

# Comment on “Magnetism and structure of $\text{Li}_x\text{CoO}_2$ and comparison to $\text{Na}_x\text{CoO}_2$ ”

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In the article [Phys. Rev. B **77**, 075119 (2008)], effective magnetic moments  $p_{\text{eff}}$  per Co site have been determined from magnetic susceptibility measurements for a series of  $\text{Li}_x\text{CoO}_2$  samples. The nominal Li content  $x$  (ranging theoretically from 1 to 0) determines the concentration of  $\text{Co}^{3+}$  ( $x$ ) and  $\text{Co}^{4+}$  ( $1 - x$ ) that occupy the Co sites. Assuming that the  $\text{Co}^{3+}$  species is diamagnetic, the effective magnetic moment  $p_{\text{eff}}$  per  $\text{Co}^{4+}$  ion has been calculated from the experimentally determined effective magnetic moment  $p_{\text{eff}}$  per Co site and the nominal  $\text{Co}^{4+}$  concentration in the cited publication [Phys. Rev. B **77**, 075119 (2008)] by assuming a purely linear relation between the effective magnetic moments and the concentrations. From the viewpoint of this comment’s author, this linear approach is not valid and the linear effective magnetic moments  $p_{\text{eff}}$  utilized in the formulas in the cited article should be replaced by their squared values  $p_{\text{eff}}^2$ . This comment provides the correct formulas for the calculations aimed to be performed in [Phys. Rev. B **77**, 075119 (2008)] and recalculates the published values. Since for low  $\text{Co}^{4+}$  concentrations a high-spin species is discussed to exist in  $\text{Li}_x\text{CoO}_2$  in numerous publications, correct handling of effective magnetic moments is of general interest in this discipline of research.

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Firstly, the part from Ref. [1] that gave rise to write this comment will shortly be recapitulated: For a series of  $\text{Li}_x\text{CoO}_2$  samples the effective magnetic moment  $p_{\text{eff}}$  per Co site has been obtained from Curie-Weiss fits to inverse magnetic susceptibility data. These effective moments per Co site have been used (i) for two samples with very small  $1 - x$  (means small nominal  $\text{Co}^{4+}$  concentration) to determine the concentration of  $\text{Co}^{4+}$  that has been assumed to be present as a high-spin species with an effective magnetic moment of  $5.9\mu_B$  and (ii) in general to determine the effective magnetic moment for the  $\text{Co}^{4+}$  species that was assumed to be present at a concentration according to the nominal delithiation  $1 - x$  for all samples. The results have been presented in Table I in Ref. [1] and the most important values are summarized in Table I of this comment. For the fully lithiated sample ( $x = 1.00$ ), the nominal oxidation state of Co was assumed to be +3 (assuming that oxygen is present as  $\text{O}^{2-}$  and Li as  $\text{Li}^+$ ). For each  $\text{Li}^+$  that was extracted from the structure, it was assumed that one  $\text{Co}^{3+}$  is oxidized to  $\text{Co}^{4+}$ . Therefore, the Li content  $x$  equals the nominal concentration of  $\text{Co}^{3+}$  and correspondingly  $1 - x$  equals the nominal concentration of  $\text{Co}^{4+}$ .  $\text{Co}^{3+}$  was assumed to be present in a diamagnetic state (zero effective magnetic moment) while  $\text{Co}^{4+}$  was supposed to possess a magnetic moment that is either suggested to be in a high-spin state for very small amounts of delithiation

(effective magnetic moment close to  $5.9\mu_B$ ), but changes to a low-spin configuration for higher degrees of delithiation.

The authors of Ref. [1] then falsely used the following linear equation to determine the  $\text{Co}^{4+}$  concentration ( $1 - x'$ ) from the measured effective moment  $p_{\text{eff}[\text{Co}]}$  per Co assuming that each  $\text{Co}^{4+}$  would be present in its high-spin state with  $p_{\text{eff}[\text{Co}^{4+}]} = 5.9\mu_B$  for the first two samples:

$$x'p_{\text{eff}[\text{Co}^{3+}]} + (1 - x')p_{\text{eff}[\text{Co}^{4+}]} = p_{\text{eff}[\text{Co}]}, \quad (1)$$

$$(1 - x') = \frac{p_{\text{eff}[\text{Co}]}}{5.9\mu_B}, \quad (2)$$

where  $x'$  represents the Lithium content calculated by this method in contrast to the nominal Li content  $x$ . The values of the so calculated Li contents  $x'$  for the first two samples are listed in round brackets in the first column of Table I in Ref. [1] and are repeated for convenience again in the first column of Table I of this comment.

Further, for all samples except for the first one with  $x = 1.00$ , the effective magnetic moments  $p_{\text{eff}[\text{Co}^{4+}]}$  per  $\text{Co}^{4+}$  have been calculated according to the following inappropriate linear equation:

$$xp_{\text{eff}[\text{Co}^{3+}]} + (1 - x)p_{\text{eff}[\text{Co}^{4+}]} = p_{\text{eff}[\text{Co}]}, \quad (3)$$

$$p_{\text{eff}[\text{Co}^{4+}]} = \frac{p_{\text{eff}[\text{Co}]}}{(1 - x)}, \quad (4)$$

where the nominal  $\text{Co}^{4+}$  concentration has been calculated as  $1 - x$  from the Li content  $x$ . The values of the so calculated values of  $p_{\text{eff}[\text{Co}^{4+}]}$  per  $\text{Co}^{4+}$  are listed in the sixth column of Table I in Ref. [1] and are repeated for convenience again in the third column of Table I of this comment.

This comment questions the mathematical procedure how (i) the concentration of a  $\text{Co}^{4+}$  species with assigned effective magnetic moment of  $5.9\mu_B$  was calculated from the

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TABLE I. Nominal Li content  $x$ , effective magnetic moment  $p_{\text{eff}}$  per Co site, effective magnetic moment  $p_{\text{eff}}$  per  $\text{Co}^{4+}$ , and calculated Li content  $x'$  given in round brackets as published in Ref. [1] (first three columns). All values that have been recalculated within this comment are marked with an asterisk sign “\*”: the two recalculated Li contents  $x'$  in squared brackets and the recalculated effective magnetic moments  $p_{\text{eff}}^*$  per  $\text{Co}^{4+}$ . See text for details.

| Li content $x$         | $p_{\text{eff}}$ ( $\mu_B/\text{Co}$ ) | $p_{\text{eff}}$ ( $\mu_B/\text{Co}^{4+}$ ) | $p_{\text{eff}}^*$ ( $\mu_B/\text{Co}^{4+}$ ) |
|------------------------|----------------------------------------|---------------------------------------------|-----------------------------------------------|
| 1.00 (0.985)[0.9998*]  | 0.09                                   | (5.9)                                       | [5.9]                                         |
| 0.97 (0.961) [0.9985*] | 0.23                                   | 7.67(5.9)                                   | 1.33*[5.9]                                    |
| 0.78                   | 0.38                                   | 1.72                                        | 0.81*                                         |
| 0.72                   | 0.43                                   | 1.54                                        | 0.81*                                         |
| 0.70                   | 0.49                                   | 1.63                                        | 0.89*                                         |
| 0.62                   | 0.48                                   | 1.26                                        | 0.78*                                         |
| 0.59                   | 0.37                                   | 0.90                                        | 0.58*                                         |
| 0.58                   | 0.36                                   | 0.86                                        | 0.56*                                         |
| 0.51                   | 0.37                                   | 0.76                                        | 0.53*                                         |

determined effective magnetic moment per Co in a linear approach [Eq. (2)] and (ii) how the effective magnetic moment of the  $\text{Co}^{4+}$  species has been calculated from the effective magnetic moment per Co and the  $\text{Co}^{4+}$  concentration  $(1 - x)$  also in a purely linear approach [Eq. (4)]. From the viewpoint of this comment’s author, a “quadratic” approach is necessary to treat the effective magnetic moments correctly. Especially for  $\text{Li}_x\text{CoO}_2$  with the hypothesis of a high-spin  $\text{Co}^{4+}$  species for the very low delithiation states, a correct effective moment assignment is mandatory. For the calculations in this comment the Gaussian and cgs emu unit system [2] has been used and the following definition of the Curie constant can be found in standard text books or journal publications about solid state physics and magnetism [3–6].

When a number of  $N_A$  (Avogadro’s number) Co lattice points  $i$  are all occupied by identical Co ions that are at a certain oxidation state with a corresponding total spin  $S$ , the molar Curie constant  $C_{\text{mol[Co]}}$  is calculated as

$$C_{\text{mol[Co]}} = \frac{\mu_B^2 g_e^2 S(S+1) N_A}{3k_B} \quad (5)$$

$$= \frac{p_{\text{eff[Co]}}^2 \mu_B^2 N_A}{3k_B}, \quad (6)$$

with the effective magnetic moment  $p_{\text{eff[Co]}} = g_e \sqrt{S(S+1)} \mu_B$  for the single Co site with spin  $S$ , the  $g$ -factor  $g_e \approx 2$  for a free electron, the Boltzmann constant  $k_B$ , and the Bohr magneton  $\mu_B$ . In the case that the Co lattice points  $i$  are not all occupied by identical but by individual effective magnetic moments  $p_{\text{eff[Co]}}$ , the expression for  $p_{\text{eff[Co]}}^2$  from Eq. (6) becomes

$$p_{\text{eff[Co]}}^2 = \frac{1}{N_A} \sum_{i=1}^{N_A} p_{\text{eff[Co]}}^2. \quad (7)$$

The special case that only two different species Co(1) and Co(2) would occupy the Co sites with the concentrations  $c_1$  and  $c_2$  (with  $c_1 + c_2 = 1$ ) can then be written as

$$p_{\text{eff[Co]}}^2 = \frac{1}{N_A} (c_1 N_A p_{\text{eff[Co(1)]}}^2 + c_2 N_A p_{\text{eff[Co(2)]}}^2) \quad (8)$$

$$= c_1 p_{\text{eff[Co(1)]}}^2 + c_2 p_{\text{eff[Co(2)]}}^2, \quad (9)$$

To further adopt the equation to the scenario considered in Ref. [1],  $\text{Co}^{3+}$  represents the first Co species that is diamagnetic with spin  $S = 0$  and  $p_{\text{eff[Co3+]}} = 0$ , and  $\text{Co}^{4+}$  represents the second Co species that has an effective magnetic moment  $p_{\text{eff[Co4+]}} > 0$ . As outlined above, the  $\text{Co}^{3+}$  species is present at a concentration equal to the Li content  $x$  and the  $\text{Co}^{4+}$  species correspondingly at a concentration  $1 - x$  with the sum always equal to one.

$$p_{\text{eff[Co]}}^2 = x p_{\text{eff[Co3+]}}^2 + (1 - x) p_{\text{eff[Co4+]}}^2. \quad (10)$$

Since  $p_{\text{eff[Co3+]}} = 0$  the expression can further be simplified to

$$p_{\text{eff[Co]}}^2 = (1 - x) p_{\text{eff[Co4+]}}^2. \quad (11)$$

Equation (11) is the fundamental relation that can be used to recalculate the values from Ref. [1] in the following.

In analogy to what has been calculated in Ref. [1] by applying Eq. (2) (see above), this comment suggest instead to calculate the  $\text{Co}^{4+}$  concentration  $(1 - x')$  by the measured effective magnetic moment per Co by the following equation derived from Eq. (11) by setting the effective paramagnetic moment of the  $\text{Co}^{4+}$  in high-spin again to  $p_{\text{eff[Co4+]}} = 5.9 \mu_B$ :

$$(1 - x') = \frac{p_{\text{eff[Co]}}^2}{p_{\text{eff[Co4+]}}^2} \quad (12)$$

$$= \frac{p_{\text{eff[Co]}}^2}{(5.9 \mu_B)^2}. \quad (13)$$

The correct values obtained for the calculated  $\text{Co}^{4+}$  concentrations  $(1 - x')$  by applying Eq. (13) are added in square brackets to the first column of Table I of this comment.

For the recalculation of the effective magnetic moments per  $\text{Co}^{4+}$  according to this comment, Eq. (11) is rearranged appropriately:

$$p_{\text{eff[Co]}}^2 = (1 - x) p_{\text{eff[Co4+]}}^2, \quad (14)$$

$$p_{\text{eff[Co4+]}} = p_{\text{eff[Co]}} / \sqrt{1 - x}. \quad (15)$$

The correct values for the effective magnetic moment  $p_{\text{eff[Co4+]}}$  per  $\text{Co}^{4+}$  were calculated from the experimentally determined values of the effective magnetic moment  $p_{\text{eff[Co]}}$  per Co and the nominal  $\text{Co}^{4+}$  concentration  $(1 - x)$  according

to Eq. (15) and the corrected values are listed in the fourth column of Table I of this comment. For the slightly delithated sample with nominal Li content  $x = 0.97$ , the correct magnetic moment per  $\text{Co}^{4+}$  is recalculated to  $\mu_{\text{eff}[\text{Co}^{4+}]} = 1.33\mu_{\text{B}}$ , i.e., a value even slightly below the low-spin value for  $\text{Co}^{4+}$  of  $1.73\mu_{\text{B}}$ . In contrast, by applying inappropriate formulas, a very high effective magnetic moment per  $\text{Co}^{4+}$  of  $\mu_{\text{eff}[\text{Co}^{4+}]} = 7.67\mu_{\text{B}}$  was published in Ref. [1] for this sample, a value that

is even well above the theoretical spin-only magnetic moment for a  $\text{Co}^{4+}$  high-spin of  $5.9\mu_{\text{B}}$ .

#### DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the author.

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