

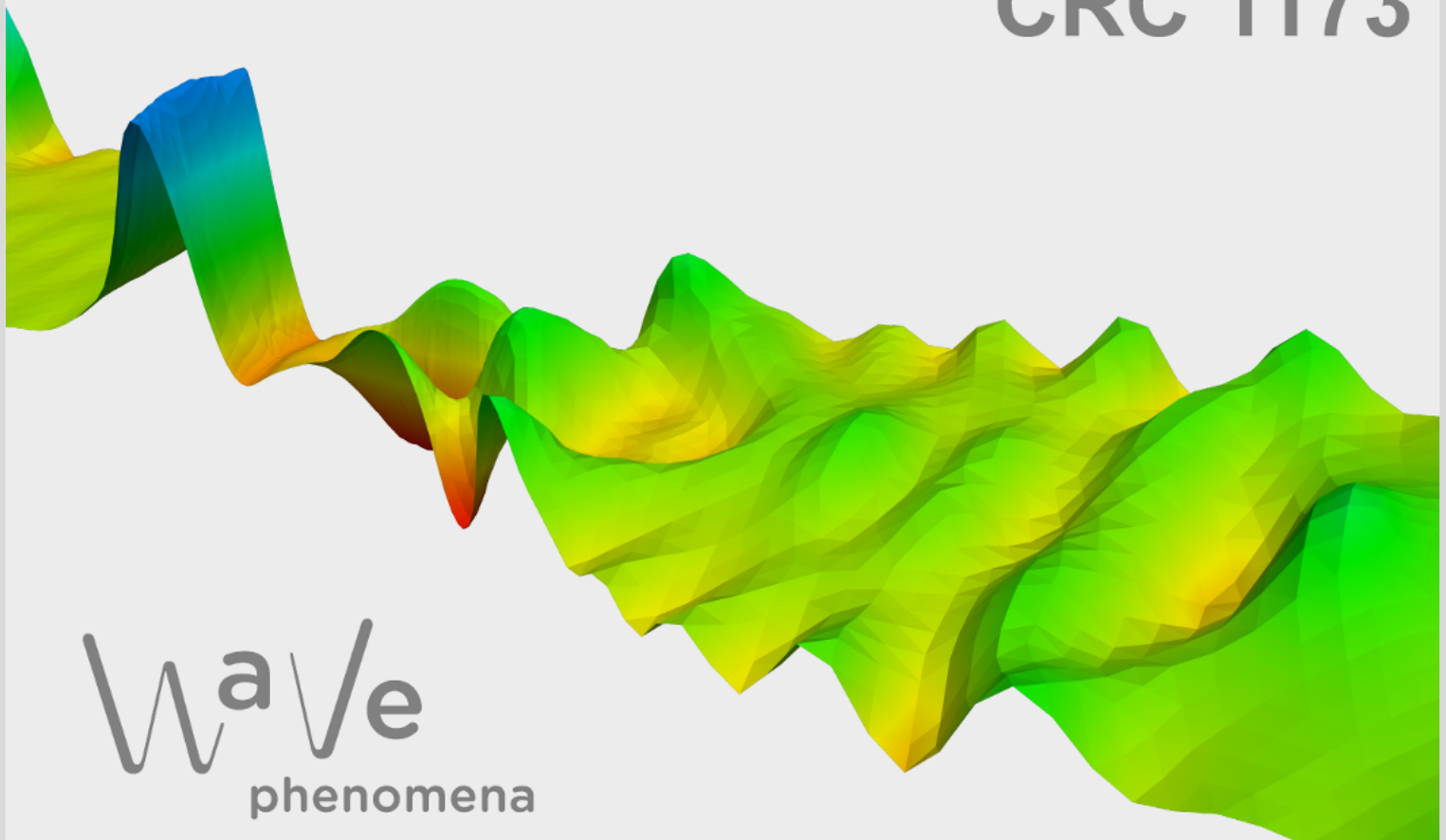
Photonic time crystals assisted by quasi-bound states in the continuum

P. Garg, E. Almpanis, L. Zimmer, J. D. Fischbach,
X. Wang, M. S. Mirmoosa, M. Nyman, N. Stefanou,
N. Papnikolaou, V. Asadchy, C. Rockstuhl

CRC Preprint 2025/59, December 2025

KARLSRUHE INSTITUTE OF TECHNOLOGY

CRC 1173



Wave
phenomena

Participating universities



Funded by



Photonic time crystals assisted by quasi-bound states in the continuum

P. Garg^{1*}, E. Almpenis^{2,3}, L. Zimmer¹, J. D. Fischbach⁴, X. Wang⁵, M. S. Mirmoosa⁶, M. Nyman¹, N. Stefanou², N. Papanikolaou³, V. Asadchy^{7*}, C. Rockstuhl^{1,4*}

¹Institute of Theoretical Solid State Physics, Karlsruhe Institute of Technology, Kaiserstrasse 12, Karlsruhe, 76131, Germany

²Section of Condensed Matter Physics, National and Kapodistrian University of Athens, Panepistimioupolis, Athens, 15784, Greece

³Institute of Nanoscience and Nanotechnology, NCSR “Demokritos”, Patriarchou Gregoriou and Neapoleos Str., Ag. Paraskevi, Athens, 15310, Greece

⁴Institute of Nanotechnology, Karlsruhe Institute of Technology, Kaiserstrasse 12, Karlsruhe, 76131, Germany

⁵College of Physics and Optoelectronic Engineering, Harbin Engineering University, Nantong Street 145, Qingdao, 150001, China

⁶Department of Physics and Mathematics, University of Eastern Finland, Yliopistokatu 7, Joensuu, 80101, Finland

⁷Department of Electronics and Nanoengineering, Aalto University, Maarintie 8, Espoo, 02150, Finland

***Corresponding author(s). E-mail(s):** puneet.garg@kit.edu; viktar.asadchy@aalto.fi; carsten.rockstuhl@kit.edu;

Abstract

Photonic time crystals are a class of artificial materials that have only recently been explored. They are characterized by the ultrafast modulation of the material properties in time, causing a momentum bandgap for light that propagates through such novel states of matter. However, the observation of these unique properties at optical frequencies remains elusive, as the necessary modulation amplitudes of the permittivity to show notable momentum bandgaps are relatively high, inaccessible with available materials. While it has been known that

structuring photonic time crystals at the sub-wavelength scale can enhance the momentum bandgap, we push this concept to the extreme by leveraging the nanophotonic toolbox. Specifically, we demonstrate that nanophotonic structures composed of scatterers supporting quasi-bound states in the continuum can significantly reduce the required amplitude of temporal permittivity modulation by enhancing the interaction time between light and time-varying matter. This allows us to observe extremely wide momentum bandgaps despite the material properties having tiny modulation amplitudes. Our approach bridges the concepts of bound states in the continuum and time-varying metamaterials, paving the way toward realizable photonic time crystals at optical frequencies.

Keywords: Bound states in the continuum, momentum bandgaps, metasurfaces, photonic time crystals, time-varying photonic systems

Recently, there has been a surge of interest in the physics of time-varying photonic media [1–12]. In particular, photonic time crystals (PTCs), bulk materials characterized by periodic modulation of the permittivity in time, have been at the forefront of this research domain [13, 14]. The salient property of PTCs is that they exhibit momentum bandgaps. Inside these momentum bandgaps, eigenmodes exist with complex frequencies for a certain range of wavenumbers. Depending on the sign of the imaginary part, the complex frequency modes amplify the incident radiation that couples to them [13, 15]. Photonic time crystals show promise for intensifying light-matter interactions, much like spatial photonic crystals enhance light confinement [16, 17]. Applications of momentum bandgaps in PTCs include amplified spontaneous emission [18], subluminal Cherenkov radiation [19], and temporal localization effects [20–22], among many others.

Although PTCs have been recently experimentally demonstrated at microwave frequencies thanks to the availability of fast variable electronic and radio frequency (RF) components [23–25], their realization at even higher frequencies remains a significant challenge. Achieving the required temporal modulation demands extremely fast material changes – typically at twice the oscillation frequency of the probing light – and a relative permittivity change close to unity when considering a bulk, *i.e.*, a spatially homogeneous, PTC [26, 27].

The optical modulation of transparent conducting oxides (TCOs) by an external pump has emerged as a viable path to study effects related to time-varying media. In particular, indium tin oxide (ITO) and aluminum doped zinc oxide (AZO) offer relative permittivity changes of up to 100% in their epsilon-near-zero (ENZ) frequency regime [28–31]. However, achieving such modulation requires extremely high pumping power densities [27]. Combined with substantial material dissipation, these requirements pose a risk of thermal damage, making practical implementation of PTCs with TCOs highly challenging [32]. Moreover, achieving a “pure PTC” regime using time-varying bulk TCO platforms may prove difficult [27, 33].

A loophole to achieve a PTC not obscured by other effects is to use third-order nonlinearity in transparent materials, such as Si, Ge, or GaAs [27]. For example, the process of third-order nonlinearity (under the assumption of an undepleted pump) can

be modeled using linear coupled-mode equations of a spatially uniform material with time-varying permittivity. The modulation frequency in this case can be extremely high, virtually unlimited [27]. However, in low-loss materials, nonlinear effects are inherently weak [34], restricting the relative permittivity change to a maximum of not more than 1%. This tiny modulation amplitude is insufficient to observe the main features of PTCs. Indeed, inevitable scattering losses in realistic material systems and other imperfections may easily obstruct the weak signal amplification.

Nevertheless, a recent theoretical study demonstrated that momentum bandgaps in low-loss PTCs can be expanded even with weak permittivity modulation amplitudes by leveraging structural resonances in a metasurface [35]. The proposed metasurface comprised an array of subwavelength resonant silicon spheres that support dipolar Mie resonances. However, this design suffers from limitations in that the resonances had relatively low quality factors (Q-factors), which necessitates an operation too close to the thermal damage threshold of realistic materials [36–41].

To overcome this limitation, we leverage here the concept of photonic quasi-bound states in the continuum (qBICs) and combine it with PTCs. Photonic qBICs are known to exhibit arbitrarily high Q-factors, limited only by material losses and fabrication tolerances [42–44]. We demonstrate that metasurfaces made of nanoresonators supporting qBICs pave the way for the realization of practical PTCs using conventional materials and accounting for realistic losses. Specifically, we propose two metasurface geometries based on multilayered spherical and homogeneous cylindrical nanoresonators. The study of multilayered sphere-based metasurfaces sheds light on the theoretical aspects of expanding momentum bandgaps with qBICs. Meanwhile, the cylinder-based metasurface demonstrates that wide momentum bandgaps are achievable despite realistic losses with reasonable designs. We show that these bandgaps are achieved while operating at pump powers more than an order of magnitude below the thermal threshold.

Concept

To explain our concept, Fig. 1 illustrates the physical idea. A conventional PTC is spatially homogeneous and consists of a medium whose material properties, such as the electron density $N(t)$, change periodically in time, *i.e.*, $N(t) = N_0 [1 + M_s \cos(\omega_m t)]$. We employ the Drude-Lorentz model to account for the time-varying electron density (see Supplementary Section S1). Here, N_0 is the electron density of the corresponding static medium, M_s is the modulation amplitude, and ω_m is the modulation frequency. The dispersion relation of light in the static medium in its low material loss regime is a straight line. The periodic time modulation, in a temporal empty lattice approximation (*i.e.*, $M_s \rightarrow 0$, and $\omega_m \neq 0$), causes the periodic repetition of the dispersion relation in the frequency domain. Each branch is displaced by a multiple of ω_m . These are the Floquet bands, and they cross at the edges of the frequency Brillouin zone. When switching on a finite amplitude for the time modulation (*i.e.*, $M_s \neq 0$), a momentum bandgap opens around the crossing point. Within the momentum bandgap, the eigenfrequencies can have positive or negative imaginary parts. Assuming a time convention of $e^{-i\omega t}$, these two solutions exponentially grow or decay in time, respectively,

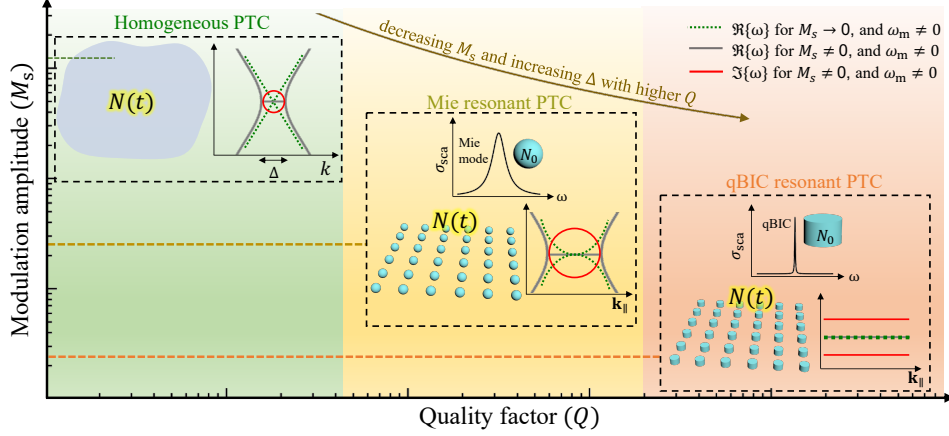


Fig. 1 An illustration that discusses the modulation amplitude M_s of the material properties necessary to open momentum bandgaps as a function of the quality factor Q of the resonances supported by meta-atoms in various PTC geometries. We observe that, as Q increases, M_s decreases for increasing gapsize Δ . The illustration depicts a spatially homogeneous PTC (green background), a PTC constructed from meta-atoms that sustain Mie resonances (yellow background), and the proposed PTC that hosts a high- Q qBIC (orange background). The Mie and qBIC resonances of the static meta-atoms having electron density N_0 are depicted in the plots of their respective scattering cross sections σ_{sca} . Here, $N(t)$ represents the time-periodic electron density of the underlying media of the PTCs, ω is the frequency of the eigenmodes, k is the wavenumber of the eigenmodes of homogeneous PTCs, and $\mathbf{k}_{\parallel}$ is the Bloch wave vector of the eigenmodes of structured PTCs. Besides the basic illustration of the geometry and resonances, the band structure is shown for zero (dotted green) and non-zero (solid gray and solid red) modulation amplitude M_s . Note that the high- Q resonance of an isolated static meta-atom leads to a strong localization of the field inside the meta-atom. This localization reduces the interaction between the neighboring meta-atoms when they are placed in a lattice, leading to reduced spatial dispersion in the lattice. Therefore, a flat band emerges in the band structure of the corresponding static metasurface.

upon excitation. However, as already discussed above and studied in literature [27], the achievable momentum bandgap remains rather tiny, even for substantial modulation amplitudes.

To overcome this limitation, we can spatially structure the PTC and exploit basic photonic resonances supported in such a structured material. A prototypical example of a photonic resonance is that of a Mie resonance. It occurs whenever a multipole moment sustained in the spatially localized scatterer is resonantly excited. Thanks to such a resonance, light will remain trapped inside the scatterer for an extended duration, where it experiences the time-varying material properties.

When periodically arranging such Mie scatterers to form a metasurface, the bands in the static material will experience a rather strong curvature as a consequence of the resonant modulation of the otherwise linear dispersion relation (see Fig. 1). Switching on the time modulation with a modulation frequency twice the resonance frequency causes a repetition of the dispersion relation in the frequency domain, a crossing at the spectral location of the resonance, and a larger momentum bandgap. However, the approach has its limitations. The achievable Q -factors for ordinary Mie resonances

remain rather low for the lowest-order multipole moments. These resonances couple efficiently to free space and therefore suffer from high radiation losses.

However, nanophotonics provides us with the tools to perceive scatterers with little to no radiation losses. These structures host bound states in the continuum (BICs) and are designed to have entirely suppressed radiative losses [42–45]. Admittedly, the complete suppression of radiation losses is not desirable, as we are still required to excite these resonances. Nevertheless, by deliberately breaking the symmetry or detuning the system incrementally from the BIC condition, we can deterministically adjust the Q-factor that the scatterer sustains. The resulting high-Q resonances are qBICs.

When placing these scatterers with qBICs in a metasurface lattice, a photonic flat band emerges for the stationary medium (see Fig. 1). Now, by choosing ω_m to be twice the frequency of the qBIC resonance frequency, an extremely wide momentum bandgap emerges even for a tiny modulation amplitude M_s . The crossing of the two flat bands, split by time modulation, eventually covers the entire Brillouin zone. By using our understanding of how to structure materials to induce resonances with ultrahigh Q-factors, we can pave the way for the realistic implementation of PTCs and their associated momentum bandgaps.

It is important to note that, in this work, we exploit the flat bands of the metasurfaces to open large momentum bandgaps. The flatness of the bands is predominantly controlled by the Q-factors of the resonances of the isolated meta-atoms. Therefore, we optimize the individual meta-atoms (and not the metasurface) to host high-Q qBICs (see Supplementary Section S2).

In the following, we illustrate this concept with two different examples that achieve such scatterers with ultrahigh Q-factors by leveraging different nanophotonic approaches. The first example concerns a metasurface made from ENZ-material-based multilayered spheres. The second example concerns a metasurface composed of germanium cylinders. In both examples, we capitalize on high-Q qBICs to show that the modulation amplitude M_s required to obtain large momentum bandgaps can be considerably reduced as compared to homogeneous PTCs and metasurfaces supporting ordinary Mie resonances.

Results

PTCs based on metasurfaces made from multilayered spheres

To begin with, we analyze the BICs of static isolated multilayered spheres. We consider a sphere that consists of a core and two layers (see the inset in Fig. 2(a)). The core and the outer layer of the sphere are made from AZO. The dispersive permittivity of AZO is expressed using the Drude-Lorentz model (see Supplementary Section S1), where the corresponding parameters were taken from [46]. The central layer is made from silica with relative permittivity $\varepsilon = 2.15$ [47]. We denote the radii of the core, central, and outer layers of the sphere by r_0 , r_1 , and r_2 , respectively. We assume $r_0 = 120$ nm, $r_1 = \eta r_2$, and $r_2 = 200$ nm, where η is a variable. To demonstrate the existence of a true BIC, we first turn off the material losses in the AZO layers by setting the Drude-Lorentz loss parameter γ to 0 (see Eq. (S1) in Supplementary Section S1). Next, we

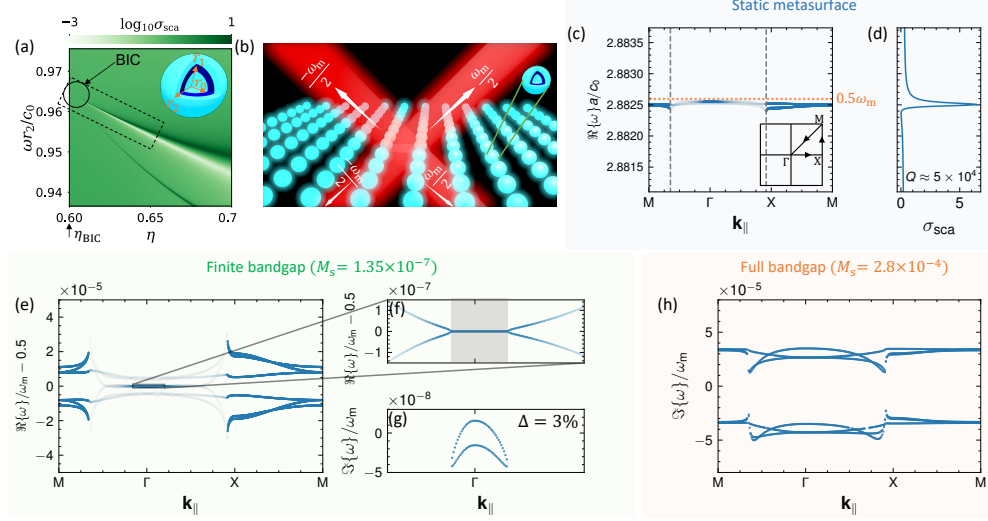


Fig. 2 (a) The normalized scattering cross section σ_{sca} of the static multilayered sphere (see inset) as a function of the frequency ω and the geometry parameter $\eta = r_1/r_2$. The black circle marks the spectral location of the true BIC. Note that material losses are neglected here to clearly illustrate the effects. (b) A metasurface-based PTC made from a square lattice of multilayered spheres supporting qBICs as shown in (a). (c) The band structure of the metasurface from (b) under static conditions, i.e., $M_s = 0$ in the spectral domain of interest. (d) The normalized scattering cross section σ_{sca} of the static isolated multilayered sphere shown in (b) as a function of frequency. The sphere hosts a qBIC with a quality factor of $Q \approx 5 \times 10^4$. (e) The band structure of the time-varying metasurface zoomed at the very narrow frequency range centered at $\Re\{\omega\} = \omega_m/2$. An extremely low modulation amplitude $M_s = 1.35 \times 10^{-7}$ is used. (f) The further zoomed band structure in the highlighted region of (e) illustrates a finite momentum bandgap. (g) The corresponding imaginary part of the frequency inside the bandgap in (f) for a fixed $\Re(\omega)/\omega_m = 0.5$. The displayed modes have a symmetry compatible with that of p -polarized plane waves. (h) The imaginary part of the frequency inside the full bandgap for a fixed $\Re(\omega)/\omega_m = 0.5$. Here, $M_s = 2.8 \times 10^{-4}$ is used. In (g) and (h), the plots are not symmetric about $\Im(\omega) = 0$ due to the finite (although small) radiation losses in the metasurfaces. The gray-dotted lines in (c) represent the light lines. The transparency of the bands in (c), (e), and (f) is directly proportional to the radiative losses that the underlying modes possess. Furthermore, a is the lattice period of the metasurface, and c_0 is the speed of light in a vacuum.

plot the normalized scattering cross section σ_{sca} of such a lossless sphere as a function of the frequency ω and the geometric parameter η (see Fig. 2(a)). The bright and dark regions in the color plot correspond to scattering zeros and resonances of the sphere, respectively. In the dashed box, the scattering zeros and the resonances approach each other, eventually merging at a point at $\eta = \eta_{\text{BIC}} \approx 0.6$. This coalescing of the zeros and the resonances is the unmistakable signature for the existence of a true BIC at that point in parameter space [45]. This BIC appears at the frequency where $\varepsilon = 0$ for the AZO layers [45, 48]. In the following, we study how this BIC affects the band structures of metasurfaces made from these multilayered spheres.

The considered metasurface is made from a square lattice of multilayered spheres (see Fig. 2(b)). The lattice period of the metasurface is $a = 3r_2$. We assume the electron density N in the Drude-Lorentz model of both AZO layers of the spheres to

be time-harmonic, *i.e.*, $N(t) = N_A [1 + M_s \cos(\omega_m t)]$. Here, N_A is the electron density of AZO under static conditions. The silica layer continues to be static. At first, we turn off the temporal modulation of AZO and consider the static case, *i.e.*, $M_s = 0$. Furthermore, we operate at $\eta = 1.018\eta_{\text{BIC}} = 0.611$ so that the spheres support a qBIC (see Fig. 2(a)). The band structure of the corresponding static metasurface is shown in Fig. 2(c). We use the T-matrix method to compute the band structure [49, 50] (see Methods). Here, we observe the emergence of flat bands due to the qBIC of the sphere. For comparison, we plot σ_{sca} of the sphere as a function of the frequency ω in Fig. 2(d). We note the existence of the qBIC of the sphere with the Q-factor approximately 5×10^4 in Fig. 2(d), which explains the appearance of the flat bands in Fig. 2(c). Note that the qBIC shown in Fig. 2(d) has an electric dipolar character. Due to the spatial symmetry of the sphere, this resonance is threefold degenerate. Upon placing the sphere in the lattice, this degeneracy is lifted. Therefore, three distinct bands exist in Fig. 2(c).

Next, we turn on the time modulation of the spheres. In particular, we choose ω_m such that $0.5\omega_m$ coincides with the spectral location of the flat bands and hence that of the qBIC. The numerical value of ω_m is $\omega_m = 2\pi \times 459$ THz. Furthermore, we assume a vanishingly small M_s , *i.e.*, $M_s = 1.35 \times 10^{-7}$. The band structure of the corresponding time-varying metasurface is shown in Fig. 2(e). We observe that the band structure of the time-varying metasurface is formed by folding the band structure of the static metasurface about $\Re(\omega) = 0.5\omega_m$. In the time-varying case, the eigenmodes at $\Re(\omega) = 0.5\omega_m$ interact with each other. Choosing $0.5\omega_m$ close to the qBIC frequency enhances this interaction due to the large lifetime of the qBIC. Therefore, light interacts with the time-varying matter for a longer duration, leading to the emergence of a large momentum bandgap at $0.5\omega_m$ even for vanishingly small M_s . We show the existence of a relatively large momentum bandgap in the magnified region around the Γ -point (Fig. 2(f)). In particular, the relative gapsize is $\Delta = 3\%$ (see Eq. (3) in Methods). This gapsize is referred to as large because to open a similarly-sized bandgap using conventional homogeneous PTC, one needs M_s of the order of unity [27, Fig. 3]. Here, we operate at a modulation amplitude seven orders of magnitude smaller. Furthermore, we show the imaginary part of the frequency $\Im(\omega)$ inside the momentum bandgap for a fixed $\Re(\omega)/\omega_m = 0.5$ in Fig. 2(g). As expected, we observe that for the $\mathbf{k}_{\parallel}$ values falling inside the gap, there exist modes with $\Im(\omega) > 0$ responsible for wave amplification.

Another interesting characteristic of the qBIC-assisted PTCs is that they can support full momentum bandgaps spanning over all $\mathbf{k}_{\parallel}$ values. To open such a full bandgap, we increase the modulation amplitude to $M_s = 2.8 \times 10^{-4}$ while keeping all other parameters unchanged. The corresponding imaginary part of the frequency $\Im(\omega)$ inside the momentum bandgap for a fixed $\Re(\omega)/\omega_m = 0.5$ is shown in Fig. 2(h). We find that there exist modes with $\Im(\omega) > 0$ for all possible $\mathbf{k}_{\parallel}$ values, leading to a full bandgap. Note that, in Fig. 2(h), we show that $\Im(\omega) > 0$ on the edges of the irreducible Brillouin zone (IBZ). However, here, as well as in the following examples with full bandgaps, we verified that $\Im(\omega) > 0$ inside the entire IBZ.

Next, we show the effects of varying the Q-factor on the required modulation amplitude to maintain the large bandgap. For a fair comparison, we always require

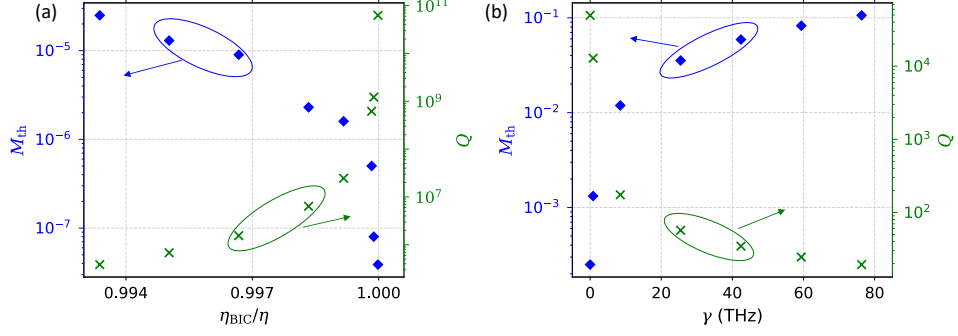


Fig. 3 The threshold modulation amplitude M_{th} of the multilayered sphere metasurface required to exhibit a full bandgap as a function of the geometry parameter $\eta = r_1/r_2$ (blue diamonds in (a)) and the Drude-Lorentz loss parameter γ (blue diamonds in (b)). The Q-factor of the qBIC supported by the isolated static multilayered sphere as a function of η (green crosses in (a)) and γ (green crosses in (b)). Note that $\eta/\eta_{BIC} = 1.018$ is used in (b).

a full bandgap. First, we vary the Q-factor by changing the geometry parameter η . We note that, to open a full bandgap using the time-varying metasurface, a threshold modulation amplitude $M_s = M_{th}$ is required. In Fig. 3(a), we show M_{th} as a function of η (blue diamonds). We find that as $\eta \rightarrow \eta_{BIC}$, $M_{th} \rightarrow 0$. Such behavior is observed because, as $\eta \rightarrow \eta_{BIC}$, the Q-factor of the qBIC approaches infinity, *i.e.*, $Q \rightarrow \infty$ (green crosses in Fig. 3(a)). The high Q-factors reduce the radiation losses in the otherwise lossless system. Therefore, an arbitrarily small M_{th} is sufficient to maintain a full bandgap.

So far, for a theoretical demonstration, we assumed the absence of material losses in the AZO layers. Next, we proceed to analyze the effect of material losses on M_{th} to open a full bandgap. We plot M_{th} as a function of the Drude-Lorentz loss parameter γ of the AZO layers in Fig. 3(b) (blue diamonds). Note that η/η_{BIC} is fixed as $\eta/\eta_{BIC} = 1.018$. We observe that upon increasing γ , the required M_{th} increases. Such behavior is expected because increasing the material losses decreases the Q-factor of the qBICs, leading to a decreased interaction time of light with time-varying matter (green crosses in Fig. 3(b)). Therefore, a higher M_{th} is needed to open a full bandgap.

In particular, the experimentally reported value of γ in the spectral region of interest (the ENZ regime of AZO) is $\gamma \approx 80$ THz [46]. From Fig. 3(b), we note that $M_{th} \approx 0.12$ is required to open a full bandgap using this value of γ . In comparison, homogeneous PTCs made from ENZ materials require $M_{th} \sim 1$ to even open finite bandgaps [27].

It is worth mentioning that, while the qBICs of the multilayered spheres assist in decreasing M_{th} , assuming realistic losses, the corresponding metasurfaces still require rather high modulation amplitudes to open full bandgaps. In literature, metamaterial-based approaches have shown that the losses in the effective ENZ medium can be considerably reduced [51]. Such an approach could be beneficial while using multilayered sphere qBICs to attain large momentum bandgaps while maintaining low modulation amplitudes. Another challenge with multilayered sphere geometry is that

it is difficult to fabricate. Keeping these difficulties in mind, we present a more practical design of the PTCs that utilizes the qBICs of dielectric cylinders as follows.

PTCs based on metasurfaces made from germanium cylinders

Now, we begin with an isolated static germanium (Ge) cylinder supporting a high-Q qBIC. The material dispersion in the Ge cylinder is considered using the Drude-Lorentz model (see Supplementary Section S1). The Drude-Lorentz parameters of Ge are taken from [52]. Note that we consider realistic material losses in the Ge cylinder from the beginning. The cylinder, with a radius of $r = 465$ nm and an aspect ratio of $r/h \approx 0.698$ (where h is the cylinder height), has been geometrically optimized following the strategy discussed in [42, 43]. This optimization leverages an avoided crossing between a Mie-type mode of azimuthal index $m = 0$ and principal (radial) index $n = 1$ and a neighboring $m = 0$ Mie-type mode of different principal order ($n = 2$), where their interaction suppresses the radiative losses in the resulting hybrid mode.

Group theory analysis reveals that these $m = 0$ modes belong to the A_{2g} irreducible representation of the $D_{\infty h}$ point group that is relevant here. As a result, the hybrid mode is symmetry-protected and dark at axial incidence but emerges as a sharp qBIC under non-axial s -polarized plane wave excitation [53]. This qBIC is formed by the Friedrich-Wintgen mechanism [42] (see Supplementary Section S3). Note that, because the qBIC has a symmetry compatible only with that of s -polarized plane waves, no resonance peak is observed for the p -polarized plane wave excitation.

First, we analyze metasurfaces made from such qBIC supporting cylinders (see Fig. 4(a)). We assume the lattice period of the metasurface to be $a = 3.5r$. The band structure of the metasurface under static conditions is shown in Fig. 4(b). We observe the emergence of a flat band. This flat band arises due to the existence of the aforementioned qBIC that the isolated static Ge cylinder hosts. For comparison, we show the rotationally averaged normalized scattering cross section σ_{sca} of the cylinder as a function of frequency ω in Fig. 4(c). From Fig. 4(c), we note the existence of the qBIC with $Q \approx 500$, which explains the existence of the flat band in Fig. 4(b). Note that the Q-factor of the qBIC of the Ge cylinders is considerably lower than that of the multilayered spheres. This is because the spheres were assumed to be lossless. With realistic material losses, as here, finite dielectric nanoresonators cannot achieve arbitrarily high Q-factors [43]. The Q-factor of the qBIC of a dielectric nanoresonator diverges in the limit of vanishing material losses and for a large real part of the relative permittivity (*i.e.*, $\varepsilon \gg 1$). However, as we demonstrate below, the Q-factor of the considered cylinders with realistic material parameters is sufficiently high to significantly reduce the required modulation amplitude for opening large momentum bandgaps, compared to homogeneous PTCs and Mie-resonant metasurfaces.

Next, we turn on the time modulation of the metasurface. We assume the electron density N of the Ge cylinders to be time-periodic, *i.e.*, $N(t) = N_G [1 + M_s \cos(\omega_m t)]$. Here, N_G is the electron density of Ge under static conditions. We choose the modulation frequency ω_m such that $0.5\omega_m$ coincides with the spectral location of the flat band and hence that of the qBIC. Numerically, $\omega_m = 2\pi \times 300$ THz. The modulation amplitude is assumed to be $M_s = 5 \times 10^{-5}$. We show the large momentum bandgap hosted by the metasurface in the region around the Γ -point in Fig. 4(d). We also plot

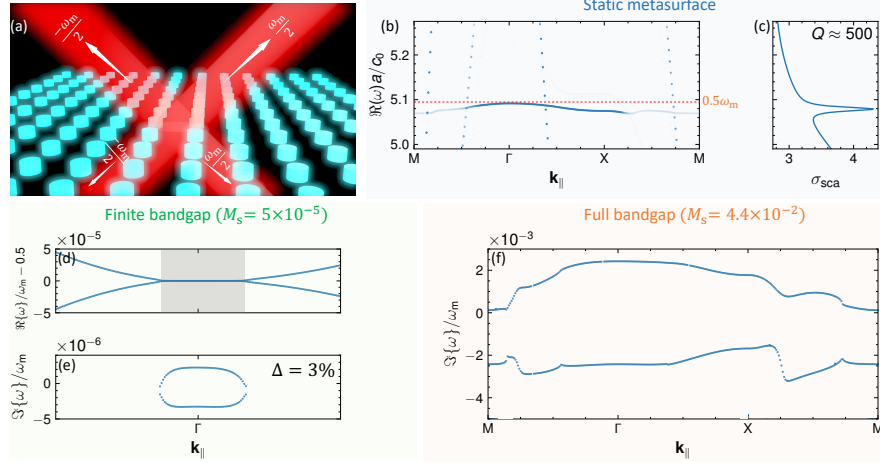


Fig. 4 (a) A metasurface-based PTC made from a square lattice of resonant cylinders. (b) The band structure of the metasurface shown in (a) under static conditions, *i.e.*, $M_s = 0$. Here, all the shown modes lie within the light cone. (c) The rotationally averaged normalized scattering cross section σ_{sca} of the isolated cylinder used in (a) as a function of frequency. The cylinder hosts a qBIC with the quality factor $Q \approx 500$. (d) The band structure of the time-varying metasurface zoomed at the very narrow frequency range centered at $\Re\{\omega\} = \omega_m/2$ near the Γ -point, illustrating a finite bandgap. Here, $M_s = 5 \times 10^{-5}$ is used. (e) The corresponding imaginary part of the frequency inside the bandgap in (d) for a fixed $\Re(\omega)/\omega_m = 0.5$. The displayed mode branch has a symmetry compatible with that of *s*-polarized plane waves. (f) The imaginary part of the frequency inside the full bandgap for a fixed $\Re(\omega)/\omega_m = 0.5$. Here, $M_s = 4.4 \times 10^{-2}$ is used.

the imaginary part of the frequency inside the bandgap in Fig. 4(e). The size of the displayed bandgap is $\Delta = 3\%$. We find that the cylinder-based metasurface requires 16 times lower modulation amplitude than the recently reported Mie resonant structure to achieve a bandgap of the same size (see Supplementary Section S4) [35]. Moreover, it needs orders of magnitude lower M_s compared to homogeneous PTCs to open the same-sized bandgaps [27]. Importantly, the damage threshold of bulk Ge is $M_s \approx 10^{-2}$ [37, 38]. Furthermore, the nanostructuring of Ge reduces its damage threshold by an order of magnitude, *i.e.*, $M_s \approx 10^{-3}$ [40, 41]. Therefore, the modulation amplitude of $M_s = 5 \times 10^{-5}$ is well reachable in Ge using the all-optical Kerr effect without causing thermal damage to the underlying material [37].

Finally, we show that, similar to the previous example, time-varying metasurfaces based on cylinders can also support full bandgaps. To illustrate this, we increase the modulation amplitude of the metasurface to $M_s = 4.4 \times 10^{-2}$, keeping all the other parameters unchanged. We plot the imaginary part of the frequency inside the bandgap in Fig. 4(f). We observe the existence of modes with $\Im(\omega) > 0$ for all $k_{\parallel}$, leading to the demonstration of a full bandgap. Note that the M_s value needed here for opening a full bandgap remains slightly above the damage threshold of Ge. Therefore, it remains a challenge to open full bandgaps using the qBICs of Ge cylinders in the near infrared range of frequencies (as considered here). Nevertheless, our theory predicts that one can utilize the much higher-Q qBICs of low-loss materials available in the

radio frequency range with a much larger real part of the permittivity to open such full bandgaps with a vanishingly small M_s [54].

Conclusion and Outlook

In conclusion, we have shown that by capitalizing on qBICs sustained in meta-atoms, we can significantly expand the momentum bandgaps in the corresponding metasurface-based PTCs. Our theoretical analysis, based on metasurfaces composed of time-varying multilayered spheres, demonstrates that, by suppressing radiative losses in the meta-atoms, it is possible to achieve large momentum bandgaps for arbitrarily small modulation amplitudes, provided that absorption losses are negligible. Furthermore, we explored a viable route to realize a PTC based on metasurfaces that exploit qBICs supported by Ge cylinders. Using these metasurfaces, we demonstrate large momentum bandgaps for modulation amplitudes as small as $M_s = 5 \times 10^{-5}$. This modulation amplitude is many orders of magnitude lower than that required in conventional homogeneous PTCs to open similarly-sized bandgaps [27]. This value of M_s is achievable using the all-optical Kerr effect without causing laser-induced damage to the Ge cylinders [37–41]. We anticipate that our results will pave the way for the first experimental realization of PTCs at optical frequencies.

Supplementary information. See the supplementary material for more information

Acknowledgements. The authors would like to thank Dr. Kirill Koshelev for the fruitful discussions about qBICs. V.A. acknowledges the Finnish Foundation for Technology Promotion, and Research Council of Finland Flagship Programme, Photonics Research and Innovation (PREIN), decision number 346529, Aalto University. X.W. acknowledges the Fundamental Research Funds for the Central Universities, China (project no. 3072024WD2603). P.G. and C.R. are part of the Max Planck School of Photonics, supported by the Bundesministerium für Bildung und Forschung, the Max Planck Society, and the Fraunhofer Society. P.G. and C.R. acknowledge support by the German Research Foundation within the SFB 1173 (project ID no. 258734477). P.G. and J.D.F. acknowledge support from the Karlsruhe School of Optics and Photonics (KSOP). M.N. and C.R. acknowledge support by the KIT through the “Virtual Materials Design” (VIRTMAT) project. J.D.F. and C.R. acknowledge financial support by the Helmholtz Association in the framework of the innovation platform “Solar TAP”.

Methods

Band structure calculation

We employ the T-matrix method to evaluate the band structures of the time-varying metasurfaces [50, 55]. The T-matrix is closely linked to the well-known S-matrix used in scattering theory (See Supplementary Section S5)[56]. According to the method, the incident field coefficients $\mathbf{A}_{\text{inc}}$ and the scattered field coefficients $\mathbf{A}_{\text{sca}}$, computed

in a basis of vector spherical harmonics (VSHs), satisfy

$$\mathbf{A}_{\text{sca}} = \left(\hat{\mathbf{I}} - \hat{\mathbf{T}}_0(\omega) \sum_{\mathbf{R} \neq 0} \hat{\mathbf{C}}^{(3)}(-\mathbf{R}) e^{i\mathbf{k}_{\parallel} \cdot \mathbf{R}} \right)^{-1} \hat{\mathbf{T}}_0(\omega) \mathbf{A}_{\text{inc}}. \quad (1)$$

Here, $\hat{\mathbf{I}}$ is the identity matrix, $\mathbf{R}$ is a lattice vector, and $\hat{\mathbf{C}}^{(3)}$ is the translation matrix of VSHs [50, 57–59]. Furthermore, $\hat{\mathbf{T}}_0$ is the T-matrix of an isolated time-varying scatterer. The T-matrices of isolated time-varying multilayered spheres can be evaluated using the Floquet-Mie theory, while that of isolated time-varying cylinders can be computed using the dynamic extended boundary condition method (EBCM) [60–62]. Moreover, the lattice summation shown in Eq. 1 is calculated using the software `treams` [59]. To allow evaluating the lattice sums in the lower half of the complex frequency plane (i.e., for $\Im(\omega) < 0$), the branch cut of the incomplete gamma function is rotated down from the typical negative real axis by $\pi/2$. The incomplete gamma function with shifted branch cut is evaluated via its relation to the unshifted incomplete gamma function (see Eq. (8.2.9) of reference [63]). Here, we use J as the total number of frequencies that the time-varying structure scatters upon a monochromatic incidence to construct $\hat{\mathbf{C}}^{(3)}$ and $\hat{\mathbf{T}}_0$. Furthermore, these matrices consist of terms up to the maximum multipolar order l_{max} . In particular, for computing the T-matrix of cylinders, using the dynamic EBCM, we use the maximum multipolar order l_{cut} . Here, $l_{\text{cut}} > l_{\text{max}}$ is chosen to ensure sufficient numerical convergence. However, after computing the T-matrix, we retain its terms only up to the order l_{max} . Therefore, the dimension of the matrices $\hat{\mathbf{C}}^{(3)}$ and $\hat{\mathbf{T}}_0$ is $2Jl_{\text{max}}(l_{\text{max}} + 2)$. We provide the used J , l_{max} , and l_{cut} values for generating the results in this article in the Supplementary Section S6.

For calculating the spectral location of an eigenmode, we require $\mathbf{A}_{\text{sca}} \neq 0$ while $\mathbf{A}_{\text{inc}} = 0$ [64]. Using Eq. (1), we find that this requirement is satisfied if the following holds

$$\underbrace{\left| \hat{\mathbf{I}} - \hat{\mathbf{T}}_0(\omega) \sum_{\mathbf{R} \neq 0} \hat{\mathbf{C}}^{(3)}(-\mathbf{R}) e^{i\mathbf{k}_{\parallel} \cdot \mathbf{R}} \right|}_{|\Psi(\mathbf{k}_{\parallel}, \omega)|} = 0. \quad (2)$$

Therefore, those ω and $\mathbf{k}_{\parallel}$ values that satisfy Eq. (2) characterize the spectral location of an eigenmode in the band structure of the metasurface. The band structure is plotted as a scatter plot of the spectral locations of the eigenmodes of the metasurface. Due to radiative loss and the gain/loss introduced by the time modulation, the bands generally possess complex-valued ω . Locating the resonance frequencies of eigenmodes is a well-known computational problem for static photonic resonators. Recently, the adaptive Antoulas–Anderson (AAA) algorithm [65] has been introduced to the field of resonant nanophotonics [66] to locate the eigenfrequencies as poles of some optical response function. The AAA algorithm was shown to be well-suited for locating the resonances of coupled clusters with a finite number of meta-atoms, leveraging the

T-matrix method [67]. Here we generalize this approach to periodic lattices of time-varying meta-atoms. The AAA algorithm is used to find a good rational approximation $r_{\mathbf{k}_{\parallel}}(\omega)$ given by

$$r_{\mathbf{k}_{\parallel}}(\omega) \approx f(\mathbf{k}_{\parallel}, \omega) = \frac{1}{|\Psi(\mathbf{k}_{\parallel}, \omega)|},$$

based on a sufficient number of sample evaluations of $|\Psi(\mathbf{k}_{\parallel}, \omega)|$ for a fixed $\mathbf{k}_{\parallel}$. Then, performing a pole search of the function $r_{\mathbf{k}_{\parallel}}(\omega)$ allows us to find the ω and $\mathbf{k}_{\parallel}$ values that satisfy Eq. (2). The band structures in the static and finite bandgap cases in Figs. 2 and 4 were calculated using only samples at real frequencies, while the full bandgap cases were addressed using frequency samples with fixed $\Re\{\omega\}/\omega_m = 0.5$ and changing imaginary part.

Gapsize calculation

We define the relative size Δ of a momentum bandgap along the M- Γ -X directions of the Brillouin zone as

$$\Delta = \frac{|k_{\parallel}^M + k_{\parallel}^X|}{L}. \quad (3)$$

Here, $\mathbf{k}_{\parallel}^M$ and $\mathbf{k}_{\parallel}^X$ are Bloch wavevectors at which $\Im(\omega) = 0$ for $\Re(\omega) = 0.5\omega_m$ in the M- Γ and Γ -X directions, respectively. Therefore, modes with $\Im(\omega) > 0$ lie between $\mathbf{k}_{\parallel}^M$ and $\mathbf{k}_{\parallel}^X$ for $\Re(\omega) = 0.5\omega_m$. Moreover, L is the total momentum extent of M- Γ -X region, which equals to $(\sqrt{2} + 1)\frac{\pi}{a}$.

References

- [1] Galiffi, E. *et al.* Photonics of time-varying media. *Adv. Photonics* **4**, 014002 (2022). URL <https://doi.org/10.1117/1.AP.4.1.014002>.
- [2] Hayran, Z. & Monticone, F. Capturing broadband light in a compact bound state in the continuum. *ACS Photonics* **8**, 813 (2021). URL <https://pubs.acs.org/doi/10.1021/acsp Photonics.0c01696>.
- [3] Yang, Z. *et al.* Ultrafast q-boosting in semiconductor metasurfaces. *Nanophotonics* **13**, 2173 (2024). URL <https://doi.org/10.1515/nanoph-2023-0718>.
- [4] Schiller, O., Plotnik, Y., Segal, O., Lyubarov, M. & Segev, M. Time-domain bound states in the continuum. *Phys. Rev. Lett.* **133**, 263802 (2024). URL <https://link.aps.org/doi/10.1103/PhysRevLett.133.263802>.
- [5] Engheta, N. Four-dimensional optics using time-varying metamaterials. *Science* **379**, 1190 (2023). URL <https://www.science.org/doi/abs/10.1126/science.adf1094>.
- [6] Engheta, N. *Metamaterials with high degrees of freedom: space, time, and more*, 657 (De Gruyter, Berlin, Boston, 2021). URL <https://doi.org/10.1515/9783110710687-051>.

- [7] Prudêncio, F. R. & Silveirinha, M. G. Synthetic axion response with space-time crystals. *Phys. Rev. Appl.* **19**, 024031 (2023). URL <https://link.aps.org/doi/10.1103/PhysRevApplied.19.024031>.
- [8] Huidobro, P., Silveirinha, M., Galiffi, E. & Pendry, J. Homogenization theory of space-time metamaterials. *Phys. Rev. Appl.* **16**, 014044 (2021). URL <https://link.aps.org/doi/10.1103/PhysRevApplied.16.014044>.
- [9] Galiffi, E. *et al.* Optical coherent perfect absorption and amplification in a time-varying medium. *arXiv preprint arXiv:2410.16426* (2024). URL <https://doi.org/10.48550/arXiv.2410.16426>.
- [10] Koufdis, S., Koutserimpas, T., Monticone, F. & McCall, M. Electromagnetic wave propagation in time-periodic chiral media. *Optical Materials Express* **14**, 3006 (2024). URL <https://doi.org/10.1364/OME.543181>.
- [11] Pacheco-Peña, V. & Engheta, N. Antireflection temporal coatings. *Optica* **7**, 323 (2020). URL <https://doi.org/10.1364/OPTICA.381175>.
- [12] Mirmoosa, M. S., Setälä, T. & Norrman, A. Quantum state engineering and photon statistics at electromagnetic time interfaces. *Phys. Rev. Res.* **7**, 013120 (2025). URL <https://link.aps.org/doi/10.1103/PhysRevResearch.7.013120>.
- [13] Zurita-Sánchez, J. R., Halevi, P. & Cervantes-González, J. C. Reflection and transmission of a wave incident on a slab with a time-periodic dielectric function $\varepsilon(t)$. *Phys. Rev. A* **79**, 053821 (2009). URL <https://link.aps.org/doi/10.1103/PhysRevA.79.053821>.
- [14] Asgari, M. M. *et al.* Theory and applications of photonic time crystals: a tutorial. *Adv. Opt. Photon.* **16**, 958 (2024). URL <https://doi.org/10.1364/AOP.525163>.
- [15] Zhang, S., Dong, J., Li, H., Xu, J. & Shapiro, B. Longitudinal optical phonons in photonic time crystals containing a stationary charge. *Phys. Rev. B* **110**, L100306 (2024). URL <https://link.aps.org/doi/10.1103/PhysRevB.110.L100306>.
- [16] John, S. Strong localization of photons in certain disordered dielectric superlattices. *Phys. Rev. Lett.* **58**, 2486 (1987). URL <https://doi.org/10.1103/PhysRevLett.58.2486>.
- [17] Yablonovitch, E. Inhibited spontaneous emission in solid-state physics and electronics. *Phys. Rev. Lett.* **58**, 2059 (1987). URL <https://doi.org/10.1103/PhysRevLett.58.2059>.
- [18] Lyubarov, M. *et al.* Amplified emission and lasing in photonic time crystals. *Science* **377**, 425 (2022). URL <https://www.science.org/doi/10.1126/science.abo3324>.

- [19] Dikopoltsev, A. *et al.* Light emission by free electrons in photonic time-crystals. *Proc. Natl. Acad. Sci.* **119**, e2119705119 (2022). URL <https://doi.org/10.1073/pnas.2119705119>.
- [20] Carminati, R., Chen, H., Pierrat, R. & Shapiro, B. Universal statistics of waves in a random time-varying medium. *Phys. Rev. Lett.* **127**, 094101 (2021). URL <https://doi.org/10.1103/PhysRevLett.127.094101>.
- [21] Sharabi, Y., Lustig, E. & Segev, M. Disordered photonic time crystals. *Phys. Rev. Lett.* **126**, 163902 (2021). URL <https://doi.org/10.1103/PhysRevLett.126.163902>.
- [22] Apffel, B., Wildeman, S., Eddi, A. & Fort, E. Time localization of energy in disordered time-modulated systems. *arXiv preprint* (2021). URL <https://doi.org/10.48550/arXiv.2102.07583>.
- [23] Reyes-Ayona, J. R. & Halevi, P. Observation of genuine wave vector (k or β) gap in a dynamic transmission line and temporal photonic crystals. *Appl. Phys. Lett.* **107**, 074101 (2015). URL <https://doi.org/10.1063/1.4928659>.
- [24] Park, J. *et al.* Revealing non-hermitian band structure of photonic floquet media. *Sci. Advances* **8**, eabo6220 (2022). URL <https://www.science.org/doi/10.1126/sciadv.abo6220>.
- [25] Wang, X. *et al.* Metasurface-based realization of photonic time crystals. *Sci. Advances* **9**, eadg7541 (2023). URL <http://science.org/doi/10.1126/sciadv.adg7541>.
- [26] Zurita-Sánchez, J. R., Halevi, P. & Cervantes-González, J. C. Reflection and transmission of a wave incident on a slab with a time-periodic dielectric function $\varepsilon(t)$. *Phys. Rev. A* **79**, 053821 (2009). URL <https://doi.org/10.1103/PhysRevA.79.053821>.
- [27] Hayran, Z., Khurgin, J. B. & Monticone, F. $\hbar\omega$ versus $\hbar k$: dispersion and energy constraints on time-varying photonic materials and time crystals. *Opt. Mater. Express* **12**, 3904 (2022). URL <https://doi.org/10.1364/OME.471672>.
- [28] Alam, M. Z., Leon, I. D. & Boyd, R. W. Large optical nonlinearity of indium tin oxide in its epsilon-near-zero region. *Science* **352**, 795 (2016). URL <https://science.org/doi/10.1126/science.aae0330>.
- [29] Bohn, J., Luk, T. S., Horsley, S. & Hendry, E. Spatiotemporal refraction of light in an epsilon-near-zero indium tin oxide layer: frequency shifting effects arising from interfaces. *Optica* **8**, 1532 (2021). URL <https://opg.optica.org/optica/abstract.cfm?URI=optica-8-12-1532>.

- [30] Tirole, R. *et al.* Double-slit time diffraction at optical frequencies. *Nat. Physics* **19**, 999 (2023). URL <https://doi.org/10.1038/s41567-023-01993-w>.
- [31] Zhou, Y. *et al.* Broadband frequency translation through time refraction in an epsilon-near-zero material. *Nat. Commun.* **11**, 2180 (2020). URL <https://doi.org/10.1038/s41467-020-15682-2>.
- [32] Lustig, E. *et al.* Time-refraction optics with single cycle modulation. *Nanophotonics* **12**, 2221 (2023). URL <https://doi.org/10.1515/nanoph-2023-0126>.
- [33] Khurgin, J. B. Photonic time crystals and parametric amplification: Similarity and distinction. *ACS Photonics* **11**, 2150 (2024). URL <https://pubs.acs.org/doi/10.1021/acsphotonics.4c00607>.
- [34] Borchers, B., Brée, C., Birkholz, S., Demircan, A. & Steinmeyer, G. Saturation of the all-optical kerr effect in solids. *Opt. Lett.* **37**, 1541 (2012). URL <https://doi.org/10.1364/OL.37.001541>.
- [35] Wang, X. *et al.* Expanding momentum bandgaps in photonic time crystals through resonances. *Nat. Photonics* **19**, 149 (2025). URL <https://doi.org/10.1038/s41566-024-01563-3>.
- [36] Cowan, B. *Optical damage threshold of silicon for ultrafast infrared pulses* Vol. 877 (AIP Conf. Proc., 2006). URL <https://www.slac.stanford.edu/pubs/slacpubs/12000/slac-pub-12089.pdf>.
- [37] Polyanskiy, M. N., Pogorelsky, I. V., Babzien, M., Vodopyanov, K. L. & Palmer, M. A. Nonlinear refraction and absorption properties of optical materials for high-peak-power long-wave-infrared lasers. *Opt. Mater. Express* **14**, 696 (2024). URL <https://opg.optica.org/ome/abstract.cfm?URI=ome-14-3-696>.
- [38] Agustsson, R. *et al.* Measuring single-shot, picosecond optical damage threshold in ge, si, and sapphire with a 5.1 μ m laser. *Opt. Mater. Express* **5**, 2835–2842 (2015). URL <https://opg.optica.org/ome/abstract.cfm?URI=ome-5-12-2835>.
- [39] Hon, N. K., Soref, R. & Jalali, B. The third-order nonlinear optical coefficients of si, ge, and Si_{1-x}Ge_x in the midwave and longwave infrared. *J. Appl. Phys.* **110**, 011301 (2011). URL <https://doi.org/10.1063/1.3592270>.
- [40] Kern, C. *et al.* Comparison of femtosecond laser-induced damage on unstructured vs. nano-structured au-targets. *Appl. Phys. A* **104**, 15 (2011). URL <https://doi.org/10.1007/s00339-011-6449-2>.
- [41] Grinblat, G., Li, Y., Nielsen, M. P., Oulton, R. F. & Maier, S. A. Efficient third harmonic generation and nonlinear subwavelength imaging at a higher-order anapole mode in a single germanium nanodisk. *ACS Nano* **11**, 953–960 (2017). URL <https://pubs.acs.org/doi/10.1021/acsnano.6b07568>.

- [42] Rybin, M. V. *et al.* High- q supercavity modes in subwavelength dielectric resonators. *Phys. Rev. Lett.* **119**, 243901 (2017). URL <https://link.aps.org/doi/10.1103/PhysRevLett.119.243901>.
- [43] Koshelev, K. *et al.* Subwavelength dielectric resonators for nonlinear nanophotonics. *Science* **367**, 288 (2020). URL <http://science.org/doi/10.1126/science.aaz3985>.
- [44] Bogdanov, A. A. *et al.* Bound states in the continuum and Fano resonances in the strong mode coupling regime. *Adv. Photonics* **1**, 016001 (2019). URL <https://doi.org/10.1117/1.AP.1.1.016001>.
- [45] Monticone, F. & Alù, A. Embedded photonic eigenvalues in 3d nanostructures. *Phys. Rev. Lett.* **112**, 213903 (2014). URL <https://link.aps.org/doi/10.1103/PhysRevLett.112.213903>.
- [46] Caspani, L. *et al.* Enhanced nonlinear refractive index in ϵ -near-zero materials. *Phys. Rev. Lett.* **116**, 233901 (2016). URL <https://link.aps.org/doi/10.1103/PhysRevLett.116.233901>.
- [47] Gao, L., Lemarchand, F. & Lequime, M. Exploitation of multiple incidences spectrometric measurements for thin film reverse engineering. *Opt. Express* **20**, 15734 (2012). URL <https://doi.org/10.1364/OE.20.015734>.
- [48] Silveirinha, M. G. Trapping light in open plasmonic nanostructures. *Phys. Rev. A* **89**, 023813 (2014). URL <https://link.aps.org/doi/10.1103/PhysRevA.89.023813>.
- [49] Waterman, P. Matrix formulation of electromagnetic scattering. *Proc. of the IEEE* **53**, 805 (1965). URL <https://ieeexplore.ieee.org/document/1445988/>.
- [50] Garg, P. *et al.* Modeling four-dimensional metamaterials: a t-matrix approach to describe time-varying metasurfaces. *Opt. Express* **30**, 45832–45847 (2022). URL <https://opg.optica.org/oe/abstract.cfm?URI=oe-30-25-45832>.
- [51] Kelly, P., Zhang, W., Liu, M. & Kuznetsova, L. Engineering the structural, plasmonic, and optical properties of multilayered aluminum-doped zinc oxide metamaterial grown by pulsed laser deposition. *Appl. Opt.* **58**, 5681 (2019). URL <https://doi.org/10.1364/AO.58.005681>.
- [52] Nunley, T. N. *et al.* Optical constants of germanium and thermally grown germanium dioxide from 0.5 to 6.6eV via a multisample ellipsometry investigation. *J. Vac. Sci. Technol. B* **34**, 061205 (2016). URL <https://doi.org/10.1116/1.4963075>.
- [53] Gantounis, G. Plasmon modes of axisymmetric metallic nanoparticles: A group theory analysis. *J. Phys. Chem. C* **113**, 21560 (2009). URL <https://pubs.acs.org/doi/10.1021/jp908019s>.

- [54] Odit, M. *et al.* Observation of supercavity modes in subwavelength dielectric resonators. *Adv. Mater.* **33**, 2003804 (2021). URL <https://doi.org/10.1002/adma.202003804>.
- [55] Panagiotidis, E., Almpanis, E., Papanikolaou, N. & Stefanou, N. Optical transitions and nonreciprocity in spatio-temporally periodic layers of spherical particles. *Adv. Opt. Mater.* **11**, 2202812 (2023). URL <https://onlinelibrary.wiley.com/doi/abs/10.1002/adom.202202812>.
- [56] Waterman, P. C. Matrix theory of elastic wave scattering. *J. Acoust. Soc. Am.* **60**, 567 (1976). URL <https://doi.org/10.1121/1.381130>.
- [57] Mackowski, D. W. Exact solution for the scattering and absorption properties of sphere clusters on a plane surface. *J. Quant. Spectrosc. Radiat. Transfer* **109**, 770 (2008). URL <https://doi.org/10.1016/j.jqsrt.2007.08.024>.
- [58] Beutel, D., Groner, A., Rockstuhl, C. & Fernandez-Corbaton, I. Efficient simulation of biperiodic, layered structures based on the t-matrix method. *J. Opt. Soc. Am. B* **38**, 1782 (2021). URL <https://doi.org/10.1364/JOSAB.419645>.
- [59] Beutel, D., Fernandez-Corbaton, I. & Rockstuhl, C. treams – a t-matrix-based scattering code for nanophotonics. *Comput. Phys. Commun.* **297**, 109076 (2024). URL <https://www.sciencedirect.com/science/article/pii/S0010465523004216>.
- [60] Lamprianidis, A. G. *Generalized Transition Matrix Methods for the Analysis of Linear Nanophotonic Systems*. Ph.D. thesis, Karlsruher Institut für Technologie (KIT) (2023). URL <http://publikationen.bibliothek.kit.edu/1000171446>.
- [61] Ptitcyn, G. *et al.* Floquet–mie theory for time-varying dispersive spheres. *Laser Photonics Rev.* **17**, 2100683 (2023). URL <https://doi.org/10.1002/lpor.202100683>.
- [62] Stefanou, N. *et al.* Light scattering by a periodically time-modulated object of arbitrary shape: the extended boundary condition method. *J. Opt. Soc. Am. B* **40**, 2842 (2023). URL <https://opg.optica.org/josab/abstract.cfm?URI=josab-40-11-2842>.
- [63] *NIST Digital Library of Mathematical Functions*. <https://dlmf.nist.gov/>, Release 1.2.4 of 2025-03-15. URL <https://dlmf.nist.gov/>. F. W. J. Olver, A. B. Olde Daalhuis, D. W. Lozier, B. I. Schneider, R. F. Boisvert, C. W. Clark, B. R. Miller, B. V. Saunders, H. S. Cohl, and M. A. McClain, eds.
- [64] Rahimzadegan, A. *et al.* A comprehensive multipolar theory for periodic meta-surfaces. *Adv. Opt. Mater.* **10**, 2102059 (2022). URL <https://doi.org/10.1002/adom.202102059>.

- [65] Nakatsukasa, Y., Sète, O. & Trefethen, L. N. The AAA Algorithm for Rational Approximation. *SIAM J. Sci. Comput.* **40**, A1494 (2018). URL <https://epubs.siam.org/doi/10.1137/16M1106122>. Publisher: Society for Industrial and Applied Mathematics.
- [66] Betz, F., Hammerschmidt, M., Zschiedrich, L., Burger, S. & Binkowski, F. Efficient Rational Approximation of Optical Response Functions with the AAA Algorithm. *Laser Photonics Rev.* **18**, 2400584 (2024). URL <https://onlinelibrary.wiley.com/doi/10.1002/lpor.202400584>.
- [67] Fischbach, J. D. *et al.* A Framework to Compute Resonances Arising from Multiple Scattering. *Adv. Theor. Simul.* **8**, 2400989 (2025). URL <https://onlinelibrary.wiley.com/doi/abs/10.1002/adts.202400989>.
- [68] Ustimenko, N., Rockstuhl, C. & Evlyukhin, A. B. Resonances in finite-size all-dielectric metasurfaces for light trapping and propagation control. *Phys. Rev. B* **109**, 115436 (2024). URL <https://link.aps.org/doi/10.1103/PhysRevB.109.115436>.
- [69] Waterman, P. T-matrix methods in acoustic scattering. *J. Acoust. Soc. Am.* **125**, 42 (2009). URL <https://doi.org/10.1121/1.3035839>.

Supplementary Material

S1 Drude-Lorentz model

We use the Drude-Lorentz model to consider dispersion in time-varying media. The polarization density of the media can be written as $\mathbf{P}(\mathbf{r}, t) = \varepsilon_0(\varepsilon_\infty - 1)\mathbf{E}(\mathbf{r}, t) + \mathbf{P}_{\text{dyn}}(\mathbf{r}, t)$. Here, ε_0 is the permittivity of a vacuum, ε_∞ is the background permittivity contribution, and $\mathbf{E}(\mathbf{r}, t)$ is the electric field. The polarization density contribution $\mathbf{P}_{\text{dyn}}(\mathbf{r}, t)$ satisfies the differential equation [61]

$$\left[\frac{\partial^2}{\partial t^2} + \gamma \frac{\partial}{\partial t} + \omega_0^2 \right] \mathbf{P}_{\text{dyn}}(\mathbf{r}, t) = \frac{N(t)e^2}{m_e} \mathbf{E}(\mathbf{r}, t). \quad (\text{S1})$$

Here, e and m_e correspond to the charge and mass of an electron, respectively. Furthermore, γ is the damping factor, and ω_0 is the oscillator resonance frequency. The time modulation is considered in the model by assuming a time-periodic electron density $N(t)$. In this work, we use $N(t) = N_0(1 + M_s \cos(\omega_m t))$. The initial phase ϕ of $N(t)$ is set to zero without loss of generality, as any nonzero phase merely amounts to the time-translation of the function $N(t)$ by $\Delta t = \phi/\omega_m$. This time translation does not affect the band structures of the PTCs.

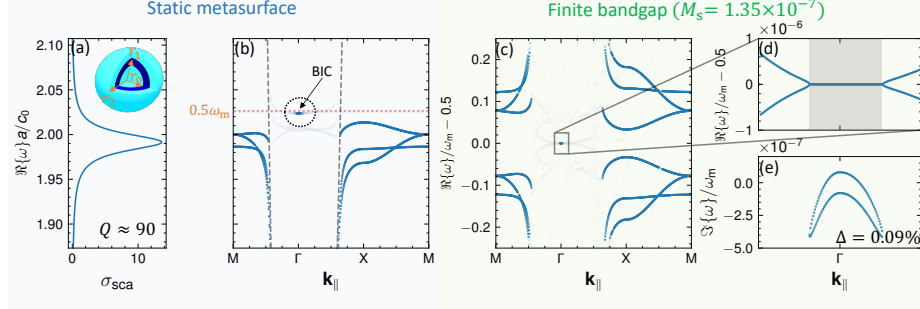


Fig. S1 (a) The normalized scattering cross section σ_{sca} of the static multilayered sphere (see inset) as a function of the frequency ω . The sphere consists of a core and two layers. The core and the outer layer of the sphere are made from AZO. The middle layer is made from silica. Note that material losses are neglected here to clearly illustrate the effects. (b) The band structure of the metasurface formed by a square lattice of multilayered spheres under static conditions, *i.e.*, $M_s = 0$ in the spectral domain of interest. The black circle marks the spectral location of the true lattice BIC. (c) The band structure of the time-varying metasurface. An extremely low modulation amplitude $M_s = 1.35 \times 10^{-7}$ is used. (d) The further zoomed band structure in the highlighted region of (c) illustrates a finite momentum bandgap. The displayed mode has a symmetry compatible with that of *p*-polarized plane waves. (e) The corresponding imaginary part of the frequency inside the bandgap in (d) for a fixed $\Re(\omega)/\omega_m = 0.5$.

S2 Comparison of the meta-atom and lattice qBICs

In this section, we show the influence of the qBICs of the metasurfaces formed by lattice interactions on momentum bandgap sizes. We also compare the results obtained thanks to these lattice qBICs with those of the meta-atom qBICs used in the main text. We consider a metasurface made from a square lattice of lossless multilayered spheres (see the inset of Fig. S1(a)). The core and the outer layer of the spheres are made from lossless AZO. The middle layer is made from lossless silica. To clearly illustrate the impact of the lattice qBICs, the isolated multilayered sphere is designed such that it sustains a relatively low-Q resonance. We use, $r_2 = 200$ nm, $r_0 = 66.67$ nm, and $\eta = 0.9$.

The normalized scattering cross section σ_{sca} of the static multilayered sphere is shown in Fig. S1(a). Here, we observe that the multilayered sphere hosts an electric dipolar Mie resonance with $Q \approx 90$. This Q-factor is three orders of magnitude lower than that of the qBIC resonance used in Fig. 2(d). Next, we plot the band structure of the corresponding static metasurface in Fig. S1(b). The metasurface is formed by a square lattice of multilayered spheres with lattice period $a = 3r_2$. We observe the existence of flat bands in Fig. S1(b). However, due to the relatively low Q-factors of the multilayered spheres, the bands are significantly less flat than those shown in Fig. 2(c).

This metasurface also supports a true symmetry-protected lattice BIC at the Γ -point. Such a BIC is formed due to the lattice interactions in the metasurface [68]. We note that, despite the presence of a lattice BIC, the flatness of the bands of the metasurface is controlled by the Q-factors of the individual meta-atoms.

To study the impact of the lattice qBICs on the sizes of the momentum bandgaps, we turn on the time modulation of the meta-atoms. We choose $0.5\omega_m$ such that its

numerical value is extremely close to the spectral location of the lattice BIC (see the orange dotted line Fig. S1(b)). The numerical value of ω_m is $2\pi \times 161$ THz. Furthermore, $M_s = 1.35 \times 10^{-7}$. Here, M_s is chosen identical to that used in Fig. 2(e). The band structure of the corresponding time-varying metasurface is shown in Fig. S1(c). Here, we note that due to the lower flatness of the bands, upon time modulation, a smaller number of modes interact with each other as compared to those in Fig. 2(e) (see the gray box). Therefore, a relatively smaller gapsize opens despite the same modulation amplitude M_s . We show the corresponding momentum bandgap in Fig. S1(d). The imaginary part of the frequency $\Im(\omega)$ inside the bandgap is shown in Fig. S1(e). Here, the gapsize is $\Delta = 0.09\%$. This gapsize is approximately 30 times lower than that shown in Fig. 2(f)–(g).

Remarkably, the lattice qBICs of the metasurfaces allow for the existence of narrow momentum bandgaps with extremely low M_s . However, it is the qBICs of the individual meta-atoms that assist in expanding the momentum bandgap sizes while simultaneously lowering the modulation amplitude requirements. Physically, the strong localization of the fields inside the individual meta-atoms reduces the spatial dispersion in the lattice while simultaneously allowing for a large interaction time of light with time-varying matter.

S3 qBICs of germanium cylinders

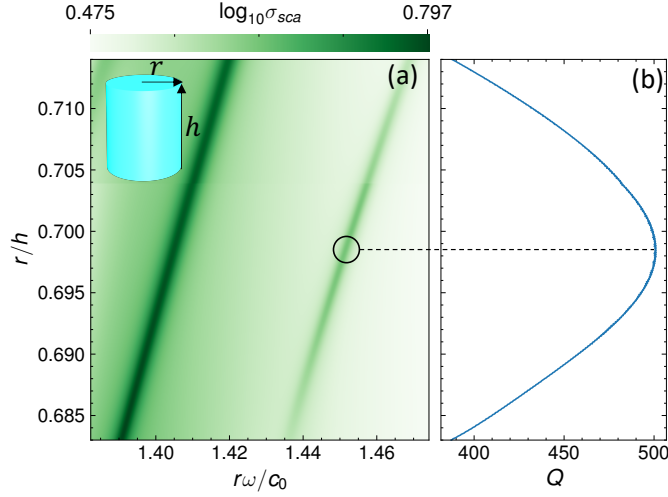


Fig. S2 (a) The rotationally averaged normalized scattering cross section σ_{sca} of the germanium cylinder (see inset) as a function of the aspect ratio r/h and frequency ω . We change the height h of the cylinder to vary the aspect ratio. The radius r of the cylinder is fixed as $r = 465$ nm. The modes in the right resonance branch represent qBICs formed by the Friedrich-Wintgen mechanism. (b) The quality factor Q of the qBICs in (a) in the right branch as a function of the aspect ratio r/h .

In this section, we discuss the qBIC formation mechanism of finite Ge cylinders. This mechanism was first discussed in [42]. We plot the rotationally averaged normalized scattering cross section σ_{sca} of a Ge cylinder as a function of its aspect ratio r/h and frequency ω in Fig. S2(a). Here, we change h to vary the aspect ratio. The radius r of the cylinder is fixed to $r = 465$ nm (as in the main text). In Fig. S2(a), we observe the emergence of two resonance branches. These branches represent hybrid modes formed by the interaction of Mie-type modes with indices $m = 0, n = 1$ and neighboring Mie-type modes with indices $m = 0, n = 2$. In particular, the far fields of the hybrid modes in the right branch are formed by a destructive interference of the far fields of the parent Mie modes. Due to the destructive interference, these modes possess relatively high Q-factors. In fact, these modes are qBICs formed by the Friedrich-Wintgen mechanism.

The Q-factors of these qBIC modes in the right branch are plotted in Fig. S2(b). Here, we observe that the maximum Q-factor is achieved for the aspect ratio $r/h \approx 0.698$. In the main text, we use Ge cylinders with $r/h \approx 0.698$ that host a qBIC with $Q \approx 500$. Note that in the limit of vanishing material losses and large real part of the relative permittivity ε of the cylinders, the Q-factor of the qBIC diverges to infinity, forming a true BIC.

S4 PTCs based on metasurfaces made from homogeneous germanium spheres

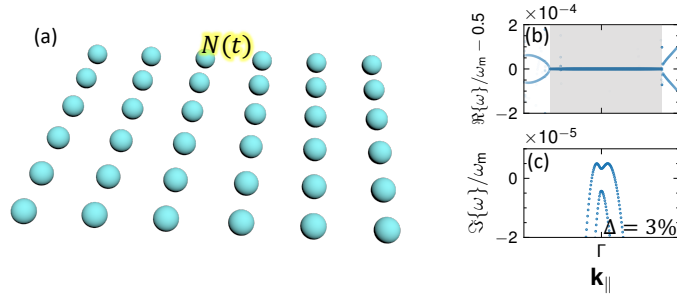


Fig. S3 (a) A metasurface-based PTC made from a square lattice of homogeneous spheres supporting the magnetic dipolar Mie resonance. (b) The band structure of the time-varying metasurface made from Mie resonant homogeneous spheres illustrating a finite bandgap with gapsize $\Delta = 3\%$. The gapsize is identical to that obtained for qBIC resonant cylinder metasurfaces in Fig. 4(d). Here, $M_s = 8.3 \times 10^{-4}$ is used, which is 16 times higher than that needed by the cylinder metasurface to open the same-sized bandgap. (c) The corresponding imaginary part of the frequency inside the bandgap shown in (b) for a fixed $\text{Re}(\omega)/\omega_m = 0.5$. The displayed mode branch has a symmetry compatible with that of p -polarized plane waves. We use the lattice period $a = 3.5R_0$ for the computations. Here, R_0 is the radius of the homogeneous sphere.

A recent study showed that using metasurfaces made from low-loss homogeneous spheres reduces the required modulation amplitude to attain large momentum

bandgaps [35]. The study exploits magnetic dipolar resonances of the spheres. We compare our results shown in Figs. 4(d)-(e) for the qBIC resonant cylinder metasurface to those reported in [35].

For a fair comparison, we consider a metasurface composed of a square lattice of homogeneous Ge spheres (see Fig. S3(a)). The radius R_0 of the spheres is optimized such that the flat bands of the metasurface due to the magnetic dipolar resonance occur at the frequency $\omega = 2\pi \times 150$ THz. Here, the frequency of the flat bands of the spheres is identical to that of the cylinders. This choice ensures that the homogeneous sphere metasurface experiences a similar material dispersion as that of the cylinder metasurface at the spectral location of the momentum bandgap. Therefore, we use $\omega_m = 2\pi \times 300$ THz. The optimized radius of the spheres is $R_0 = 245$ nm. Furthermore, the quality factor of the magnetic dipolar resonance of the optimized homogeneous static sphere is $Q \approx 20$.

We plot the band structure of the time-varying metasurface in Fig. S3(b). Moreover, we also show the imaginary part of the frequency inside the momentum bandgap (see Fig. S3(c)). We find that to open a bandgap of size $\Delta = 3\%$, a modulation amplitude of $M_s = 8.3 \times 10^{-4}$ is required. This value of M_s is too close to the damage threshold of structured Ge [37–41]. On the other hand, to open the same-sized bandgap for the metasurface with cylinders, $M_s = 5 \times 10^{-5}$ is needed, which is more than an order of magnitude below the damage threshold of structured Ge. In fact, the qBIC resonant cylinder metasurface requires 16 times lower modulation amplitude as compared to the homogeneous sphere metasurface to open the same-sized bandgap. Furthermore, at $M_s = 5 \times 10^{-5}$, no bandgap is observed in the homogeneous sphere metasurfaces due to the relatively high radiative losses caused by low-Q magnetic dipolar Mie resonances.

S5 Link between the T-matrix and S-matrix formalism

The S-matrix $\hat{\mathbf{S}}_0$ of an isolated time-varying scatterer can be calculated from its T-matrix $\hat{\mathbf{T}}_0$ as [69]

$$\hat{\mathbf{S}}_0 = (\hat{\mathbf{I}} + 2\hat{\mathbf{T}}_0) . \quad (\text{S2})$$

Furthermore, for a metasurface made from such scatterers with the effective T-matrix $\hat{\mathbf{T}}_{\text{eff}}$, the corresponding S-matrix $\hat{\mathbf{S}}$ is given by

$$\hat{\mathbf{S}} = \hat{\mathbf{I}} + \hat{\mathbf{V}} \hat{\mathbf{T}}_{\text{eff}} \hat{\mathbf{U}} . \quad (\text{S3})$$

Here, $\hat{\mathbf{V}}$ is the coordinate transformation matrix of the scattered field from the spherical wave to plane wave basis, and $\hat{\mathbf{U}}$ is the coordinate transformation matrix of the incident field from the plane wave to the spherical wave basis [50].

S6 Details of the convergence parameters used during simulations

For performing the simulations throughout the article, we used various convergence parameters. These parameters are the maximum multipolar order $l_{\max}$, and the total number of scattered frequencies J . Here, we list the numerical values of these parameters.

Figures 2(a),(c),(d): $l_{\max} = 4$, and $J = 1$.

Figures 2(e)–(h): $l_{\max} = 4$, and $J = 7$.

Figures 3(a)–(b): $l_{\max} = 4$, and $J = 7$.

Figures 4(b)–(c): $l_{\text{cut}} = 7$, $l_{\max} = 6$, and $J = 1$.

Figures 4(d)–(f): $l_{\text{cut}} = 7$, $l_{\max} = 6$, and $J = 7$.

Figures S1(a)–(b): $l_{\max} = 4$, and $J = 1$.

Figures S1(c)–(e): $l_{\max} = 4$, and $J = 7$.

Figures S2(a)–(b): $l_{\text{cut}} = 7$, $l_{\max} = 6$, and $J = 1$.

Figures S3(b)–(c): $l_{\max} = 4$, and $J = 7$.