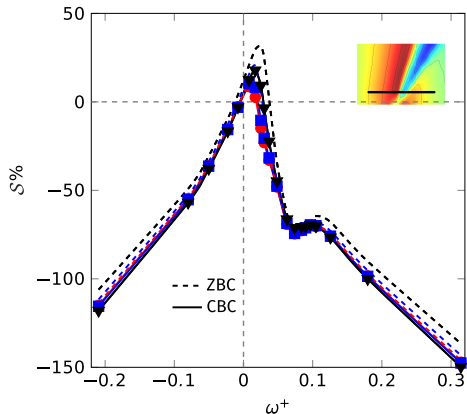
ZBC versus CBC



$$\mathcal{R}_{M=0.3} \approx 40\% \quad \mathcal{R}_{M=1.5}^{ZBC} \approx 52\% \quad \mathcal{R}_{M=1.5}^{CBC} \approx 43\%$$

Net power saving \mathcal{S}

ZBC versus CBC



$$\mathcal{S} = \mathcal{R} - \frac{P_{in}}{P_0}$$

$$\mathcal{S}_{M=0.3} \approx 10\%$$

$$\mathcal{S}_{M=1.5}^{ZBC} \approx 31\%$$

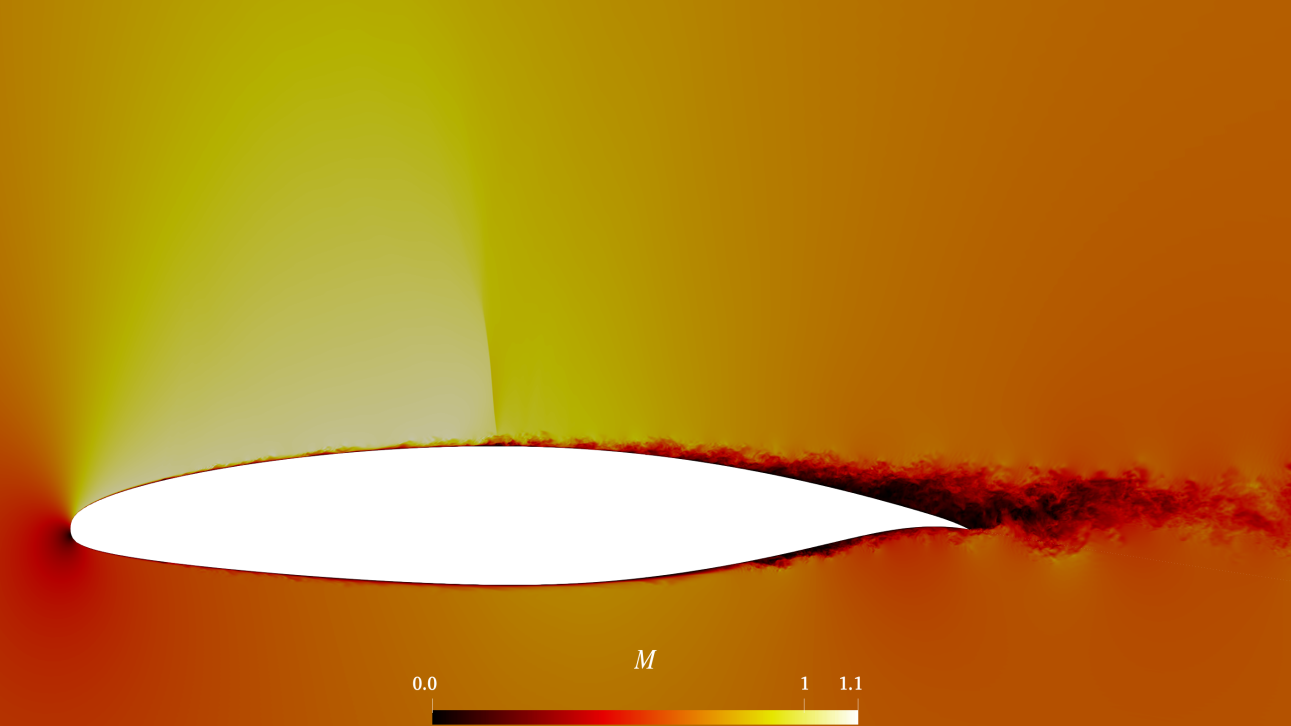
$$\mathcal{S}_{M=1.5}^{CBC} \approx 18\%$$

The Ma -dependency of drag reduction

What do we know now?

- The effects of **compressibility** (Ma) and **temperature** should be **addressed separately**
- Two different comparison strategies:
 - Zero Bulk Cooling (Θ =variable)
 - Constrained Bulk Cooling (Θ =const.): decouples compressibility and temperature
- **Compressibility** has **little effects** on \mathcal{R}
- **Temperature** has **large effects** on \mathcal{R}
- Results published in Gattere *et al.*, JFM (2024)

What is still left
to be done?



0.0

M

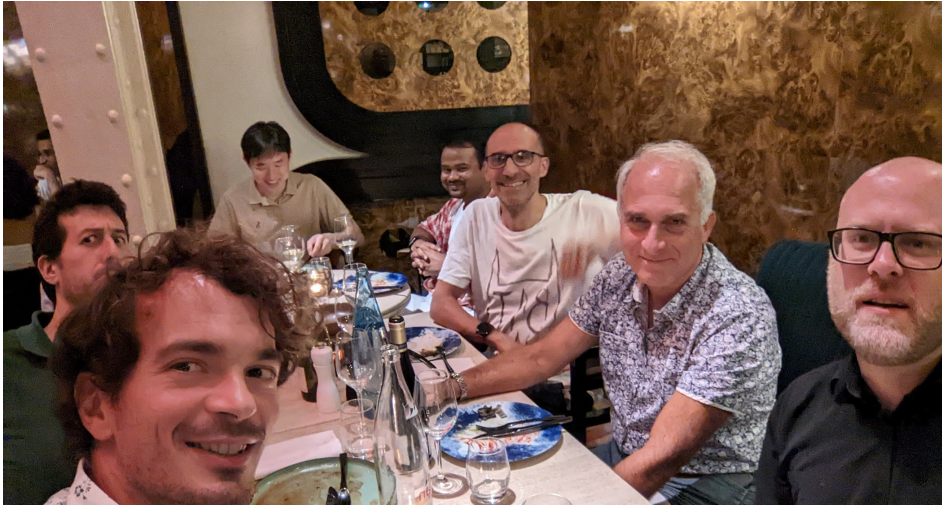
1

1.1

Thanks!



Thanks!



Parameters of Chandran et al., JFM 2023

Not constant in viscous units!

