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Assessing modeling choices for distributed energy resources in demand side management based on the Shapley value

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Abstract

Distributed Energy Resources (DERs), i.e., small-scale electricity systems composed of local generators, storage systems and flexible demand, play a crucial role in Demand Side Management (DSM). In order for DSM measures to control a huge number of DERs in an optimal way, a balance between their model complexity and level of detail needs to be found. To deal with this challenge, we propose the evaluation of different white-box model variants, consisting of a different set of submodels, based on the Shapley value and the maximal power of the DER. The resulting Shapley value is used to assess which submodels contribute most to the model's utility, which we measure by its accuracy. The approach is applied to a heat pump case study using a white-box modeling framework. In the modeling framework, four submodels are defined and assessed by utilizing the proposed approach. Their contributions are quantified based on the corresponding Shapley values, which are 10.58, 0.68, 0.25 and 0.52 *kW* respectively. Specifically, the contribution of the larger hot water storage (HWS), i.e. 0.52, is quantified as 2.08 times than that of the smaller HWS, i.e. 0.25, despite the larger HWS having only 1.39 times the storage capacity of the smaller HWS. It indicates that the contribution of the storage submodel is not linearly proportional to its physical capacity. The results demonstrate the feasibility of the proposed approach for white-box DER models and extend its applicability beyond black-box models in which it has primarily been studied. Therefore, this work provides a new perspective on utilizing the properties of the Shapley value to assess DER modeling choices.

Keywords Distributed energy resource, Quantification, The Shapley value, Modeling

Introduction

Advancing towards a sustainable energy system, enhancing energy flexibility has become increasingly important to accommodate the growing integration of renewable energy sources. According to the federal statistical office of Germany [1], the total energy consumption of private households in Germany has only slightly increased by 7.8% from 2015 to 2021. During the same period, however, the share of renewable energy experienced a substantial growth of 23.3%, reaching over 14% of the primary energy usage in 2021. Energy flexibility refers not only to the ability to adjust power generation, which is challenging with renewables due to their intermittent nature, but also to adapting electricity load on the demand side, which is a measure for Demand Side Management (DSM) [2]. To adapt electrical load on the demand side, load-changing ability is required and this ability is usually provided by Distributed Energy Resources (DERs). DERs are small-scale electricity generation, storage and consumption systems that are able to be controlled. For instance, in the residential sector, assets such as heat pumps for heating and supplying domestic hot water, rooftop solar panels for self-generating electricity, battery storages, home charging stations for e-mobility can be classified as DERs. To investigate the effects of DERs, an indispensable step is to develop DER models. In [3], we investigate the relationship between model complexity and model utility in the context of DER for a heat pump system. The modeling of the heat pump system is implemented based on a white-box approach and some models can be divided into several submodels. In order to understand the individual effects or contribution of each submodel, it's necessary, but also challenging, to quantify its contribution to the overall model utility. For this purpose, we propose a new quantification approach in the present paper based on the research results in our previous work [3], to measure the contribution of the submodels.

The main contributions of the present paper are as follows:

- A new approach based on the Shapley value, for quantifying the contribution of submodels in the context of white-box DER models, is proposed.
- The approach is validated based on a heat pump case study using a white-box modeling framework.
- The approach can serve as a complementary component to another proposed hypothesis in [4], offering additional support for decision-makers in better understanding and effectively utilizing different white-box DER models and submodels.

The remainder of the paper is divided into the following five parts. In Section "Theoretical Foundation" the theoretical foundation of the Shapley value in game theory is given, which serves as the basis for the introduced quantification approach. Section "Related Work" presents an overview of related work on the application of the Shapley value across different domains and highlights the novelty of the approach introduced within the framework of this study. Section "Approach and Case Study" applies the approach to a heat pump case study and evaluates the feasibility of this quantification method. Finally, the discussions and conclusions of this work as well as suggestions for future improvements are highlighted in Section "Discussion" and Section "Conclusion and Outlook".

Theoretical foundation

In 1951, the mathematician Lloyd Shapley has introduced a solution concept in an n-Person Game, i.e., cooperative game theory, for quantifying each player's contribution by evaluating the marginal impact they make when added to all possible subsets of players, and computing the average of these impacts [5]. This quantified value is also known as the Shapley value. His perspective on the solution concept has proved to be the only solution that satisfies all four foundational properties for contribution measures, namely efficiency, symmetry, linearity and null player, in 1953 [6]. This establishes the theoretical groundwork for the method's future use in a wide range of disciplines.

According to [7], the Shapley value φ of player *i* is calculated by using the following equations, where *S* is a subset or a coalition of the whole set *N* of *n* players and *v* is the value function to determine the value or worth of a coalition. $|S|$ represents the number of players in this coalition *S*, whose cardinality is between 0 and $n-1$.

$$\varphi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(n-|S|-1)!}{n!}(v(S \cup \{i\}) - v(S)) \tag{1}$$

$$= \frac{1}{n} \sum_{S \subseteq N \setminus \{i\}} \binom{n-1}{|S|}^{-1} (v(S \cup \{i\}) - v(S)) \tag{2}$$

As mentioned above, the Shapley value satisfies the four foundational properties for contribution measures. The first property is efficiency, which means the sum of Shapley values of all players equals the value of the whole set *N*. This property guarantees that the gain is distributed among all players. Equation (3) [6] formulates this property.

$$\sum_{i \in N} \varphi_i(v) = v(N) \tag{3}$$

The second property is symmetry, which means that if two different players *i* and *j* contribute equally to all possible coalitions, then their Shapley values are also equal. This property guarantees that the players are treated equally if contributing equally. Equation (4) [6] formulates this property, where *S* is an arbitrary coalition without *i* and *j*.

$$v(S \cup \{i\}) = v(S \cup \{j\}) \Rightarrow \varphi_i(v) = \varphi_j(v) \tag{4}$$

The third property is linearity, which means the Shapley values of the player *i* in different value functions are additive (Equation (5)) and homogeneous (Equation (6)). This property considers mainly the situation where more value functions are involved in the analysis. The following equations [6] formulate this property, where *v* and *w* are different value functions and *a* is any real number.

$$\varphi_i(v + w) = \varphi_i(v) + \varphi_i(w) \tag{5}$$

$$\varphi_i(av) = a\varphi_i(v) \tag{6}$$

The last foundational property is the so-called null player or dummy player, which means that if player *i* does not contribute to any coalition in terms of the value function *v*, then the Shapley value of *i* is zero and this player *i* is the so-called dummy player or null player. This property further guarantees the fair contribution of each player when evaluating the marginal impact they make. Equation (7) [6] formulates this property, where *S* is an arbitrary coalition without *i*.

$$v(S \cup \{i\}) = v(S) \Rightarrow \varphi_i(v) = 0 \tag{7}$$

It is precisely because the Shapley value satisfies these four foundational properties for contribution measures that it has found applications not only in game theory, but also in a wide range of other fields with the idea of "players" being replaced with other domain concepts. Some of them are discussed in the following section.

Related work

In recent years, the concept of the Shapley value, initially proposed within game theory, has found widespread applications in various domains beyond its original context, such as explainable Artificial Intelligence (AI), cost allocation and energy systems [8]. For instance, Lundberg et al. [9] have proposed a unified framework named as SHapley Additive exPlanations (SHAP), which is based on Shapley value, for interpreting predictions. In 2020, they further improved the interpretability of tree-based models, such as random forests, decision trees, and gradient boosted trees, by presenting TreeExplainer, an explanation method for trees in these models based on Shapley value [10]. However, Kumar et al. [11] point out that some further assumptions, such as the feature independence, are needed to use this framework for interpretation. This could lead to an inaccurate approximation of the true Shapley values. To deal with the dependence of features in machine learning models, [12] have improved the Kernel SHAP method by estimating the conditional distribution to incorporate dependence. This improvement has led to a more accurate approximations of the true Shapley values compared to the original Kernel SHAP approach. In 2023, the team that developed SHAP has published a review article further discussing algorithms to generate Shapley value explanations [13]. By comparing 24 distinct algorithms, the authors highlight the key innovations in recent approaches for estimating Shapley value explanations and give recommendations, such as the importance of the feature-removal approach and the specific estimation strategy, for the utilization of Shapley value explanations in both industry and academia [13]. Another survey carried out by [14] focuses on methods for computing the Shapley value in the domain of databases and machine learning, where they also point out the idea of "players" are being tuples, tables, features, data samples and models in specific applications. To deal with the interpretability problem, which exist in some data Shapley valuation methods, Li et al. [15] propose a learning framework that maps sample characteristics directly to their Shapley values. The framework contains an innovative neural regression tree as its core component and is tailored to deep learning tasks.

In the domain of energy systems, the Shapley value has also proven to be highly applicable as well. For approximating the Shapley value in realistic community energy settings, a team in the Netherlands has compared several existing methods and proposed a more efficient one by clustering consumers into a smaller number of demand profiles [16]. In other application, Cai et al. [17] have presented an Energy Management (EM) strategy for residential microgrid systems using Model Predictive Control (MPC)-based Reinforcement Learning (RL) and Shapley value, where the Shapley value approach is applied as a feasible solution for distributing the collective cost. Considering energy system operators, a distributed approach of Shapley value calculation for the redispatch congestion cost allocation to deal with data privacy concerns with respect to real-world implementation is briefly discussed in [18]. For device flexibility in DSM, the authors [8] have proposed a method to determine the value of flexible assets using average marginal contributions, based on the Shapley value. In their scenario, the optimization goal is to minimize the Euclidean norm of a single smart house profile using DSM.

As summarized in Table 1, most of these recent publications are focused either on the methods for computing the Shapley value or on the application in black-box models [9] - [15]. However, in the context of white-box models for DERs in DSM, the potential of the Shapley value remains to be explored. In the next section, a novel approach for quantifying the contribution of DERs submodels based on the Shapley value and a given use case will be presented and discussed.

Approach and case study

The goal of this section is to propose a new approach for quantifying the contribution of DERs submodels in the context of white-box DER models based on the Shapley value and to show how the approach can be applied in the context of DSM with the use case in [3]. For this purpose we conduct a case study for a heat pump with heat storage systems.

Heat pump and heat pump storage case study

To facilitate the subsequent introduction and discussion of the quantification approach, it is necessary to briefly explain the modeling objective and assumptions, as well as the processes involved in the case study. The considered system is a heat pump system with two different hot water storages (HWS) (Fig. 1). The system is installed in a stand-alone house in real-world scenarios in Switzerland [19], which is equipped with corresponding automatic data measurement and data storage equipment.

The heat transfer within the heat pump system takes place mainly in three parts, namely the borehole ground heat exchanger, the heat pump itself and the HWS. The heat transfer in the borehole is modeled in (8), where T^{in} and T^{out} are the inlet and outlet temperature of the borehole heat exchanger. c_b is the specific heat capacity of the brine and $\dot{m}_b$ is the mass flow of the brine. Besides, P_Q^{abs} is the absorbed thermal power, which is also the difference between thermal power P_Q and electrical power P, as modeled in (9). Based on the first law of thermodynamics, thermal power P_Q of the heat pump itself can also be modeled in (10), where c_w is the specific heat capacity of water, $\dot{V}_w$ is the flow rate and ρ_w is the density of water. Moreover, T^{supply} and T^{return} are the supply and return temperature of the heat pump respectively.

$$T^{out} = T^{in} + \frac{P_Q^{abs}}{c_b \cdot \dot{m}_b} \tag{8}$$

$$P_Q^{abs} = P_Q - P \tag{9}$$

$$P_Q = c_w \cdot \dot{V}_w \cdot \rho_w \cdot (T^{supply} - T^{return}) \tag{10}$$

The thermal energy change ΔQ_s in both hot water storages are modeled based on the first law of thermodynamics in (11), (12)and (13), where different HWS have a different

Table 1 Summary of several existing applications

Type of Model	Application	Reference
Black-box model	Determination of values of flexible assets in DSM	[8]
Black-box model	Interpretation of predictions and tree-based models	[9, 10]
Black-box model	Improvement of interpretation of predictions	[11]
Black-box model	Improvement of approximation's accuracy	[12]
Black-box model	Addressing interpretability problem in several data Shapley valuation methods	[15]
Black-box model	A feasible solution for distributing the collective cost in an EM strategy	[17]

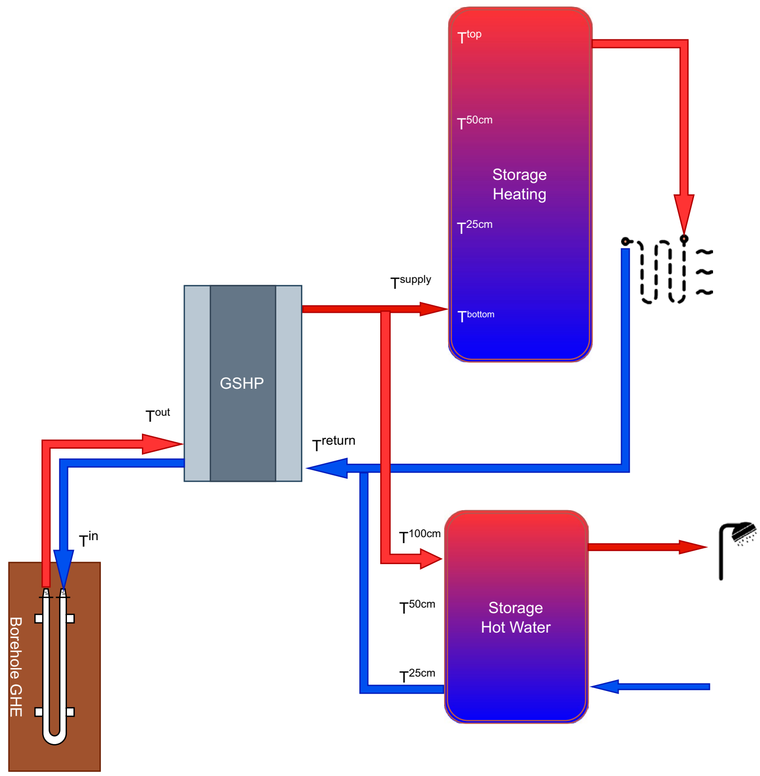

Fig. 1 The schematic structure of the considered HP system [3]

number of temperature sensors in each layer and the number such as 25*cm* or description such as *bottom* means the position of temperature sensors. V_s in Equation (11) means the volume of a storage system.

$$\Delta Q_s = c_w \cdot V_s \cdot \rho_w \cdot (T_t^{mean} - T_{t-1}^{mean}) \tag{11}$$

$$T_t^{mean,s} = \frac{T_t^{25cm} + T_t^{50cm} + T_t^{100cm}}{3} \tag{12}$$

$$T_t^{mean,l} = \frac{T_t^{bottom} + T_t^{25cm} + T_t^{50cm} + T_t^{top}}{4} \tag{13}$$

Building on the heat transfer models of the individual components, system models of varying complexity are developed, and the corresponding model utility *U* are computed. As the utility of a model is strongly dependent on the specific use case, we quantify model utility via the accuracy of a model, since model accuracy is generally important, no matter the exact use case. To compute model utility *U*, the simulated load profile over a time horizon of 7 days or 168 hours derived from different system models are compared with the actual measurements by using normalized Mean Absolute Error (nMAE). Equation (14) presents the definition of model utility *U*:

$$U = (1 - (\text{nMAE})) \cdot 100[\%] \tag{14}$$

Table 2 presents an overview of the results and Fig. 1 illustrates the schematic structure of the system. Based on this structure, Model A considers the energy changes in both storages, Model B and Model C neglect the impact of the small and the large hot water tank respectively. Moreover, Model D is further simplified by ignoring the energy changes in both storage. Further details can be found in [3]. It is worth noting that, as the model complexity is varied, the overall structure of the system model changes accordingly.

Definition of the value function

In order to quantify the contribution of submodels based on the Shapley value in the following analysis, an essential prerequisite lies in how the value function *v* is defined. Considering that, a device's ability to contribute effectively to load profile adjustment, i.e., flexibility adjustment, is closely related to its maximum power P_{max}. Therefore, the following definition for the value function *v* is first proposed in our scenario, where *S* is the coalition of $submodel_i$. It is important to note that the $submodel_i$ has a different meaning than system models A to D listed in Table 2. It represents different mathematical models which are used to build a system model. In other words, Equations such as (11) and (12) form the $submodel_i$ and S is a set of submodels, which is used for system models. Thus, system models A to D are formed by different sets S. Under this definition, the value function *v* represents the maximum adjustable power of a coalition *S* to load adjustment in DSM. Additionally, it is worth noting that different definitions of the value function may have an impact on the final results in different applications.

$$v(S) = v(\sum_{S} submodel_i) = P_{max} \cdot U(S) \tag{15}$$

Then the Shapley value of a coalition *S*, i.e., a white-box model with a certain set of submodels in our scenario, can be calculated according to Equation (2).

The Shapley value for heat pump and hot water storage submodels

In this section, we show how the proposed approach can be applied to the use case with white-box DERs models. As discussed in Section "Introduction" and Section "Related Work", the idea of "player" in game theory can be replaced with other concepts depending on the use case. As shown in Table 2, it is evident that Equations (8, 9, 10) are required for all four system models. Based on the proposed concept, the system models can be conceptualized as an n-Person Game, where n is equal to 4. The detailed mapping is presented in Table 3. Table 4 then shows the conceptualization of system model as a coalition and its corresponding value based on Equation (15).

According to Equation (2), the Shapley value of each $submodel_i$ can be computed step by step. Given that n is equal to 4 in this use case, the number of computational steps remains manageable, thereby enabling a visualization of the full calculation procedure. For instance, Table 5 presents the full calculation procedure of the Shapley value of $submodel_1$, where $\varphi_{submodel_1} = 10.58kW$ when the final result is rounded with two decimal precision. Similarly, the Shapley value of the rest *submodels* can be computed

Table 2 Model classification and the corresponding model utility *U*

Model	Combination	Model Utility
Model A	(8) (9) (10) (11) (12) (13)	96.23%
Model B	(8) (9) (10) (11) (13)	89.38%
Model C	(8) (9) (10) (11) (12)	83.01%
Model D	(8) (9) (10)	79.22%

Table 3 Mapping of $submodel_i$ based on the proposed approach

Equations	"Player" $submodel_i$
(8) (9) (10)	$submodel_1$
(11)	$submodel_2$
(12)	$submodel_3$
(13)	$submodel_4$

Table 4 Conceptualization of system model based on the proposed concept

System model	Coalition	Value of coalition *v*(*S*)
Model A	$submodel_1, submodel_2, submodel_3, submodel_4$	12.03 *kW*
Model B	$submodel_1, submodel_2, submodel_4$	11.17 *kW*
Model C	$submodel_1, submodel_2, submodel_3$	10.38 *kW*
Model D	$submodel_1$	9.90 *kW*

Table 5 The Shapley value *kW* of $submodel_1$

Subset S	$v(S \cup \{i\})$	*v*(*S*)	$\lvert S\rvert$	Weight	Weighted Contribution
ϕ	9.90	0	0	1/4	2.475 *kW*
$submodel_2$	9.90	0	1	1/12	0.825 *kW*
$submodel_3$	9.90	0	1	1/12	0.825 *kW*
$submodel_4$	9.90	0	1	1/12	0.825 *kW*
$submodel_2, submodel_3$	10.38	0	2	1/12	0.865 *kW*
$submodel_2, submodel_4$	11.17	0	2	1/12	0.931 *kW*
$submodel_3, submodel_4$	9.90	0	2	1/12	0.825 *kW*
$submodel_2, submodel_3, submodel_4$	12.03	0	3	1/4	3.008 *kW*
$\varphi_{submodel_1}$					10.58 *kW*

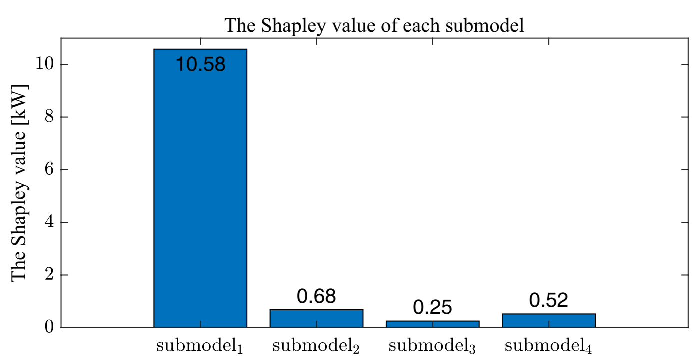

Fig. 2 The Shapley value of each model in *kW*

and the full steps will not be repeated here. The results of the Shapley value of all four *submodels* are printed in Fig. 2.

With the calculated Shapley value, the contribution of each *submodel* in the coalition is quantified. Based on the results, the satisfaction of the four foundational properties mentioned in Section "Introduction" needs to be verified first. The first and most important property, namely efficiency, can directly be verified by aggregating the Shapley value of all submodels, which is equal to 12.03 *kW* exactly. The second and the third property, symmetry and linearity, do not apply to this use case, as no two submodels' Shapley values are equal and the value function is uniquely defined. The last property is also not applicable to this case since each *submodel* ("player") has a certain contribution to the coalition. More detailed discussions are presented in the following section.

Discussion

Based on the results in Fig. 2, it's clear that the Shapley value of $submodel_1$ is markedly higher than that of the remaining *submodels* in the whole set, i.e., Model A. This finding indicates that $submodel_1$ has a dominant influence in the formulation of Model A when using Model A to evaluate the flexibility of the investigated DSM assets. This result is also expected, as this submodel is essential across all system models. Furthermore, the Shapley value of $submodel_4$ is more than twice that of $submodel_3$. And in this scenario, $submodel_3$ and $submodel_4$ interpret the impact of the small and the large HWS respectively. Based on the approach, the contribution of $submodel_4$, that is, the large HWS, could be quantified as 2.08 times than that of $submodel_3$, the small HWS. Combined with their Shapley values, their impacts on the whole coalition - which is defined considering the maximal power of the heat pump system - are also quantified. Furthermore, the large HWS and the small HWS has a volume of 500*l* and 360*l* respectively. This indicates that, despite the larger HWS having only 1.39 times the storage capacity of the smaller HWS, its contribution reaches a factor of 2.08. It means that the contribution of the storage submodel is not linearly proportional to its physical capacity. This suggests that when selecting submodels to construct the system model while controlling model complexity, the influence of smaller storage units can be selectively neglected.

In the modeling process, it is important to note that the system models A to D take different hardware configurations into account. This implies that whether model complexity variation is possible without altering the system structure is a question that is not addressed in this paper. Addressing this limitation represents an important avenue for future improvement. Moreover, if the overall system structure remains unchanged, whether the maximum power or another factor should be incorporated into the definition of the value function is another issue that warrants further investigation.

The feasibility of the proposed approach is proven to be effective for the heat pump systems within a white-box modeling framework. However, it remains to be investigated whether the same approach is applicable to other white-box energy system models and system configurations. In other words, an additional valuable point to be considered is its transferability.

In addition, it is important to highlight how this methodology differs from another approach, namely sensitivity analysis, which studies how the uncertainty in the output of a mathematical model or system can be apportioned to different sources of uncertainty in its inputs [20]. It should be more considered as a pre-requisite for statistical models [21] or used to test the robustness of the results of a model or system [22].

Conclusion and outlook

This paper introduces a new quantification method to measure the contribution of submodels within the framework of white-box DER models, which can help in assessing modeling choices for DERs in DSM. The quantification approach builds upon the concept of the Shapley value in game theory. The feasibility of the proposed method is then demonstrated by a heat pump use case, which is also investigated in [3].

Overall, the evaluation with the use case shows the feasibility of the proposed methodology to quantify the contribution of submodels within white-box DER models in DSM based on the Shapley value. The methodology can be used together with the proposed hypothesis, which states that, in general the complexity-utility relationship in the field of DSM modeling could be represented by a diminishing marginal utility curve [4] to further assist developers, engineers or operators, not only in selecting a model with an appropriate level of complexity based on utility requirements, but also facilitating a deeper understanding of which *submodel* within the system model contributes most to the overall utility in a model for DSM, particularly when the system model can be viewed as a coalition.

Given the results of this case study, there are different areas that future works can improve on. In defining the value function in this case study, the maximum power of the heat pump system was taken into account. This is because the maximum power determines the theoretical upper limit of the adjustable capacity that a DER can provide when participating in DSM. However, the value function could be redefined according to different scenarios, allowing it to be integrated with other variables such as energy costs. This could further extend the applicability of the approach. Besides, it would be valuable to repeat the evaluation under identical system conditions in future work. Maintaining a consistent system configuration during model simplification could help reduce confounding influences and support a more reliable evaluation of the experimental outcomes. Furthermore, an extension of this approach to encompass other model types, such as lumped resistance-capacity (RC) models or gray-box models, might serve as a potential improvement.

Author contributions

CL: Conceptualization, Methodology, Programming, Validation, Visualization and Writing of the original manuscript. KF, JM, VH: Review and Editing.

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Data availability

No datasets were generated or analysed during the current study.

Declarations

Ethics approval and consent to participate

Not applicable.

Consent for publication

Not applicable.

Competing interests

The authors declare no competing interests.

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References

1. Statistisches Bundesamt: energy consumption of households. https://www.destatis.de/EN/Themes/Society-Environment/Environment/Environmental-Economic-Accounting/private-households/Tables/energy-consumption-households.html. Accessed 10 Sep 2025 (2023)
2. Impram S, Varbak Nese S, Oral B (2020) Challenges of renewable energy penetration on power system flexibility: a survey. Energ Strat Rev 31:100539
3. Li C, Brecher G, Kovačević J, Çakmak HK, Förderer K, Matthes J, Hagenmeyer V (2024) Gossen's first law in the modeling for demand side management: a thorough heat pump case study with deep learning based partial time series data generation. Energy Inform 7(1):47
4. Li C, Förderer K, Moser T, Spatafora L, Hagenmeyer V (2023) Gossen's first law in the modeling for demand side management: a first heat pump case study. Energy Informatics Academy Conference. Springer, pp 111–125
5. Shapley LS (1951) Notes on the n-person game-ii: the value of an n-person game.(RAND memorandum)
6. Kuhn HW, Tucker AW (1953) Contributions to the Theory of Games, vol vol 28. Princeton University Press, Princeton, NJ
7. Winter E (2002) The shapley value. Handbook of game theory with economic applications 3:2025–2054
8. Varenhorst IA, Hurink J, Gerards ME (2024) Quantifying device flexibility with shapley values in demand side management. In: Proceedings of the 15th ACM International Conference on Future and Sustainable Energy Systems. pp 147–157

9. Lundberg SM, Lee S-I (2017) A unified approach to interpreting model predictions. Adv Neural Inf Process Syst. p 30
10. Lundberg SM, Erion G, Chen H, DeGrave A, Prutkin JM, Nair B, Katz R, Himmelfarb J, Bansal N, Lee S-I (2020) From local explanations to global understanding with explainable ai for trees. Nat Mach Intell 2(1):56–67
11. Kumar IE, Venkatasubramanian S, Scheidegger C, Friedler S (2020) Problems with shapley-value-based explanations as feature importance measures. Int Conf Mach Learn. pp 5491–5500 (PMLR)
12. Aas K, Jullum M, Løland A (2021) Explaining individual predictions when features are dependent: more accurate approximations to shapley values. Artif Intell 298:103502
13. Chen H, Covert IC, Lundberg SM, Lee S-I (2023) Algorithms to estimate shapley value feature attributions. Nat Mach Intell 5(6):590–601
14. Luo X, Pei J (2024) Applications and computation of the shapley value in databases and machine learning. In: Companion of the 2024 International Conference on Management of Data. pp 630–635
15. Li M, Zhu W, Wu O (2025) Toward learnable and interpretable data shapley valuation for deep learning. Knowl Based Syst. https://doi.org/10.1016/j.knosys.2025.114002
16. Cremers S, Robu V, Hofman D, Naber T, Zheng K, Norbu S. Efficient methods for approximating the shapley value for asset sharing in energy communities. In: Proceedings of the Thirteenth ACM International Conference on Future Energy Systems, Association for Computing Machinery, New York, NY, USA, pp. 320–324. (2022). https://doi.org/10.1145/3538637.3538861
17. Cai W, Kordabad AB, Gros S (2023) Energy management in residential microgrid using model predictive control-based reinforcement learning and shapley value. Eng Appl Artif Intell 119:105793. https://doi.org/10.1016/j.engappai.2022.105793
18. Bauer R, Dai X, Hagenmeyer V. A shapley value-based distributed ac opf approach for redispatch congestion cost allocation. In: Proceedings of the 14th ACM International Conference on Future Energy Systems. e-Energy '23, Association for Computing Machinery, New York, NY, USA, pp. 109–113. (2023). https://doi.org/10.1145/355813.3576881
19. Meyer J. Aktuelle Messwerte der Sole-Wasser Wärmepumpen Anlag. Accessed 26 Jan 2026 (2024). https://www.effiziente-waermepumpe.ch/messdaten/index.php
20. The Joint Research Centre: EU Science Hub: sensitivity Analysis: SAMO. https://joint-research-centre.ec.europa.eu/sensitivity-analysis-samo_en. Accessed 01 Jul 2025 (2025)
21. Saltelli A, Tarantola S, Campolongo F, Ratto M (2005) Sensitivity analysis in practice: a guide to assessing scientific models. J R Stat Soc A Stat Soc 168(2)
22. Gutierrez-Garcia F, Arcos-Vargas A, Gomez-Exposito A (2022) Robustness of electricity systems with nearly 100% share of renewables: a worst-case study. Renew Sustain Energy Rev 155:111932

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