European Economic Review 189 (2026) 105436

Contents lists available at ScienceDirect

European Economic Review

journal homepage: www.elsevier.com/locate/eer

A small volume reduction that melts down the market: Auctions with endogenous rationing[☆]

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ARTICLE INFO

JEL classification:
D47
D44
Q48

Keywords:
Market design
Auctions
Participation
Endogenous rationing
Renewables

ABSTRACT

Auctions with endogenous rationing are motivated by the regulatory requirement of so-called proportionality of public support and are used to allocate support for renewable energy. They reduce the auctioned volume below the tender volume when competition is low. We show theoretically that endogenous rationing has the extreme effect of deterring any participation when single-unit bidders have participation costs. Our experiment provides support for a strong negative effect on participation. We derive optimal mechanisms for three objectives and several designs for their implementation. They do not use endogenous rationing and exhibit the usual tradeoffs between objectives. However, reducing participation costs, as opposed to increasing the number of potential bidders, contributes to all three objectives and helps ensure proportionality.

1. Introduction

Carbon-free electricity is key to achieving the ambitious climate goals.[1] The fast expansion of carbon-free generation and building of new renewable power plants is supported by public policies, which include tax credits, renewable portfolio standards, commitments to procure carbon-free electricity, and auctions (tenders).

Auctions for renewable energy support are a growing field of application of multi-unit auctions, with total awards reaching twelve-figure dollar amounts each year.[2,3] The motivation for using auctions to allocate support is to select firms with the lowest costs for providing renewable energy, i.e., efficiency is an objective. Auctions are also used to address regulatory constraints: state aid has to be allocated at the lowest necessary support payments. In line with the State Aid Guidelines of the European Commission (2014, 2022), we term this requirement the "proportionality" of public support payments. Using auctions is a means to prove this requirement is met.[4]

☆ This article is part of a Special issue entitled: 'Nora Szech' published in European Economic Review.

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[1] For example, the US aims to reduce emissions by 50%–52% by 2030 compared to 2005 levels (US, 2021). The EU's goals are to reduce emissions by at least 55% by 2030 compared to 1990 levels, to double the share of renewable electricity production compared to 2020 levels to reach about 65%, and to become climate neutral by 2050 (European Commission, 2020).

[2] For example, the State Aid Guidelines of the European Commission (2014, 2022) make auctions obligatory for all new support schemes for which member states wish to obtain state aid approval.

[3] Worldwide, an estimated total capacity of 111 gigawatts of renewable energy was auctioned in 2017–2018 (IRENA, 2019). For estimating the monetary value, we set a price of USD 50 per MWh, a duration of support of 20 years, and 2000 full load hours per year (mix of different renewable energy sources). This gives a monetary value of USD 222 billion in 2017–2018.

https://doi.org/10.1016/j.euroecorev.2026.105436
Received 13 December 2024; Received in revised form 17 July 2026; Accepted 18 July 2026
Available online 25 July 2026

However, the idea of ensuring proportionality through auctions has been challenged by undersubscribed tenders with payments close to or equal or the reserve price, that is, the highest accepted support set by the auctioneer. In this case, the auctioneer rather than competition determines the size of the support, and the regulation does not consider such an auction to be competitive.[5] To address this challenge, auctions with *endogenous rationing (ER)* have been applied, in which the original auction volume may be reduced based on the total volume of the bids in order to assure that not all submitted bids win.[6,7] The question arises, how strongly this approach to ensure proportionality of support interferes with the other objectives of the auctioneer.

This paper addresses this question by analyzing auctions with ER theoretically and experimentally and comparing them with auctions without rationing, which we refer to as *basic auctions*, and with optimal mechanisms with respect to the objectives welfare maximization, surplus maximization, and achieving the volume goals.[8]

We analyze multi-unit procurement (reverse) auctions with single-unit bidders and costly participation. The single-unit bidder approach is common in analyses of multi-unit auctions (e.g., Schweizer and Szech, 2017) and aligns well with renewable energy auctions, where the percentage of bidders offering a single project is typically high, often exceeding 90% (BNetzA, 2022). One reason for this is that a legally independent project company is usually established for each project (e.g., Luther Law Firm, 2025).

In renewable energy auctions, the volume put out to tender is usually capacity, or, less frequently, energy. A bid for a project includes the *bid volume*, i.e., the plant's capacity, measured by its peak power (or, in case of energy, the energy production), and the *bid price* for the energy fed into the grid by the planned plant during operation. Based on the award price, support is provided for the energy produced by the realized project in some time period, e.g., the first 20 years. The project specification determines the bid volume, which is usually mandatory and can only be adjusted slightly, if at all, after the auction (e.g., Anatolitis et al., 2022). Therefore, we assume fixed project sizes and thus fixed bid volumes, which we normalize to one.

Participation costs arise from fulfilling the auctioneer's participation requirements and are not paid to the auctioneer (in contrast to entry fees, e.g., Szech, 2011). Participation requirements are common in procurement auctions.[9] They are typical in renewable energy auctions and primarily address physical requirements that bidders must meet, such as submitting an approval to build a plant on a specific site before the auction (Wigand et al., 2016; Mora et al., 2017). Participation costs may be substantial. For onshore wind projects, these costs amount to between two and ten percent of the total investment (Wallasch et al., 2015; Pietrowicz and Quentin, 2015; Kitzing and Wendring, 2016). As a consequence, only firms whose expected profit from the auction covers their participation costs will participate in the auction.

ER interferes with this necessity to cover the participation costs, not only by reducing the participation incentives per se but also by spurring a downward spiral on participation. By ensuring that not all submitted bids win, ER eliminates the incentive for the weakest bidder to participate, causing any bidder who expects to be the weakest to stay out. As a result, there may be no participation incentive for any firm. The first main result of this paper reveals this strong negative effect of ER on participation: the market completely unravels and no bids are submitted. While it is obvious that ER makes participation less attractive, the result reveals an extreme and unintended consequence that only becomes apparent once equilibrium behavior is accounted for.

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[4] European Commission (2022): "When assessing whether environmental protection and energy aid can be considered compatible with the internal market under Article 107(3), point (c) of the Treaty, the Commission will analyse the following aspects: [...] (iii) the proportionality of the aid (aid limited to the minimum necessary to attain its objective) ". "A detailed assessment [...] will not be required if the aid amounts are determined through a competitive bidding process, because it provides a reliable estimate of the minimum aid required by potential beneficiaries".

[5] European Commission (2014): " 'competitive bidding process' means [...] the budget or volume related to the bidding process is a binding constraint leading to a situation where not all bidders can receive aid". For examples of undersubscribed auctions, see auctions in Germany (BNetzA, 2022), Brazil and France (Robert and Simon, 2019), and Italy (Enerdata, 2021).

[6] For example, the auction may stipulate that only a share (e.g., 80%) of the total bid volume submitted to the auction will be allotted to bidders if the total bid volume does not exceed the original auction volume. This kind of rationing (with an *endogenous auction volume*) is used in Germany (BReg, 2019), France (European Commission, 2018), Ukraine (Legislation of Ukraine, 2019), and Switzerland Bundesamt für Energie (2019). (See Appendix B for details on this specific design.) Similar measures are applied in Brazil (IRENA, 2015), Greece (Papachristou et al., 2017), and Mexico (Jiménez, 2016). For another concept of ER, the *endogenous reserve price*, see Appendix C.

[7] We use the term "endogenous rationing" in line with the use of "rationing" in the European regulation (European Commission, 2022, Para. 49). We refer to rationing as "endogenous" to differentiate it from the term "rationing" in economic literature, where it is used in a different context and with a different meaning to address rationing decisions to enable market clearing. For example, in Bertrand competition with capacity constraints rationing addresses the question of which consumers buy where, if the low price firm's capacity is not sufficient to serve all consumers at this price (Levitan and Shubik, 1972; Kreps and Scheinkman, 1983). Furthermore, our use of "endogenous rationing" as an auction design feature differs from Boyer and Moreaux (1989), where "endogenous rationing" is an equilibrium property in a leader-follower duopoly.

[8] These objectives are mentioned in guidelines and directives. For example, the European Commission (2014) and BReg (2017) stipulate, without prioritization, the minimization of the support payments and the minimization of the overall costs to achieve the renewables expansion targets (Kreiss et al., 2021). The minimization of the support payments is stipulated, e.g., in the Netherlands and the United Kingdom, and proposed for developing countries. The minimization of the overall costs to achieve the expansion target is stated, e.g., in California (US) and Mexico (Kreiss et al., 2021; IRENA, 2013). Minimizing the overall (social) costs, which is related to maximizing the social welfare, is another goal in national laws (e.g., BMU, 2016; BReg, 2017; Kazakh Government, 2009). The maximization of the consumer surplus or low prices for the customers are also postulated (e.g., BMU, 2016; IRENA, 2013; Hochberg and Poudineh, 2018; Kreiss et al., 2021). The goal to achieve the targeted expansion of renewable energy, which implies allotting the volume put out to tender in the renewable energy auctions, is stated, e.g., in Germany, Kazakhstan, Brazil, Mexico, and proposed for developing countries (BReg, 2017; Kazakh Government, 2009; Hochberg and Poudineh, 2018; IRENA, 2013).

[9] The auctioneer often requires a certain kind of project preparation as a condition of participation in order to ensure the offers' quality or seriousness. To meet these requirements, bidders have to undertake costly actions and investments in their project before they know whether it can be realized. Typically, the measures required are also necessary for the realization of a project. Examples of such actions are the development of a prototype in the industrial sector or the obtaining of building permits in the construction sector.

Based on this result, the analysis proceeds in two directions. First, in a laboratory experiment, auctions with ER are compared with basic auctions to establish and evaluate the causal effect of rationing on participation with human decision makers. Second, optimal mechanisms tailored to different auctioneer's objectives are used to identify alternate approaches for improvements in all objective functions.

The experimental results reveal that ER indeed can melt down the market. Participation in the auctions with ER is lower than in the basic auctions and decreases sharply when subjects gain experience.[10] Moreover, social welfare is lower with ER and also decreases over the rounds of the experiment. Although ER leads to lower payments per good than the basic auction, the auctioneer does not achieve a higher surplus.

The analysis of optimal mechanisms shows that the objectives of maximizing surplus, maximizing welfare, and achieving volume goals can each be achieved through auctions that reimburse all bidders or all losing bidders for their full participation costs in combination with an appropriately adjusted reserve price. Alternatively, if the auctioneer prefers not to reimburse participation costs in full, the two objectives of maximizing surplus and maximizing welfare can also be achieved through designs that partially or fully reimburse no bidder, all bidders, all losing bidders, or all winning bidders. In general, the optimal mechanisms for the objectives differ.

Nevertheless, measures exist that contribute to all three objectives and to ensuring proportionality (i.e., competitive bids rather than the reserve price determine payments). Lowering the participation costs (unlike increasing the number of bidders) increases the auctioneer's surplus, social welfare, the allotted auction volume, and competition by boosting participation. The effects of lowering participation costs on these objectives apply not only to the respective optimal mechanisms but also to any basic auction with a reserve price below the auctioneer's valuation. Current discussions about renewable energy auctions suggest that lowering participation costs can be achieved by reducing permitting and zoning challenges, particularly for wind farm projects.

In accordance with the results of this paper, the European Commission added a rule to its State Aid Guidelines which stipulates that auctions with ER may not be used to establish proportionality of support: "the Commission considers that the proportionality of the aid is ensured if the following criteria are fulfilled: [...] (d) ex post adjustments to the bidding process outcome (such as subsequent negotiations on bid results or rationing) are avoided as they may undermine the efficiency of the process's outcome". (European Commission, 2022, Para. 49).[11]

Related literature. To the best of our knowledge, this is the first study to analyze auctions with ER and participation costs, both theoretically and experimentally. ER is, by its potential reduction of the allotted volume below the original auction volume, related to the literature on auctions with variable volume, in which the auction volume or the rule to determine the auction volume is not fixed before the auction. This is a common practice in treasury auctions (Nyborg et al., 2002). The idea is to avoid low-price equilibria in these forward auctions when uniform pricing is used. Since bidders in treasury auctions have multi-unit demand and are allowed to submit non-increasing demand functions, they have an incentive to coordinate on low-price equilibria by strategically reducing their demand. Back and Zender (2001), Damianov (2005), McAdams (2007), and Damianov and Becker (2010) show that the seller can reduce or even eliminate low-price equilibria if it has the right to adjust the volume after collecting bids, e.g., by choosing the optimal volume given the submitted bids. LiCalzi and Pavan (2005) find that the seller can reduce incentives for low bid prices (in forward auctions) by committing to an increasing volume schedule before observing the bids. Ausubel and Cramton (2004) find truthful bidding in a divisible good auction with Vickrey pricing (also with resale) and an endogenous auction volume or reserve price that is monotonic in the bids. These models do not consider participation costs.

There are crucial differences between treasury auctions and the auctions analyzed in this paper as well as between a variable auction volume, such as in treasury auctions, and ER. The variable volume auctions allow for all bids to be successful, whereas the motivation for ER is to ensure that not all submitted bids win. Treasury auctions are offered frequently, some even on a daily basis. Bidders repeatedly demand multiple units of the good, and their participation costs are insignificant. In contrast, renewable energy auctions and other procurement auctions are offered much less frequently and there is often only one or very few chances for a bidder to win.[12] Bidders' participation costs are crucial, and projects are awarded that will be realized exactly once. Moreover, since most bidders participate with only one project, we analyze a setting with single-unit supply bidders, in which the collusive equilibria that are addressed by the variable volume approaches do not exist. Note that, with multi-unit supply bidders, ER does not replicate the auctioneer's decreasing demand if bid prices (in a reverse auction) increase in LiCalzi and Pavan (2005). ER does not react with an adjusted volume when bid prices are increased, but it may decrease demand if fewer bids are submitted.

Our experimental control treatment with a basic multi-unit auction with costly participation relates to the literature on single-unit

[10] The development of the auction with endogenous volume adaption used for the support of energy efficiency projects and programs in Switzerland Bundesamt für Energie (2019) is in line with our experimental result. That auction experienced a relative decrease in the bid volume after the introduction of the rationing rule (Ehrhart et al., 2020). However, there are confounding effects that hinder a causal interpretation, e.g., the announced budget has increased over the years.

[11] In the context of the revision of the State Aid Guidelines, the authors presented and discussed the findings of this paper with the European Commission's competition department.

[12] Renewable energy auctions usually take place at most a few times a year and re-participation entails further costs due to, e.g., rescheduling of the project or renewal of permissions.

(forward) auctions with costly entry. Aycinena and Rentschler (2018) find over-entry compared to risk-neutral prediction, with no evidence of a difference in entry between pricing rules. Brosig and Reiss (2007) find changes in participation in line with theory when the opportunity cost of participation increases.[13] Li and Zheng (2009) find empirical evidence of predictions for basic auctions, e.g., lower entry probability when the number of potential bidders increases.

Our theoretical analysis of basic auctions and optimal mechanisms with costly participation extends existing work on single-unit auctions to our setting with an arbitrary number of goods, including cases with no competition, and an objective not yet considered in the literature, which is necessary to answer our research question on ER.[14] For the single-unit setting, Stegemann (1996) shows payoff equivalence of the symmetric equilibria of the first- and second-price auction, which extends to our multi-unit setting and all basic auctions. He, as well as Tan and Yilankaya (2006), Celik and Yilankaya (2009), and Lu (2009a), unlike us, also analyze asymmetric equilibria and asymmetric mechanisms. Samuelson (1985) determines the socially optimal and the auctioneer's surplus-maximizing reserve prices of optimal auctions, which have later been shown to implement the optimal mechanisms. Further implementations of surplus- or revenue-maximizing mechanisms have been identified by Lu (2009a) and Menezes and Monteiro (2000) and of welfare-maximizing mechanisms by Lu (2009b), including a second-price auction with an optimally chosen reserve price, in which all participating bidders are reimbursed for their participation costs. In addition to extending these findings to the multi-unit setting, we show that auctions in which bidders receive reimbursement for any share of their participation costs can be used to implement welfare-optimal auctions (as Lu (2009a) has shown for surplus-optimal single-unit auctions). Moreover, we find that the auctioneer has further options for implementing the welfare- or surplus-optimal mechanism by choosing designs in which only winning bidders or only losing bidders receive full or partial reimbursement of their participation costs. However, the mechanism for maximizing the allotted volume can be implemented only with either all bidders or all losing bidders receiving full reimbursement for their participation costs.

The paper proceeds as follows. Section 2 presents the theoretical analysis of basic auctions and of auctions with ER. Section 3 examines the impact of ER experimentally. Section 4 derives optimal auctions for three objectives and discusses a policy measure suggested by their design. Section 5 concludes. All proofs are in Appendix A.

2. Theoretical analysis

Consider a multi-unit procurement auction for k units of a good, $k \geq 1$. The set of potential bidders contains n risk-neutral firms each with single-unit supply, $n \geq 1$. We consider both $n \leq k$ and $n > k$ because ER has been suggested in particular for auctions with low or no competition. Firms are symmetric and have independent private costs for supplying the good. The firms' private costs $x_1, x_2, \ldots, x_n$ are independently drawn from the distribution F with density f and full support on $[\underline{x}, \bar{x}]$, $0 \leq \underline{x} < \bar{x}$. Let $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ and $\mathbf{x}_{-i} = (x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$. The associated random variable of the kth lowest of n independent signals is denoted by $X_{(k,n)}$ and its distribution function by $F_{(k,n)}$. With this notation, $F_{(k,n-1)}(x) = \sum_{i=k}^{n-1} \binom{n-1}{i} F(x)^i (1 - F(x))^{n-1-i}$ if $n > k$, and we define $F_{(k,n-1)}(x) = 0$ if $n \leq k$.

Our model of a multi-unit procurement auction follows the approach of Samuelson (1985) for a single-unit procurement auction. Firms simultaneously decide on their participation in the auction and on their bidding strategy in case of participation. Only participating firms can bid and win in the auction. We consider only pure strategies. The number of firms that participate in the auction is denoted by m, $m \leq n$.

To participate in the auction, each firm has to incur participation costs $c > 0$, e.g., to meet qualification requirements set by the auctioneer. These are sunk costs for the firms when the auction starts and they are not paid to the auctioneer. A firm knows c and her private costs x_i when she decides about her participation.[15] Conditional on participating in the auction, she expects a profit $\pi(x_i)$ from the auction. Her profit from the auction is $p - x_i$ if she wins a good at the payment p and is zero, otherwise. Firm i aims to maximize her overall payoff $\Pi(x_i)$, where $\Pi(x_i) = \pi(x_i) - c$ if she participates in the auction and $\Pi(x_i) = 0$ if she does not participate.

2.1. Basic auction

We analyze basic auctions as a benchmark for our analysis of auctions with ER.

[13] Experiments in which bidders learn their private values only after their entry by Meyer (1993) and by Palfrey and Pevnitskaya (2008) find a decrease in participation if entry costs increase; Palfrey and Pevnitskaya (2008) find over-entry.

[14] Our approach has common participation costs and bidders know their private costs when deciding on participation. Further studies with this basic model, e.g., on heterogeneous bidders and uniqueness of equilibria, are by Campbell (1998), Cao and Tian (2010), Miralles (2008), and Cao and Tian (2013). Models with common participation costs where bidders learn their private costs only after deciding on participation and models with private participation costs have also been analyzed (e.g., McAfee and McMillan, 1987; Tan, 1992; Levin and Smith, 1994; Li and Zheng, 2009; Cao et al., 2018).

[15] This approach where firms know x_i when deciding on participation captures the renewables setting and other procurement settings with mature projects or (modifications of) standard products better than a model in which firms have no information about x_i and their expected position relative to their opponents (cp. Footnote 14).

2.1.1. Definition

Basic auctions are characterized by the following properties:

(P1) The n firms simultaneously decide whether or not to participate in the auction and bid if they participate. Thus, when bidding, the firms know n but do not know m, the number of participating bidders.[16]
(P2) Bids may not exceed a reserve price $r \in \mathbb{R}_+$, $r > \underline{x} + c$, set by the auctioneer.[17]

The following two properties of basic auctions do not apply to auctions with ER:

(P3) The k lowest bids win if $m \geq k$ (ties at the kth lowest bid are broken randomly); all other firms obtain nothing. If $m < k$, all m bids win.
(P4) The reserve price r is the maximum payment from the auction, and for each firm there exists a bid such that the firm's payment with this bid is r if $m \leq k$.

In addition, there is an existence and monotonicity property concerning equilibrium.

(P5) The participating firms have an equilibrium bidding strategy $\beta(x_i, r)$ that is strictly increasing in x_i if $n > k$ and weakly increasing in x_i if $n \leq k$.[18]

We call an auction with properties (P1)–(P5) a basic auction. Basic auctions have exogenous demand k and no ER. If auctions have multiple equilibria, we will focus on equilibria with (P5). (P3) and (P5) imply that the goods are supplied by the firms with the lowest costs, i.e., the allocation is efficient conditional on the participating firms.

2.1.2. Analysis

Participation costs c may prevent firms with high costs x_i from participating in the auction. More precisely, since (P5) implies $\pi(x_i) > \pi(x_j)$ for $x_i < x_j$ in equilibrium, there exist *cutoff costs* $\hat{x}$ that separate participating firms with costs $x_i \leq \hat{x}$ from non-participating firms with $x_i > \hat{x}$.[19] Thus, it is possible that not all k goods are supplied, even if the costs of k or more firms are below r. The firm with the cutoff costs $\hat{x}$ of equilibrium participation is denoted by $\hat{\imath}$. Firm $\hat{\imath}$ submits the highest bid, which wins only if a maximum of $k-1$ other firms bid. Therefore, $\hat{\imath}$ adjusts her bid to this case and bids to ensure the payment r in the event that she wins. If $\hat{x} < \bar{x}$, firm $\hat{\imath}$ is indifferent between participating and not participating, and the cutoff costs $\hat{x}$ are uniquely determined by $\Pi(\hat{x}, r, c, n) = 0$, where

$$\Pi(\hat{x}, r, c, n) = (r - \hat{x})\left(1 - F_{(k,n-1)}(\hat{x})\right) - c = (r - \hat{x}) \sum_{i=0}^{\min\{k,n\}-1} \binom{n-1}{i} F(\hat{x})^i \left(1 - F(\hat{x})\right)^{n-1-i} - c\,. \tag{1}$$

The following lemma collects properties of the cutoff costs $\hat{x}$.

Lemma 1. *For a firm's cutoff costs $\hat{x}$ the following hold:*

- $\frac{d\hat{x}}{dr} > 0$ *and* $\frac{d\hat{x}}{dc} < 0$ *if* $n > k$ *or* $r < \bar{x} + c$. $\hat{x}$ *increases in* k *if* $n > k$. $\hat{x}$ *decreases in* n *if* $n \geq k$.
- $\hat{x} \begin{cases} \in (\underline{x}, \bar{x}) & \text{if } n > k \text{ or } r < \bar{x} + c\,, \\ = \bar{x} & \text{otherwise.} \end{cases}$

Note that $\hat{x}$ determines a firm's ex-ante participation probability $F(\hat{x})$, which increases with $\hat{x}$; $F(\hat{x}) \in (0, 1)$ if $n > k$ or $r < \bar{x} + c$, and $F(\hat{x}) = 1$ otherwise.

The intuition behind Lemma 1 is as follows. Increasing the reserve price r or the number of goods k (if $n > k$) increases the marginal bidder's expected profit from the auction via an increased profit in case of winning or an increased winning probability. This results in higher cutoff costs $\hat{x}$, which implies a higher expected number of participants. Participation costs c have to be absorbed by the expected profits from the auction. Thus, a higher c lowers $\hat{x}$. A growth of the pool of firms reduces the marginal bidder's expected profits from the auction if $n \geq k$. Thus, $\hat{x}$ decreases in n.[20]

Under mild conditions ($n > k$ or $r < \bar{x} + c$) we have $\hat{x} < \bar{x}$, that is, high-cost firms do not participate. If $n > k$, there is no r that can induce full participation because type $\bar{x}$'s profit from the auction is zero if all firms participate. If $n \leq k$, each bid wins but high-cost firms forgo the auction if r does not cover their total costs $x_i + c$.

The cutoff type $\hat{x}$ is the same in all basic auctions and is the worst-off type among the participants. Payoff equivalence holds for all basic auctions.[21]

[16] This assumption is inconsequential in the case of independent private values, where revenue equivalence holds between auctions with known and unknown number of bidders (Harstad et al., 1990; Menezes and Monteiro, 2000).
[17] If $r < \underline{x} + c$, no firm will participate in the auction.
[18] Compare, e.g., Levin and Smith (1994) for a similar assumption.
[19] If this did not hold, firm i could profitably deviate by bidding like the higher-cost firm j.
[20] Li and Zheng (2009) find empirical evidence of this "entry effect": a negative relationship between the number of potential bidders n and the participation probability.
[21] See Stegemann (1996, Theorem 4) and Menezes and Monteiro (2000) for payoff equivalence between symmetric equilibria of single-unit first- and second-price auctions with participation costs.

Lemma 2. *Basic auctions are payoff equivalent.*

That is, a firm with costs x has the same expected profit in all basic auctions and the auctioneer has the same expected surplus. The basic model permits multiple payment rules which include *pay-as-bid pricing*, where all winning bidders receive their bids, and *uniform pricing*, where the lowest rejected bid, or the reserve price in case $n \leq k$, determines the payment to all winning bidders.

Lemma 3. *The symmetric equilibrium bidding strategy in the pay-as-bid auction is given by, for all $x \in [\underline{x}, \hat{x}]$:*

$$\beta^{PaB}(x,r) = \begin{cases} x + \dfrac{(1 - F_{(k,n-1)}(\hat{x}))(r - \hat{x}) + \int_x^{\hat{x}} 1 - F_{(k,n-1)}(y)dy}{1 - F_{(k,n-1)}(x)} & \text{if } n > k \\ r & \text{if } n \leq k\,. \end{cases}$$

The symmetric equilibrium bidding strategy in the uniform-price auction is given by, for all $x \in [\underline{x}, \hat{x}]$ and $n \gtreqless k$:

$$\beta^{UP}(x,r) = x.$$

If there is no competition, $n \leq k$, the equilibrium payment of basic auctions equals r, either because all bidders bid r in anticipation of the low number of bids or because the payment rule determines a uniform payment of r.

2.2. Auction with endogenous rationing

Instruments of ER have been suggested as a means to avoid high payments in cases of low competition and to assure proportionality of support (i.e., that competitive bids and not the reserve price determine the payments). The prevalent variant of ER is the *endogenous auction volume*, that is, the adaption of the auction volume to the supply volume (number of bids).[22] In such an auction, the auctioned volume is reduced below the tender volume if the number of submitted bids m is low. As an example, an ER rule may determine that the number of winning bids is k if $m > k$, i.e., when there is competition, and $m - 1$ if $m \leq k$, i.e., when there is no competition. With a tender volume of $k = 10$, there will be ten winning bids if $m > 10$, but only 9, 8, 7, ... winning bids if $m = 10, 9, 8, \ldots$ bids are submitted.

For the comparison of ER auctions with basic auctions, we use the same reserve price r, number of goods k, number of firms n, and payment rule. ER auctions are defined as follows.

Definition 1. An ER auction is an auction in which, if $m \geq 0$ bids are submitted, the number of winning bids $\kappa(m) \geq 0$ is determined according to a commonly known function

$$\kappa(m) \begin{cases} = k & \text{if } m > k, \\ < m & \text{if } 0 < m \leq k, \\ = 0 & \text{if } m = 0\,. \end{cases}$$

The $\kappa(m)$ winning bids are the lowest bids, and payments are determined as in the related basic auction.

That is, $\kappa(m) \leq k$, and if $0 < m \leq k$, the original auction volume k is reduced by at least one unit. Payments in the ER auction are determined as in the related basic auction. For example, in a pay-as-bid auction, the $\kappa(m)$ winning bidders are paid their bid, and in a uniform-price auction, they are paid the $(\kappa(m) + 1)$th lowest bid.

ER auctions meet (P1) (participation and bidding) and (P2) (bids are limited by the reserve price r). They violate (P3) ($\min\{m, k\}$ lowest bids win): albeit the lowest bids win, fewer than m bids win if $m \leq k$. ER auctions are designed to prevent payment of a reserve price r to all bidders when $n \leq k$ and thus also violate (P4). ER auctions assign goods differently than basic auctions and give a different payment to the worst-off type. Therefore, payoff equivalence to the basic auctions is not given.

ER auctions share some basic properties of basic auctions, e.g., a bidder's probability of winning decreases in her bid. They differ from basic auctions in the variability of the auction volume: not all submitted bids win in the ER auction, irrespective of the relationship between k and n.

ER intends to keep the payments low in case of low supply. However, it creates a strong adverse effect on participation.

Proposition 1. *In an ER auction, the number of participating bidders is zero, $m = 0$.*

Intuitively, a highest type $\hat{x}$ that participates cannot exist with $c > 0$ because this bidder will submit the highest bid among all participating bidders, and this bid never wins due to competition or rationing. The result hinges on positive participation costs, but it is robust to changes in other assumptions.

Corollary 1. *The result of Proposition 1 extends to a setting where n is unknown, and includes randomized participation decisions.*

The first result follows from the proof of Proposition 1 and the second follows from the fact that there cannot exist a highest type $\hat{x}$ with positive participation probability because this type would never win the auction.

[22] Another variant is the *endogenous reserve price*, which is analyzed in Appendix C with the same main finding. In auctions with endogenous reserve price, decreasing a bid price can even reduce the auctioned volume, whereas with endogenous volume, the auctioned volume depends on the bid volume but not on the bid price. However, some rules for endogenous reserve price adjustment, like quantile rules, can be translated into endogenous volume rules.

2.3. Auctions with ER and additional, participation-cost-free bidders

Since the extreme result of Proposition 1 is caused by positive participation costs, we now consider the case where $\tilde{n}$ firms with no participation costs, $c = 0$, are added to the auction. These firms may have no qualification requirements or different participation conditions than the firms with participation costs $c > 0$.[23] Their private costs x_i for supplying the good are also drawn from the distribution F. This extension modifies the basic model outlined at the beginning of Section 2 from symmetric to asymmetric bidders with different participation incentives. There are two groups of bidders: n bidders with $c > 0$ and $\tilde{n}$ bidders with $c = 0$. To keep this setting with rationing tractable, we assume uniform pricing. In the uniform price auction (with or without rationing), as in Lemma 3, the firms' optimal bids are equal to their private costs x_i, regardless of whether $c = 0$ or $c > 0$. The two groups of bidders differ in their decision to participate in the auction, which for the bidders with $c > 0$ depends on their x_i whereas all bidders with $c = 0$ will participate, because their expected payoff from the auction is weakly larger than zero. Due to monotonicity of the bidding function, a cutoff $\hat{x}$ on costs for participation by the bidders with $c > 0$ exists.

There are two countervailing effects of the firms with $c = 0$ on the n potential bidders with $c > 0$. They are competitors for winning the auction, and they may protect from rationing – in particular, the entrant with the highest costs –, thus enabling positive expected payoffs from the auction. The question is, thus, whether and under what conditions their presence improves the performance of ER auctions.

For simplicity, the analysis with $\tilde{n} > 0$ focuses on endogenous rationing with a fixed number R, $0 < R < k$, of rationed goods:

$$\kappa(\tilde{n} + m) \begin{cases} = k & \text{if } \tilde{n} + m > k, \\ = \max\{0, \tilde{n} + m - R\} & \text{if } \tilde{n} + m \leq k\,. \end{cases}$$

Proposition 2.

(i) If $\tilde{n} + n \leq k$ and $\tilde{n} < R$, then $m = 0$ firms with $c > 0$ enter the ER auction. There are no winning bids.
(ii) If either $\tilde{n} + n \leq k$ and $\tilde{n} \geq R$ or $\tilde{n} + n > k \geq \tilde{n}$, then the cutoff for participation in the ER auction is $\hat{x} \in (\underline{x}, \bar{x})$. In expectation, rationing occurs and there are winning bids.
(iii) If $\tilde{n} > k$, then rationing never occurs and the ER auction is identical to the corresponding uniform-price auction without rationing.

In Case (i), the deterrence effect from Proposition 1 applies, so that $m = 0$. The $\tilde{n}$ bids in the auction do not win due to rationing. In Case (ii), the number of firms with no participation costs is sufficiently high to reduce the deterrence effect of rationing on firms with participation costs; however, their presence implies competition for those who enter the auction. In Case (iii), the number of firms with no participation costs is sufficiently large to preclude the application of rationing.

Not surprisingly, in cases (i) and (ii), the expected number of bidders and of winning bids are strictly lower than in the corresponding basic auction. In Case (i), this is obvious. In Case (ii), let $\hat{x}$ be the cutoff for participation in the ER auction, i.e., a bidder with type $\hat{x}$ has zero expected payoff. Assume $\hat{x}$ was also the cutoff in the basic auction. A bidder with costs $\hat{x}$ would then face additional winning cases and weakly higher prices in the basic auction, providing a positive expected payoff. Thus, the cutoff in the basic auction must be higher, implying more bidders. More bidders and no rationing imply a higher number of winning bids.

As an unusual property, adding competitors can increase the expected payoff of firms. Concretely, adding sufficiently many bidders with $c = 0$ to an ER auction (such that $\tilde{n} \geq R$ or $\tilde{n} + n > k$) increases payoffs for firms with $c > 0$. This follows directly from propositions 1 and 2.

Does an ER auction meet the objective of proportionality? The price is below the reserve price by construction, and it is determined by a bid. However, if ER is effective, it creates a tradeoff between volume and price, as it directly reduces both, and it reduces participation, which further reduces the allotted volume. In propositions 1 and 2(i), the reduction in participation and allotted volume clearly overshoots and eliminates any potential price advantage. Also in Case (ii), the lower participation due to ER creates cases where the basic auction allots k goods at a price determined by a competitive bid whereas the ER auction allots fewer goods (at a lower price) such that both auctions fulfill proportionality but the auction with ER sacrifices allotted volume.

3. Experimental study

To evaluate auctions with ER, we test them under the conditions of Section 2.2 and compare them to basic auctions in the experimental lab.[24]

3.1. Experimental design

The laboratory experiment consists of an *ER Treatment* and a *Control Treatment*. In the ER Treatment, the subjects play a pay-as-bid ER auction, as described in Section 2.2 and Appendix B. The subjects in the Control Treatment play a basic pay-as-bid auction (see Section 2.1).

[23] One example is the so-called citizens' energy companies, which are subject to much weaker qualification requirements than typical energy companies in the auctions for the promotion of renewable energies in Germany (BReg, 2017, Para. 36g).

[24] The experiment was conducted at the KD2lab (Karlsruhe Decision and Design Lab) in Karlsruhe, Germany. The KD2lab is DFG-funded and situated at the Karlsruhe Institute of Technology. For further information, see https://www.kd2lab.kit.edu/english/index.php.

Table 1
Experimental design parameters.

Number of goods per auction	$k = 6$
Reserve price (maximum bid price)	$r = 77$ ExCU
Pricing rule	Pay as bid
Number of potential bidders per auction	$n = 9$
Private costs	$x_i \sim U[50, 75]$ ExCU
Participation costs	$c = 5$ ExCU

The parameters in Table 1 apply to both treatments. We refer to the game that nine subjects play in a round (including the participation decision) as an auction. In each auction, six goods (projects) are demanded. Each subject can supply one good. The auction is organized as follows: First, the nine subjects receive their private cost signals x_i (with two decimals) i.i.d. drawn from the uniform distribution on the interval [50, 75] experimental currency units (ExCU). One ExCU is equal to 0.5 Euro. Second, the nine subjects simultaneously decide whether to participate in the auction. Participation costs 5 ExCU. We refer to those subjects who participate in the auction as bidders. The number of bidders is not revealed. Third, the auction is conducted and each bidder submits one bid (with two decimals). The lowest bids are awarded according to the rules of the respective treatment. Pay-as-bid pricing applies. Bidders whose bids are awarded receive the difference between their bid and their costs minus the participation costs. Bidders whose bids are not awarded make a loss equal to the participation costs.

In the auctions in the ER Treatment, endogenous volume adaption is implemented by the following rule: If eight or nine bids are submitted, the six lowest bids are awarded. If seven or fewer bids are submitted, two bids less are awarded. That is, if seven bids are submitted, the five lowest bids are awarded; if six bids are submitted, the four lowest bids are awarded and so on. If one or two bids are submitted, no bid is awarded.[25] In the basic auctions in the Control Treatment, if seven or more bids are submitted, the six lowest bids are awarded. If six or fewer bids are submitted, all bids are awarded.

A subject is assigned to either the ER treatment or the control treatment. 72 subjects participate in each treatment and are divided into four matching groups of 18 subjects each.[26] Each matching group participates in 15 consecutive rounds. In each round, two groups of nine participants from the matching group are formed at random. Each group plays one auction. Thus, 120 auctions are played in each treatment and each subject participates in 15 consecutive auctions with alternating opponents.[27]

3.2. Experimental hypothesis

In the experiment, Proposition 1 predicts no participation in the equilibrium of the one-shot ER auction. Since this is an extreme result requiring sophisticated coordination of beliefs, we derive a weaker hypothesis involving learning. That is, we do not expect low participation right at the beginning, but rather a downward spiral, i.e., participation that gradually decreases over the course of repeated auctions.[28]

3.3. Experimental results

Table 2 shows the means (over all groups) of different variables in all rounds (1–15), in the first round (1), and in the last round (15). The calculation of the auctioneer's surplus and the social welfare requires assigning the auctioneer a valuation v. We use $v = r$, which is the valuation for which r is the welfare-maximizing reserve price in the Control Treatment (see Proposition 3).

First, the results of the Control Treatment match the theoretical predictions in Section 2. That is, the subjects' behavior is in line with the equilibrium strategy, both with respect to participation and bids (for detailed results see Appendix D.2).

Next, to test the effects of the treatments and the rounds on the variables in Table 2, we apply linear mixed-effects models and additionally Wilcoxon rank sum tests for inter-treatment comparisons in the first and the last round. The tests are presented in Tables D.7 and D.8 in Appendix D.3. We use a significance level of 5%.

Fig. 1 shows that the number of submitted bids (i.e., the participation level) and the number of awarded bids are lower in the ER Treatment than in the Control Treatment and decrease in the ER Treatment, while they show no trend in the Control Treatment. The results of the statistical tests support these observations.

[25] Compare Appendix B for this rule that ensures a linear reduction of the number of winning bids in the number of submitted bids.

[26] At the beginning of the experiment, written instructions were read out aloud. Before the experiment started, subjects had to answer questions on the instructions. (A translation of the instructions and the questionnaire is provided as additional materials.) At the end of the experiment, the subjects execute the Holt-Laury task to measure their risk aversion (Holt and Laury, 2002). Each session lasted around 45 min. The experiment was programmed in oTree (Chen et al., 2016) and the experiment was organized and recruited with the software hroot (Bock et al., 2014).

[27] A subject's final payment consists of a show-up fee of 8 Euro, the subject's average profits of five randomly drawn auction rounds and a payment from the risk-aversion task. The average payment was 11.00 EUR. The lowest and highest payment were 5.00 EUR and 16.30 EUR.

[28] This hypothesis of gradual development is in line with experimental results for example on the beauty contest game (e.g., Nagel, 1995). The model by Camerer et al. (2004) captures such non-equilibrium behavior: players use different depths of iterative reasoning for deriving their strategies and their beliefs about their competitors' behavior might not represent the equilibrium strategy.

Table 2
Experimental results of the auctions.

Treatment	ER (endogenous rationing)			Control (basic auction)		
Rounds	1–15	1	15	1–15	1	15
Avg. number of submitted bids	4.03	5.13	2.75	6.31	5.75	6.13
Avg. number of awarded bids	2.13	3.13	1.00	5.63	5.75	5.50
Avg. value of submitted bids	62.13	57.74	63.10	69.91	65.23	71.41
Avg. value of awarded bids	56.65	47.84	55.91	69.43	65.23	71.03
Avg. auctioneer's surplus	41.51	83.63	18.38	42.88	67.63	32.73
Avg. social welfare	105.55	167.45	34.31	366.90	376.91	358.21

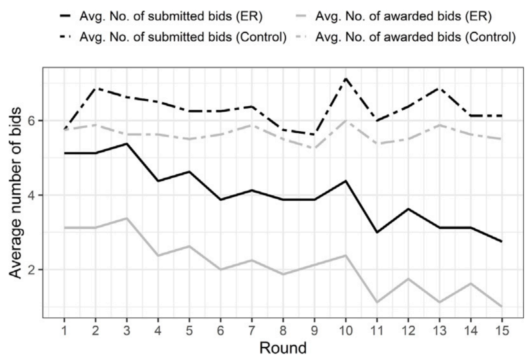

Fig. 1. Development of the numbers of submitted and awarded bids.

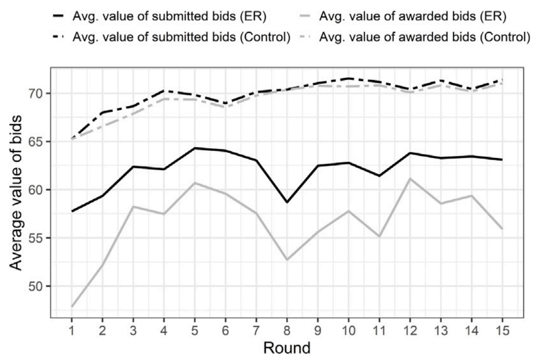

Fig. 2. Development of the means of submitted and awarded bids.

Result 1. *In the ER Treatment, the number of submitted bids is significantly lower than in the Control Treatment (overall and last round) and significantly decreases in the ER Treatment. This also applies to the number of awarded bids, where there is already a significant difference in the first round.*

This result supports the weakened hypothesis in Section 3.2 and reveals the expected strong negative effect of ER on participation.

Result 2. *The submitted bids and the awarded bids are significantly lower in the ER Treatment than in the Control Treatment, which also applies to the last round and for the awarded bids also to the first round (Fig. 2).*

The lower bids are associated with a selection effect since the lower participation in the ER Treatment is mainly due to higher-cost types staying out.[29]

Result 3. *The auctioneer's surplus significantly decreases over the rounds in both treatments, where the decrease in the ER Treatment is stronger than in the Control Treatment (Fig. 3).*

[29] The private costs of the bidders who submit bids and of those whose bids are awarded are significantly lower in the ER Treatment than in the Control Treatment and significantly decrease over the rounds (see Table D.7 in Appendix D.3).

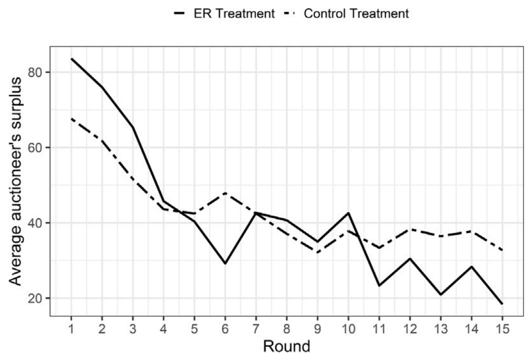

Fig. 3. Development of the auctioneer's surplus.

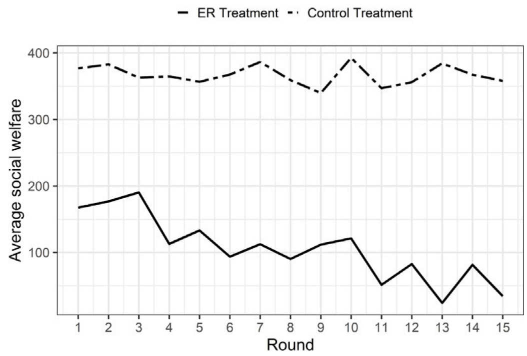

Fig. 4. Development of the social welfare.

There is no significant treatment difference in the level of the auctioneer's surplus.

Result 4. *Social welfare is significantly lower in the ER Treatment than in the Control Treatment, which also applies to the first and to the last round. Moreover, social welfare significantly decreases in the ER Treatment (Fig. 4).*

Thus, the experiment supports the theoretical predictions: the participation level and the social welfare under ER are lower than in the corresponding basic auction and strongly decrease over the rounds.[30] Although ER generates lower payments than the basic auction, a higher auctioneer's surplus cannot be identified.

4. Conflicting objectives and measures that contribute to all of them

ER is motivated by the desire to ensure proportionality of support payments. Indeed, if the experimental auctions with ER were judged based on observed support payments, they would appear advantageous because payments are lower than in the basic auction and, by construction, there is competition for the last unit allotted. However, as the experimental analysis has revealed, even though participation is higher than theory predicts, the ER auction performs worse than the basic auction with respect to multiple performance measures because of its negative effect on participation and number of goods allotted.

Auctioneer objectives. The theoretical and experimental analysis has highlighted conflicts between ensuring proportionality through ER auctions and other performance measures. To shed further light on these conflicts, we derive optimal mechanisms for three auctioneer's objectives which are prevalent in procurement auctions, particularly those organized by the government or other public institutions, as discussed in the introduction:

O1 Maximize the auctioneer's expected surplus
O2 Maximize the expected social welfare (at lowest payments)
O3 Maximize the expected number of goods allotted, given k (at lowest payments)

[30] With empirical auction data, such comparisons may be hampered by the unobservability of the number of potential bidders, e.g., in the context of renewable energy, the firms that could potentially develop a project and register for an auction.

O1 and O2 require that prior to the auction, the auctioneer assigns a value to acquiring goods. For example, in an auction for renewable energy support, the government's value for a good is the social value of the energy produced by the renewable energy plants. We assume that every acquired unit of the good has the same value v and that $c+\underline{x}<v$, so that the production of the goods can increase social welfare.

Optimal mechanisms for the three objectives. An optimal mechanism for the objective O1 maximizes the auctioneer's expected surplus, and thus trades off the value generated by acquiring goods with the payments to the firms. Switching the objective to social welfare O2 shifts the focus from support payments to firms' total costs. According to objective O3, the auctioneer aims to acquire as many as possible of the demanded goods k. In the context of renewable energy support, this objective corresponds to a commitment to achieve the development goals. For objectives O2 and O3, we break ties among optimal mechanisms in favor of those with the lowest auctioneer's payments.

Proposition 3 presents auctions that implement optimal mechanisms for objectives O1 to O3. These mechanisms include an optimal combination of two interdependent decision variables for the auctioneer: a reserve price r and a partial or full reimbursement s that all participating firms receive for their costs c. Any basic auction could replace the uniform-price auction in the proposition due to payoff equivalence (Lemma 2).

Proposition 3. *Assume $x+\frac{F(x)}{f(x)}$ is increasing.*[31] *An optimal symmetric mechanism for objective Oj, $j\in\{1,2,3\}$, implements a unique optimal cutoff type $\hat{x}_j$. A continuum of optimal symmetric mechanisms for Oj can be implemented by a uniform-price auction with different combinations (r_j,s_j), with the exception of a unique optimal combination (r_3,s_3) in case $n>k$.*

O1 *If $n>k$ or $v<\bar{x}+\frac{1}{f(\bar{x})}+c$, then $\hat{x}_1$ is given by $\left(v-\hat{x}_1-\frac{F(\hat{x}_1)}{f(\hat{x}_1)}\right)\left(1-F_{(k,n-1)}(\hat{x}_1)\right)=c$, and $r_1=v-\frac{s_1}{1-F_{(k,n-1)}(\hat{x}_1)}-\frac{F(\hat{x}_1)}{f(\hat{x}_1)}$, $s_1\in[0,c]$. Otherwise, if $n\le k$ and $v\ge\bar{x}+\frac{1}{f(\bar{x})}+c$, then $\hat{x}_1=\bar{x}$ and $r_1=\hat{x}_1+c-s_1$, $s_1\in[0,c]$.*

O2 *If $n>k$ or $v<\bar{x}+c$, then $\hat{x}_2$ is given by $(v-\hat{x}_2)\left(1-F_{(k,n-1)}(\hat{x}_2)\right)=c$, and $r_2=v-\frac{s_2}{1-F_{(k,n-1)}(\hat{x}_2)}$, $s_2\in[0,c]$. Otherwise, if $n\le k$ and $v\ge\bar{x}+c$, then $\hat{x}_2=\bar{x}$ and $r_2=\hat{x}_2+c-s_2$, $s_2\in[0,c]$.*

O3 *If $n>k$, then only the auction with $s_3=c$ is optimal, and $\hat{x}_3=\bar{x}$ and $r_3=\hat{x}_3$. If $n\le k$, then $\hat{x}_3=\bar{x}$ and $r_3=\hat{x}_3+c-s_3$, $s_3\in[0,c]$.*

In the following text, we will sometimes refer to the optimal reserve price corresponding to a special case of s_j as $r_j(s_j)$ for simplicity.

The optimal mechanism is determined by the optimal cutoff type and the payments to implement this cutoff. In case of no competition ($n\le k$) and sufficiently high value v, the optimal mechanism is the same for all three objectives and maximizes participation. All firms participate ($\hat{x}_j=\bar{x}$ for $j\in\{1,2,3\}$) because every firm wins and receives the reserve price $r_j=\bar{x}+c-s_j$ plus the reimbursement s_j. Every bid is awarded and generates value. This contributes to O3 and, if $v\ge\bar{x}+c$, to O2. If $v\ge\bar{x}+1/f(\bar{x})+c$, the tradeoff between efficiency and surplus disappears, and incentivizing full participation is optimal also for O1.

In contrast, in the case of competition ($n>k$) or sufficiently low value v, optimal mechanisms for the three objectives differ and involve tradeoffs between participation, total participation costs, and total payments.[32] Unlike in auctions without participation costs, the optimal reserve prices for O1 and O2 depend on $\hat{x}$, and, therefore, on n (except for $r_2(0)$, discussed below). To get some intuition for the optimal mechanisms, consider the cases $s_j=c$ and $s_j=0$, i.e., a uniform-price auction with an optimal reserve price $r_j(c)$ and a reimbursement c to all participating firms and a uniform-price auction with an optimal reserve price $r_j(0)$ and no reimbursement. Participants in these auctions have a weakly dominant strategy to bid their costs x_i. The cutoff type $\hat{x}_j$ is the same in the optimal auction with $s_j=c$ and $s_j=0$ for the respective objective $j\in\{1,2\}$. In the optimal auction with $s_j=c$, the reimbursement of c makes participation costless, and the reserve price $r_j(c)=\hat{x}_j$ guarantees that firms with $x>\hat{x}_j$ do not participate. In the optimal auction with $s_j=0$, the incentive to participate is provided by higher expected payments in the auction due to a higher reserve price $r_j(0)>r_j(c)$ that gives the same expected payoffs to all bidder types as in the auction with $s_j=c$.[33] Each objective $j\in\{1,2,3\}$ may be achieved with an auction with $s_j=c$, whereas with an auction with $s_j<c$ it is impossible to achieve objective O3. This is because O3 requires full participation, which cannot be achieved with $s_3<c$ because type $\bar{x}$ receives the payoff zero from the auction if all firms participate and thus cannot cover the participation costs (compare Lemma 1).

For any $s\in[0,c]$, the objectives' optimal cutoffs can be ordered by size.

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31 This assumption is necessary for a unique optimal cutoff for objective O1, but not for O2 and O3.

32 Of course, the designer does not need to fully decide in favor of one of the three conflicting objectives by choosing the respective optimal mechanism but could use, e.g., convex combinations of the objectives. By monotonicity of the objective functions in $\hat{x}$ and as $\hat{x}$ increases in r (see Lemma 1), a reserve price between r_j and r_k is optimal for some convex combination of Oj and Ok, $j,k\in\{1,2,3\}$ (cp. Corollary 2 for orders of optimal $\hat{x}$ and r). The objective function for O3 increases in r because $\hat{x}$ increases in r; the objective function for O2 increases in $\hat{x}$ for $\hat{x}<\hat{x}_2$ and decreases for $\hat{x}>\hat{x}_2$ by the derivative $nf(\hat{x})((v-\hat{x})(1-F_{(k,n-1)})-c)$ being $\lesseqgtr 0$ for $\hat{x}\gtreqless\hat{x}_2$; the objective function for O1 increases in $\hat{x}$ because the expected surplus decreases with every valuable cost type excluded or any too high costs type incentivized to participate, see Eq. (A.11). For optimal auctions for more general forms of combined objectives see, e.g., Likhodedov and Sandholm (2004), Diakonikolas et al. (2012).

33 For $k=1$, several studies identified optimal auctions and mechanisms for objectives O1 and O2. Samuelson (1985) derives the optimal auction with $s=0$ for O1 and O2 and Menezes and Monteiro (2000) show that the optimal auction with $s=c$ is an optimal mechanism for O1. Lu (2009a) derives the implementation of the optimal mechanisms as an auction with $s\in[0,c]$ for O1 and Lu (2009b) the implementation with $s=c$ for O2. Stegemann (1996) shows that equilibria with asymmetric participation can improve efficiency. Tan and Yilankaya (2006) provide conditions under which only symmetric participation occurs in the welfare-maximizing auction.

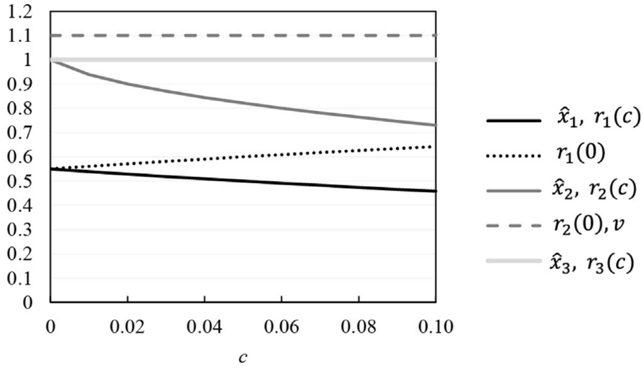

Fig. 5. Cutoff costs $\hat{x}_1$, $\hat{x}_2$, and $\hat{x}_3$, reserve prices $r_1(c)$, $r_2(c)$, and $r_3(c)$, and reserve prices $r_1(0)$ and $r_2(0)$ for $k = 1$, $n = 2$, $v = 1.1$, and for different levels of c with a uniform distribution of private costs on $[0, 1]$.

Corollary 2. *Assume $x + \frac{F(x)}{f(x)}$ is increasing. If $n > k$ or $v < \bar{x} + c$, then optimal cutoff types and reserve prices for the objectives are ordered as follows: $\hat{x}_3 > \hat{x}_2 > \hat{x}_1$, $r_3 > r_2 > r_1$ for $s = c$, and $r_2 > r_1$ for all $s \in [0, c)$. If $n \leq k$ and $v \geq \bar{x} + \frac{1}{f(\bar{x})} + c$, then optimal cutoff types and reserve prices are the same for all objectives: $\hat{x}_j = \bar{x}$ and $r_j = \bar{x} + c - s_j$ for $s_j \in [0, c]$, $j \in \{1, 2, 3\}$.*

Fig. 5 illustrates these rankings using a uniform distribution F on $[0, 1]$.[34] It also illustrates some general properties of how optimal cutoffs and reserve prices change with c. If c increases, the surplus-maximizing cutoff $\hat{x}_1$ and reserve price $r_1(c)$ (which are equal) decrease and are always below the valuation v. The decreasing cutoff $\hat{x}_1$ and the reserve price $r_1(0)$ diverge when c increases by the assumption that $\hat{x}_1 + F(\hat{x}_1)/f(\hat{x}_1)$ decreases as $\hat{x}_1$ decreases (whereas $r_1(0)$, which increases in Fig. 5, in general increases or decreases in c depending on whether the reverse hazard rate $F(\hat{x}_1)/f(\hat{x}_1)$ decreases or increases as $\hat{x}_1$ decreases).[35] Furthermore, independent of c and F we have that $\hat{x}_3 = \bar{x}$ and $r_2(0) = v$, and for all F we have that $\hat{x}_2$ and $r_2(c)$ decrease in c.

Informational robustness. On the one hand, among the optimal auctions, the welfare-maximizing auction without reimbursement stands out because the auctioneer can implement it by setting the reserve price equal to the auctioneer's valuation for a good, $r_2(0) = v$. Therefore, the auctioneer needs to know neither n nor F. Just as in an auction without participation costs, $r_2(0) = v$ is socially optimal because a firm participates if and only if her expected social contribution ($v - \hat{x}$ times winning probability minus participation costs) is positive.

On the other hand, the auctions with reimbursement $s = c$ distinguish themselves by providing weakly dominant strategies for firms' participation and bidding decisions (cp. Lu, 2009b): Firms participate iff $x_i \leq \hat{x}$ and bid their costs x_i, independent of the other firms' participation or bidding decisions.

Optimal mechanisms with reimbursement to subsets of bidders. The designs in Proposition 3 are not the only ways to implement the optimal mechanisms.[36]

Corollary 3. *The optimal mechanism for Oj, $j \in \{1, 2\}$, can be implemented (i) by a uniform-price auction and a reimbursement $s_j \in [0, c]$ to all winning bidders; (ii) by a uniform-price auction and a reimbursement $s_j \in [0, c]$ to all losing bidders. The optimal mechanism for O3 can be implemented by an auction of type (ii) with $s_3 = c$.*

If only winning or only losing bidders receive a reimbursement, a firm's bid is lower or higher, respectively, than its cost by the amount of the reimbursement. As a result, the optimal auction in which only winning bidders receive the reimbursement has the reserve price $r_j^w(s) = r_j(0) - s$ and is ex-post payoff equivalent to the optimal auction in which no one receives a reimbursement, while the optimal auction in which only losing bidders receive the reimbursement has the reserve price $r_j^l(s) = r_j(s) + s$ and is ex-post payoff equivalent to the optimal auction in which all bidders receive a reimbursement. Hence, these designs give the auctioneer the ability to shift payments between award and reimbursement. However, they cannot address the conflict of objective.

Policies that support all objectives. The question arises if there are policies that support all objectives. A measure that improves all three dimensions in a setting without participation costs is to increase the number of bidders n. With participation costs, however, increasing n can increase or decrease surplus and welfare depending on F, c, and v.[37]

The optimal auction formats with full reimbursement $s = c$ highlight the relevance of the participation costs c. Reducing c is a measure that supports all three objectives.

[34] Note that with this distribution, the optimal reserve price $r_1(0)$ and cutoff value $\hat{x}_1$ add up to the auctioneer's valuation: $r_1(0) + \hat{x}_1 = v - F(\hat{x}_1)/f(\hat{x}_1) + \hat{x}_1 = v$.

[35] The effect of the number of potential bidders n on cutoff type and reserve price is the same as the effect of c. Proposition 4 will, however, show that the influence of increasing n and c on the value of the objective function can differ.

[36] Technically, auctions with entry fees (i.e., $s < 0$) are another option (Menezes and Monteiro, 2000).

[37] Samuelson (1985) and Menezes and Monteiro (2000) provide examples for both objectives.

Proposition 4. *Consider participation costs c and c' with $c' < c$. The expected auctioneer's surplus (O1), expected social welfare (O2), and the expected number of goods allotted (O3) in a basic auction with reserve price $r \leq v$ and no reimbursement are with c' strictly higher than with c if $n > k$.*

Proposition 4 states that, with any basic auction with $r \leq v$, surplus, welfare, and number of allotted goods benefit from a decrease in participation costs.[38] For a given reserve price, decreasing c has the direct effect of decreasing participation costs for all firms that participate with c but in addition it increases the cutoff costs and more (higher) cost types participate (Lemma 1). Therefore, the expected number of goods allotted increases, the expected payment per good decreases, and the welfare contribution of all types increases. Proposition 4 implies that the positive effect of decreasing c is robust to non-optimal specifications of r and that all three objectives benefit from decreasing c even if the original reserve price is chosen optimally for only one of them.

From Proposition 4 follows directly that surplus and welfare in the respective optimal mechanism for $n > k$ increase if participation costs decrease because a basic auction with optimal reserve price below the valuation is an optimal mechanism for these objectives. Further improvements in surplus and welfare are achieved by optimally adjusting the reserve price to the higher optimal cutoff. If $n \leq k$, welfare in the optimal mechanism increases directly due to the lower participation costs and surplus increases by the reduction of the optimal reserve price. Corollary 4 summarizes this result. The optimal mechanism for O3 achieves full participation, and therefore the expected number of goods allotted does not change in the participation costs.

Corollary 4. *Consider participation costs c and c' with $c' < c$. The expected auctioneer's surplus (O1) and expected social welfare (O2) in the respective optimal mechanism with c' are strictly higher than with c.*

Policy implications. A straightforward policy implication of Proposition 4 is to reduce the participation costs. Even if the objectives are in conflict, such a measure contributes to three different objectives. It also contributes to ensuring proportionality because the likelihood that competition determines the auction price increases. Thus, any factors that unnecessarily increase participation costs should be eliminated. In the context of auctions for renewables support these include administrative barriers, limited availability of zones for plant construction, lack of public acceptance, species and conservation laws, and litigation that may lengthen or impede the planning and permitting processes. Speeding up administrative processes, preplanning sites, changing laws to expand available zones, using more exemptions under species protection laws or adjusting bird protection (e.g., from an individual analysis to a species aggregate), and involving or even compensating citizens in order to reduce their opposition to renewable energy projects near their homes are among the measures being discussed to address barriers. Of course, any measure that reduces the firms' participation costs may involve implementation costs. Furthermore, any measure that reduces participation costs by mitigating participation requirements must take into account adverse effects on project quality. Therefore, it is advisable not to eliminate the participation requirements that cause participation costs, but to strike a reasonable balance between entry barriers that ensure the seriousness of bids and attractive conditions for participation in the auction.

5. Conclusion

ER has been proposed as a means of ensuring proportionality and increasing competition in the case of low participation in auctions. However, both in theory and in the experiment the primary effect of ER in auctions with costly participation is a large reduction in participation, which impairs welfare and volume goals.

Guidelines and directives for renewable energy auctions stipulate proportionality and a number of additional or alternative objectives. This paper proposes a variety of auction designs to achieve these objectives. These designs have in common that they do not apply ER.

In general, the objectives to maximize the auctioneer's surplus, social welfare, or the number of goods allotted, and ensuring proportionality conflict. However, one policy measure that contributes to all three objectives and to ensuring proportionality for any number of potential bidders – assuming it absorbs its implementation costs and does not adversely affect project quality – is to reduce the costs of participation.

This paper identifies low participation as a detrimental effect of ER. If the strategy space is expanded, auctions with ER may also be plagued by strategic manipulations. ER auctions are susceptible to actions that artificially increase supply in order to profit from a higher probability of an award. A firm may participate with a serious bid and with an additional bid (either because multi-unit supply bidding is feasible or under a different identity) that equals the reserve price to increase the probability of an award for the serious bid. Participation costs reduce the attractiveness or availability of such strategic supply expansion. Moreover, a relevant parameter in multi-unit bidding settings is whether all of a bidder's bids can be eliminated due to rationing in an ER auction. Analyzing how such extended options play out in more general settings is an avenue for future research.

Declaration of generative AI and AI-assisted technologies

During the preparation of this work the authors used DeepL in order to improve the readability and language of the manuscript and Refine.ink to detect inconsistencies and improve the paper. After using these tools, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

[38] The restriction $r \leq v$ is sufficient but not necessary. However, for $r > v$ there exist examples where the result does not hold. The restriction to $n > k$ is because for $n \leq k$, number of goods allotted and surplus (for sufficiently high r) do not change in c.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Marion Ott reports financial support was provided by Federal Ministry of Research, Technology and Space, Germany. Karl-Martin Ehrhart reports financial support was provided by European Commission. Ann-Katrin Fleck reports financial support was provided by European Commission. Karl-Martin Ehrhart reports a relationship with Federal Ministry for Economic Affairs and Climate Action that includes: consulting or advisory. Ann-Katrin Fleck reports a relationship with Federal Ministry for Economic Affairs and Climate Action that includes: consulting or advisory.

Acknowledgments

This work was supported by the European Commission [grant number 817619]; and the BMFTR (Federal Ministry of Research, Technology and Space, Germany) through the Kopernikus-Project SynErgie [grant numbers 03SFK3Y2-2, 03SFK3Y2-3]. It contains results of the Horizon 2020 project AURES II. Karl-Martin Ehrhart and Ann-Katrin Fleck are involved in AURES II through Takon (https://www.takon.com/en). Any dissemination of results reflects only the authors' view, and the European Commission Horizon 2020 is not responsible for any use that may be made of the information this study contains. We thank members of the BMWK (Federal Ministry for Economic Affairs and Climate Action, Germany) and of the AURES II consortium as well as Verena Dorner, Vitali Gretschko, Tonda Roder, Silvana Tiedemann for helpful discussions.

Appendix A. Proofs

The proofs of Lemma 1 and Proposition 3 will make use of the following lemma.

Lemma 4.

$$\sum_{i=0}^{k}\binom{n}{i}F(x)^{i-1}(1-F(x))^{n-i-1}(i-nF(x)) = -n\binom{n-1}{k}F(x)^{k}(1-F(x))^{n-k-1} \tag{A.1}$$

$$\sum_{i=0}^{k}\binom{n}{i}(i-k)F(x)^{i-1}(1-F(x))^{n-i-1}(i-nF(x)) = \sum_{i=1}^{k}\binom{n}{i}iF(x)^{i-1}(1-F(x))^{n-i} \tag{A.2}$$

Proof of Lemma 4. We will use the identities

$$\binom{n}{i}i = n\binom{n-1}{i-1} \qquad \text{and} \qquad \binom{n}{i}(n-i) = n\binom{n-1}{i} \tag{A.3}$$

and the binomial theorem

$$\sum_{i=0}^{n}\binom{n}{i}F(x)^{i}(1-F(x))^{n-i} = (F(x)+1-F(x))^{n} = 1\,. \tag{A.4}$$

Proof of (A.1) via simplifying a telescoping sum.

$$\begin{aligned}
\sum_{i=0}^{k}\binom{n}{i}F(x)^{i-1}(1-F(x))^{n-i-1}(i-nF(x)) &= \sum_{i=0}^{k}\binom{n}{i}\left(iF(x)^{i-1}(1-F(x))^{n-i}-(n-i)F(x)^{i}(1-F(x))^{n-i-1}\right)\\
&\underset{(A.3)}{=} n\left(\sum_{i=1}^{k}\binom{n-1}{i-1}F(x)^{i-1}(1-F(x))^{n-i}-\sum_{i=0}^{k}\binom{n-1}{i}F(x)^{i}(1-F(x))^{n-i-1}\right)\\
&= n\left(\sum_{i=0}^{k-1}\binom{n-1}{i}F(x)^{i}(1-F(x))^{n-i-1}-\sum_{i=0}^{k}\binom{n-1}{i}F(x)^{i}(1-F(x))^{n-i-1}\right)\\
&= -n\binom{n-1}{k}F(x)^{k}(1-F(x))^{n-k-1}
\end{aligned}$$

Proof of (A.2) via induction. For $k=1$, $n(1-F(x))^{n-1} = n(1-F(x))^{n-1}$, which proves the base case. For the induction step assume (A.2) holds for k (IH). Now look at $k+1$.

$$\begin{aligned}
&\sum_{i=0}^{k+1}\binom{n}{i}F(x)^{i-1}(1-F(x))^{n-i-1}(i-nF(x))(i-k-1)\\
&= \sum_{i=0}^{k}\binom{n}{i}F(x)^{i-1}(1-F(x))^{n-i-1}(i-nF(x))(i-k) - \sum_{i=0}^{k}\binom{n}{i}F(x)^{i-1}(1-F(x))^{n-i-1}(i-nF(x))\\
&\underset{IH}{=} \sum_{i=0}^{k}\binom{n}{i}iF(x)^{i-1}(1-F(x))^{n-i} - \sum_{i=0}^{k}\binom{n}{i}iF(x)^{i-1}(1-F(x))^{n-i-1} + \sum_{i=0}^{k}\binom{n}{i}nF(x)^{i}(1-F(x))^{n-i-1}\\
&= -\sum_{i=0}^{k}\binom{n}{i}iF(x)^{i}(1-F(x))^{n-i-1} + \sum_{i=0}^{k}\binom{n}{i}nF(x)^{i}(1-F(x))^{n-i-1}
\end{aligned}$$

$$= \sum_{i=0}^{k} \binom{n}{i}(n-i)F(x)^i(1-F(x))^{n-i-1}$$
$$= \sum_{i=1}^{k+1} \binom{n}{i-1}(n-i+1)F(x)^{i-1}(1-F(x))^{n-i}$$
$$= \sum_{i=1}^{k+1} \binom{n}{i} i F(x)^{i-1}(1-F(x))^{n-i}$$

■

Proof of Lemma 1. First we show that $\hat{x} < \bar{x}$ if $n > k$ or $r < \bar{x} + c$. If a firm with costs $\bar{x}$ participates, the other $n-1$ firms also participate. If a firm with costs $\bar{x}$ is wins a good, it receives a payment of r by (P4). A firm with $\bar{x}$ wins a good with probability 0 if $n > k$ and with probability 1 if $n \leq k$. A firm participates iff its expected profit from participating is non-negative. Thus, if $n > k$, a firm with $\bar{x}$ does not participate because $(r-\bar{x}) \cdot 0 - c < 0$. If $n \leq k$, a firm with $\bar{x}$ does not participate iff $(r-\bar{x}) \cdot 1 - c < 0$.

If, to the contrary, $n \leq k$ and $r \geq \bar{x} + c$, then the expected payoff of the worst-off type $\bar{x}$ is positive, $r - \bar{x} - c \geq 0$, and all firms participate, $\hat{x} = \bar{x}$.

To prove the remaining properties, consider the expected profit of the firm $\hat{i}$ who receives a good only if no more than $k-1$ other firms participate if $n > k$ or who receives a good for sure if she participates if $n \geq k$. Her expected profit is (see (1))

$$\Pi(\hat{x}, r, c, n) = (r-\hat{x})(1 - F_{(k,n-1)}(\hat{x})) - c = \begin{cases} (r-\hat{x}) \sum_{i=0}^{k-1} \binom{n-1}{i} F(\hat{x})^i \left(1-F(\hat{x})\right)^{n-i-1} - c & \text{if } n > k \\ r - \hat{x} - c & \text{if } n \leq k. \end{cases} \tag{A.5}$$

Since firm $\hat{i}$ participates only if $\Pi(\hat{x}, r, c, n) \geq 0$ and since $c > 0$, it follows that $\hat{x} < r$.

If not all firms participate ($\hat{x} < \bar{x}$), firm $\hat{i}$ is indifferent between participating and not participating in the auction. Since firm $\hat{i}$ is indifferent if and only if her expected profit from participating is zero, the cutoff costs $\hat{x}$ are determined by

$$\Pi(\hat{x}, r, c, n) = 0\,. \tag{A.6}$$

There exists a unique $\hat{x}$ that fulfills (A.6), since the derivative of (A.5) with respect to $\hat{x}$ is negative for all $\hat{x} \in [\underline{x}, r]$[39]:

$$\frac{\partial \Pi(\hat{x}, r, c, n)}{\partial \hat{x}} = \begin{cases} -\sum_{i=0}^{k-1} \binom{n-1}{i} F(\hat{x})^i \left(1-F(\hat{x})\right)^{n-i-1} - \\ (r-\hat{x})k\binom{n-1}{k} f(\hat{x}) F(\hat{x})^{k-1}\left(1-F(\hat{x})\right)^{n-k-1} & \text{if } n > k \\ -1 & \text{if } n \leq k \end{cases}$$
$$< 0\,. \tag{A.7}$$

The payoff $\Pi(\hat{x}, r, c, n)$ decreases in $\hat{x}$ but is positive for $\hat{x} = \underline{x}$: $r - \underline{x} - c > 0$ by the assumption on r in (P2). Thus, $\hat{x} > \underline{x}$.

To determine how $\hat{x}$ depends on r and c, we apply the implicit function theorem. With (A.6) and (A.7) we get

$$\frac{d\hat{x}}{dr} = -\frac{\frac{\partial \Pi(\hat{x},r,c,n)}{\partial r}}{\frac{\partial \Pi(\hat{x},r,c,n)}{\partial \hat{x}}} = -\frac{\sum_{i=0}^{\min\{k,n\}-1} \binom{n-1}{i} F(\hat{x})^i \left(1-F(\hat{x})\right)^{n-i-1}}{\frac{\partial \Pi(\hat{x},r,c,n)}{\partial \hat{x}}} > 0\,,$$
$$\frac{d\hat{x}}{dc} = -\frac{\frac{\partial \Pi(\hat{x},r,c,n)}{\partial c}}{\frac{\partial \Pi(\hat{x},r,c,n)}{\partial \hat{x}}} = \frac{1}{\frac{\partial \Pi(\hat{x},r,c,n)}{\partial \hat{x}}} < 0\,.$$

The cutoff costs $\hat{x}$ increase in k if $n > k$ because the probability in (A.5) increases in k.

Further, $\hat{x}$ decreases in n if $n \geq k$. For given cutoff of participation $\tilde{x}$, the probability of getting a good in (A.5) decreases in n because $F_{(k,n-1)}(\tilde{x}) < F_{(k,n)}(\tilde{x})$. Take the cutoff $\hat{x}$ that fulfills (A.6) for n firms. If there were $n+1$ firms and the firm with costs $\hat{x}$ was the cutoff type, its probability of getting a good would be lower and the payoff in (A.5) would be negative. Thus, the cutoff costs with $n+1$ firms must be smaller than with n firms.

Proof of Lemma 2. Firms participate in the auction iff $x_i \leq \hat{x}$. If $n > k$, type $\hat{x}$ wins iff $m \leq k$ and, by (P4), type $\hat{x}$ can then bid to receive the maximum payment r. Type $\hat{x}$'s expected profit from the auction is, with $G(x) = F_{(k,n-1)}(x)$, $\pi(\hat{x}, r, c, n) = (r-\hat{x})(1-G(\hat{x})) = c$ (see (1) and the proof of Lemma 1). If $n \leq k$, $\pi(\hat{x}, r, c, n) = r - \hat{x}$.

If $n > k$, we assume symmetric, strictly increasing equilibrium bidding functions (P5). Denote by $\pi(x, z)$ the expected profit of a firm of type $x \leq \hat{x}$ who bids as if her type was z. Let $p(z)$ denote the payment to a firm who bids like type z. To maximize

$$\pi(x, z) = p(z) - (1 - G(z))x \qquad \text{for all } x, z \in [\underline{x}, \hat{x}],$$

we derive the first-order condition

$$\frac{\mathrm{d}}{\mathrm{d}z}\pi(x, z) = \frac{d}{dz}p(z) + g(z)x = 0 \qquad \text{for all } x, z \in [\underline{x}, \hat{x}].$$

[39] The second term of the derivative stems from simplifying a telescoping sum (like the steps to get Lemma 4(A.1)).

In equilibrium, $z = x$, and, thus

$$\frac{\mathrm{d}}{\mathrm{d}y} p(y) = -g(y)y \qquad \text{for all } y \in [\underline{x}, \hat{x}]$$

$$\Longrightarrow\ p(x) = \text{const} + \int_x^{\hat{x}} g(y)y\,\mathrm{d}y = (1 - G(\hat{x}))\,r + \int_x^{\hat{x}} g(y)y\,\mathrm{d}y \qquad \text{for all } x \in [\underline{x}, \hat{x}]\,.$$

Therefore, the expected profit

$$\pi(x, r, c, n) = (1 - G(\hat{x}))\,r + \int_x^{\hat{x}} g(y)y\,\mathrm{d}y - (1 - G(x))\,x \ \text{ for all } x \in [\underline{x}, \hat{x}]\,, \tag{A.8}$$

is the same for all auctions that assign goods to the same types (P5) and in which a firm can bid to receive r if $m \le k$ (P4).

If $n \le k$, $\pi(x, r, c, n) = r - x$ for all $x \in [\underline{x}, \hat{x}]$ in all auctions with property (P4). ■

Proof of Lemma 3. The proof uses payoff equivalence (Lemma 2).

Consider first the case $n > k$. In a pay-as-bid auction, if all firms choose β^{PaB}, a firm with type $x \le \hat{x}$ wins, has costs x, and receives the payment $\beta^{PaB}(x, r)$ iff she is among the k lowest types, which has probability $1 - G(x)$ with $G(x) = F_{(k,n-1)}(x)$. Therefore, her expected payment is

$$(1 - G(x))\beta^{PaB}(x, r) = (1 - G(x))\,x + (1 - G(\hat{x}))(r - \hat{x}) + \int_x^{\hat{x}} (1 - G(y))\,\mathrm{d}y = (1 - G(\hat{x}))\,r + \int_x^{\hat{x}} g(y)y\,\mathrm{d}y\,,$$

using partial integration $\int_x^{\hat{x}} (1 - G(y))\,\mathrm{d}y = [(1 - G(y))y]_x^{\hat{x}} + \int_x^{\hat{x}} g(y)y\,\mathrm{d}y$. Thus, her expected profit is equal to (A.8) in the proof of Lemma 2.

In a uniform-price auction, if all firms choose $\beta^{UP}(x, r) = x$, a firm with type $x \le \hat{x}$ wins and has costs x if she is among the k lowest types, which has probability $1 - G(x)$. She receives the payment r if no more than k firms participate, which has probability $1 - G(\hat{x})$. Her payment is equal to her opponents' kth lowest bid if more than k firms participate and she is among the k lowest types. Her expected payment from these two cases is $(1 - G(\hat{x}))r + \int_x^{\hat{x}} g(y)y\,\mathrm{d}y$. Thus, her expected profit is equal to (A.8).

If $n \le k$, $\pi(x, r, c, n) = r - x$ for all $x \in [\underline{x}, \hat{x}]$ by payoff equivalence. Bidders receive the payment r by bidding r in a pay-as-bid auction and by bidding x (or r) in a uniform-price auction. ■

Proof of Proposition 1. With symmetric monotonic bidding strategies (see (P5)), the expected payoff from an ER auction is strictly monotonic and continuously decreasing in the type x: As in a basic auction, both the payoff in case of winning and the winning probability continuously decrease in x. Thus, if a type x participates, any type lower than x will also participate, and if a type x does not participate, any type above x will not participate.

Assume there is a cutoff type $\hat{x}$ for participation such that all types $x \le \hat{x}$ participate and all types $x > \hat{x}$ do not participate. In the auction, a bidder of type $\hat{x}$ will submit the highest bid. This bidder does not win: either because there is competition (if $m > k$) or because of rationing (if $m \le k$). A bidder with no chance of winning will not participate due to the participation costs $c > 0$. Thus, by contradiction, such cutoff type $\hat{x} \ge 0$ does not exist.

Assume there is a cutoff type $\hat{x}$ for participation such that all types $x < \hat{x}$ participate and all types $x \ge \hat{x}$ do not participate. Type $\hat{x}$ does not participate, so the overall expected payoff in case of participation must be negative. As the expected payoff is continuously decreasing in x, there must be a type $\hat{\hat{x}} < \hat{x}$ such that all types $x \in [\hat{\hat{x}}, \hat{x})$ have negative payoff. By contradiction, this excludes the existence of the assumed cutoff type $\hat{x}$.

Therefore, no bidder will participate. ■

Proof of Proposition 2. Remember, the $\tilde{n}$ bidders with $c = 0$ always participate and, due to uniform pricing, all participating bidders bid their costs x_i.

Case (i): If $\tilde{n} + n \le k$, then ER must occur. If $R > \tilde{n}$, the highest type x of the up to n participants with $c > 0$ would never win. Thus, by contradiction, a cutoff type $\hat{x} \ge 0$ for participation of these n potential bidders cannot exist (cp. the arguments in the proof of Proposition 1). With $\tilde{n} \le k$ participants and $R > \tilde{n}$, rationing implies $\max\{0, \tilde{n} + m - R\} = 0$ winning bids.

Case (ii): We divide this case in three subcases. First, consider $\tilde{n} + n \le k$ and $R \le \tilde{n}$. Rationing always occurs. Assume $\hat{x} \in (\underline{x}, \bar{x})$. Then $m \in \{0, \dots, n\}$ bidders with $c > 0$ participate. The bidder with the highest costs among these wins if the realized costs of at least R bidders with $c = 0$ are above the considered bidder's costs, which is feasible even in the worst case $R = \tilde{n}$. Thus, the expected payoff from the auction is positive. With $r > \underline{x} + c$, type $\underline{x}$ has a strictly positive overall payoff. As this expected payoff continuously decreases in x, there exists a $\hat{x} > \underline{x}$ such that the expected payoff is positive for all $x \le \hat{x}$. Thus, $\hat{x} > \underline{x}$. Moreover, $\hat{x} < \bar{x}$ because type $\bar{x}$ cannot have lower costs than R bidders with $c = 0$. As $\tilde{n} + m \le k$ and $R \le \tilde{n}$ rationing implies $\max\{0, \tilde{n} + m - R\} = \tilde{n} + m - R$ winning bids.

Second, consider $\tilde{n} + n > k > R > \tilde{n}$. Let $\hat{x} \in (\underline{x}, \bar{x})$. Then $m \in \{0, \dots, n\}$ bidders with $c > 0$ participate. The bidder with the highest costs among these wins if and only if $\tilde{n} + m > k$ and the costs are among the k lowest costs; if $\tilde{n} + m \le k$, this bidder is among those whose bids are rationed because $R > \tilde{n}$. The costs are among the k lowest costs if $m \le k$ and no more than $k - m$ costs of the $\tilde{n}$ bidders with $c = 0$ are below. Such costs and m exist and, thus, the expected payoff from the auction is positive. By the arguments in the case in the previous paragraph, $\hat{x} \in (\underline{x}, \bar{x})$ exists, and $\hat{x} \notin \{\underline{x}, \bar{x}\}$.

Third, consider $\tilde{n} + n > k \ge \tilde{n} \ge R$ (with $R < k$). Let $\hat{x} \in (\underline{x}, \bar{x})$. Then $m \in \{0, \dots, n\}$ bidders with $c > 0$ participate. The bidder with the highest costs among these wins if (i) $\tilde{n} + m > k$ and the costs are among the k lowest costs or (ii) $\tilde{n} + m \le k$ and the bidder's costs are not among the R highest costs. In case (i), the costs are among the k lowest costs if $m \leq k$ and no more than $k-m$ costs of the $\tilde{n}$ bidders with $c=0$ are below. In case (ii), the costs are not among the R highest costs if at least R of the $\tilde{n}$ bidders with $c=0$ have higher costs. Such costs and m exist and, thus, the expected payoff from the auction is positive. By the arguments in the case in the previous paragraph, $\hat{x} \in (\underline{x}, \bar{x})$ exists, and $\hat{x} \notin \{\underline{x}, \bar{x}\}$.

Case (iii): Consider $\tilde{n} > k$. Then more than k bidders participate and rationing is not applied. By definition, the ER auction is then identical to the corresponding auction without ER. ■

Proof of Proposition 3. We will prove parts O1 to O3 of Proposition 3 consecutively.

O1 Auctioneer's surplus We use standard mechanism design arguments (e.g., Myerson, 1981; Krishna, 2010) to derive optimal mechanisms when firms have to bear participation costs in order to bid. Let $q^p(x_i)$, $q^p : [\underline{x}, \bar{x}] \to [0,1]$, define a firm's participation probability as a function of her costs x_i, let $q_i^g(\mathbf{x})$, $q_i^g : [\underline{x}, \bar{x}]^n \to [0,1]$ denote firm i's probability of getting an item when i does participate and the costs $\mathbf{x}$ are announced, and let $p_i(\mathbf{x})$, $p_i : [\underline{x}, \bar{x}]^n \to \mathbb{R}$ denote the payment to firm i when the costs $\mathbf{x}$ are announced. The firms send messages $\mathbf{x}$ to the mechanism. The mechanism designer chooses a mechanism $(q^p, (q_1^g, q_2^g, \dots, q_n^g), (p_1, p_2, \dots, p_n))$ to maximize his expected surplus, taking the firms' (interim) individual rationality (IR) and incentive compatibility (IC) constraints into account. His problem is

$$\max_{(q^p,(q_1^g,q_2^g,\dots,q_n^g),(p_1,p_2,\dots,p_n))} \int \sum_{i=1}^{n} \left[q^p(x_i) q_i^g(\mathbf{x}) v - p_i(\mathbf{x})\right] \, \mathrm{d}H(\mathbf{x})$$

s.t. (IR), (IC)

$$0 \leq q^p(x_i) \leq 1, 0 \leq q_i^g(\mathbf{x}) \leq 1 \quad \forall i = 1,2,\dots,n, \sum_{i=1}^{n} q_i^g(\mathbf{x}) \leq k$$

where $H(\mathbf{x})$ denotes the joint distribution of the individual cost distributions, $H(\mathbf{x}) = \Pi_{i=1}^{n} F(x_i)$ and $H_{-i}(\mathbf{x}_{-i}) = \Pi_{j \neq i} F(x_j)$.

A firm i's (interim) expected payoff from reporting x_i when costs are x_i is

$$\Pi_i(x_i) = \int \left[p_i(\mathbf{x}) - q^p(x_i)\left(q_i^g(\mathbf{x}) x_i + c\right)\right] \mathrm{d}H_{-i}(\mathbf{x}_{-i}). \tag{A.9}$$

The IR constraint and the IC constraint, which ensure truthful reporting of x_i, are

$$\Pi_i(x_i) \geq 0 \quad \forall x_i \in [\underline{x}, \bar{x}] \tag{IR}$$

$$\Pi_i(x_i) \geq \int \left[p_i(x_i', \mathbf{x}_{-i}) - q^p(x_i')\left(q_i^g(x_i', \mathbf{x}_{-i}) x_i + c\right)\right] \mathrm{d}H_{-i}(\mathbf{x}_{-i}) \; \forall x_i, x_i' \in [\underline{x}, \bar{x}]. \tag{IC}$$

Furthermore, define firm i's (interim) expected probability of getting a good conditional on i's participation

$$Q_i(x_i) := \int q_i^g(\mathbf{x}) \mathrm{d}H_{-i}(\mathbf{x}_{-i}).$$

Condition (IC) implies, for all x_i and x_i'

$$\begin{aligned}
\Pi_i(x_i) &= \int \left[p_i(x_i, \mathbf{x}_{-i}) - q^p(x_i)\left(q_i^g(x_i, \mathbf{x}_{-i}) x_i + c\right)\right] \mathrm{d}H_{-i}(\mathbf{x}_{-i}) \\
&\geq \int \left[p_i(x_i', \mathbf{x}_{-i}) - q^p(x_i')\left(q_i^g(x_i', \mathbf{x}_{-i}) x_i + c\right)\right] \mathrm{d}H_{-i}(\mathbf{x}_{-i}) \\
&= \int \left[p_i(x_i', \mathbf{x}_{-i}) - q^p(x_i')\left(q_i^g(x_i', \mathbf{x}_{-i}) x_i' + c\right)\right] \mathrm{d}H_{-i}(\mathbf{x}_{-i}) + \int \left[q^p(x_i')\, q_i^g(x_i', \mathbf{x}_{-i})(x_i' - x_i)\right] \mathrm{d}H_{-i}(\mathbf{x}_{-i}) \\
&= \Pi_i(x_i') + q^p(x_i') Q_i(x_i')(x_i' - x_i).
\end{aligned}$$

Thus, for $x_i > x_i'$,

$$\frac{\Pi_i(x_i) - \Pi_i(x_i')}{x_i - x_i'} \geq -q^p(x_i') Q_i(x_i')$$

and for $x_i < x_i'$,

$$\frac{\Pi_i(x_i) - \Pi_i(x_i')}{x_i - x_i'} \leq -q^p(x_i') Q_i(x_i').$$

Therefore, (IC) implies that

$$\frac{\mathrm{d}\Pi_i(x_i)}{\mathrm{d}x_i} = -q^p(x_i) Q_i(x_i)$$

and, by integration,

$$\Pi_i(x_i) = \mathrm{const}_i + \int_{x_i}^{\bar{x}} q^p(z) Q_i(z) \, \mathrm{d}z. \tag{A.10}$$

Using (A.9) and (A.10) we can rewrite the auctioneer's expected surplus with incentive compatible payoffs of firms as

$$\begin{aligned}
\Pi_0 &= \int \sum_{i=1}^{n} q^p(x_i) q_i^g(\mathbf{x}) v - p_i(\mathbf{x}) \, \mathrm{d}H(\mathbf{x}) \\
&= \int \left[\sum_{i=1}^{n} q^p(x_i) q_i^g(\mathbf{x}) v - p_i(\mathbf{x}) + \int p_i(\mathbf{x}) - q^p(x_i)\left(q_i^g(\mathbf{x}) x_i + c\right) \mathrm{d}H_{-i}(\mathbf{x}_{-i}) \right] \mathrm{d}H(\mathbf{x}) - \int \sum_{i=1}^{n} \Pi_i(x_i) \, \mathrm{d}H(\mathbf{x}) \\
&= \int \sum_{i=1}^{n} q^p(x_i) \left[q_i^g(\mathbf{x})(v - x_i) - c \right] \mathrm{d}H(\mathbf{x}) - \sum_{i=1}^{n} \mathrm{const}_i - \int \sum_{i=1}^{n} \int_{x_i}^{\bar{x}} q^p(z) Q_i(z) \, \mathrm{d}z \, \mathrm{d}H(\mathbf{x}) \\
&= \int \sum_{i=1}^{n} q^p(x_i) \left[q_i^g(\mathbf{x})(v - x_i) - c \right] \mathrm{d}H(\mathbf{x}) - \sum_{i=1}^{n} \mathrm{const}_i - \sum_{i=1}^{n} \int \frac{F(x_i)}{f(x_i)} q^p(x_i) Q_i(x_i) \, \mathrm{d}H(\mathbf{x}) \\
&= \int \sum_{i=1}^{n} q^p(x_i) \left[q_i^g(\mathbf{x}) \left(v - x_i - \frac{F(x_i)}{f(x_i)} \right) - c \right] \mathrm{d}H(\mathbf{x}) - \sum_{i=1}^{n} \mathrm{const}_i \,,
\end{aligned}$$

where in the next-to-last step we interchanged the order of integration in the hindmost integral.[40] To maximize his surplus, the auctioneer will choose const_i as low as possible, which is zero because (IC) (i.e., (A.10)) and (IR) bound const_i to zero from below.

Thus, the auctioneer's problem can be written as

$$\max_{(q^p,(q_1^g,q_2^g,\ldots,q_n^g),(p_1,p_2,\ldots,p_n))} \int \sum_{i=1}^{n} q^p(x_i) \left[q_i^g(\mathbf{x}) \left(v - x_i - \frac{F(x_i)}{f(x_i)} \right) - c \right] \mathrm{d}H(\mathbf{x})$$

$$\text{s.t.} \quad 0 \le q^p(x_i) \le 1, \quad 0 \le q_i^g(\mathbf{x}) \le 1 \quad \forall i = 1,2,\ldots,n, \quad \sum_{i=1}^{n} q_i^g(\mathbf{x}) \le k$$

We assume that $x_i + \frac{F(x_i)}{f(x_i)}$ is increasing in x_i. Thus, either there exists a unique $\tilde{x} < \bar{x}$ such that $v - \tilde{x} - \frac{F(\tilde{x})}{f(\tilde{x})} = 0$ and $v - x_i - \frac{F(x_i)}{f(x_i)} \ge 0$ for all $x_i \le \tilde{x}$, or we have that $v - x_i - \frac{F(x_i)}{f(x_i)} \ge 0$ for all $x_i \in [\underline{x}, \bar{x}]$, in which case $\tilde{x} = \bar{x}$. Conditional on participation, the auctioneer will choose $q_i^g(\mathbf{x}) = 1$ for the at most $\min\{n,k\}$ firms with the lowest $x_i \le \tilde{x}$, and $q_i^g(\mathbf{x}) = 0$ for the remaining firms. Thus, for firms that participate and have $x_i \le \tilde{x}$, we get

$$Q_i(x_i) = \int q_i^g(\mathbf{x}) \mathrm{d}H_{-i}(\mathbf{x}_{-i}) = \mathrm{Prob}\{x_i \text{ is among the } \min\{n,k\} \text{ lowest costs}\} = \sum_{j=0}^{\min\{k,n\}-1} \binom{n-1}{j} F(x_i)^j (1 - F(x_i))^{n-j-1} \,.$$

The auctioneer maximizes

$$\max_{(q^p,(p_1,p_2,\ldots,p_n))} \sum_{i=1}^{n} \int q^p(x_i) \left[Q_i(x_i) \left(v - x_i - \frac{F(x_i)}{f(x_i)} \right) - c \right] \mathrm{d}F(x_i) \tag{A.11}$$

$$\text{s.t.} \quad 0 \le q^p(x_i) \le 1 \tag{A.12}$$

by choosing $q^p(x_i) = 1$ if $Q_i(x_i)\left(v - x_i - \frac{F(x_i)}{f(x_i)}\right) \ge c$ and $q^p(x_i) = 0$ if the inverse holds. Because $Q_i(x_i)$ is decreasing or constant in x_i and $x_i + \frac{F(x_i)}{f(x_i)}$ is increasing in x_i for all $x_i < \tilde{x}$, there exists a unique $\hat{x} \le \tilde{x}$ such that $q^p(x_i) = 1$ if $x_i \le \hat{x}$ and $q^p(x_i) = 0$ if $x_i > \hat{x}$. For $n > k$, $\hat{x}$ is determined by $Q_i(\hat{x})\left(v - \hat{x} - \frac{F(\hat{x})}{f(\hat{x})}\right) = c$. For $n \le k$, $Q_i(x_i) = 1$ for all $x_i \in [\underline{x}, \bar{x}]$ (by the binomial theorem (A.4)). Then, either $\hat{x}$ is the solution of $v - \hat{x} - \frac{F(\hat{x})}{f(\hat{x})} = c$, in which case $\hat{x} \le \bar{x}$, or we have that $v - \bar{x} - \frac{1}{f(\bar{x})} > c$, in which case $\hat{x} = \bar{x}$. Summarizing, and, for convenience, setting $Q_i(x_i) = 0$ if $q^p(x_i) = 0$, we have

$$q^p(x_i) = \begin{cases} 0 & \text{if } x_i > \hat{x} \\ 1 & \text{if } x_i \le \hat{x} \end{cases} \tag{A.13}$$

$$Q_i(x_i) = \begin{cases} 0 & \text{if } x_i > \hat{x} \\ \sum_{j=0}^{\min\{k,n\}-1} \binom{n-1}{j} F(x_i)^j (1 - F(x_i))^{n-j-1} & \text{if } x_i \le \hat{x} \,. \end{cases} \tag{A.14}$$

It remains to determine payment functions $p_1, p_2, \ldots, p_n$ such that the payoff (A.9) satisfies incentive compatibility (A.10):

$$\Pi_i(x_i) = \int p_i(\mathbf{x}) - q^p(x_i)\left(q_i^g(\mathbf{x}) x_i + c\right) \mathrm{d}H_{-i}(\mathbf{x}_{-i}) = \int_{x_i}^{\bar{x}} q^p(z) Q_i(z) \, \mathrm{d}z.$$

Plugging in (A.13) and (A.14) gives

$$\Pi_i(x_i) = \begin{cases} \int p_i(\mathbf{x}) \mathrm{d}H_{-i}(\mathbf{x}_{-i}) = 0 & \text{if } x_i > \hat{x} \\ \int p_i(\mathbf{x}) \mathrm{d}H_{-i}(\mathbf{x}_{-i}) - Q_i(x_i) x_i - c = \int_{x_i}^{\hat{x}} Q_i(z) \, \mathrm{d}z & \text{if } x_i \le \hat{x} \end{cases}$$

[40] $\int \int_{x_i}^{\bar{x}} q^p(z) Q_i(z) \, \mathrm{d}z \, \mathrm{d}H(\mathbf{x}) = \int \int_{\underline{x}}^{\bar{x}} \int_{x_i}^{\bar{x}} q^p(z) Q_i(z) \, \mathrm{d}z \, \mathrm{d}F(x_i) \, \mathrm{d}H_{-i}(\mathbf{x}_{-i}) = \int \int_{\underline{x}}^{\bar{x}} \int_{\underline{x}}^{z} q^p(z) Q_i(z) \mathrm{d}F(x_i) \, \mathrm{d}z \, \mathrm{d}H_{-i}(\mathbf{x}_{-i}) = \int \int_{\underline{x}}^{\bar{x}} q^p(z) Q_i(z) F(z) \, \mathrm{d}z \, \mathrm{d}H_{-i}(\mathbf{x}_{-i}) = \int \int_{\underline{x}}^{\bar{x}} \frac{q^p(z) Q_i(z) F(z)}{f(z)} \, \mathrm{d}F(z) \, \mathrm{d}H_{-i}(\mathbf{x}_{-i}) = \int \frac{q^p(x_i) Q_i(x_i) F(x_i)}{f(x_i)} \, \mathrm{d}H(\mathbf{x})$.

and we get

$$\int p_i(\mathbf{x})\mathrm{d}H_{-i}(\mathbf{x}_{-i}) = \begin{cases} 0 & \text{if } x_i > \hat{x} \\ \int_{x_i}^{\hat{x}} Q_i(z)\,\mathrm{d}z + Q_i(x_i)x_i + c & \text{if } x_i \le \hat{x}\,. \end{cases} \tag{A.15}$$

In the case of $n > k$, let $y_{(k,n-1)}$ denote the kth lowest of i's opponents' realized costs $\mathbf{x}_{-i}$ if $k < n$ and let $F_{(k,n-1)}$ denote the distribution of the random variable $X_{(k,n-1)}$. Note that $F_{(k,n-1)}(z) = 1 - Q_i(z)$. (The zero probability event $y_{(k,n-1)} = x_i$ is ignored in what follows and can be resolved by random tie-breaking.) In the case of $n \le k$, for ease of notation define $y_{(k,n-1)} > \hat{x}$ and $F_{(k,n-1)}(z) = 1-Q_i(z) = 0$ for all $z \in [\underline{x}, \bar{x}]$.

Payment functions that fulfill (A.15) if $\hat{x} < \bar{x}$ are, for all $s \in [0, c]$,

$$p_i(\mathbf{x}) = \begin{cases} 0 & \text{if } x_i > \hat{x} \\ v - \frac{F(\hat{x})}{f(\hat{x})} - \frac{s}{Q_i(\hat{x})} + s & \text{if } x_i \le \hat{x} < y_{(k,n-1)} \\ y_{(k,n-1)} + s & \text{if } x_i < y_{(k,n-1)} \le \hat{x} \\ s & \text{if } y_{(k,n-1)} < x_i \le \hat{x} \end{cases} \tag{A.16}$$

Obviously, (A.16) fulfills (A.15) for $x_i > \hat{x}$. For $x_i \le \hat{x}$ and (A.16), we get

$$\begin{aligned} \int p_i(\mathbf{x})\mathrm{d}H_{-i}(\mathbf{x}_{-i}) &= s + (1 - F_{(k,n-1)}(\hat{x}))\left(v - \frac{F(\hat{x})}{f(\hat{x})} - \frac{s}{Q_i(\hat{x})}\right) + \int_{x_i}^{\hat{x}} z\mathrm{d}F_{(k,n-1)}(z) \\ &= s + Q_i(\hat{x})\left(v - \frac{F(\hat{x})}{f(\hat{x})} - \frac{s}{Q_i(\hat{x})}\right) + \left[zF_{(k,n-1)}(z)\right]_{x_i}^{\hat{x}} - \int_{x_i}^{\hat{x}} F_{(k,n-1)}(z)\mathrm{d}z \\ &= Q_i(\hat{x})\left(v - \frac{F(\hat{x})}{f(\hat{x})}\right) - Q_i(\hat{x})\hat{x} + Q_i(x_i)x_i + \int_{x_i}^{\hat{x}} Q_i(z)\mathrm{d}z \\ &= c + x_i Q_i(x_i) + \int_{x_i}^{\hat{x}} Q_i(z)\mathrm{d}z\,. \end{aligned}$$

According to Stegemann (1996, Lemma 1), every direct mechanism in which each firm announces its type is associated with an outcome-equivalent semi-direct mechanism in which only participating firms announce their type.

In the semi-direct mechanism associated with (A.16), firms participate in the auction iff $x_i \le \hat{x}$, participating firms reveal their true costs x_i and receive a fixed payment of s, and the assignment and payments are determined by a uniform-price rule with the reserve price $v - \frac{F(\hat{x})}{f(\hat{x})} - \frac{s}{1-F_{(k,n-1)}(\hat{x})}$. By independence of the bidding strategy from the reimbursement and by payoff equivalence (Lemma 2), the pricing rule could be replaced by any pricing rule with a monotonic symmetric equilibrium, i.e., the uniform-price auction can be replaced by any basic auction.

If $\hat{x} = \bar{x}$, a payment function that fulfills (A.15) is $p_i(\mathbf{x}) = c + \bar{x}$, which can be implemented by a uniform-price auction with reserve price $r = c + \bar{x} - s$ and reimbursement s for $s \in [0, c]$. Then, all firms participate and receive the reserve price $r = c + \bar{x} - s$ and the reimbursement s.

O2 Social welfare We first determine the socially optimal cutoff value $\hat{x}$, which by the assumption of symmetry is the same for all firms. Second, we determine the reserve price $r(0)$ in a uniform-price auction without reimbursement that induces socially optimal participation. Third, we describe auctions with a reimbursement $s \in (0, c]$ that generate the same expected welfare.

The expected social welfare W given a cutoff $\hat{x}$ is

$$W = -\,ncF(\hat{x}) - \sum_{i=1}^{\min\{k,n\}} E[X_{(i,n)}|X_{(i,n)} \le \hat{x}]\,F_{(i,n)}(\hat{x}) + v\Big(\min\{k,n\} - \sum_{i=0}^{\min\{k,n\}-1} (\min\{k,n\} - i)\tbinom{n}{i} F(\hat{x})^i (1 - F(\hat{x}))^{n-i}\Big)\,.$$

First, assume $v \ge \bar{x} + c$ and $n \le k$. Then, participation by all firms is socially optimal and the auctioneer attracts participation by all firms with a reserve price $r(0) \ge \bar{x} + c$. The same participation can be achieved by a reserve price $r \ge \bar{x} + c - s$ and a reimbursement $s \in (0, c]$ to all participants. The auctioneer's payments are lowest with the reserve prices $r = \bar{x} + c - s$ for all $s \in [0, c]$.

Second, assume $v < \bar{x} + c$ or $n > k$. The first-order condition of the problem to maximize W is $\partial W/\partial \hat{x} = 0$, where

$$\frac{\partial W}{\partial \hat{x}} = -\,ncf(\hat{x}) - \sum_{i=1}^{\min\{k,n\}} \hat{x} f_{(i,n)}(\hat{x}) - vf(\hat{x}) \sum_{i=0}^{\min\{k,n\}-1} \tbinom{n}{i}(\min\{k,n\} - i)F(\hat{x})^{i-1}(1 - F(\hat{x}))^{n-i-1}(i - nF(\hat{x}))\,,$$

which with $f_{(i,n)}(\hat{x}) = nf(\hat{x})\binom{n-1}{i-1}F(\hat{x})^{i-1}(1 - F(\hat{x}))^{n-i}$ and Lemma 4 (A.2) and (A.3) leads to

$$(v - \hat{x}) \sum_{i=1}^{\min\{k,n\}} \tbinom{n-1}{i-1} F(\hat{x})^{i-1}(1 - F(\hat{x}))^{n-i} - c = 0\,. \tag{A.17}$$

For $v = r$, Condition (A.17) equals Condition (1) to determine the cutoff costs $\hat{x}$ of equilibrium participation. Thus, $r(0) = v$ attracts the participation that generates the social optimum. (Second-order conditions can be straightforwardly checked.)

One socially optimal mechanism is therefore a uniform-price auction with the reserve price $r(0) = v$. Other auctions that achieve socially optimal participation of all types $x \le \hat{x}$ are a uniform-price auction with $r = v - s/(1 - F_{(k,n-1)}(\hat{x}))$ and an additional payment of s to all participants, where $s \in (0, c]$. In such an auction, type $\hat{x}$'s expected payoff is $(v - s/(1 - F_{(k,n-1)}(\hat{x})) - \hat{x})(1 - F_{(k,n-1)}(\hat{x})) + s - c = (v - \hat{x})(1 - F_{(k,n-1)}(\hat{x})) - c$, which is zero by (A.17). $\hat{x}$ is thus indeed the highest type that will participate.

O3 Number of goods allotted The maximum number of goods allotted is k if $n > k$ or n if $n \leq k$. A mechanism that guarantees $\min\{n, k\}$ participants, which with symmetric entry requires $\hat{x} = \bar{x}$, is therefore optimal. If $n \leq k$, full participation can be achieved by a uniform-price auction with reserve price $r \geq \bar{x} + c - s$ and a reimbursement $s \in [0, c]$ to all participants, as with O1. Among these options, the one with $r = \bar{x} + c - s$ has the lowest payments. If $n > k$ and all firms participate, type $\bar{x}$ will not win in the auction. Thus, with a uniform-price auction and reimbursement $s \in [0, c)$, type $\bar{x}$ will not participate, independent of the reserve price (Lemma 1). However, with a uniform-price auction with $r \geq \bar{x}$ and a reimbursement c, the auctioneer achieves full participation. Among these options, the one with $r = \bar{x}$ has the lowest payments. ■

Proof of Corollary 2. The part on the case $n \leq k$ and $v \geq \bar{x} + \frac{1}{f(\bar{x})} + c$ follows directly from Proposition 3. The part on the case $n > k$ or $v < \bar{x} + c$ follows from direct comparisons using the results in Proposition 3. The cutoff $\hat{x}_3$ is equal to $\bar{x}$, and $\bar{x} > \hat{x}_2$ because $\bar{x}$ violates the equation to determine $\hat{x}_2$: $(v - \bar{x})(1 - F_{(k,n-1)}(\bar{x})) = (v - \bar{x}) \cdot 0 < c$. Further, we have $\hat{x}_2 > \hat{x}_1$ because otherwise the equations to determine $\hat{x}_2$ and $\hat{x}_1$ are violated. Assume to the contrary that $\hat{x}_2 \leq \hat{x}_1$. Then $1 - F_{(k,n-1)}(\hat{x}_2) \geq 1 - F_{(k,n-1)}(\hat{x}_1)$, and fulfilling $(v - \hat{x}_2)(1 - F_{(k,n-1)}(\hat{x}_2)) = c = (v - \hat{x}_1 - F(\hat{x}_1)/f(\hat{x}_1))(1 - F_{(k,n-1)}(\hat{x}_1))$ requires $(v - \hat{x}_2) \leq (v - \hat{x}_1 - F(\hat{x}_1)/f(\hat{x}_1))$. This implies $F(\hat{x}_1)/f(\hat{x}_1) \leq \hat{x}_2 - \hat{x}_1 \leq 0$, a contradiction. We get that $r_2 > r_1$ for all $s \in [0, c)$ again from the equations to determine $\hat{x}_2$ and $\hat{x}_1$. Assume to the contrary that $r_2 \leq r_1$, i.e., $v - s/(1 - F_{(k,n-1)}(\hat{x}_2)) \leq v - F(\hat{x}_1)/f(\hat{x}_1) - s/(1 - F_{(k,n-1)}(\hat{x}_1))$. Replacing $(1 - F_{(k,n-1)}(\hat{x}_2))$ and $(1 - F_{(k,n-1)}(\hat{x}_1))$ using the equations to determine the optimal cutoffs yields $v - s(v - \hat{x}_2)/c \leq v - F(\hat{x}_1)/f(\hat{x}_1) - s(v - F(\hat{x}_1)/f(\hat{x}_1) - \hat{x}_1)/c$. Rearranging gives $(1 - s/c)F(\hat{x}_1)/f(\hat{x}_1) \leq s(\hat{x}_1 - \hat{x}_2)/c < 0$, a contradiction. Finally, $r_3 > r_2 > r_1$ for $s = c$ because in this case $r_j = \hat{x}_j$ for all $j \in \{1, 2, 3\}$ and we have already proven the order of the cutoffs. ■

Proof of Corollary 3. Let r_j denote the optimal reserve price for objective j when all participants receive a reimbursement s. The optimal mechanism for Oj with $j \in \{1, 2\}$ can be implemented (i) by a uniform-price auction with a reserve price $r_j^w(\tilde{s}) = r_j(0) - \tilde{s}$, and a reimbursement $\tilde{s} \in [0, c]$ to all winning bidders; (ii) by a uniform-price auction with a reserve price $r_j^l(\tilde{s}) = r_j(\tilde{s}) + \tilde{s}$, and a reimbursement $\tilde{s} \in [0, c]$ to all losing bidders. The optimal mechanism for O3 can be implemented by an auction of type (ii) with $\tilde{s} = c$. We show that with these auctions, participating firms receive the same payments as with the optimal auctions in Proposition 3.

In a uniform-price auction with a reimbursement $\tilde{s}$ only to winners, the weakly dominant strategy is to bid $x_i - \tilde{s}$, because at a price equal to this bid, a firm is indifferent between winning and losing. If the reserve price is $r_j^w(\tilde{s}) = r_j(0) - \tilde{s}$, then participating firms receive the total payment $r_j(0)$ if they win and the reserve price determines the price, $y_{(k,n-1)}$ if they win and a competitor determines the price, and zero if they do not win. These are the same payments as with $r_j(0)$ and no reimbursement.

In a uniform-price auction with a reimbursement $\tilde{s}$ only to losers, the weakly dominant strategy is to bid $x_i + \tilde{s}$, because at a price equal to this bid, a firm is indifferent between winning and losing. If the reserve price is $r_j^l(\tilde{s}) = r_j(\tilde{s}) + \tilde{s}$, then participating firms receive the total payment $r_j(\tilde{s}) + \tilde{s}$ if they win and the reserve price determines the price, $y_{(k,n-1)} + \tilde{s}$ if a competitor determines the price, and $\tilde{s}$ if they do not win. These are the same payments as with $r_j(\tilde{s})$ and a reimbursement $\tilde{s}$ to all bidders. ■

Proof of Proposition 4. Consider $n > k$, participation costs c, and a basic auction with reserve price $r \leq v$. Note that $\hat{x} < \bar{x}$ by Lemma 1 and $\hat{x} > \underline{x}$ by property (P2) of a basic auction ($r > \underline{x} + c$). Therefore, and by Lemma 1, $\hat{x}$ strictly increases if c decreases to c'. Denote the equilibrium cutoff costs associated with c' by $\hat{x}'$. Thus, the additional types that participate with c' are the types in $(\hat{x}, \hat{x}']$.

Consider any profile of types $\mathbf{x}$. Given $\mathbf{x}$, surplus is weakly higher with c' than with c because weakly more goods are allotted at a weakly lower price. First, if with c we have that $m \leq k$, then with c' either the same types participate and the price remains at level r or more types enter, the number of allotted goods increases, and the price is weakly below r (strictly below r if with c' we have $m' > k$). Second, if with c we have that $m > k$, then with c' we have the same allocation and price (as only worse types may enter). Taking the expectation over all profiles $\mathbf{x}$, the cases with strict increase of surplus have positive probability, and, thus, surplus is strictly higher with c'.

Expected welfare is strictly higher with c' than with c because all firms have a weakly higher contribution to welfare and some have a strictly higher contribution. First, types $x \in (\hat{x}, \hat{x}']$ contribute nothing to welfare if participation costs are c. With c' their expected payoff from participation is positive and thus, with $r \leq v$, their expected contribution to social welfare is positive. Second, types $x \in [\underline{x}, \hat{x}]$ for a given profile $\mathbf{x}$ either are auction winners both with c and with c' or do not win in the auction both with c and with c'. Thus, their contribution to welfare after participation is either $v - x$ or 0 before and after the decrease in participation costs but their total contribution to welfare is strictly higher after the decrease because their participation costs are strictly lower.

Finally, the expected number of goods allotted increases strictly if c decreases to c' because the cutoff costs increase strictly. ■

Appendix B. Discreteness of the number of winners

The number of winning bids is discrete. Our model can incorporate issues that due to discreteness arise in practice, by extending the range of endogenous rationing in Definition 1 upward to $\overline{m} > k$ and restricting it from below by $\underline{m}$, $0 \leq \underline{m} < k$. Then, $\kappa(m) = k$ if $m > \overline{m}$, $\kappa(m) \in \{\underline{m}, \ldots, \min\{k, m\} - 1\}$ if $m \in \{\underline{m} + 1, \ldots, \overline{m}\}$, and $\kappa(m) = m$ if $m \leq \underline{m}$.

The role of $\overline{m} > k$ is to provide a "smooth" transition to rationing. Consider rationing by two units in an auction of ten units. If $\overline{m} = k$, the number of winning bids with twelve, eleven, ten, or nine submitted bids is ten, ten, eight, seven. If, instead, rationing started at eleven (i.e., $\overline{m} = 11$), the numbers of winning bids were ten, nine, eight, seven, avoiding the jump from ten to eight. Proposition 1 holds for any $\overline{m} > k$.

The constraint $\underline{m}$ provides a lower bound where rationing stops. However, choosing $\underline{m} > 0$ implies accepting outcomes in which all bidders win, which contradicts the main motivation for using ER. Thus, $\underline{m} > 0$ does not appear as an explicit design element of ER auctions, but it is implicit in some rules. For example, a rule that rations to 80% of the submitted bids (see Footnote 6) with $\kappa(m) = \lceil 0.8\,m\rceil$ stops rationing when $\lceil 0.8\,m\rceil = m$. For such rules with $\underline{m} \in \{0, 1, \dots, k-1\}$, by a similar line of reasoning as in the proof of Proposition 1, the number m of bids submitted in the ER auction with tender volume k is the same as in a basic auction with tender volume $\underline{m}$ (i.e., zero if $\underline{m} = 0$), even though the ER auction puts a larger volume k out to tender: The bidder with $\hat{x}$ in an ER auction with $\underline{m} \in \{0, 1, \dots, k-1\}$ wins only if neither competition nor rationing prevent her from winning, which happens with probability $\text{Prob}\{m \le \underline{m}\} = 1 - F_{(\underline{m},n-1)}(\hat{x})$. Therefore, her expected profit in the ER auction with volume k is the same as in a basic auction with volume $\underline{m}$ (see (1)) and she participates if and only if

$$(r - \hat{x})\,(1 - F_{(\underline{m},n-1)}(\hat{x})) - c \ge 0\,. \tag{B.1}$$

Proposition 1 reveals a strong negative effect of ER, as there is no participation. With $\underline{m} > 0$, participation is reduced but not eliminated.

The optimal mechanism with participation costs in Section 4 for auctioneer's surplus or welfare can be implemented as basic auctions with optimal reserve price (Proposition 3). These auctions provide higher surplus or welfare than any auction with ER including those with $\underline{m} > 0$.

Appendix C. Endogenous reserve price

Endogenous rationing (ER) can not only be conducted by an *endogenous auction volume (EAV)* also by an *endogenous reserve price (ERP)*, in which the submitted bids determine a reserve price below the original reserve price such that not all submitted bids win. Only bids below the ERP are successful. Variations differ in the way the bids determine the ERP, e.g., by the bids' mean or median. ERPs are applied, e.g., in France (European Commission, 2019).

An ERP is derived from the submitted bids. In the first step, the firms decide about their participation. Then, participating firms submit their bids, which may not exceed the default reserve price r. The bids of the $m \le n$ participating firms are denoted by $\mathbf{b} = (b_1, b_2, \dots, b_m)$. Next, the ERP $\varrho(\mathbf{b})$ is calculated on the basis of $\mathbf{b}$, where $\min_{i\in\{1,\dots,m\}}\{b_i\} \le \varrho(\mathbf{b}) \le \max_{i\in\{1,\dots,m\}}\{b_i\}$ and $1 \ge \frac{\partial\varrho(\mathbf{b})}{\partial b_i} \ge 0$ for all $i \in \{1, \dots, m\}$. For the award, only bids $b < \varrho(\mathbf{b})$ are taken into consideration (alternatively, requiring $b \le \varrho(\mathbf{b})$ combined with $b < \max_{i\in\{1,\dots,m\}}\{b_i\}$ would give the same conclusion).[41] We call these *accepted bids* and denote their number by m', $m' \le m$. The payment rule is applied as if only the accepted bids were submitted and as if the ERP was the reserve price.[42]

We complement the ERP rule by a floor $\underline{m}$ and a ceiling $\overline{m}$, with $0 \le \underline{m} < k \le \overline{m}$ and $\underline{m} < n$ (compare Appendix B). The cases of no floor or ceiling are included by $\underline{m} = 0$ or $\overline{m} = \infty$, respectively. Floor and ceiling override rationing. If $m \ge \underline{m} > m'$, then the $\underline{m}$ lowest bids win (with random tie-breaking) and the reserve price equals the highest of the winning bids. If $m \le \underline{m}$ or $m > \overline{m}$, all bids or k bids are accepted and the payment rule is applied with the reserve price $\max_{i\in\{1,\dots,m\}}\{b_i\}$ or r, respectively.

We focus on ERP rules with the property that for all $b_i \in (\underline{x}, \bar{x})$ there exist combinations of the other bidders' bids $\mathbf{b}_{-i}$ such that $\frac{\partial\varrho(\mathbf{b})}{\partial b_i} > 0$. Examples for ERP rules $\varrho(\mathbf{b})$ are the mean rules that determine the ERP based on the mean of the submitted bids, $\varrho(\mathbf{b}) = \alpha\frac{1}{m}\sum_{i=1}^{m} b_i$ with $\alpha \in (0, 1]$, and the median rules that determine the ERP based on the median of the submitted bids, $\varrho(\mathbf{b}) = \alpha \cdot \text{median}(\mathbf{b})$ with $\alpha \in (0, 1]$.[43]

With an ERP, it is not an equilibrium that all bidders bid r.[44] Therefore, we focus on symmetric and strictly increasing bidding functions and assume the existence of an equilibrium in symmetric pure strategies (property (P5)).

Proposition 5. *In an auction with ERP $\varrho(\mathbf{b})$, tender volume k and floor $\underline{m}$ on rationing, the cutoff costs $\hat{x}$ and the participants are the same as in a basic auction with tender volume $\underline{m}$; thus, if $\underline{m} = 0$, the number of participating bidders is zero, $m = 0$.*

A firm $\hat{\imath}$ with the cutoff costs $\hat{x}$ receives a good if and only if the number of bidders m is $\underline{m}$ or less. If $m > \underline{m}$, her bid will not win because the other bidders submit lower bids, and thus, firm $\hat{\imath}$'s bid $b_{\hat{\imath}} = \max_{i\in\{1,\dots,m\}}\{b_i\}$ is not below $\varrho(\mathbf{b})$ if $m \le \overline{m}$ or not among the

[41] To enforce rationing if $n \le k$ or $m \le k$, the ERP winner-determination rule must prevent that all bids win if all bidders bid the same, e.g., r. This is met by both forms of the ERP rule.

[42] That means, if $m' \le k$, in the uniform-price auction, the price is equal to $\varrho(\mathbf{b})$.

[43] A different auction with median-price rule is the Medicare auction analyzed by Cramton et al. (2015) and widely criticized by auction experts (see http://www.cramton.umd.edu/papers2010-2014/further-comments-of-concerned-auction-experts-on-medicare-bidding.pdf, accessed 09/16/2019). One of the reasons why the Medicare auction was predicted to perform badly is that it is not ex post individually rational in that winning bidders' payments can be below their bids: the k lowest bids win and the median of the winning bids is the payment to each winner. In contrast, all auctions that we analyze are ex post individually rational. Further differences to our setting are that in the Medicare auction bids are non-binding, bids have to be above a bid floor (assumed to be below $\underline{x}$ and necessitated by the incentives evoked by the design), and participation costs are absent.

[44] Only if the bidders knew that $m \le \underline{m}$, all bidders bidding r would be an equilibrium.

Table D.3
Overview of sessions.

Date and time of session	Treatment	Matching group label
07-02-2019, 10.30 a.m.	Control	Control-1
07-02-2019, 10.30 a.m.	ER	ER-1
07-30-2019, 10.30 a.m.	Control	Control-2
07-30-2019, 10.30 a.m.	ER	ER-2
07-30-2019, 02.00 p.m.	ER	ER-3
10-08-2019, 10.30 a.m.	Control	Control-3
10-08-2019, 10.30 a.m.	ER	ER-4
10-09-2019, 10.30 a.m.	Control	Control-4

Table D.4
Results of the auctions in the matching groups of the ER Treatment.

Matching group	ER-1			ER-2			ER-3			ER-4		
Rounds	1–15	1	15	1–15	1	15	1–15	1	15	1–15	1	15
Avg. number of submitted bids	2.63	5.00	2.00	4.33	5.00	2.50	3.97	5.50	3.00	5.17	5.00	3.50
Avg. number of awarded bids	0.93	3.00	0.50	2.33	3.00	0.50	2.03	3.50	1.00	3.20	3.00	2.00
Avg. value of submitted bids	62.63	67.60	60.15	56.36	53.42	55.28	63.95	49.07	68.65	65.52	60.88	68.31
Avg. value of awarded bids	60.41	65.00	60.00	45.10	39.41	30.50	59.05	35.46	62.95	62.23	51.50	63.14
Avg. auctioneer's surplus	14.04	36.00	8.50	64.41	100.85	23.25	39.64	125.19	14.06	47.94	72.50	27.72
Avg. social welfare	35.71	160.41	4.85	112.25	153.74	5.49	96.02	196.57	7.95	178.23	159.07	118.94

k lowest bids if $m > \overline{m}$. If $0 < m \leq \underline{m}$, firm $\hat{i}$'s bid wins and, furthermore, influences the reserve price $\max_{i\in\{1,\ldots,m\}}\{b_i\}$ and, therefore, her payment. Thus, firm $\hat{i}$ participates and bids $b_{\hat{i}} = r$ iff (B.1) holds. This proves Proposition 5.

The fact that firm $\hat{i}$'s decision problem is the same in an EAV auction as in an ERP auction reveals a parallelism between the two instruments. If $\underline{m} > 0$, then $\hat{x} > \underline{x}$, and all firms with $\hat{x} \geq x_i \geq \underline{x}$ participate in the auction. If the ERP is a quantile of the bids, then there is an EAV rule under which the same firms win and their payments are the same. For example, the median rule for the ERP corresponds to an EAV of 50%.[45] However, there are also differences, where an ERP rule does not correspond to any EAV rule. Consider, for example, the ERP that is equal to the mean. With this rule, adding a bid can increase or decrease the number of winning bidders, which is impossible under an EAV rule. Furthermore, in the uniform-price auction, bidders have an incentive to bid above their costs because an increase in their bid may increase the ERP and may therefore increase their payment. In contrast, bidders cannot influence their payment in the uniform-price EAV auction, in which it is optimal for the bidders to bid their costs x_i. Payoff equivalence does not hold for different auctions (e.g., pay-as-bid vs. uniform pricing) with the same ERP rule because different payment rules may determine different sets of winners due to the differences in the bidding strategies.

Appendix D. Additional experimental results and statistical tests

D.1. Detailed experimental results

In this section, the results from Section 3 are presented in more detail. Table D.3 shows for each matching group of each treatment when the session was conducted and how the matching groups are labeled.

In each matching group of 18 subjects, in 15 consecutive rounds two randomly assigned groups of nine subjects play the same type of auction (either the ER auction in the ER Treatment or the basic auction in the Control Treatment). Tables D.4 and D.5 present for each matching group the results for all rounds (1–15), the first round (1) and the last round (15) per matching group (Table 2 aggregates these results).

In the two ER matching groups ER-2 and ER-3 some bids are smaller than 50, i.e., the lowest possible cost signal. As a consequence, in several rounds of these two matching groups the average submitted bid and the average awarded bid are lower than 50 (Fig. D.6 and Fig. D.7). This is also the reason why the average awarded bid in the first round of the ER Treatment in Table 2 is below 50. Fig. D.7 shows that in the matching groups ER-1 and ER-3 there are rounds where no bid is awarded because in both groups of the matching group fewer than three bids are submitted (ER-1: rounds 11, 12 and 15; ER-3: round 11). In all matching groups of the Control Treatment, the average number of awarded bids is close to six, i.e., the highest possible number (Table D.5). Figs. D.8 and D.9 show that in all matching groups, the average submitted bid and the average awarded bid (after an increase in the first rounds in some matching groups) remain on the same level of about 70 (see also Appendix D.2).

[45] Concretely, the median rule with $\alpha = 1$ that chooses the higher of the two middle values in case of an even number of bids as the ERP (i.e., $\rho(\mathbf{b}) = b_{(\lceil 0.5(m+1)\rceil)}$ where $b_{(j)}$ denotes the jth lowest bid) corresponds to the 50%-EAV rule that rejects the bid at the 50% boundary in case of an odd number of bids (i.e., $\kappa(m) = \lfloor 0.5m \rfloor$).

Table D.5
Results of the auctions in the matching groups of the Control Treatment.

Matching group	Control-1			Control-2			Control-3			Control-4		
Rounds	1–15	1	15	1–15	1	15	1–15	1	15	1–15	1	15
Avg. number of submitted bids	5.90	6.00	4.50	6.40	6.00	5.50	6.27	5.00	7.50	6.67	6.00	7.00
Avg. number of awarded bids	5.37	6.00	4.50	5.73	6.00	5.50	5.57	5.00	6.00	5.87	6.00	6.00
Avg. value of submitted bids	70.03	67.37	71.86	68.62	63.96	68.63	70.33	64.96	72.61	70.67	64.61	72.53
Avg. value of awarded bids	69.68	67.37	71.86	68.04	63.96	68.63	69.83	64.96	71.67	70.16	64.61	71.95
Avg. auctioneer's surplus	39.29	57.78	22.86	51.83	78.23	45.80	39.96	60.19	32.00	40.42	74.32	30.32
Avg. social welfare	348.22	399.69	285.51	375.02	396.51	362.75	361.32	317.04	390.25	383.03	394.39	394.33

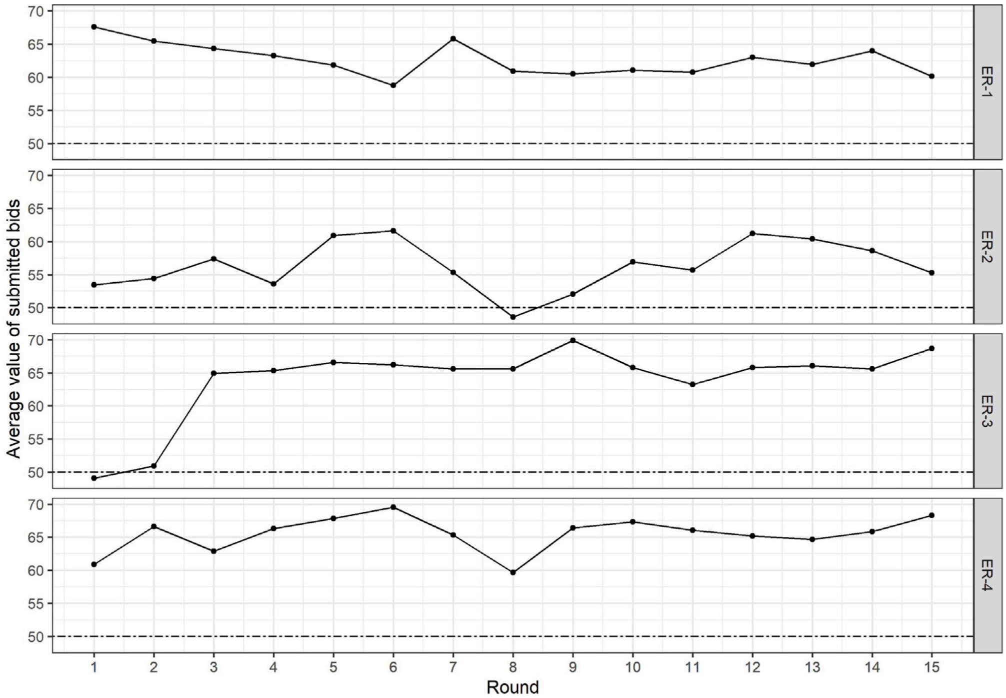

Fig. D.6. Development of the average value of submitted bids in the matching groups of the ER Treatment.

D.2. Behavior in the control treatment

The experimental parameters in Table 1 in Section 3.1 yield, by Eq. (A.6), the cutoff $\hat{x} = 66.99$. That is, theory predicts that subjects with private costs x below the cutoff participate in the auction and submit the equilibrium bid $\beta^{PaB}(x)$ in Lemma 3, whereas subjects with higher private costs do not participate.

Table D.6 shows how subjects decide on their participation in the auction with respect to the relation between their private costs x and the theoretical cutoff $\hat{x}$. In 746 (of 1080) cases, the private costs x are lower than $\hat{x}$. In these cases, we observe 686 (92.0%) participations. In 334 (of 1080) cases the private costs x are higher than $\hat{x}$ and we observe 263 (78.7%) non-participations. Thus, a total of 949 (87.9%) of the 1080 participation decisions are in line with theory, i.e., either a subject with $x \le \hat{x}$ participates in the auction or a subject with $x > \hat{x}$ does not participate. A sign test on over- or under-entry does not reveal a significant difference (37 subjects had a higher share of over-entry than under-entry – participation when $x > \hat{x}$ vs. no participation when $x \le \hat{x}$ –, 13 had identical shares, and 22 had a lower share of over-entry than under-entry, $p = 0.07$). Fig. D.10 indicates for both cases an increase of the share of participation decisions in line with theory over the rounds.

The average submitted bid (after participation in line with theory) is 69.54, while the average corresponding equilibrium bid is 71.29. Thus, the submitted bids deviate on average (median) from the equilibrium bids by -1.75 (-1.16), which amounts to 6.48% (4.30%) of the range from the lower interval limit to the reserve price $[\underline{x}, r]$. Fig. D.11 shows the development of these differences over the rounds. After negative deviations in the first rounds, the bid level reaches the equilibrium level and remains there in the following rounds (see also Fig. 2 in Section 3.3).

We conclude that the subjects' behavior in the Control Treatment is largely in line with the equilibrium strategy derived in Section 2.

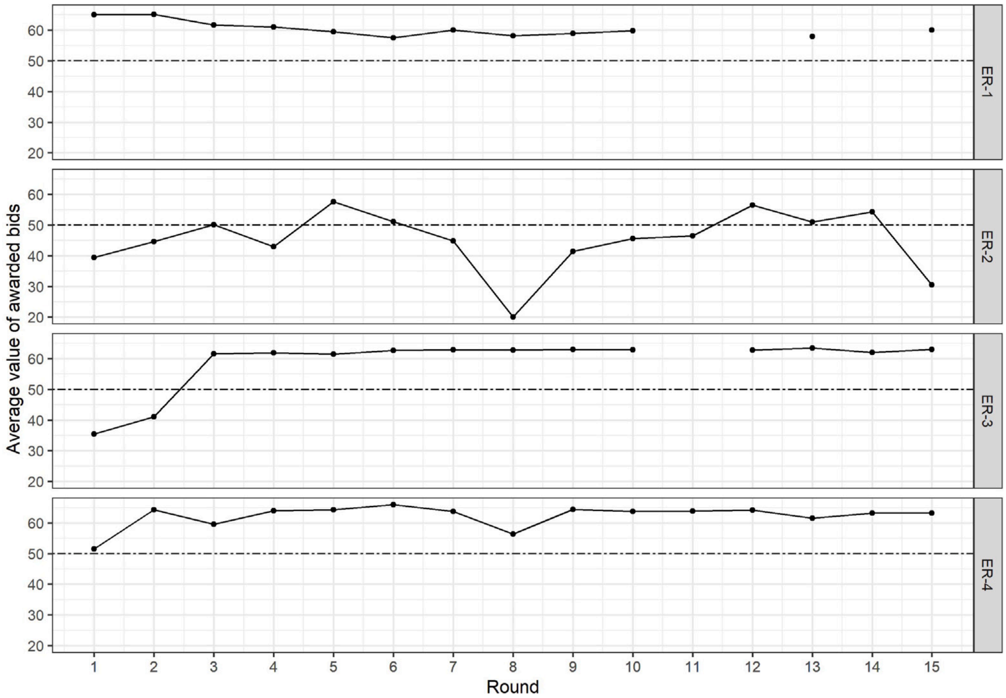

Fig. D.7. Development of the average value of awarded bids in the matching groups of the ER Treatment.

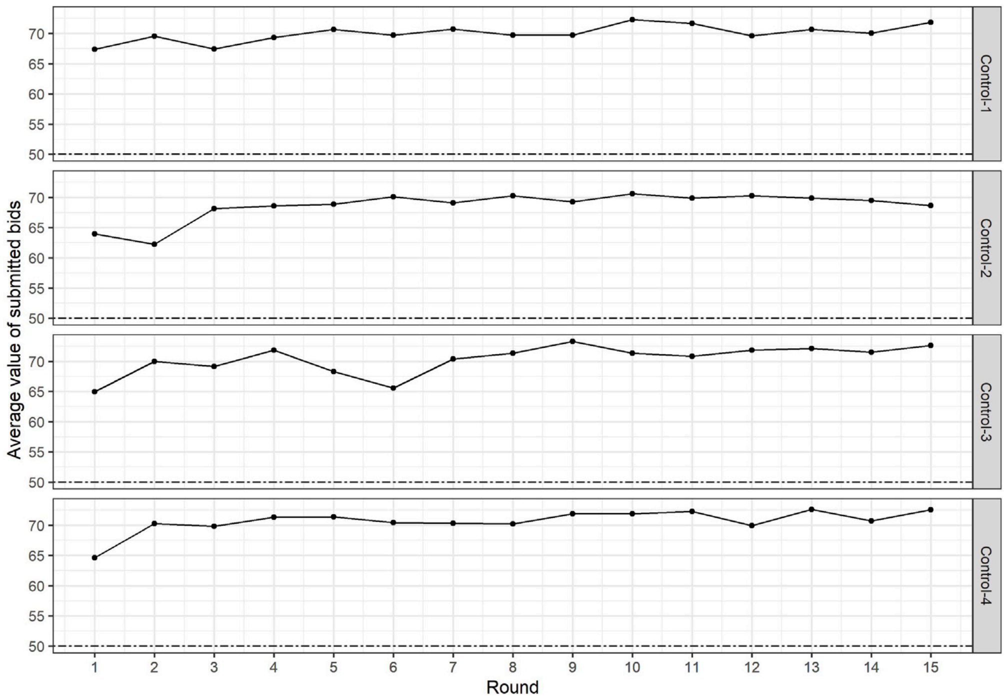

Fig. D.8. Development of the average value of submitted bids in the matching groups of the Control Treatment.

D.3. Statistical analysis

Throughout the statistical analysis we use the significance level $\alpha = 0.05$.

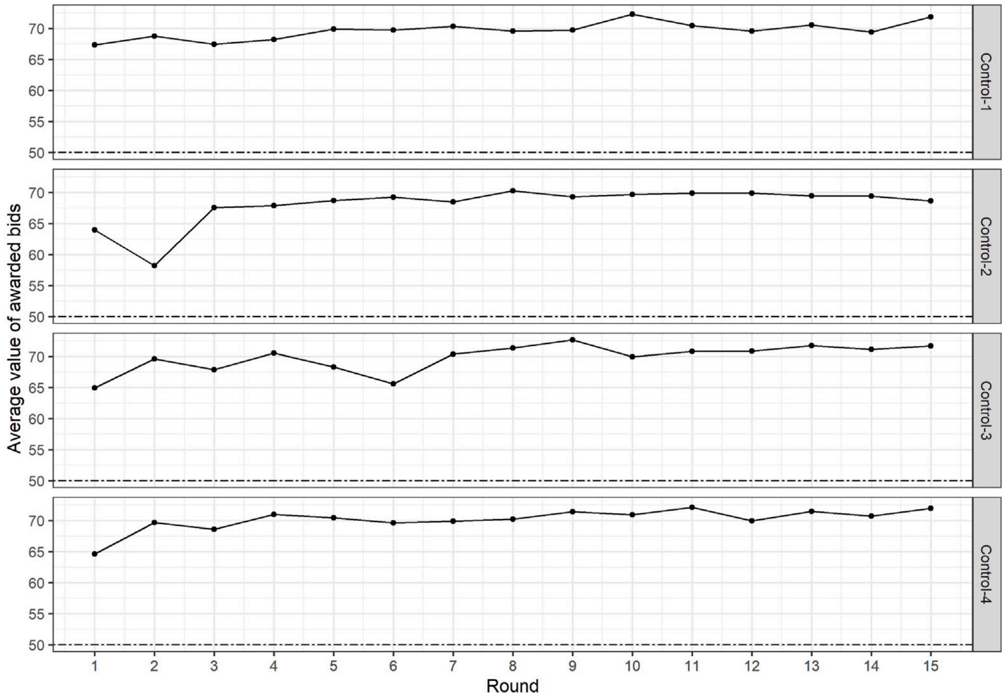

Fig. D.9. Development of the average value of awarded bids in the matching groups of the Control Treatment.

Table D.6
Distribution of the participation decisions in the Control Treatment.

		Private costs x		
		$x \leq \hat{x}$	$x > \hat{x}$	Sum
Participation	Yes	686 (63.5%)	71 (6.6%)	757 (70.1%)
	No	60 (5.6%)	263 (24.4%)	323 (29.9%)
	Sum	746 (69.1%)	334 (30.9%)	1080 (100%)

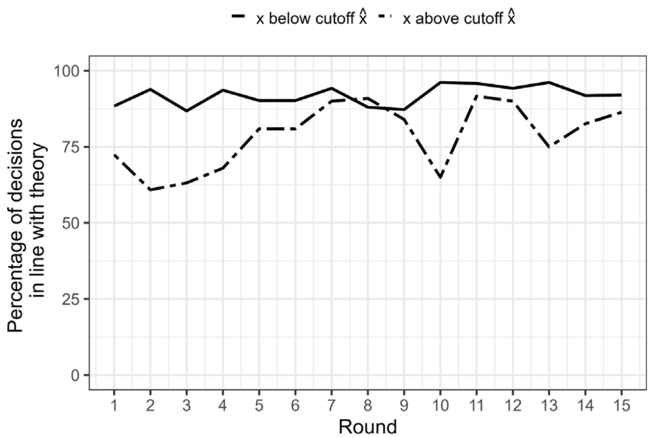

Fig. D.10. Development of the percentage of participation decisions in line with theory in the Control Treatment: percentage of participation for $x \leq \hat{x}$ (solid line) and percentage of non-participation for $x > \hat{x}$ (dotted line).

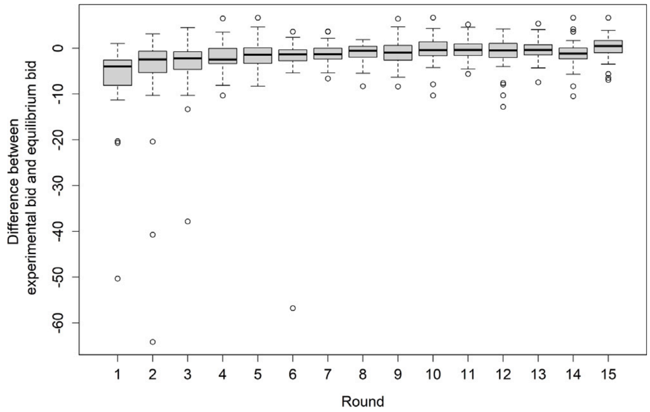

Fig. D.11. Box-and-whisker plots of the difference between equilibrium bids and submitted bids in the rounds of the Control Treatment conditional on participation in line with theory: median, interquartile range IQR, whiskers (outliers at max 1.5 × IQR), and outliers more than 1.5 × IQR.

Table D.7
Effect of treatment and round on different variables.

Dependent variable	Number of observations	TreatER	Round	TreatER · Round	Constant
Number of submitted bids	240	−0.965 (0.633)	−0.003 (0.028)	−0.165*** (0.039)	6.333*** (0.448)
Number of awarded bids	240	−2.401** (0.531)	−0.010 (0.020)	−0.138*** (0.028)	5.712*** (0.376)
Value of submitted bids	1240	−7.160** (2.084)	0.298*** (0.074)	−0.071 (0.120)	67.601*** (1.445)
Value of awarded bids	931	−12.343** (3.251)	0.328*** (0.077)	−0.176 (0.154)	66.779*** (2.234)
Private costs of submitted bids	1240	−2.230* (0.954)	−0.142** (0.046)	0.024 (0.075)	60.296*** (0.650)
Private costs of awarded bids	931	−3.198* (1.120)	−0.156*** (0.045)	−0.024 (0.088)	59.732*** (0.728)
Auctioneer's surplus	240	13.333 (11.993)	−1.939*** (0.438)	−1.838** (0.619)	58.387*** (8.480)
Social welfare	240	−190.723*** (34.366)	−0.627 (1.437)	−8.828*** (2.032)	371.909*** (24.301)

*p<0.05; **p<0.01; ***p<0.001

First, we apply a linear mixed-effects model (Gałecki and Burzykowski, 2013) to test the influence of the treatment and the round on the dependent variables in Table 2. We use the R-packages *lme4* (Bates et al., 2015) and *lmerTest* (Kunzetsova et al., 2017) on R 4.0.3.

The vector of the dependent variables is denoted by y. We include the fixed effects treatment and round, and an interaction effect between treatment and round to test whether the influence of these variables has an interdependence. The parameters for the constant, the two main effects, and the interaction effect are denoted by β_j for $j \in \{0, 1, 2, 3\}$. An advantage of linear mixed-effects models is the inclusion of dependencies in the data (Müller et al., 2013). We include this in our model by taking the matching group as a random effect with parameter g (Brown, 2021), since subjects are randomly put into these groups at the start of the experiment and do not change their matching group in the course of the experiment. With the vector of residual errors ε, the model is as follows:

$$y = \beta_0 + \beta_1 \cdot TreatER + \beta_2 \cdot Round + \beta_3 \cdot TreatER \cdot Round + g \cdot Matchinggroup + \varepsilon.$$

The results of the statistical calculations are presented in Table D.7. The parameter estimates for the fixed effects are given, with information on the p-value of the t-test. The basic errors are in brackets.

Table D.7 reveals the following results: There is no evidence of a trend in the number of submitted bids in the Control Treatment, while the round has a significant negative effect on the number of submitted bids in the ER treatment. The number of awarded bids is significantly lower in the ER Treatment than in the Control Treatment, with an increasing divergence over the rounds. The values of the submitted bids and of the awarded bids significantly increase over the rounds and are significantly lower in the ER Treatment than in the Control Treatment. The private costs of the submitted bids significantly decrease over the rounds and are significantly

Table D.8
Wilcoxon tests for the comparison of ER and Control Treatment in first and last round.

	Round 1 (first round)			Round 15 (last round)		
	Number of observations	W	p-value	Number of observations	W	p-value
Number of submitted bids	16	44	0.192	16	60.5	0.003
Number of awarded bids	16	63	< 0.001	16	63.5	< 0.001
Value of submitted bids	87	1138	0.098	71	934.5	< 0.001
Value of awarded bids	71	874.5	< 0.001	52	374.5	< 0.001
Private costs of submitted bids	87	1099.5	0.185	71	597	0.478
Private costs of awarded bids	71	747.5	0.038	52	254	0.048
Auctioneer's surplus	16	22	0.328	16	49	0.082
Social welfare	16	63	< 0.001	16	64	< 0.001

lower in the ER Treatment than in the Control Treatment. This also applies to the private costs of the awarded bids. The auctioneer's surplus significantly decreases over the rounds, with a significantly stronger decrease in the ER Treatment. The social welfare is significantly lower in the ER Treatment than in the Control Treatment, with an increasing divergence over the rounds.

We additionally apply Wilcoxon rank sum tests (Wilcoxon, 1945) to compare the two treatments' variables in Table 2 both in the first round and in the last round. For the calculation of the Wilcoxon rank sum tests, the R-package *stats* is used. We provide the p-values for two-tailed tests. Thus, in all tests, the null hypothesis is that there is no difference between the treatments.

The test results are presented in Table D.8. The first column presents the variable, the second to fourth columns the test results for the first round (*round 1*), and the fifth to last columns the test results for the last round (*round 15*). The number of observations, the test statistics W, and the p-value are given. We find a significant difference between the treatments both in the first and last round for the number of awarded bids, the value of the awarded bids, the average private costs of the awarded bids, and the social welfare. We find a significant difference between the treatments in the last but not the first round for the number of submitted bids and the value of submitted bids.[46] For the auctioneer's surplus and the average private costs of the submitted bids, there is neither a significant difference in the first round nor in the last round.

The two approaches of testing complement each other. We describe the overall picture in Section 3.3, where we also provide figures to illustrate the findings.

Appendix E. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.euroecorev.2026.105436.

Data availability

The data is available in the zip-file with the other replication files.

References

Anatolitis, V., Azanbayev, A., Fleck, A.K., 2022. How to design efficient renewable energy auctions? Empirical insights from Europe. Energy Policy 166, 112982. http://dx.doi.org/10.1016/j.enpol.2022.112982.

Ausubel, L.M., Cramton, P., 2004. Vickrey auctions with reserve pricing. Econom. Theory 23, 493–505. http://dx.doi.org/10.1007/s00199-003-0398-8.

Aycinena, D., Rentschler, L., 2018. Auctions with endogenous participation and an uncertain number of bidders: experimental evidence. Exp. Econ. 21 (4), 924–949. http://dx.doi.org/10.1007/s10683-017-9558-8.

Back, K., Zender, J.F., 2001. Auctions of divisible goods with endogenous supply. Econom. Lett. 73, 29–34. http://dx.doi.org/10.1016/S0165-1765(01)00465-7.

Bates, D., Mächler, M., Bolker, B., Walker, S., 2015. Fitting linear mixed-effects models using lme4. J. Stat. Softw. 67 (1), 1–48. http://dx.doi.org/10.18637/jss.v067.i01.

BMU, 2016. Bundesministerium für Umwelt, Naturschutz und nukleare Sicherheit (BMU): Klimaschutzplan 2050: Klimaschutzpolitische Grundsätze und Ziele der Bundesregierung. URL: https://www.bundeswirtschaftsministerium.de/Redaktion/DE/Artikel/Industrie/klimaschutz-klimaschutzplan-2050.html. (Accessed 12 July 2025).

BNetzA, 2022. Bundesnetzagentur (BNetzA): Ausschreibungen für EE- und KWK-Anlagen. URL: https://www.bundesnetzagentur.de/DE/Fachthemen/ElektrizitaetundGas/Ausschreibungen/artikel.html. (Accessed 12 July 2025).

Bock, O., Baetge, I., Nicklisch, A., 2014. Hroot: Hamburg registration and organization online tool. Eur. Econ. Rev. 71, 117–120. http://dx.doi.org/10.1016/j.euroecorev.2014.07.003.

Boyer, M., Moreaux, M., 1989. Endogenous rationing in a differentiated product duopoly. Internat. Econom. Rev. 30 (4), 877–888, URL: https://www.jstor.org/stable/2526757.

BReg, 2017. Deutsche Bundesregierung (BReg): Gesetz zur Einführung von Ausschreibungen für Strom aus erneuerbaren Energien und zu weiteren Änderungen des Rechts der erneuerbaren Energien. URL: https://www.clearingstelle-eeg-kwkg.de/eeg2017. (Accessed 12 July 2025).

[46] When we apply the rather conservative Bonferroni correction to address the multiple comparisons problem, all differences except those for the private costs remain significant.

BReg, 2019. Deutsche Bundesregierung (BReg): Verordnung zu den Innovationsausschreibungen und zur Änderung weiterer energiewirtschaftlicher Verordnungen, Drucksache 19/14065, Artikel 1. URL: https://dserver.bundestag.de/btd/19/140/1914065.pdf. (Accessed 12 July 2025).

Brosig, J., Reiss, J.P., 2007. Entry decisions and bidding behavior in sequential first-price procurement auctions: An experimental study. Games Econom. Behav. 58 (1), 50–74. http://dx.doi.org/10.1016/j.geb.2006.03.011.

Brown, V.A., 2021. An introduction to linear mixed-effects modeling in R. Adv. Methods Pr. Psychol. Sci. 4 (1), http://dx.doi.org/10.1177/2515245920960351.

Bundesamt für Energie, 2019. Wettbewerbliche Ausschreibungen – ProKilowatt. URL: https://www.bfe.admin.ch/bfe/de/home/foerderung/energieeffizienz/wettbewerbliche-ausschreibungen-prokilowatt.html#kw-83176. (Accessed 12 July 2025).

Camerer, C.F., Ho, T.H., Chong, J.K., 2004. A cognitive hierarchy model of games. Q. J. Econ. 119 (3), 861–898. http://dx.doi.org/10.1162/0033553041502225.

Campbell, C.M., 1998. Coordination in auctions with entry. J. Econom. Theory 82, 425–450. http://dx.doi.org/10.1006/jeth.1995.2452.

Cao, X., Tan, G., Tian, G., Yilankaya, O., 2018. Equilibria in second-price auctions with private participation costs. Econom. Theory 65, 231–249. http://dx.doi.org/10.1007/s00199-016-1028-6.

Cao, X., Tian, G., 2010. Equilibria in first price auctions with participation costs. Games Econom. Behav. 69, 258–273. http://dx.doi.org/10.1016/j.geb.2009.11.006.

Cao, X., Tian, G., 2013. Second price auctions with different participation costs. J. Econ. Manag. Strat. 22, 184–205. http://dx.doi.org/10.1111/jems.12004.

Celik, G., Yilankaya, O., 2009. Optimal auctions with simultaneous and costly participation. B.E. J. Theor. Econ. 9 (1), http://dx.doi.org/10.2202/1935-1704.1522.

Chen, D.L., Schonger, M., Wickens, C., 2016. oTree – An open-source platform for laboratory, online, and field experiments. J. Behav. Exp. Financ. 9, 88–97. http://dx.doi.org/10.1016/j.jbef.2015.12.001.

Cramton, P., Ellermeyer, S., Katzman, B., 2015. Designed to fail: The medicare auction for durable medical equipment. Econ. Inq. 53 (1), 469–485. http://dx.doi.org/10.1111/ecin.12101.

Damianov, D.S., 2005. The uniform price auction with endogenous supply. Econom. Lett. 88, 152–158. http://dx.doi.org/10.1016/j.econlet.2005.02.004.

Damianov, D.S., Becker, J.G., 2010. Auctions with variable supply: Uniform price versus discriminatory. Eur. Econ. Rev. 54 (4), 571–593. http://dx.doi.org/10.1016/j.euroecorev.2009.09.002.

Diakonikolas, I., Papadimitriou, C., Pierrakos, G., Singer, Y., 2012. Efficiency-revenue trade-offs in auctions. In: Czumaj, A., Mehlhorn, K., Pitts, A., Wattenhofer, R. (Eds.), Automata, Languages, and Programming. Springer Berlin Heidelberg, pp. 488–499. http://dx.doi.org/10.1007/978-3-642-31585-5_44.

Ehrhart, K.M., Hanke, A.K., Ott, M., 2020. A small volume reduction that melts down the market: Auctions with endogenous rationing. URL: http://hdl.handle.net/10419/215673. ZEW Discussion Paper 20-014.

Enerdata, 2021. Italy awards only 821 MW of renewable capacity in a 3.3 GW tender. URL: https://www.enerdata.net/publications/daily-energy-news/italy-awards-only-821-mw-renewable-capacity-33-gw-tender.html. (Accessed 12 July 2025).

European Commission, 2014. Guidelines on state aid for environmental protection and energy 2014–2020 (2014/c 200/01). URL: http://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:52014XC0628(01). (Accessed 12 July 2025).

European Commission, 2018. Specific demand response tender in France, SA.48490. URL: https://competition-cases.ec.europa.eu/cases/SA.48490. (Accessed 12 July 2025).

European Commission, 2019. France - solar support in Fessenheim, SA.51190. URL: https://competition-cases.ec.europa.eu/cases/SA.51190. (Accessed 12 July 2025).

European Commission, 2020. Stepping up europe's 2030 climate ambition. Investing in a climate-neutral future for the benefit of our people. Communication from the commission to the European parliament, the council, the European economic and social committee and the committee of the regions. URL: https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:52020DC0562&from=EN. (Accessed 12 July 2025).

European Commission, 2022. Guidelines on state aid for climate, environmental protection and energy 2022 (2022/c 80/01). URL: https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:52022XC0218(03). (Accessed 12 July 2025).

Gałecki, A., Burzykowski, T., 2013. Linear mixed-effects model. In: Linear Mixed-Effects Models using R. Springer, pp. 245–273.

Harstad, R.M., Kagel, J.H., Levin, D., 1990. Equilibrium bid functions for auctions with an uncertain number of bidders. Econom. Lett. 33 (1), 35–40. http://dx.doi.org/10.1016/0165-1765(90)90197-9.

Hochberg, M., Poudineh, R., 2018. Renewable auction design in theory and practice: Lessons from the experiences of Brazil and Mexico. URL: https://www.oxfordenergy.org/wpcms/wp-content/uploads/2018/04/Renewable-Auction-Design-in-Theory-and-Practice-Lessons-from-the-Experiences-of-Brazil-and-Mexico-EL-28.pdf. Oxford Institute for Energy Studies Paper: EL 28, (Accessed 12 July 2025).

Holt, C.A., Laury, S.K., 2002. Risk aversion and incentive effects. Am. Econ. Rev. 92 (5), 1644–1655. http://dx.doi.org/10.1257/000282802762024700.

IRENA, 2013. Renewable energy auctions in developing countries. URL: https://www.irena.org/Publications/2013/Jun/Renewable-Energy-Auctions-in-Developing-Countries. (Accessed 12 July 2025).

IRENA, 2015. Renewable energy auctions: A guide to design. URL: https://www.irena.org/publications/2015/Jun/Renewable-Energy-Auctions-A-Guide-to-Design. (Accessed 12 July 2025).

IRENA, 2019. Renewable energy auctions: Status and trends beyond price. URL: https://www.irena.org/publications/2019/Dec/Renewable-energy-auctions-Status-and-trends-beyond-price. (Accessed 12 July 2025).

Jiménez, M., 2016. CENACE obtains definitive results for the first long-term electricity tender. URL: https://www.linkedin.com/pulse/cenace-obtains-definitive-results-first-long-term-tender-jimenez. (Accessed 12 July 2025).

Kazakh Government, 2009. Government of Kazakhstan: On support for the use of renewable energy source, the law of the Republic of Kazakhstan dated 4 July 2009 no. 165-IV. URL: http://adilet.zan.kz/eng/docs/Z090000165_. (Accessed 12 July 2025).

Kitzing, L., Wendring, P., 2016. Cash flow analysis of past RES auctions, AURES report D5.1. URL: https://backend.orbit.dtu.dk/ws/files/126608974/d5_1_report_model_application_final.pdf. (Accessed 12 July 2025).

Kreiss, J., Ehrhart, K.M., Haufe, M.C., Soysal, E.R., 2021. Different cost perspectives for renewable energy support: Assessment of technology-neutral and discriminatory auctions. Econ. Energy Environ. Policy 10 (1), http://dx.doi.org/10.5547/2160-5890.10.1.jkre.

Kreps, D.M., Scheinkman, J.A., 1983. Quantity precommitment and Bertrand competition yield Cournot outcomes. Bell J. Econ. 14 (4), 326–337, URL: https://www.jstor.org/stable/3003636.

Krishna, V., 2010. Auction Theory, second ed. Academic Press.

Kunzetsova, A., Brockhoff, P., Christensen, R., 2017. Lmertest package: tests in linear mixed effect models. J. Stat. Softw. 82, 1–26.

Legislation of Ukraine, 2019. On alternative energy sources (document 555-IV). URL: https://zakon.rada.gov.ua/laws/show/555-15?lang=en. (Accessed 12 July 2025).

Levin, D., Smith, J.L., 1994. Equilibrium in auctions with entry. Am. Econ. Rev. 84 (3), 585–599, URL: https://www.jstor.org/stable/2118069.

Levitan, R., Shubik, M., 1972. Price duopoly and capacity constraints. Internat. Econom. Rev. 13 (1), 111–122, URL: https://www.jstor.org/stable/2525908.

Li, T., Zheng, X., 2009. Entry and competition effects in first-price auctions: Theory and evidence from procurement auctions. Rev. Econ. Stud. 76, 1397–1429. http://dx.doi.org/10.1111/j.1467-937X.2009.00558.x.

LiCalzi, M., Pavan, A., 2005. Tilting the supply schedule to enhance competition in uniform-price auctions. Eur. Econ. Rev. 49, 227–250. http://dx.doi.org/10.1016/S0014-2921(02)00324-0.

Likhodedov, A., Sandholm, T., 2004. Mechanism for optimally trading off revenue and efficiency in multi-unit auctions. In: Faratin, P., Parkes, D.C., Rodríguez-Aguilar, J.A., Walsh, W.E. (Eds.), Agent-Mediated Electronic Commerce V. Designing Mechanisms and Systems. Springer Berlin Heidelberg, pp. 92–108. http://dx.doi.org/10.1007/978-3-540-25947-3_6.

Lu, J., 2009a. Auction design with opportunity cost. Econom. Theory 38, 73–103. http://dx.doi.org/10.1007/s00199-008-0331-2.

Lu, J., 2009b. Why a simple second-price auction induces efficient endogenous entry. Theory and Decision 66, 181–196. http://dx.doi.org/10.1007/s11238-007-9092-9.

Luther Law Firm, 2025. International onshore wind study. URL: https://www.luther-lawfirm.com/fileadmin/user_upload/PDF/Broschueren/International%20On-shore%20Wind%20Study.pdf. (Accessed 12 July 2025).

McAdams, D., 2007. Adjustable supply in uniform price auctions: Non-commitment as a strategic tool. Econom. Lett. 95, 48–53. http://dx.doi.org/10.1016/j.econlet.2006.08.035.

McAfee, R.P., McMillan, J., 1987. Auctions with entry. Econom. Lett. 23, 343–347.

Menezes, F.M., Monteiro, P.K., 2000. Auctions with endogenous participation. Rev. Econ. Des. 5, 71–89. http://dx.doi.org/10.1007/s100580050048.

Meyer, D.J., 1993. First price auctions with entry: An experimental investigation. Q. Rev. Econ. Financ. 33 (2), 107–122. http://dx.doi.org/10.1016/1062-9769(93)90017-E.

Miralles, A., 2008. Intuitive and noncompetitive equilibria in weakly efficient auctions with entry costs. Math. Social Sci. 56, 448–455. http://dx.doi.org/10.1016/j.mathsocsci.2008.06.004.

Mora, D., Kitzing, L., Soysal, E.R., Steinhilber, S., del Río, P., Wigand, F., Klessmann, C., Tiedemann, S., Amazo, A., Welisch, M., Kreiß, J., Roy, O.F., Woodman, B., 2017. Auctions for renewable energy support: Taming the beast of competitive bidding, AURES report D9.2. URL: https://orbit.dtu.dk/files/142941994/aures_finalreport.pdf. (Accessed 12 July 2025).

Müller, S., Scealy, J.L., Welsh, A.H., 2013. Model selection in linear mixed models. Statist. Sci. 28 (2), 135–167, URL: https://www.jstor.org/stable/43288485.

Myerson, R., 1981. Optimal auction design. Math. Oper. Res. 6 (1), 58–73. http://dx.doi.org/10.1287/moor.6.1.58.

Nagel, R., 1995. Unraveling in guessing games: An experimental study. Am. Econ. Rev. 85 (5), 1313–1326, URL: https://www.jstor.org/stable/2950991.

Nyborg, K.G., Rydqvist, K., Sundaresan, S.M., 2002. Bidder behavior in multiunit auctions: Evidence from Swedish treasury auctions. J. Political Econ. 110 (2), 394–424. http://dx.doi.org/10.1086/338748.

Palfrey, T.R., Pevnitskaya, S., 2008. Endogenous entry and self-selection in private value auctions: An experimental study. J. Econ. Behav. Organ. 66 (3–4), 731–747. http://dx.doi.org/10.1016/j.jebo.2006.06.005.

Papachristou, D., Kapetana, P., Daliouris, P., Petmezas, T., 2017. The pilot tender for PV in Greece within 2016. In: 33rd European Photovoltaic Solar Energy Conference and Exhibition.

Pietrowicz, M., Quentin, J., 2015. Dauer und Kosten des Planungs- und Genehmigungsprozesses von Windenergieanlagen an Land (Fachagentur Windenergie an Land). URL: https://www.fachagentur-windenergie.de/fileadmin/files/Veroeffentlichungen/dauer-kosten-wea-projektierung.pdf. (Accessed 12 July 2025).

Robert, A., Simon, F., 2019. French wind energy 'in limbo' for more than a year. URL: https://www.euractiv.com/section/energy/news/french-wind-energy-in-limbo-for-more-than-a-year/. (Accessed 12 July 2025).

Samuelson, W.F., 1985. Competitive bidding with entry costs. Econom. Lett. 17, 53–57. http://dx.doi.org/10.1016/0165-1765(85)90126-0.

Schweizer, N., Szech, N., 2017. Revenues and welfare in auctions with information release. J. Econom. Theory 170, 86–111. http://dx.doi.org/10.1016/j.jet.2017.04.005.

Stegemann, M., 1996. Participation costs and efficient auctions. J. Econom. Theory 71, 228–259. http://dx.doi.org/10.1006/jeth.1996.0116.

Szech, N., 2011. Optimal disclosure of costly information packages in auctions. J. Math. Econom. 47, 462–469. http://dx.doi.org/10.1016/j.jmateco.2011.06.004.

Tan, G., 1992. Entry and R & D in procurement contracting. J. Econom. Theory 58 (1), 41–60. http://dx.doi.org/10.1016/0022-0531(92)90100-V.

Tan, G., Yilankaya, O., 2006. Equilibria in second price auctions with participation costs. J. Econom. Theory 130, 205–219. http://dx.doi.org/10.1016/j.jet.2005.02.008.

US, 2021. The long-term strategy of the United States. Pathways to net-zero greenhouse gas emissions by 2050. Published by the United States Department of State and the United States Executive Office of the President, Washington DC. November 2021. URL: https://unfccc.int/sites/default/files/resource/US_accessibleLTS2021.pdf. (Accessed 12 July 2025).

Wallasch, A.K., Lüers, S., Rehfeldt, K., 2015. Kostensituation der Windenergie an Land in Deutschland – Update (Deutsche WindGuard). (Accessed 12 July 2025). URL: https://www.windguard.de/veroeffentlichungen.html?file=files/cto_layout/img/unternehmen/veroeffentlichungen/2015/Kostensituation%20der%20Windenergie%20an%20Land%20in%20Deutschland%20-%20Update.pdf.

Wigand, F., Förster, S., Amazo, A., Tiedemann, S., 2016. Auctions for renewable energy support: Lessons learnt from international experiences, AURES report D4.2. URL: https://ec.europa.eu/research/participants/documents/downloadPublic?documentIds=080166e5aa83001d&appId=PPGMS. (Accessed 12 July 2025).

Wilcoxon, F., 1945. Individual comparisons by ranking methods. Biom. Bull. 1 (6), 80–83, URL: http://www.jstor.org/stable/3001968.