



Fluxonium qubits with multiple degrees of freedom

Zur Erlangung des akademischen Grades eines
Doktors der Naturwissenschaften (Dr. rer. nat.)

von der KIT-Fakultät für Physik des
Karlsruher Instituts für Technologie (KIT)

genehmigte
Dissertation

von
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Tag der mündlichen Prüfung:
17 Juli 2026

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Declaration of Authorship

I, the author Denis Bénâtre, take full responsibility for the accuracy and originality of the work presented in this manuscript.

I acknowledge the use of generative AI tools in the form of large language models M365 Copilot, GPT-5, and Mistral Small 4 to assist with:

- Python codes for data analysis and plotting,
- LaTeX codes for editing,
- literature research,
- grammar and spelling checks,
- style refinement.

These tools were not used to generate portions of text or images. Every content generated by them was reviewed and revised by myself.

Data availability

Data, analyses and plots supporting the findings of this thesis are freely available on Zenodo [1].

Abstract

According to Greek myths, Dionysus was born from the thigh of his father Zeus and promised to a great fate. He, the son of a mortal mother, was destined to become god of wine, festivities and frenzy. Physics too has a Dionysus, heir to the Nobel lineage of condensed matter, called to a bright quantum computing future and sometimes caught up in the media frenzy: the field of superconducting qubits. Like the god of madness and folly, qubits are unpredictable and difficult to control. Thus, generations of students and advisors have worked on designing, engineering and operating single-degree-of-freedom circuits, achieving high levels of accuracy in qubit manipulation and readout. Today, research builds upon these systems to add degrees of freedom, in particular in quantum processors and noise-protected qubits. This work fits within this line of research and aims to realize a protected qubit based on a proposed four-loop fluxonium circuit. In order to work as intended, the system first needs to trap and retain a persistent current equivalent to one flux quantum in the circuit, referred to as a fluxon. Second, it needs to be as insensitive as possible to the charge offset on its central island. These two requirements constitute the two challenges tackled in this thesis.

The first challenge is to trap and retain a fluxon in the circuit. To do this, we propose to cool down the device in a magnetic field corresponding to one flux quantum in its most external loop. This flux-locking technique is demonstrated on gradiometric fluxoniums, in which an aluminum ring retains the trapped fluxon indefinitely. On average, four out of five qubits are locked simultaneously in a single waveguide. By compensating the inductive asymmetry between the two loops of the design, we are able to lock the effective flux-bias within $\Phi_{\text{eff}} = -3 \times 10^{-4} \Phi_0$ from the target, corresponding to only 15 % degradation in coherence time $T_{2,E}$ when operated in zero external field. Using such a gradiometric geometry for flux-tunable elements in quantum processors would reduce flux crosstalk and potentially eliminate the need for passive biasing.

The second challenge is the charge-offset noise arising from the central island in the design. To address it, we engineer charge-tunable fluxoniums in which the main junction is replaced by two junctions in series, separated by an island. In such a system, quasiparticles tunneling through either junction change the charge offset by one elementary charge e , a process called parity switching. By employing state-of-the-art techniques of infrared filtering and gap engineering, we increase the parity switching time to twenty seconds. We also identify other phenomena, such as a parity saturation by microwave pumping, allowing us to prepare the qubit in odd or even parity with above 80% fidelity. Finally, these devices can be used to uncover the coupling mechanism of qubits to long-lived two-level systems in their environment.

Overall, this thesis paves the way to studying multiple degrees of freedom, be it in multi-qubit processors or in a single superconducting circuit based on the fluxonium architecture. It highlights the phenomena that need to be understood and mitigated in order to translate protected qubit concepts into experimental reality.

Acknowledgements

I would like to start by thanking you, Ioan, for giving me the opportunity to work in your group, from my first stay as one of many French summer interns, to the PhD. I also want to specifically thank Hugues Pothier for agreeing to be my second reviewer.

I want to thank the whole Black Forest Quantum group for the good years and good memories, in and out of the lab. In particular, I want to thank "my generation" of PhDs: Ameya, NicoG, NicoZ, Simon2, Sören. NicoG, I am glad that my samples could be useful to your TLS research and I am very proud of our paper. Good luck on the thesis writing, I know you will do great!

Thank you to senior members, Patrick W., Francesco, Martin, Patrick P., Simon1, Dennis. A particular thank you, Patrick, for the help with the cryostats in campus north. Knowing that you would be able to help at any time with cooldown, thermal resets, etc, was a huge stress relief.

Shoutout to Mitchell, surely a pillar of the group. You're always ready to offer your help, happy to discuss and bounce new ideas, proactive. For sure the group will miss you when your time to go comes.

And thanks also to the newer members (i.e. people who started after me) Janic, Philipp, Paul, Priya, Danilo. I wish you the best in the remaining of your PhD and the future of your life and career.

Thanks as well to Sumeru, I can see that your presence is already making a difference in the group.

Many thanks Thomas for your help in campus north, with scientific or practical problems (shoutout to the CN Plassys).

I thank as well the shorter-term members of the group, bachelor students, master students and interns that have come and passed along the years. I thank particularly Tina for her work on the charge-tunable fluxoniums under my supervision. It was a great learning experience for me and I wish you the best.

Of course, special thanks to Mathieu for the supervision, discussions, ideas. This work would have been very different without you.

I also thank the NSL cleanroom staff, secretaries, IT staff, workshop workers at PHI, IQMT and INT who, directly or indirectly, made this work possible. Special shoutout to Lucas and Silvia.

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List of Publications

Authors marked with $\dagger$ have contributed equally.

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List of Symbols

A_{ring}	outer ring area	α	inductive asymmetry
$\hat{a}, \hat{a}^\dagger$	annihilation/creation operator	Γ_1, Γ_2	relaxation/coherence rate
$\mathbf{B}, B_{\text{cd}}$	magnetic field (during cooldown)	Δ	superconducting gap
C	capacitance	Δf	charge dispersion
$\mathbf{E}$	electric field	λ_L	London penetration depth
E_J	Josephson energy	ξ_0, ξ	(bulk) coherence length
E_L	inductive energy	ρ_n	normal-state resistivity
E_C	capacitive energy	ρ_s	superfluid stiffness
e	elementary charge	$\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z$	Pauli operators
f_{ij}	$i - j$ transition frequency	$\Phi, \hat{\Phi}$	flux (operator)
g	coupling strength	$\Phi_{\text{ext}}, \Phi_{\text{eff}}$	external/effective flux
$\hat{\mathcal{H}}$	Hamiltonian	Φ_0	magnetic flux quantum
$\hbar, h$	(reduced) Planck constant	$\varphi, \hat{\varphi}$	phase (operator)
I_c	critical current	χ	dispersive shift
$\mathbf{j}_s$	supercurrent density	Ψ	order parameter
k_B	Boltzmann constant	ω_q, ω_r	qubit/resonator angular frequency
$\mathcal{L}$	Lagrangian		
L	inductance		
$n, \hat{n}$	reduced charge (operator)		
n_g	charge offset		
n_s	Cooper pair density		
P_e	excited-state population		
$Q, \hat{Q}$	charge (operator)		
Q_c, Q_i	coupling/internal quality factor		
S	power spectral density (PSD)		
T_c	critical temperature		
T_p	parity switching time		
T_q	qubit temperature		
T_1	relaxation time		
$T_{2,R}$	Ramsey coherence time		
$T_{2,E}$	Hahn-echo coherence time		
t	tunneling rate		
w	circuit loop width		

1 General introduction

*“Tell me, O Muse, of **the man of many devices**, who wandered full many ways after he had sacked the sacred citadel of Troy. [...] Of these things, goddess, daughter of Zeus, beginning where thou wilt, tell thou even unto us.”*
— Homer, *Odyssey*, translation A.T. Murray

The purpose of this chapter is to give the reader a short introduction to superconductivity and a quick overview of the field of superconducting qubits. While superconducting qubits have come under much attention in the last decade, superconductivity has been a research subject for over a century, embedded in condensed matter and mesoscopic physics. Connecting recent developments to phenomena elucidated in the early twentieth century, we introduce key concepts necessary to understand the work presented in the thesis, from flux quantization to fluxonium qubits. We also give an overview of common and less-common qubit ideas and designs.

The rest of the manuscript is organized as follows. Chapter 2 dives into flux-locking of gradiometric fluxoniums, extending upon the work done in Ref. [2]. In particular, Section 2.5 presents the work done on a four-loop fluxonium device. Chapter 3 explores charge-tunable fluxonium qubits, specifically the experimental challenges attached to controlling the charge on a superconducting island.

1.1 A short history of superconductivity

1.1.1 From Kamerlingh Onnes’s experiments to the Bardeen-Cooper-Schrieffer (BCS) theory

As he rejoiced at the view of a liquid surface in his dewar on July 10, 1908, Kamerlingh Onnes knew he and his team had won the race for liquefying helium¹. But surely he did not realize yet that this scientific breakthrough was merely an open door to the discovery of superconductivity, a phenomenon of utter relevance for all quantum physicists and one that

¹ For this achievement, and many others, he was awarded the Nobel Prize in 1913 and nicknamed “*le gentleman du zéro absolu*” by French physicist Arsène d’Arsonval, at a meeting of the *Association Internationale du froid* in 1914.

sparks curiosity and excitement in the minds of many. It took less than three years and a bit of serendipity for Kamerlingh Onnes to measure superconductivity in mercury on April 8, 1911 [3], showing the first experimental data in superconductor-to-insulator transition (SIT) research. In the early twentieth century, the fields of cryogenics and low-temperature physics were already blooming, but the discovery of superconductivity sparked the interest of more physicists in many countries, especially in Germany, at the forefront of the first quantum revolution. Among them were Walther Meissner and Robert Ochsenfeld, who discovered in 1933 the effect that bears their names² and soon became a defining property of superconductors [4]. As illustrated in Fig. 1.1.a, when a superconductor is immersed in a magnetic field $\mathbf{B}$, the field is expelled out of the superconductor, regardless of whether it was cooled down in field or not. The effect is visually impressive, as it is the phenomenon that allows levitation of superconductors, and often used as a pedagogical tool in physics lectures. Another reason why this effect is important is its contribution to the London brothers' development of their theory of superconductivity.

Indeed, the story would not be complete without the input from theorists. Theories came in three waves. In 1935, the London brothers proposed the first phenomenological theory of superconductivity, in the form of the two London equations [5] which not only succeed in explaining the Meissner effect but also introduce relevant physical quantities such as the London penetration depth λ_L . Let us take a closer look at their theory. In the following, we will be using modern notations, slightly different from the ones used in the original paper. The London equations are:

$$\frac{\partial \mathbf{j}_s}{\partial t} = \frac{n_s e^2}{m_e} \mathbf{E}, \quad (1.1)$$

$$\nabla \times \mathbf{j}_s = - \frac{n_s e^2}{m_e} \mathbf{B}, \quad (1.2)$$

where $\mathbf{j}_s$ is the supercurrent density, n_s the charge carrier density, e the elementary charge, m_e the electron mass, $\mathbf{E}$ the electric field and $\mathbf{B}$ the magnetic field. The first London equation, Eq. (1.1), is the analogous to Ohm's law. In metals, it states that the current is proportional to the electric field, leading to a nonzero resistance and dissipation. Here, the current $\mathbf{j}_s$ is accelerated by the electric field $\mathbf{E}$, offering the first defining characteristic of a superconductor: zero resistance. The second London equation, Eq. (1.2), leads to the Meissner effect. Indeed, by using the Maxwell-Ampère and Maxwell-Thomson equations, it becomes

$$\nabla^2 \mathbf{B} - \frac{1}{\lambda_L^2} \mathbf{B} = 0, \quad (1.3)$$

where λ_L is the penetration length, defined as $\lambda_L \equiv \sqrt{m_e / \mu_0 n_s e^2}$, with μ_0 the vacuum permeability. λ_L is the first physical quantity used to characterize a superconductor. This equation shows that the magnetic field $B(x)$ decays exponentially in a superconductor, as shown in Fig. 1.1.b (in blue), thus providing the explanation of the Meissner effect.

² Though it is often simply called the Meissner effect.

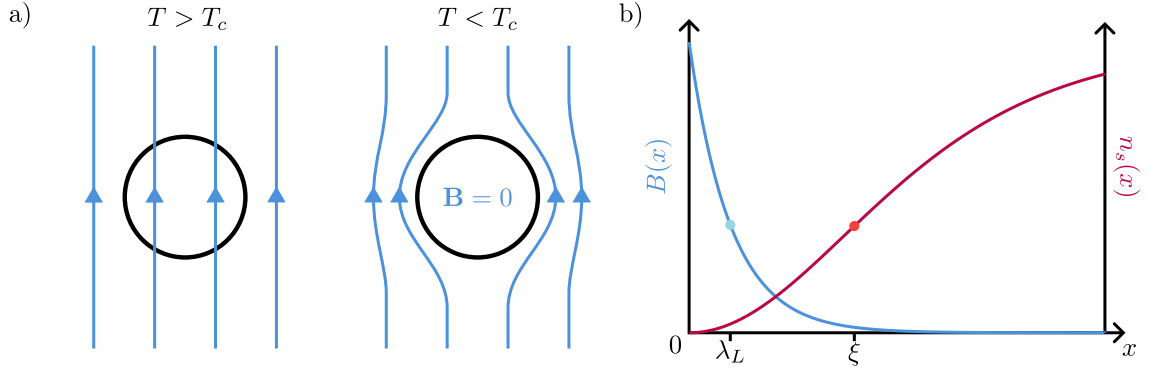


Figure 1.1: Meissner-Ochsenfeld effect and characteristic lengths in a superconductor. **a)** Illustration of the Meissner-Ochsenfeld effect. Below the critical temperature T_c , the sample becomes superconducting and expels the applied magnetic field $\mathbf{B}$, independently of whether the field was applied above T_c . **b)** Magnetic field $B(x)$ and Cooper pair density $n_s(x) = |\Psi|^2$ versus position x in a superconductor, where the surface is at $x = 0$. Penetration length λ_L and coherence length ξ are indicated on the x -axis, as they would be for a type-I superconductor, with $\xi > \lambda_L \sqrt{2}$.

In 1950, Ginzburg and Landau developed and published a phenomenological model of superconductivity, known today as the Ginzburg-Landau theory [6]. Postulating the complex order parameter $\Psi = \sqrt{n_s} e^{i\varphi}$, with n_s the density of charge carriers and φ the superconducting phase, it describes the free energy of a superconductor with two temperature-dependent parameters $\alpha(T)$ and $\beta(T)$. Defining the critical temperature T_c by $\alpha(T = T_c) = 0$, α is approximated by $\alpha(T) = \alpha_0(T - T_c)$ and $\Psi(T < T_c)$ is given by

$$|\Psi| = \sqrt{\frac{\alpha_0(T_c - T)}{\beta}}. \quad (1.4)$$

Consistent with the London equations, Ginzburg-Landau theory gives the charge-carrier density n_s as a function of parameters α and β , which leads to the penetration depth

$$\lambda(T) = \sqrt{\frac{m_e \beta}{2\mu_0 e^2 \alpha_0(T_c - T)}}, \quad (1.5)$$

with m_e the electron mass, μ_0 the vacuum permeability and e the elementary charge. Ginzburg-Landau theory also defines a second characteristic length for superconductors, the coherence length ξ , which gives the typical length for the order parameter to reach its bulk value from the surface or a defect, as pictured in Fig. 1.1.b (in carmine). It is written

$$\xi(T) = \sqrt{\frac{\hbar^2}{4m_e |\alpha(T)|}}. \quad (1.6)$$

These two characteristic lengths give a criterium to distinguish type-I from type-II superconductors. If ratio $\kappa = \lambda/\xi$ is less than $1/\sqrt{2}$, the superconductor is type-I.

In 1953, Pippard proposed to refine these models and introduced a third characteristic length for superconductors: the mean free path l [7]. In the so-called dirty limit, when the mean free path is smaller than the bulk coherence length ξ_0 , ξ is rescaled as

$$\frac{1}{\xi} = \frac{1}{\xi_0} + \frac{1}{l}. \quad (1.7)$$

In the field of superconducting circuits, virtually every superconductor is used in the form of thin film wires, i.e. the mean free path is much smaller than ξ_0 , making the rescaled ξ the relevant quantity. For the qubit applications highlighted in this thesis, aluminum is the main superconductor. This type-I superconductor has the following bulk properties [8, 9]:

$$T_c = 1.18 \text{ K}, \quad \lambda_L = 45 \text{ nm}, \quad \xi_0 = 1600 \text{ nm}. \quad (1.8)$$

Only a few years later, in 1957, Bardeen, Cooper and Schrieffer introduce the first microscopic theory of superconductivity, known today as the BCS theory, building upon the concept of electronic pairs introduced by Cooper a year before [10, 11]. It is consistent with the London equations and the Ginzburg-Landau theory. In particular, BCS states that there is a universal scaling for the superconducting gap Δ :

$$\Delta = 1.764 k_B T_c, \quad (1.9)$$

with k_B the Boltzmann constant and T_c the superconducting critical temperature.

1.1.2 Flux quantization: prediction and observations

Flux quantization was suggested by London [13] and is simply a consequence of the superconducting order parameter $\Psi = \sqrt{n_s} e^{i\varphi}$ being single-valued, which yields the following relation for its phase φ along a closed path C :

$$\oint_C \nabla \varphi \cdot d\mathbf{r} = 2\pi m, \quad (1.10)$$

with m an integer. Let us now consider a superconducting ring threaded by a magnetic field $\mathbf{B}$, as in Fig. 1.2.a. From Eq. (1.10), we can write

$$m\Phi_0 = \int_S \mathbf{B} \cdot d\mathbf{S} + \mu_0 \lambda_L^2 \oint_C \mathbf{j}_s \cdot d\mathbf{r}, \quad (1.11)$$

with $\Phi_0 = h/2e$ the magnetic flux quantum. In anticipation of the formalism of Section 1.2, let us already point out that the first term in the right-hand-side is the external flux Φ_{ext} while the second term is the flux through an inductive element $\Phi = LI_s$, where the supercurrent I_s is the integral of $\mathbf{j}_s$ over the cross-section of the wire: $I_s = \iint_A \mathbf{j}_s \cdot d\mathbf{A}$.

Interestingly, the two experiments that proved flux quantization (and Cooper pairs being the charge carriers in superconductors) were performed in parallel in Stanford, USA and Herrsching am Ammersee, Germany, in 1961. As far as we know, the two teams were

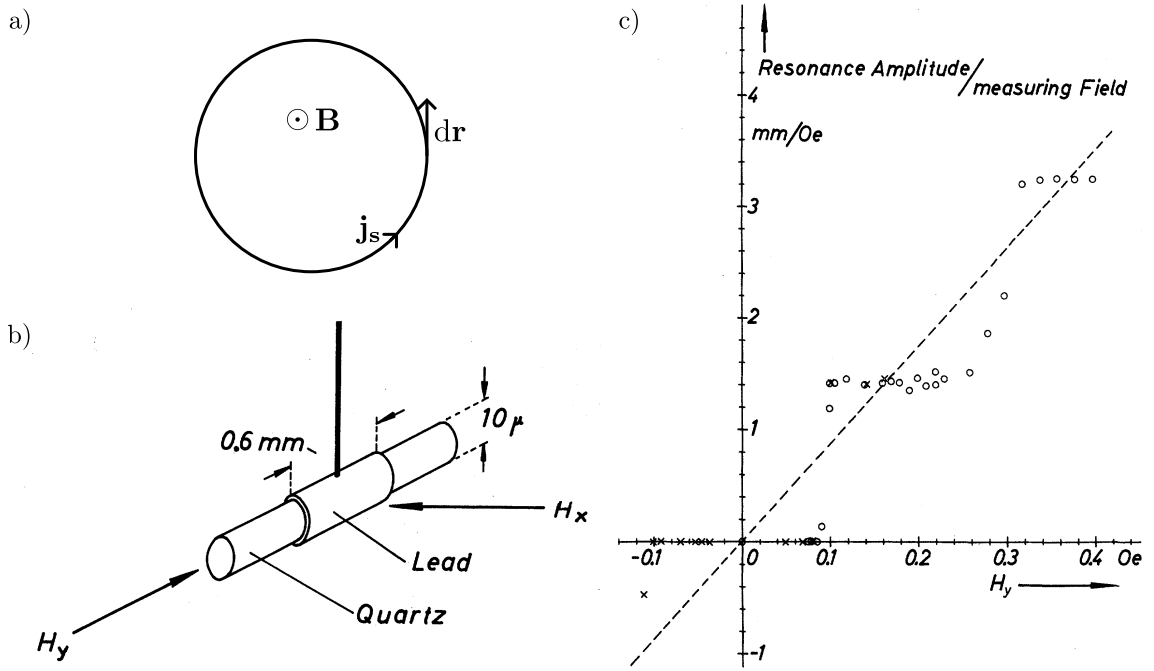


Figure 1.2: Flux quantization. a) Schematics of a superconducting ring threaded by a magnetic field B . b) Schematics of the experiment done by Doll and Näbauer in Germany. A lead cylinder is suspended to a torsion pendulum and is cooled down in an orthogonal magnetic field H_y . c) Resonance amplitude of the pendulum vs. cooling field H_y . The unit Oe on the x axis is the oersted, where $1\ \text{Oe} = \frac{10^3}{4\pi}\ \text{A/m}$. Panels b and c are taken from Ref. [12].

not aware of the other team's work, but published in the same issue of Physical Review Letters [12, 14]. Figure 1.2.b shows the experimental setup of the German team. It consists of a cylinder of quartz coated with lead ($T_c = 7.2\ \text{K}$) and suspended to a torsion pendulum. Two magnetic fields, one orthogonal to the cylinder's opening (H_y) and one in-plane (H_x) are used, respectively to trap flux in the cylinder during cooldown and to measure the magnetic moment of the cylinder at low temperature. The resonance amplitude of the pendulum is reported in Fig. 1.2.c as a function of the cooling field H_y . The resonance amplitude exhibits plateaus, the first of which is around $H_{y,0} = 0.2\ \text{Oe}$. From this, we can estimate $\Phi_0^{(exp)}$ to be

$$\Phi_0^{(exp)} = \pi \frac{d^2}{4} \mu_0 H_{y,0} = 1.7 \times 10^{-15}\ \text{Wb}. \quad (1.12)$$

This value, despite being within 10% of the real value $\Phi_0 = 2.067 \times 10^{-15}\ \text{Wb}$, surprised Doll and Näbauer who did not yet know that the charge carriers are not electrons but pairs of electrons. For a modern reader, this might come as a surprise given that Bardeen, Cooper and Schrieffer had published their theory four years earlier. Let it be a lesson for us physicists and science enthusiasts: just because a paper is published does not mean it is immediately known by the whole community.

1.1.3 Josephson effect and Josephson junction

In 1962, Brian Josephson published his prediction that zero-voltage supercurrent should flow between two superconductors separated by an insulating barrier [15], as illustrated in Fig. 1.3.a. This prediction, known as the Josephson effect, is governed by the Josephson relations:

$$I = I_c \sin \varphi, \quad (1.13)$$

$$\frac{d\varphi}{dt} = \frac{2e}{\hbar} V, \quad (1.14)$$

with I and V respectively the current and voltage across the junction, $\varphi = \varphi_2 - \varphi_1$ the superconducting phase drop across the junction, and I_c the critical current. While originally predicted and demonstrated for an insulating barrier [16–19], the effect is actually more general and holds for any weak link between two superconducting electrodes, be it a normal metal [20–22], a semiconductor [23] or a geometric constriction [24, 25]. Note however that the relation in Eq. (1.13) might not be strictly sinusoidal and can include higher harmonics.

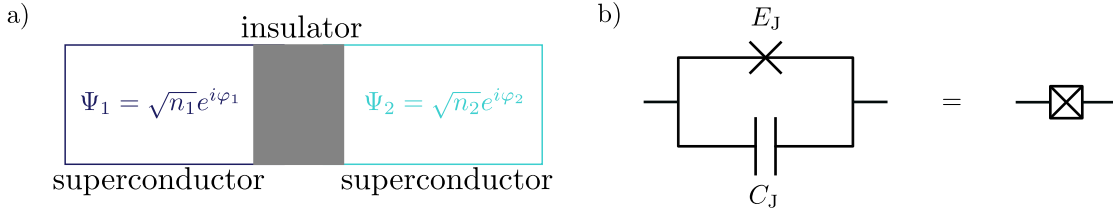


Figure 1.3: Josephson junction. **a)** Schematics of a superconductor-insulator-superconductor (SIS) Josephson junction. Superconducting order parameter Ψ_i in superconductor i defines the density of Cooper pairs n_i and the phase φ_i . **b)** Josephson junction equivalent circuit, including the non-linear inductor characterized by Josephson energy E_J symbolized by the cross, and the capacitance C_J . Both elements in parallel are represented by the cross-in-a-box symbol.

Combining Eq. (1.13) and Eq. (1.14), we can calculate the energy stored in the junction:

$$E = \int V I dt = \int \left(\frac{\hbar}{2e} \dot{\varphi} I_c \sin \varphi \right) dt = -E_J \cos \varphi \quad (1.15)$$

$$= -E_J \left(1 - \frac{\varphi^2}{2} + O(\varphi^4) \right), \quad (1.16)$$

with the Josephson energy $E_J = I_c \hbar / 2e$. By comparison with the energy stored in an inductor, $\Phi^2 / 2L$, the Josephson junction is identified to a non-linear inductor, with $L_J = (\Phi_0 / 2\pi)^2 / E_J$. In circuit diagrams, it is represented by a cross, as shown in Fig. 1.3.b. Because such SIS junctions are routinely fabricated from overlapping superconductors, they form a parallel-plate capacitor in parallel with the non-linear inductor. This intrinsic capacitance C_J is embedded in the cross-in-a-box symbol, also shown in Fig. 1.3.b.

1.1.4 Disordered superconductors: the case of granular aluminum

So far, we have described phenomena pertaining to ideal superconductors. Yet, introducing disorder in superconductors can provide improved characteristics such as higher critical temperature, critical magnetic field and kinetic inductance. Disordered superconductors such as titanium nitride (TiN), niobium nitride (NbN), titanium niobium nitride (NbTiN) are used in radio-frequency cavities [26], single-photon detectors [27], MKIDs³ [28], as well as parametric amplifiers [29, 30] and even qubits [31]. In the following, we discuss granular aluminum (grAl), a disordered superconductor made from aluminum and oxygen that has also been used for high-quality-factor linear resonators [32–34], amplifiers [35] and high-impedance superconducting qubits [36, 37].

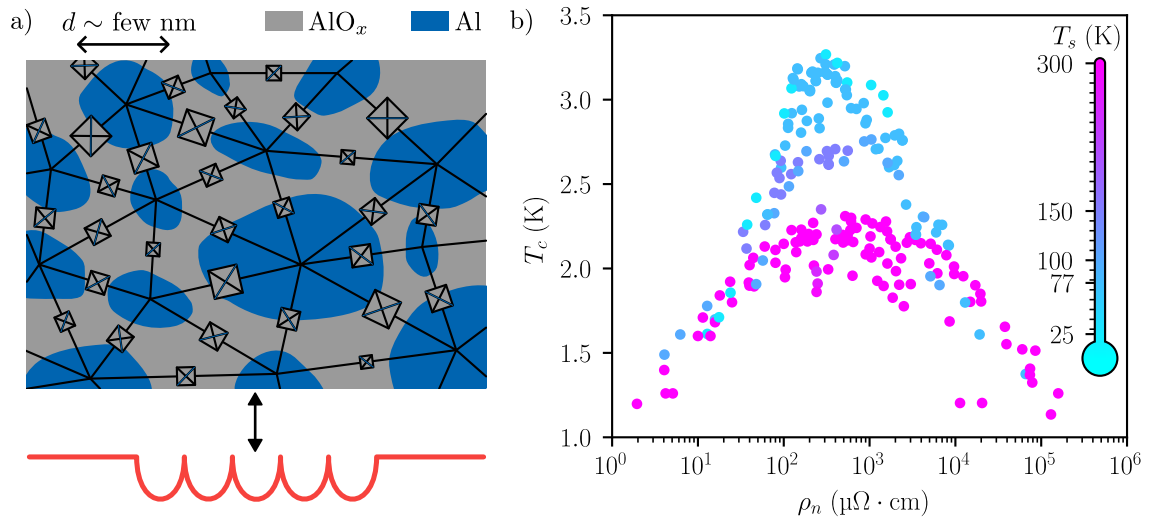


Figure 1.4: Granular aluminum. **a)** Schematic representation of granular aluminum: aluminum grains (in blue) embedded in aluminum oxide (in gray). The grains form a network of Josephson junctions that is effectively modeled as a linear inductance. **b)** Superconducting critical temperature T_c of granular aluminum versus normal-state resistivity ρ_n and substrate temperature during deposition T_s (thermometer colorbar). ρ_n is measured either at room temperature or at 4 K. 77 K is liquid nitrogen temperature. Data from Refs. [38–50], extracted using online tool WebPlotDigitizer [51].

The schematic representation of granular aluminum in Fig. 1.4.a highlights its most interesting property for our applications: high kinetic inductance. Indeed, grAl consists of nanometer-size grains of superconducting aluminum embedded in aluminum oxide. Together, they form an effective network of Josephson junctions, which can be modeled as a quasi-linear inductance. The sheet kinetic inductance is calculated by the Mattis-Bardeen formula [52]:

$$L_{\square} = R_{\square} \frac{h}{2\pi^2 \Delta \tanh(\Delta/2k_B T)}, \quad (1.17)$$

with $R_{\square} = \rho_n/d$ the normal-state sheet resistance, ρ_n the normal-state resistivity, d the thickness and Δ the superconducting gap. In the weak-electron-phonon-coupling regime, where BCS applies, $2\Delta = 3.53 k_B T_c$.

³ Microwave Kinetic Inductance Detectors

One striking property of granular aluminum is its so-called superconducting dome, represented in Fig. 1.4.b. The critical temperature T_c depends on both the normal-state resistivity ρ_n and the substrate temperature during evaporation T_s . When evaporated at room temperature ($T_s = 300$ K), maximum $T_c = 2.31$ K is obtained for $\rho_n = 5 \times 10^2 \mu\Omega \cdot \text{cm}$. Cooling the substrate down to liquid nitrogen temperature ($T_s = 77$ K) or even lower brings the maximum critical temperature to $T_c = 3.27$ K at $\rho_n = 3 \times 10^2 \mu\Omega \cdot \text{cm}$. The spread of T_c across samples from the literature, especially after the maximum, coincides with the widening of the phase transition. In this thesis, samples were fabricated from granular aluminum deposited at room temperature and normal-state resistivity around $\rho_n = 1 \times 10^3 \mu\Omega \cdot \text{cm}$. On the left side of the dome, granular aluminum is in the weak-coupling regime, with ratio $2\Delta/k_B T_c \approx 3.53$, while on the right side the ratio is above 4 [44, 45].

The history of superconductivity does not end here, instead spreading into many subfields, from the quest of high- T_c superconductors [53] to the study of exotic superconductors, cuprates, etc. For the purpose of this thesis however, we shall put our curiosity on pause and look at another of its offspring: superconducting circuits.

1.2 Superconducting circuits

The goal of this section is to introduce superconducting circuits, starting from the simplest, a linear LC resonator, and building our way up to complex circuits with multiple degrees of freedom. On our way, we will give more details on the two most common qubit circuits: transmon and fluxonium.

1.2.1 Harmonic oscillator

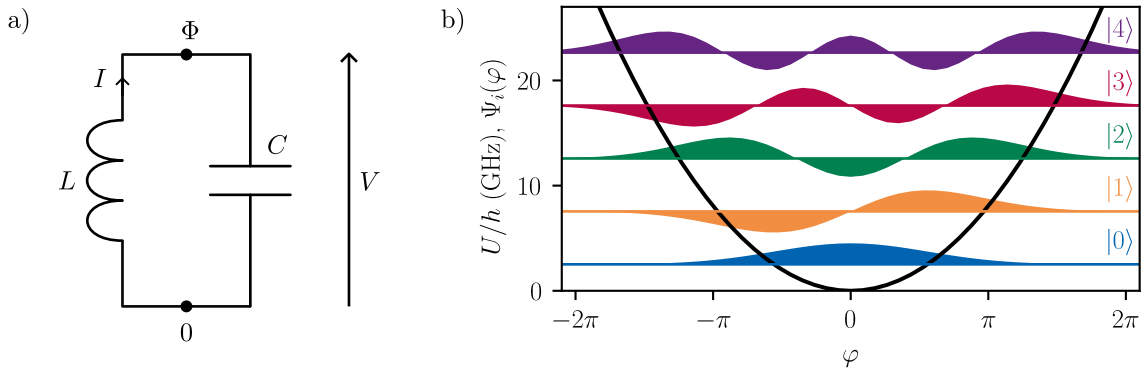


Figure 1.5: Harmonic oscillator. **a)** Lumped-element circuit of a resonator, consisting of an inductor and a capacitor. The diagram also defines current I , voltage V and flux Φ at the top node. **b)** Parabolic potential energy U and (real) wavefunctions Ψ_i for the first 5 states, versus phase $\varphi = 2\pi \times \Phi/\Phi_0$.

Starting from the circuit depicted in Fig. 1.5a, we can write the classical equations of motion:

$$V = L \frac{dI}{dt}, \quad (1.18)$$

$$I = C \frac{dV}{dt}, \quad (1.19)$$

and use them to derive the energy associated with each component, inductor and capacitor:

$$U_L = \int V I dt = \int L \frac{dI}{dt} I dt = \frac{1}{2} L I^2, \quad (1.20)$$

$$U_C = \int V I dt = \int V C \frac{dV}{dt} dt = \frac{1}{2} C V^2. \quad (1.21)$$

We now introduce the flux Φ at the top node of the circuit as

$$\Phi = \int V dt, \quad (1.22)$$

a position-like variable that allows us to rewrite U_L and U_C as

$$U_L = \frac{1}{2L} \Phi^2, \quad (1.23)$$

$$U_C = \frac{C}{2} \dot{\Phi}^2, \quad (1.24)$$

and the Lagrangian $\mathcal{L}$ of our system is given by

$$\mathcal{L}(\Phi, \dot{\Phi}) = T - U = \frac{C}{2} \dot{\Phi}^2 - \frac{1}{2L} \Phi^2, \quad (1.25)$$

with T the kinetic energy, corresponding in this system to the capacitive energy U_C , and U the potential energy, here the inductive energy U_L . The potential is purely parabolic in Φ , as represented in Fig. 1.5.b.

The Lagrangian gives us the ability to define the conjugate variable Q , a momentum-like variable corresponding to the charge in the system

$$Q = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}} = C \dot{\Phi}, \quad (1.26)$$

and finally write the Hamiltonian

$$\mathcal{H}(\Phi, Q) = \dot{\Phi} Q - \mathcal{L} = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}. \quad (1.27)$$

This Hamiltonian is more commonly expressed in terms of the reduced variables $\varphi = 2\pi \Phi / \Phi_0$ and $n = Q / 2e$, respectively the phase and the reduced charge, with $\Phi_0 = h / 2e$ the

magnetic flux quantum introduced in Section 1.1.2. Expressed as quantum variables, such that $[\hat{\phi}, \hat{n}] = i$, we get the following harmonic oscillator Hamiltonian:

$$\hat{\mathcal{H}}_{\text{HO}} = 4E_C \hat{n}^2 + \frac{E_L}{2} \hat{\phi}^2, \quad (1.28)$$

with the charging energy $E_C = e^2/2C$ and the inductive energy $E_L = (\Phi_0/2\pi)^2/L$.

The Hamiltonian in Eq. (1.28) is further rewritten in terms of the creation $a^\dagger$ and annihilation a operators as

$$\hat{\mathcal{H}}_{\text{HO}} = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right), \quad (1.29)$$

where $\omega_r = 1/\sqrt{LC} = \sqrt{8E_L E_C}/\hbar$ and such that

$$\varphi = \varphi_{\text{ZPF}}(a + a^\dagger), \quad (1.30)$$

$$n = n_{\text{ZPF}}i(a - a^\dagger), \quad (1.31)$$

with the zero-point fluctuation of the conjugate variables being, respectively:

$$\varphi_{\text{ZPF}} = 2\sqrt{\frac{\pi Z}{R_Q}} = \sqrt[4]{2E_C/E_L}, \quad (1.32)$$

$$n_{\text{ZPF}} = \frac{1}{4}\sqrt{\frac{R_Q}{\pi Z}} = \sqrt[4]{E_L/32E_C}, \quad (1.33)$$

where R_Q the resistance quantum is defined as $R_Q = h/e^2$ and Z is the impedance, $Z = \sqrt{L/C}$.

Diagonalizing the Hamiltonian in Eq. (1.28) gives the eigenenergies and wavefunctions, plotted in Fig. 1.28.b. As explicated by Eq. (1.29), the energy of Fock state $|n\rangle$ is $\hbar\omega_r(n + \frac{1}{2})$ with n the number of photons. The energies are evenly split, as expected for a harmonic oscillator.

1.2.2 Cooper-pair box and transmon

Harmonic oscillators, while being a prerequisite in quantum mechanics studies, are not sufficient for quantum computation. Indeed, implementing a qubit requires engineering a non-linearity that allows to distinguish a two-level subspace from the other levels. This subspace is called the computational subspace and comprises the ground state $|g\rangle$ or $|0\rangle$ and the excited state $|e\rangle$ or $|1\rangle$. The first demonstrated qubit, called the Cooper-pair box, was realized in 1999 [54] and its offspring, the transmon, is used ubiquitously today in both academia and industry.

The Cooper-pair box and transmon circuits are very similar to the linear resonator, as shown in Fig. 1.6.a. With the same method as in the previous section, we can write its Lagrangian

$$\mathcal{L}(\Phi, \dot{\Phi}) = \frac{C}{2} \dot{\Phi}^2 + \frac{C_J}{2} \dot{\Phi}^2 + \frac{C_g}{2} (\dot{\Phi} - V_g)^2 + E_J \cos \frac{\Phi}{\Phi_0}, \quad (1.34)$$

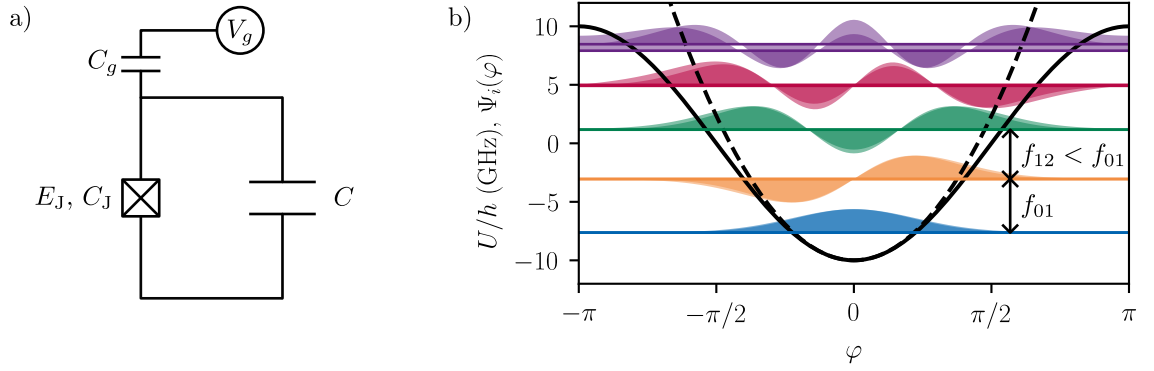


Figure 1.6: Transmon qubit. **a)** Circuit of a Cooper-pair box (CPB) or transmon qubit, consisting of a Josephson junction, a capacitance and a capacitive coupling to a voltage source that tunes the offset charge n_g . **b)** Potential energy and wavefunction of the first levels in a transmon qubit, calculated with Python package scqubits [55].

and from it derive the conjugate variable Q

$$Q = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}} = (C + C_J + C_g)\dot{\Phi} - C_g V_g. \quad (1.35)$$

Here we can already observe that the capacitance of the circuit is rescaled by the capacitance C_g to the voltage source, and that the charge Q has an offset proportional to V_g . The Hamiltonian is therefore written as:

$$\mathcal{H}(\Phi, Q) = \dot{\Phi}Q - \mathcal{L} \quad (1.36)$$

$$= \frac{C + C_J + C_g}{2} \dot{\Phi}^2 - \frac{C_g}{2} V_g^2 - E_J \cos \frac{\Phi}{\Phi_0} \quad (1.37)$$

$$= \frac{(Q + C_g V_g)^2}{2(C + C_J + C_g)} - E_J \cos \frac{\Phi}{\Phi_0}. \quad (1.38)$$

In the reduced variables defined previously, φ and n , we get the transmon Hamiltonian:

$$\hat{\mathcal{H}}_{\text{transmon}} = 4E_C(\hat{n} - n_g)^2 - E_J \cos \hat{\varphi}, \quad (1.39)$$

where $E_C = e^2/2(C + C_J + C_g)$ and $n_g = -C_g V_g/2e$ is the reduced charge offset.

The transmon Hamiltonian in Eq. (1.39) differs from the harmonic oscillator in two ways. First, the inductive contribution, coming from the Josephson junction, is non-linear. This spreads the energy levels unevenly in the potential well, which is quantified by the anharmonicity $\alpha = f_{12} - f_{01}$. The less capacitance across the junction (in other words, the bigger E_C), the more the levels are pulled up in the well and experience a non-harmonic potential wider than the harmonic one that tunes them down. For this reason, the energy levels in transmons are lower than in linear resonators. In Fig. 1.6.b, we represent the sinusoidal potential energy U of the transmon (solid black line) alongside the parabolic potential of the corresponding linear resonator (dashed black line). We also show the eigenenergies and their associated wavefunctions Ψ_i , for both $n_g = 0$ and $n_g = 0.5$. In this

plot, we see how the deviation of the potential from the parabola pushes the levels closer. Intuitively, this explains why, at high E_J/E_C , the anharmonicity fulfills $\alpha \approx -E_C$. Second, the transmon Hamiltonian is charge dependent. The charge dependence is quantified by the charge dispersion, which for a given level is defined as $\epsilon_n = E_n(n_g = 0.5) - E_n(n_g = 0)$. Analytically,

$$\epsilon_n \simeq -\frac{2^{4n+5}}{n!} \sqrt{\frac{2}{\pi}} E_C \left(\frac{E_J}{2E_C} \right)^{\frac{n}{2} + \frac{3}{4}} e^{-\sqrt{8E_J/E_C}}, \quad (1.40)$$

with n the qubit level [56]. As we commonly reason in terms of transition frequencies rather than absolute frequency, we derive the charge dispersion $\Delta f = f_{01}(n_g = 0) - f_{01}(n_g = 0.5)$ from Eq. (1.40):

$$\Delta f = 2^5 \sqrt{\frac{2}{\pi}} E_C \left(\frac{E_J}{2E_C} \right)^{\frac{3}{4}} e^{-\sqrt{8E_J/E_C}} \left(2^4 \left(\frac{E_J}{2E_C} \right)^{\frac{1}{2}} - 1 \right). \quad (1.41)$$

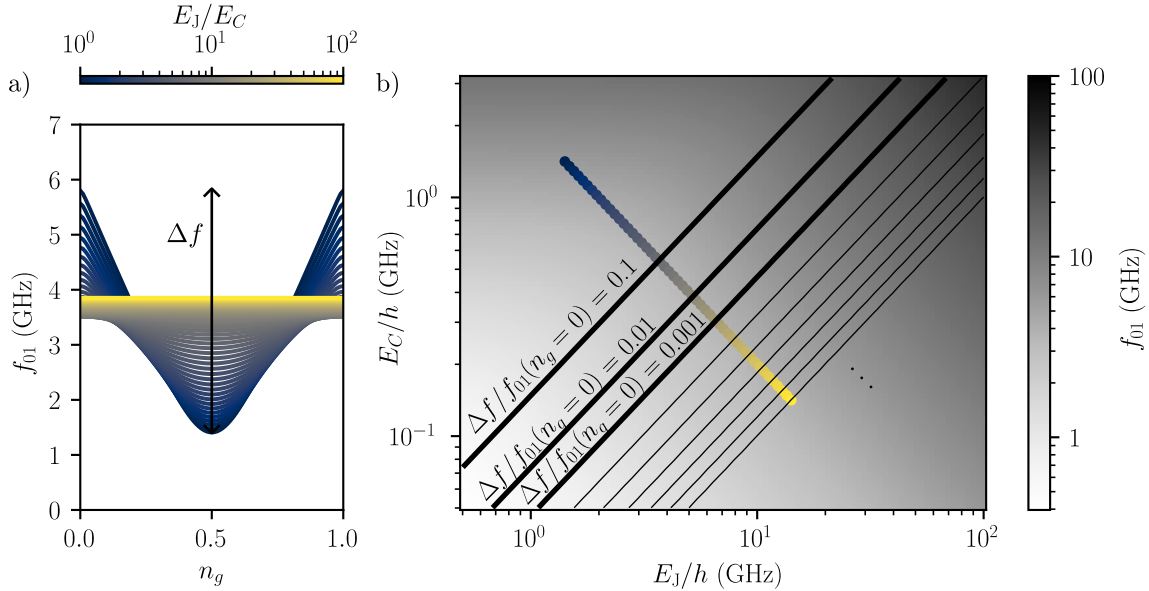


Figure 1.7: Transmon spectrum. **a)** First transition frequency f_{01} versus charge offset n_g showing the charge dispersion Δf and its dependence on the ratio E_J/E_C . The frequency was computed for E_J and E_C values corresponding to the colorbar and a constant mean frequency $\tilde{f}_{01} \approx 4$ GHz over the charge offset range. **b)** f_{01} versus E_J and E_C . Solid lines indicate constant relative dispersion $\Delta f/f_{01}(n_g = 0)$. Additionally, the anharmonicity $\alpha = -E_C$ and is thus given by the y axis.

In Fig. 1.7.a, we show the first transition frequency f_{01} when sweeping the charge offset n_g , exhibiting the charge dispersion Δf , for various values of the ratio E_J/E_C . Such a dependence is detrimental to the performance of superconducting qubits because the charge noise kills the dephasing time as $T_\phi \propto \frac{df_{01}}{dn_g}$. Hence, a lot of effort was put into reducing or eliminating this dependence. One strategy is to change the design to eliminate it intrinsically, which we will see in the next section. Another one is to tweak the parameters in such a way that the charge dispersion is suppressed by orders of magnitude [56]. From Eq. (1.41), we deduce that the most direct and efficient way is to increase E_J/E_C , mainly

by increasing C (decreasing E_C). As highlighted by Fig. 1.7.a, Δf is reduced by orders of magnitude when E_J/E_C is swept to 100. Attentive readers might have realized that this conflicts with the anharmonicity $\alpha \approx -E_C$ quoted above. In short, designing a transmon means finding the sweet spot between a large anharmonicity (but large dephasing charge noise) and low dephasing.

Finally, in Fig. 1.7.b, f_{01} and Δf are calculated numerically in the whole 2D landscape of E_J and E_C . In particular, we show the relative dispersion $\Delta f/f_{01}$ as lines of constant value, for 0.1, 0.01, etc. This illustrates that the bottom right corner is a good target, as long as the anharmonicity, given by the y axis $\alpha \approx -E_C$, is sufficiently large.

Thanks to these efforts, the transmon has become the standard of superconducting qubits in the industry, and is used across the world in single-qubit experiments as well as quantum processors [57]. Improvements in materials and design have successfully pushed the coherence of these qubits, above the one-millisecond threshold with tantalum on sapphire [58].

1.2.3 Flux qubit and fluxonium

The other solution to reduce charge noise is to remove the charge offset parameter altogether. This is done by shunting the junction with an inductance L . Doing so leads to the circuit diagram shown in Fig. 1.8.a, referred to as the flux qubit or fluxonium qubit circuit. Introducing an inductive shunt introduces a superconducting loop in the circuit, through which a magnetic flux Φ_{ext} is threaded.

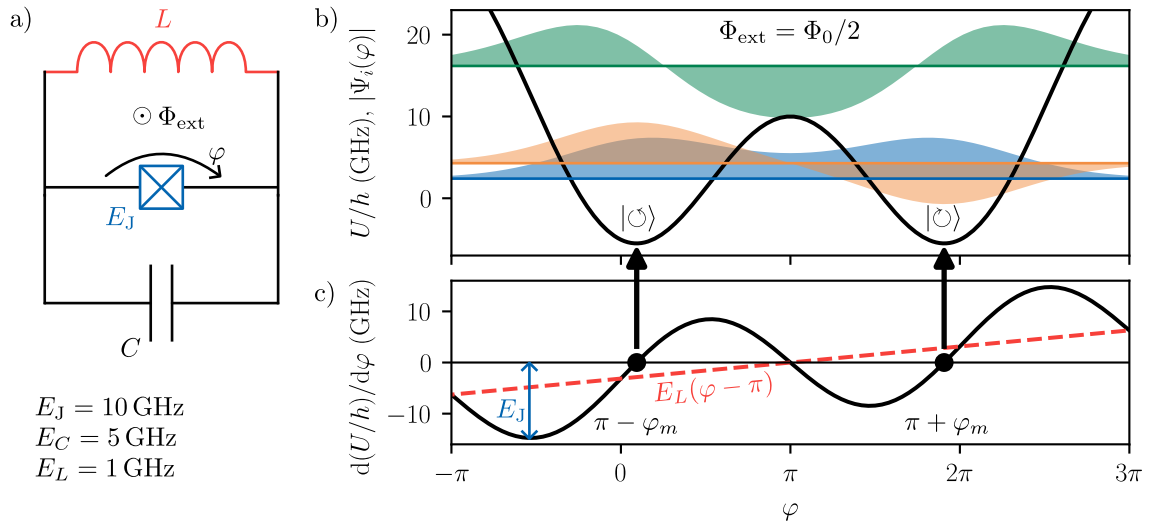


Figure 1.8: Fluxonium qubit. **a)** Fluxonium qubit circuit diagram, with inductance L in red, capacitance C and Josephson junction of energy E_J in blue. **b)** Potential U at the half-flux-quantum sweet spot $\Phi_{\text{ext}} = \Phi_0/2$ (black). In colors, wavefunctions for the first three states. **c)** Potential derivative $dU/d\varphi = E_L(\varphi - \pi) + E_J \sin \varphi$ gives the potential minima, $\pi - \varphi_m$ and $\pi + \varphi_m$, close to $\varphi = 0$ and $\varphi = 2\pi$.

From the circuit in Fig. 1.8.a, we can derive the Hamiltonian for fluxoniums. First, we write the Lagrangian

$$\mathcal{L} = \frac{C}{2}\dot{\Phi}^2 - \frac{1}{2L}\Phi_L^2 + E_J \cos \varphi. \quad (1.42)$$

Using the loop equation

$$\varphi_L - \varphi + \varphi_{\text{ext}} = 0, \quad (1.43)$$

we can remove φ_L and write $\mathcal{L}$ with only one degree of freedom:

$$\mathcal{L} = \frac{C}{2}\dot{\Phi}^2 - \frac{1}{2L}(\Phi - \Phi_{\text{ext}})^2 + E_J \cos \varphi. \quad (1.44)$$

The conjugate variable to Φ is defined as

$$Q = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}} = C\dot{\Phi}, \quad (1.45)$$

which allows to write the Hamiltonian

$$\mathcal{H} = Q\dot{\Phi} - \mathcal{L} \quad (1.46)$$

$$= \frac{1}{2C}Q^2 + \frac{1}{2L}(\Phi - \Phi_{\text{ext}})^2 - E_J \cos \varphi. \quad (1.47)$$

Finally, we define the reduced variables $\varphi = 2\pi \Phi/\Phi_0$ and $n = Q/2e$ and write the Hamiltonian with the corresponding quantum operators such that $[\hat{\varphi}, \hat{n}] = i$:

$$\hat{\mathcal{H}}_{\text{fluxonium}} = 4E_C \hat{n}^2 + \frac{E_L}{2}(\hat{\varphi} - \varphi_{\text{ext}})^2 - E_J \cos \hat{\varphi}, \quad (1.48)$$

with $E_C = e^2/2C$ and $E_L = (\Phi_0/2\pi)/L$.

From these calculations, we take the potential energy

$$U(\varphi) = \frac{E_L}{2}(\varphi - \varphi_{\text{ext}})^2 - E_J \cos \varphi, \quad (1.49)$$

which is represented at the sweet spot $\varphi_{\text{ext}} = \pi$ in Fig. 1.8.b, along with wavefunctions $|\Psi_i(\varphi)\rangle = |\langle i|\varphi\rangle\rangle$ at their corresponding eigenenergy E_i , for parameters $E_J/h = 10$ GHz, $E_C/h = 5$ GHz and $E_L/h = 1$ GHz.

At the half-flux-quantum sweet spot, the fluxonium exhibits a double-well potential. The positions φ_m of the wells are the two local minima of U , symmetric along $\varphi = \pi$. Given by $dU/d\varphi = 0$, they satisfy

$$E_L(\varphi_m - \pi) + E_J \sin \varphi_m = 0, \quad (1.50)$$

as shown in Fig. 1.8.c. These phases close to 0 and 2π , notated $\pi \pm \varphi_m$ in the following, correspond to two persistent current states, or fluxon states, eigenstates of $\hat{\varphi}$, such that:

$$\hat{\varphi} |\odot\rangle = (\pi - \varphi_m) |\odot\rangle, \quad \hat{\varphi} |\oslash\rangle = (\pi + \varphi_m) |\oslash\rangle. \quad (1.51)$$

This picture of two tunneling fluxon states $|\odot\rangle$ and $|\oslash\rangle$ allows us to write $\hat{\mathcal{H}}$ as a tight-binding Hamiltonian:

$$\hat{\mathcal{H}}_{\text{tight-binding}} = \begin{pmatrix} U(\pi - \varphi_m) & -t \\ -t & U(\pi + \varphi_m) \end{pmatrix}, \quad (1.52)$$

with t the tunneling rate [59–61]:

$$t = \frac{4}{\sqrt{\pi}} (8E_J^3 E_C)^{1/4} e^{-\sqrt{8E_J/E_C}}. \quad (1.53)$$

Removing the constant terms, the eigenenergies of $\hat{\mathcal{H}}_{\text{tight-binding}}$ are $E_g = -t$ and $E_e = +t$, such that their respective eigenstates are symmetric and antisymmetric superpositions of the persistent current states:

$$|g\rangle = \frac{|\odot\rangle + |\oslash\rangle}{\sqrt{2}}, \quad (1.54)$$

$$|e\rangle = \frac{|\odot\rangle - |\oslash\rangle}{\sqrt{2}}. \quad (1.55)$$

This approximation is only valid deep into the fluxonium regime, when $E_J/E_L \gg 1$. In Fig. 1.9, we define the different regimes of the flux qubit circuit in the literature and cite their most common names. The regimes are determined by two ratios, E_J/E_C which corresponds to the *mass* of the virtual particles in the wells and E_J/E_L which parametrizes the size of the potential barrier between the wells. For all three panels, data is taken from published articles referenced in the database from Ref. [62]. For comparison, we show a few examples of Cooper-pair box and transmon systems from historically significant papers (Refs. [54, 63–66]).

In Fig. 1.9.a, we show the qubit frequency f_q at the half-flux sweet spot. Several regimes can be identified. The flux qubit regime, at low E_J/E_L , is the historical persistent current qubit regime [67]. More often than not, the role of the inductance in the circuit (see Fig. 1.8.a) is played by as few as three Josephson junctions, making it less linear and subject to charge offset fluctuations. As shown in Fig. 1.9.b, this regime is associated with large dielectric losses, due to the large charge matrix element $|\langle 0 | \hat{n} | 1 \rangle|$. Increasing the inductance, i.e. moving upward in E_J/E_L , brings us to the fluxonium regime, crossing the quarton regime at $E_J = E_L$ [68]. As the inductance flattens the potential and E_J increases the height of the barrier, persistent current states in the fluxonium regime are better localized in the wells. As a consequence, the frequency at half-flux is lower (lower tunneling rate t) and the phase matrix element $|\langle 0 | \hat{\varphi} | 1 \rangle|$ is larger (the well separation $2\varphi_m$ is larger). On the contrary, $|\langle 0 | \hat{n} | 1 \rangle|$ is exponentially suppressed, thanks to the lower n_{ZPF} . Fluxoniums therefore experience less dielectric losses, making them a promising platform for long-lived qubits.

Localizing further in the wells can be done by increasing the E_J/E_C ratio, typically by adding a large capacitance C . By analogy with mechanics, adding capacitance is equivalent

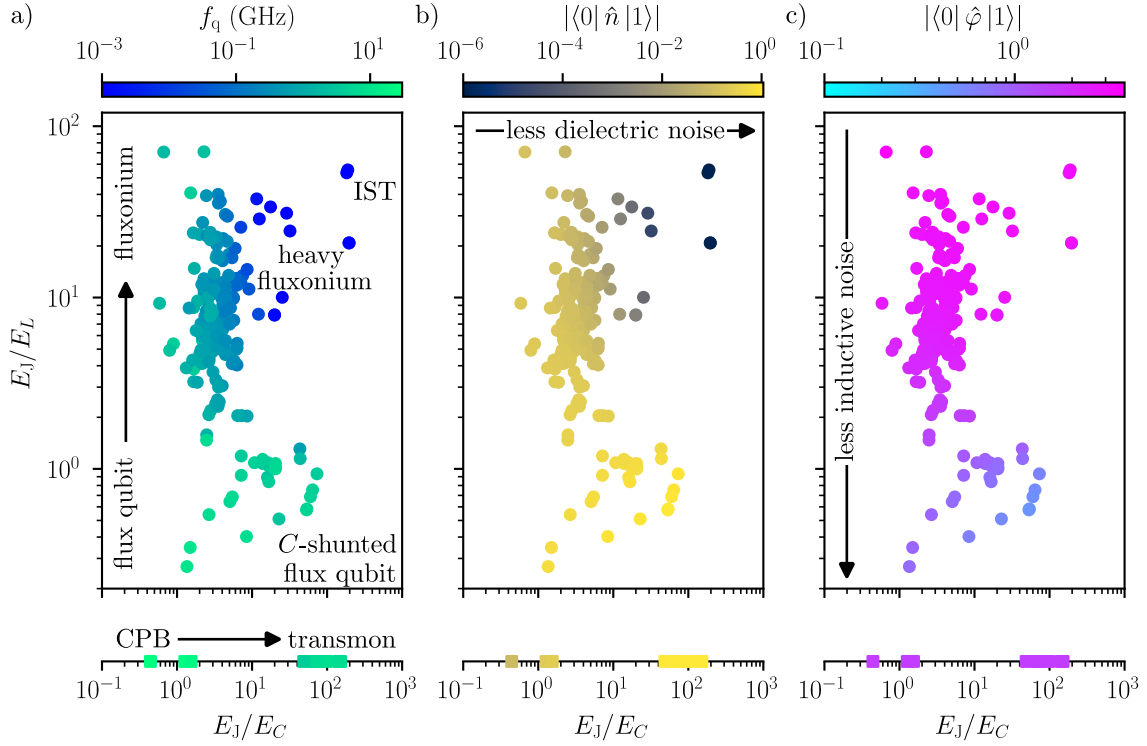


Figure 1.9: Fluxonium qubits parameter space. Parameter space E_J/E_C versus E_J/E_L of fluxonium relatives, populated with flux qubit and fluxonium devices from the literature, as well as CPB and transmons for comparison (bottom insets). *IST* stands for inductively shunted transmon, *CPB* stands for Cooper-pair box. Fluxonium data is from database in Ref. [62] while transmon data is from Refs. [54, 63–66]. **a)** In color, qubit frequency f_q at the half-flux sweet spot. **b, c)** In color, charge $\hat{n}$ (b) and phase $\hat{\phi}$ (c) matrix elements at the half-flux sweet spot.

to adding mass to the particles in the wells. In transmons, as we saw in Section 1.2.2, this results in more harmonic states and less charge offset dependence. The localization of the eigen-wavefunctions around φ unfortunately also results in larger $|\langle 0 | \hat{n} | 1 \rangle|$. In fluxoniums, thanks to the finite well separation, $|\langle 0 | \hat{\phi} | 1 \rangle|$ is increased to a maximum, while $|\langle 0 | \hat{n} | 1 \rangle|$ is further exponentially suppressed, in the so-called heavy fluxonium regime [69]. Even heavier is the inductively shunted transmon (IST), where the fluxon tunneling rate can be suppressed to lifetimes of hours [70], though in that regime the states are encoded in the plasmon states in a given well.

Recently, the heavy fluxonium regime has gained traction thanks to the demonstration of relaxation times T_1 above 1 ms [71–73]. In Fig. 1.10.a, we represent T_1 in the same parameter space, zoomed-in on samples where T_1 was measured at the half-flux sweet spot. We highlight the heavy fluxoniums cited above as well as devices from Ref. [74]. The two markers with black contours indicate two of our gradiometric fluxoniums from Ref. [2], reproduced in Chapter 2 (see coherence in Section 2.4). Interestingly, the highest quality factors $Q_i = 2\pi f_q T_1$, displayed in Fig. 1.10.b, do not exactly align with the highest relaxation times T_1 . Remarkably, many experiments have shown flux qubit and fluxonium samples with Q_i in the 10^6 range. The dephasing times $T_{\varphi,E}$, in Fig. 1.10.c, are obtained from

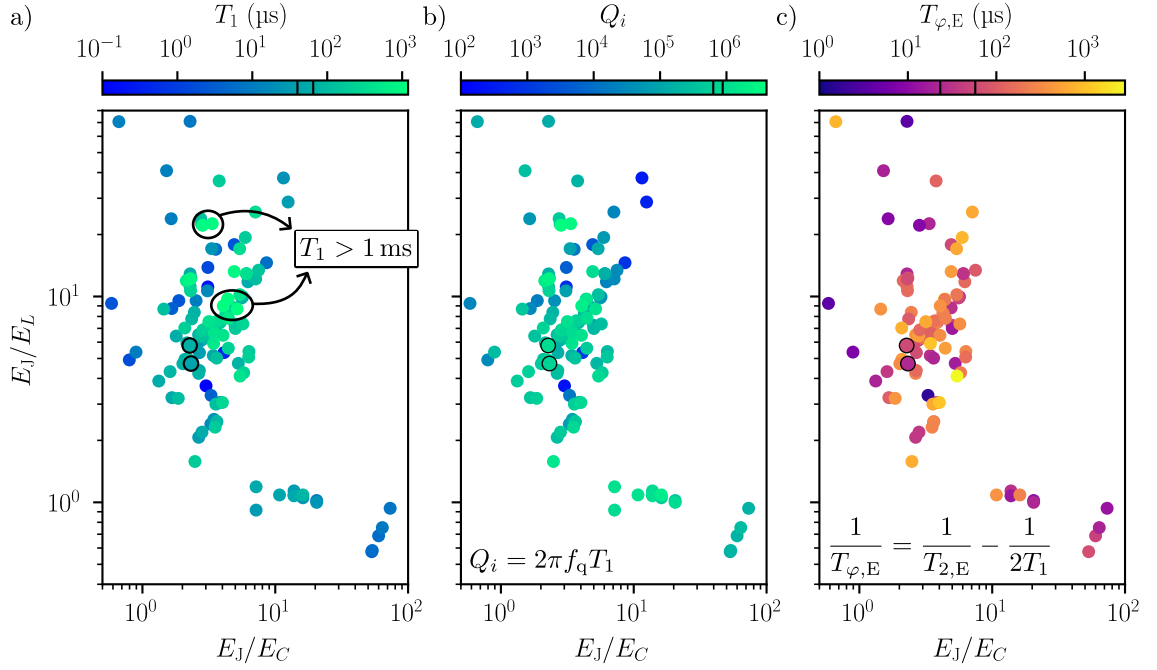


Figure 1.10: Fluxonium coherence. Parameter space E_J/E_C versus E_J/E_L of fluxonium relatives, populated with flux qubit and fluxonium devices from the literature. Data is from database in Ref. [62]. In all panels, the two markers with a black edge are samples from Ref. [2], whose coherence measurements are shown in Chapter 2 (Section 2.4). **a)** In color, relaxation time T_1 at the half-flux sweet spot. **b)** In color, qubit quality factor $Q_i = 2\pi f_q T_1$. **c)** In color, dephasing time $T_{\varphi,E}$ from Hahn echo measurements.

the coherence times $T_{2,E}$ measured by a Hahn-echo sequence. Most fluxonium coherences in the literature are not T_1 -limited, as shown by the relatively low $T_{\varphi,E}$, even for high- T_1 qubits. Identifying a good parameter regime for coherence from this parameter space is less straightforward, as $T_{\varphi,E}$ depends on other parameters such as the dispersive shift χ , governing the qubit-resonator coupling for readout. Due to this coupling, residual photons in the resonator dephase the qubit, a phenomenon referred to as photon shot noise. Dispersive readout is discussed in the next section.

1.2.4 Dispersive readout

In order to measure its state, the qubit is coupled to a linear resonator, in a scheme known as circuit quantum electrodynamics (cQED), by analogy with cavity QED [75]. Considering the qubit as an ideal quantum two-level system, we write the Hamiltonian:

$$\hat{\mathcal{H}} = -\frac{\hbar\omega_q}{2}\hat{\sigma}_z + \hbar\omega_r\hat{a}^\dagger\hat{a} + \hat{\mathcal{H}}_{\text{int}}, \quad (1.56)$$

where ω_q and ω_r are respectively the qubit and resonator angular frequencies, $\hat{\sigma}_z$ is the Pauli operator along the z axis for the qubit, $\hat{a}$ (respectively, $\hat{a}^\dagger$) is the annihilation (respectively, creation) operator for the resonator and $\hat{\mathcal{H}}_{\text{int}}$ is the interaction Hamiltonian.

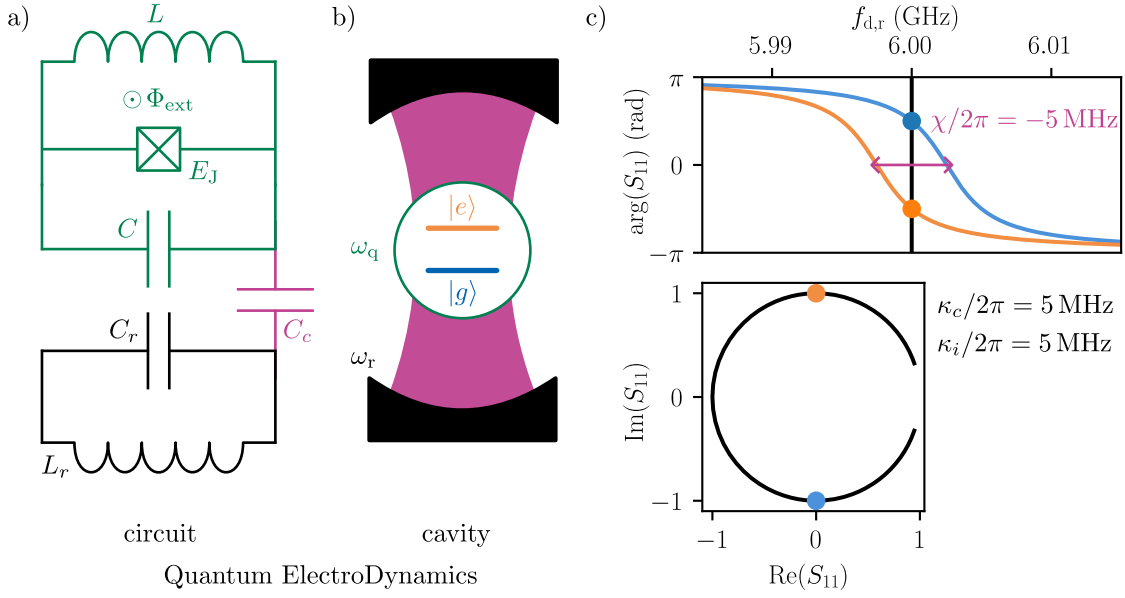


Figure 1.11: Dispersive readout. **a)** Circuit diagram of a fluxonium qubit capacitively coupled to a readout resonator. **b)** Analogy with cavity quantum electrodynamics scheme, where an atom is coupled to the modes of a cavity. **c)** Reflection coefficient S_{11} of a readout resonator coupled to a qubit with negative dispersive shift $\chi/2\pi = -5$ MHz. Top: Phase of S_{11} showing the dispersive shift between the resonator's response when the qubit is in the ground state $|g\rangle$ (blue) or excited state $|e\rangle$ (orange). Bottom: Representation in the complex plane, showing the position of the IQ cloud centers as dots.

The coupling is realized either capacitively or inductively (via mutual inductance or, equivalently, a shared inductance). As an example, we show in Fig. 1.11.a the circuit diagram of a fluxonium coupled via capacitance C_c to the readout resonator, here simply considered as a lumped-element LC resonator. The interaction Hamiltonian reads:

$$\hat{\mathcal{H}}_{\text{int}} = \begin{cases} g_C \hat{n} \text{in}_{\text{ZPF}}^r (\hat{a} - \hat{a}^\dagger) & \text{(capacitive coupling),} \\ g_L \hat{\phi} \varphi_{\text{ZPF}}^r (\hat{a} + \hat{a}^\dagger) & \text{(inductive coupling),} \end{cases} \quad (1.57)$$

with $\varphi_{\text{ZPF}}^r = \sqrt[4]{2E_C/E_L}$ and $n_{\text{ZPF}}^r = \sqrt[4]{E_L/32E_C}$ for the resonator. g_C (respectively, g_L) is the capacitive (respectively, inductive) coupling strength. Both realizations implement a transverse $g \hat{\sigma}_x (\hat{a} + \hat{a}^\dagger)$ or $g \hat{\sigma}_y (\hat{a} - \hat{a}^\dagger)$ coupling.

In the limit of low coupling, $g \ll \hbar\omega_q, \hbar\omega_r$, we can apply the rotating wave approximation (RWA), which yields the Jaynes-Cummings Hamiltonian:

$$\hat{\mathcal{H}}_{\text{JC}} = -\frac{\hbar\omega_q}{2} \hat{\sigma}_z + \hbar\omega_r \hat{a}^\dagger \hat{a} + g(\hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a}^\dagger), \quad (1.58)$$

where $\hat{\sigma}_\pm = \hat{\sigma}_x \pm i\hat{\sigma}_y$ are equivalent for the qubit to the creation and annihilation operators for the resonator (truncated to two levels). Because we removed the counter-rotating terms, the coupling term governs the exchange of only one photon between the qubit and the resonator. In cavity quantum electrodynamics, it is the Hamiltonian that describes an atom in cavity, as schematically represented in Fig. 1.11.b.

With the additional constraint that qubit and resonator are detuned (usually, $\omega_q \ll \omega_r$), the Hamiltonian is approximated by the dispersive Hamiltonian:

$$\hat{\mathcal{H}}_{\text{dispersive}} = -\frac{\hbar\omega_q}{2}\hat{\sigma}_z + \hbar\omega_r\hat{a}^\dagger\hat{a} - \frac{\hbar\chi}{2}\hat{a}^\dagger\hat{a}\hat{\sigma}_z, \quad (1.59)$$

where χ is the dispersive shift. Note that the transverse coupling in Eq. (1.57) becomes, thanks to the qubit-resonator detuning, a longitudinal coupling in $\hat{\sigma}_z\hat{a}^\dagger\hat{a}$. In the case of a transmon, which closely resembles a non-linear resonator, the dispersive shift is well approximated by $\chi \approx (g/\hbar)^2/\Delta$ with $\Delta = \omega_q - \omega_r$ the qubit-resonator detuning. For fluxoniums however, the calculation of χ has to take higher state transitions into account to be reliable. For this reason, χ is computed numerically rather than analytically when designing or characterizing a fluxonium.

The main consequence of the dispersive Hamiltonian in Eq. (1.59) is that the resonator frequency now depends on the state of the qubit. Calculating the Hamiltonian for the resonator subsystem when the qubit is in the ground state $|g\rangle$ or in the excited state $|e\rangle$ yields:

$$\langle g | \hat{\mathcal{H}}_{\text{dispersive}} | g \rangle = \hbar \left(\omega_r - \frac{\chi}{2} \right) \hat{a}^\dagger \hat{a}, \quad (1.60)$$

$$\langle e | \hat{\mathcal{H}}_{\text{dispersive}} | e \rangle = \hbar \left(\omega_r + \frac{\chi}{2} \right) \hat{a}^\dagger \hat{a}. \quad (1.61)$$

This result is illustrated in Fig. 1.11.c by the phase of the reflection coefficient S_{11} , with the phase roll-off being centered around $\omega_r - \chi/2$ (ground state, in blue) or $\omega_r + \chi/2$ (excited state, in orange). Here, χ is chosen to be negative, in accordance with fluxonium qubits at the half-flux-quantum sweet spot. Additionally, we show S_{11} in the complex plane, in the same frequency range (black line) as well as at the bare resonator frequency f_r to show the separation of the two dots representing states $|g\rangle$ and $|e\rangle$ of the qubit. S_{11} is defined and calculated for the bare resonator as:

$$S_{11}(f) = \frac{V_{\text{out}}}{V_{\text{in}}} = 1 - \frac{2\kappa_c}{\kappa_c + \kappa_i + 2i(\omega - \omega_r)}, \quad (1.62)$$

where κ_c (respectively, κ_i) is the coupling (respectively, internal) decay rate of the resonator. S_{11} can be equivalently written in terms of the quality factors $Q_i = \omega_r/\kappa_i$ and $Q_c = \omega_r/\kappa_c$. In actual experiments, the two points in Fig. 1.11.c become clouds (the *IQ clouds*), because the quantum noise and the noise coming from the output chain produce a Gaussian distribution of measurement results.

1.2.5 Other noise-protected qubits

In this section, we propose to explore other qubit designs, that aim at protecting the states from noise by separating the $|g\rangle$ and $|e\rangle$ wavefunctions in parameter space. Such circuits, often more complex than a transmon or a fluxonium, face numerous experimental challenges, in design, fabrication and measurement. We will focus on the most well-known examples, namely the $0-\pi$ qubit and the $\cos(2\varphi)$ qubit.

0- π qubit

The 0- π was originally proposed and studied theoretically in Refs. [76–79]. It was later experimentally realized in Ref. [80]. Figure 1.12 shows the 0- π circuit and each of its modes. The circuit comprises two junctions, two large inductances and two capacitances, each element being equal to the other element of the same type, ideally. It embeds three modes, each of which corresponds to a circuit already studied in Section 1.2. Following Refs. [79, 80], the modes are defined as:

- the transmon mode θ that charges the junctions and the capacitances. It is sensitive to the charge offset n_g^θ on the central superconducting island.
- the fluxonium mode ϕ that charges the junctions and the inductances. It is sensitive to the external flux ϕ_{ext} in the loop of the circuit.
- the harmonic mode ζ that charges only the linear elements, capacitances and inductances.

In case of asymmetry in the parameters, ζ couples to the qubit modes θ and ϕ , introducing noise (in the form of photon shot noise), which constitutes the first experimental challenge of this circuit.

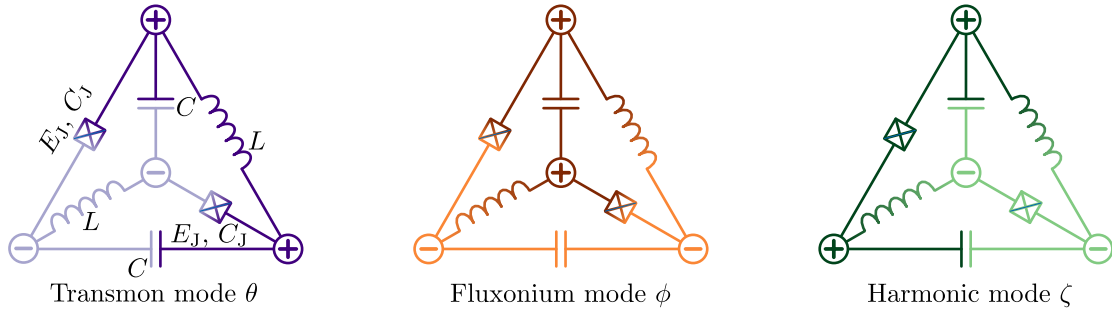


Figure 1.12: 0- π qubit modes. The three modes of the 0- π qubit: the transmon mode θ that is offset-charge sensitive, the fluxonium mode ϕ that is external-flux sensitive and the harmonic mode ζ .

Introducing the charge conjugate variables $\hat{n}_\theta$ and $\hat{n}_\phi$ of phases $\hat{\theta}$ and $\hat{\phi}$, respectively, such that $[\hat{\theta}, \hat{n}_\theta] = i$ and $[\hat{\phi}, \hat{n}_\phi] = i$, the Hamiltonian of the 0- π qubit reads:

$$\hat{\mathcal{H}}_{0-\pi} = 2E_C^\theta \left(\hat{n}_\theta - n_g^\theta \right)^2 + 2E_C^\phi \hat{n}_\phi^2 - 2E_J \cos \hat{\theta} \cos \hat{\phi} + E_L \left(\hat{\phi} + \frac{\phi_{\text{ext}}}{2} \right)^2, \quad (1.63)$$

where $E_C^\theta = e^2/2(C_J + C)$, $E_C^\phi = e^2/2C_J$ are the charging energies associated with the capacitances in the circuit. The Hamiltonian clearly displays θ as a charge mode, dependent on the charge offset n_g^θ and ϕ as a flux mode, dependent on ϕ_{ext} . These modes are coupled through the Josephson junctions.

As previously stated, the core idea behind noise-protected qubits in general is the disjoint support of logical state wavefunctions. To understand it in the context of the 0- π qubit,

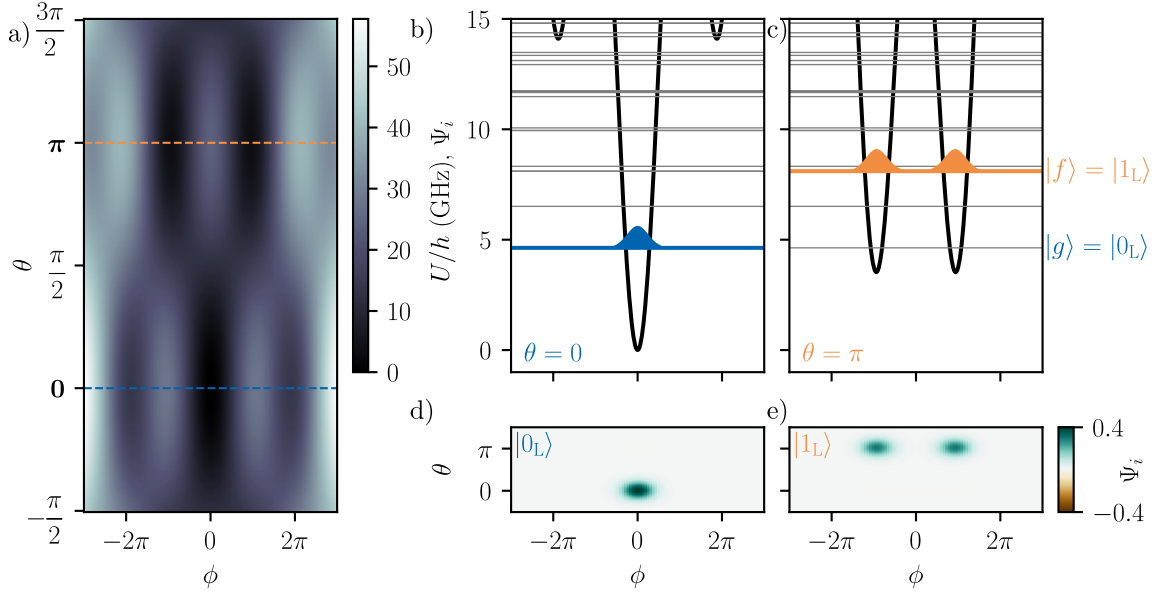


Figure 1.13: 0- π qubit potential and wavefunctions. **a)** Potential energy U versus phase variables ϕ and θ with qubit parameters in the so-called *soft regime* from Ref. [80]. **b, c)** Same potential energy versus ϕ at $\theta = 0$ (b) and $\theta = \pi$ (c), with energy levels in gray and wavefunctions for the two logical qubit states $|0_L\rangle$ and $|1_L\rangle$, corresponding respectively to the first and third states of the system ($|g\rangle$ and $|f\rangle$). **d, e)** Wavefunctions $\Psi_{|0_L\rangle}$ (d) and $\Psi_{|1_L\rangle}$ (e) in the phase parameter space, centered around 0 and π in θ , illustrating the name 0- π qubit.

we take a look at the potential energy $U(\phi, \theta) = -2E_J \cos \theta \cos \phi + E_L \left(\phi + \frac{\phi_{\text{ext}}}{2} \right)^2$, shown in Fig. 1.13. In particular, we chose circuit parameters corresponding to the so-called *soft regime* as realized experimentally in Ref. [80], at $\phi_{\text{ext}} = 0$ and $n_g^\theta = 0$. While this regime only exhibits an exponential suppression of charge noise (flux noise is only linearly suppressed), it is the device closest to the original proposal and it does show the main concepts of the circuit. In Fig. 1.13.a, U is displayed versus both ϕ and θ . Note that, because θ is a compact variable, it is restricted to an interval of length 2π . Potential U exhibits local minima at $\theta = 0$ and $\theta = \pi$, indicated by the horizontal dashed lines, repeated versus ϕ in Fig. 1.13.b ($\theta = 0$) and Fig. 1.13.c ($\theta = \pi$). There, $U(\phi)$ resembles the potential of a fluxonium qubit at the zero-flux sweet spot and the half-flux sweet spot. For this reason, the wavefunctions Ψ_i are also similar to fluxonium wavefunctions. In this soft regime, the logical states $|0_L\rangle$ and $|1_L\rangle$ correspond, respectively, to the first and third states $|g\rangle$ and $|f\rangle$, whose wavefunctions are represented at their energy level in Fig. 1.13.b and Fig. 1.13.c alongside the other energy levels (in gray).

Wavefunctions $\Psi_{|0_L\rangle}$ and $\Psi_{|1_L\rangle}$ in phase space versus ϕ and θ (Fig. 1.13.d and Fig. 1.13.e) illustrate both the name and the advantage of the 0- π qubit. Indeed, $\Psi_{|0_L\rangle}$ is localized in the single potential well at $\theta = 0$, while $\Psi_{|1_L\rangle}$ is the symmetric superposition of the fluxon states in the double-well potential formed at $\theta = \pi$. Thus, the wavefunctions have no overlap, which protects the qubit from energy relaxation. In Ref. [80], this results in relaxation time $T_1 = 1.56$ ms, thanks to the suppressed matrix elements. At the time, this was higher than any transmon or fluxonium implementation. However, the energy separation between the

levels and their dependence to both charge offset n_g^θ and external flux ϕ_{ext} translates into comparatively low dephasing times, with $T_{2,E} = 26 \mu\text{s}$.

$\cos(2\varphi)$ qubit

The $\cos(2\varphi)$ qubit is a type of qubit where the non-linear element creates a tunneling of pairs of Cooper pairs instead of the common Cooper pair tunneling in Josephson junctions. A $\cos(2\varphi)$ element can be realized by leveraging the higher harmonics in junctions made from d-wave superconductors [81], graphene [82] or semiconductors [83, 84]. However, it can also be realized effectively with superconducting interferometric devices, consisting of a superconducting loop with junctions and inductances, biased at half a flux quantum and sometimes referred to as Josephson rhombi or KITEs (Kinetic Inductance co-Tunneling Elements) [85–90]. In the following, we will focus on such implementations.

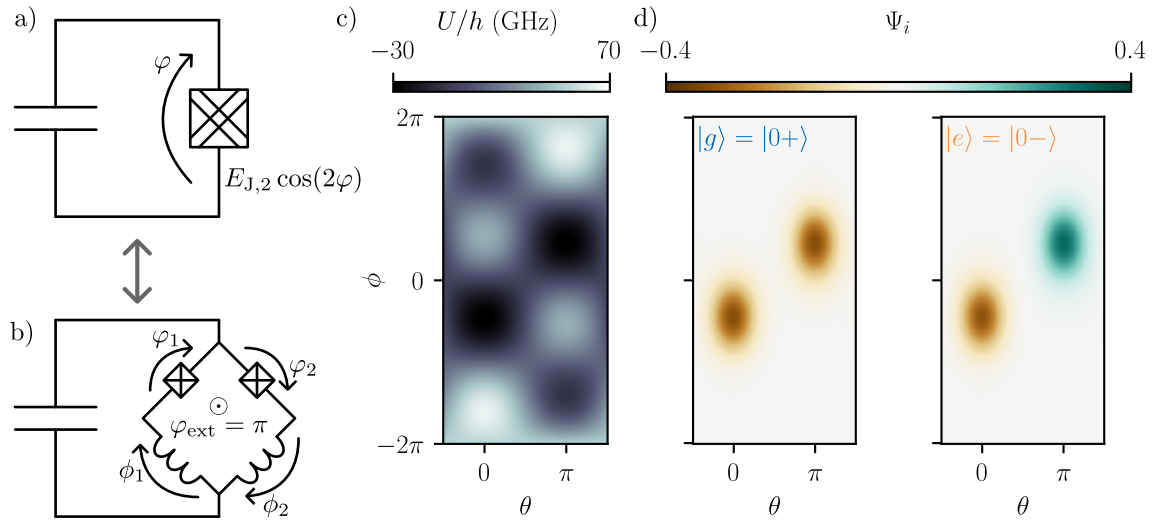


Figure 1.14: $\cos(2\varphi)$ qubit. **a)** Circuit diagram of a $\cos(2\varphi)$ qubit, where the $\cos(2\varphi)$ element is represented by a double-crossed box. **b)** Circuit diagram of a possible implementation with a KITE (Kinetic Inductance co-Tunneling Element) interferometric device, biased at $\varphi_{\text{ext}} = \pi$. **c)** Potential energy U versus variables $\theta = (\varphi_1 - \varphi_2)/2$ and $\phi = (\varphi_1 + \varphi_2)/2$, calculated with parameters from Ref. [89]. **d)** Wavefunctions of the first two states, denoted $|0+\rangle$ and $|0-\rangle$, following the conventions in Ref. [89].

The Hamiltonian these experiments aim at implementing reads

$$\hat{\mathcal{H}}_{\cos(2\varphi)} = 4E_C \hat{n}^2 - E_{J2} \cos(2\hat{\varphi}), \quad (1.64)$$

where φ is the phase across the $\cos(2\hat{\varphi})$ junction, represented Fig. 1.14.a as a double-crossed box, similarly to the standard Josephson junction symbol. The $\cos(2\varphi)$ element is associated with a protection of the circuit, which can be understood by writing it as a tunneling element,

$$\cos(2\hat{\varphi}) = \frac{1}{2}(|n\rangle\langle n+2| + |n+2\rangle\langle n|), \quad (1.65)$$

where $|n\rangle$ is an eigenstate of operator $\hat{n}$, the number of Cooper pairs. Because the tunneling only happens by pairs of Cooper pairs, the transition between states corresponding to even

and odd number of Cooper pairs is forbidden. In particular, this is true for the degenerate ground states $|0+\rangle$ and $|0-\rangle$, where $\pm$ denotes the parity of the number of Cooper pairs.

The circuit of the KITE implementation, as realized in Ref. [89], is shown in Fig. 1.14.b. At the sweet spot bias point, $\varphi = \pi$, Cooper pairs interfere destructively, making the effective $E_J \cos(\varphi)$ term smaller than the second harmonic $E_{J2} \cos(2\varphi)$. Its full Hamiltonian, which reduces to Eq. (1.64) for the lowest levels at $\varphi = \pi$, is:

$$\begin{aligned} \hat{\mathcal{H}}_{\cos(2\varphi), \text{full}} = & 2E_{C_J} \hat{n}_\phi^2 + 2E_{C_J} (\hat{n}_\theta - n_g - \hat{n}_\zeta)^2 + 4E_C \hat{n}_\zeta^2 \\ & + E_L (\hat{\phi} - \varphi_{\text{ext}}/2)^2 + E_L \hat{\zeta}^2 - 2E_J \cos \hat{\theta} \cos \hat{\phi}, \end{aligned} \quad (1.66)$$

with $E_{C_J} = e^2/2C_J$ the charging energy at each junction, E_C the charging energy of the additional capacitance, E_L the inductive energy on each inductance and E_J the Josephson energy of each junction. Degrees of freedom ϕ , θ , ζ are defined as

$$\phi = (\varphi_1 + \varphi_2)/2, \quad (1.67)$$

$$\theta = (\varphi_1 - \varphi_2)/2, \quad (1.68)$$

$$\zeta = (\phi_1 - \phi_2)/2. \quad (1.69)$$

We show in Fig. 1.14.c the potential energy U corresponding to that circuit and Hamiltonian, for the parameters used in Ref. [89] without any asymmetry, at $\zeta = 0$. U exhibits two global minima at $(\theta = 0, \phi = -\pi/2)$ and $(\theta = \pi, \phi = \pi/2)$. Note that θ is a compact variable, limited to the displayed interval, while ϕ is not. In Fig. 1.14.d, the wavefunctions of the degenerate ground states $|0+\rangle$ and $|0-\rangle$ are represented versus the same degrees of freedom. Similarly to, for example, a fluxonium, they are symmetric and antisymmetric superpositions of the states living in the two wells visible in Fig. 1.14.c. In a real device, the degeneracy is slightly lifted by asymmetries in the Josephson energies and inductances and the protection is lower than in an ideal, completely symmetric device.

Others

While most of the community focuses on transmons and fluxoniums, many other designs have been proposed or are being studied at the moment, beyond the $0-\pi$ and $\cos(2\varphi)$ qubits. Let us cite a few of them here.

In Ref. [91], a fluxonium molecule is built by coupling two fluxoniums via a large shared inductance, realizing a system with a tunable magnetic moment. At symmetry points, correlated flux noise is suppressed. In Ref. [92], a $\cos(2\varphi)$ qubit nicknamed the harmonium is proposed, where ground states degeneracy is lifted thanks to a more complex circuit, based on the KITE, allowing to tune the amplitude and sign of the first and second harmonics of the effective Josephson junction. Obviously, other qubit circuit designs have been proposed and studied, with goals other than intrinsic protection. Let us mention the efforts made toward different readout schemes, for example in Refs. [93, 94].

1.2.6 Cat qubits

To conclude this not-strictly-chronological overview of the field of superconducting qubits, let us say a word about cat qubits, which stem from a fairly different philosophy than the superconducting qubits we have considered so far. Here, we give a short primer on cat qubits, and refer the interested reader to Refs. [95–97] for details and calculations.

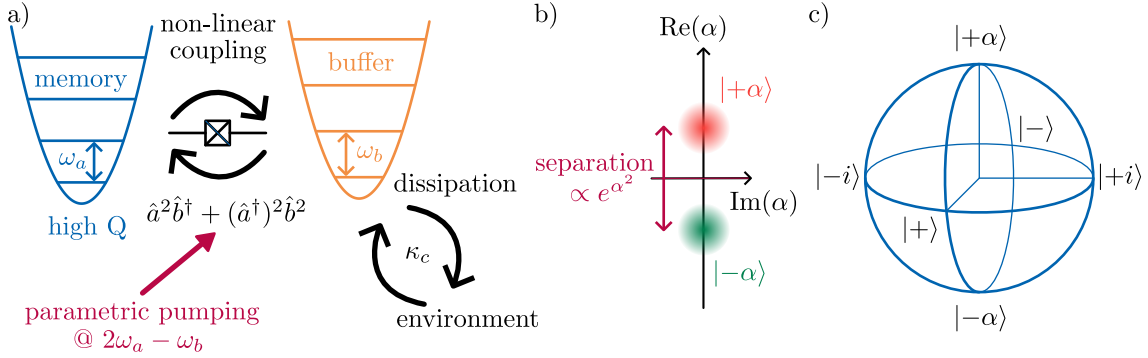


Figure 1.15: Cat qubit. **a)** Schematic representation of memory (blue), buffer (orange), their coupling and parametric pumping (carmine), all elements necessary to engineer a two-photon dissipation process to stabilize coherent states $|\pm\alpha\rangle$ in the memory oscillator. **b)** Schematic representation of coherent states $|\pm\alpha\rangle$ in the complex plane, with separation proportional to e^{α^2} . **c)** Cat qubit Bloch sphere representation.

Indeed, cat qubits rely on storing quantum information not in non-linear circuits (be it a transmon, a fluxonium or something else) but instead in harmonic resonators with high quality factors (i.e. long lifetimes), which in general is referred to as a bosonic code [95]. In particular, the environment of the resonator is engineered to realize an effective two-photon dissipation⁴ [96, 97]. In Fig. 1.15.a, we show a schematic view of a memory resonator (of frequency ω_a and with a high quality factor Q_a) coupled via a Josephson junction (or another Josephson-based non-linear element) to a buffer resonator (of frequency ω_b), which is coupled to the environment. By parametrically pumping at $2\omega_a - \omega_b$, the operator activates a photon exchange process that, thanks to the dissipation of the buffer photons to the environment, effectively implements a two-photon dissipation channel for the memory. This process stabilizes coherent states $|\pm\alpha\rangle$ and ensures that the qubit does not leave the computational basis. Coherent states are the eigenstates of the annihilation operator $\hat{a}$, such that $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$. They can be expressed in the Fock space as

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{+\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle. \quad (1.70)$$

In Fig. 1.15.b, coherent states $|\pm\alpha\rangle$ and $|\mp\alpha\rangle$ are displayed in the complex plane, with a spatial extension representing noise. These coherent states are the logical $|0\rangle$ and $|1\rangle$ states of the qubit, located at the poles of the Bloch sphere shown in Fig. 1.15.c. The separation between them scales exponentially with the number of photons α^2 in the

⁴ Engineering a four-photon dissipation gives additional protection and leads to so-called four-legged cats.

memory, which results in an exponential suppression of bit-flip errors [98], with bit-flip times now exceeding ten seconds [99]. This very favorable scaling is the advantage of cat qubits. Phase-flip errors, however, scale linearly with the number of photons. Indeed, let us consider the states on the equator $|\pm\rangle = (|+\alpha\rangle \pm |-\alpha\rangle)/\sqrt{2}$. Because of the symmetric and antisymmetric superpositions, they correspond to linear decompositions on only even or odd Fock states. Therefore, the loss of one photon (via operator $\hat{a}$) changes the parity of photon number and triggers a phase-flip error. The asymmetry between bit-flip and phase-flip error motivates the name *biased qubits* for such an implementation. In processor designs, phase-flips are corrected by repetition codes, i.e. by redundancy.

1.2.7 Fluxon-based circuits

Superconducting circuits do not generally have to be qubits. In this section, we introduce circuits based on fluxons tunneling between superconducting loops via coupling junctions and model them with quantum phase-slip (QPS) elements, similarly to Ref. [60].

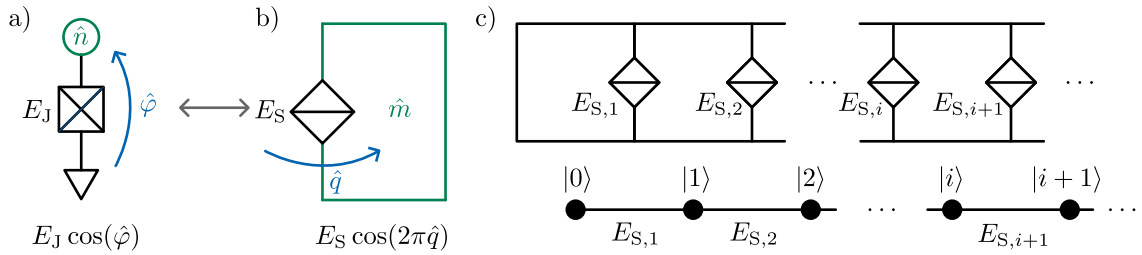


Figure 1.16: Fluxons in superconducting circuits. **a)** Equivalent circuit of a Josephson junction implementing the tunneling of Cooper pairs via Hamiltonian $E_J \cos(\hat{\phi})$. **b)** Equivalent circuit of a quantum phase-slip (QPS) junction implementing the tunneling of fluxons via Hamiltonian $E_S \cos(2\pi\hat{q})$. **c)** Chain of superconducting loops coupled by QPS junctions (top) and equivalent tight-binding model for one fluxon in the circuit (bottom).

As we come to the end of this introductory chapter, we have learned about three superconducting circuit elements: capacitance, inductance and Josephson junction. Capacitance and inductance are dual of each other, which begs the question: what is the dual of a Josephson junction? Is there such a thing as a non-linear capacitance, by analogy with the non-linear inductance given by a Josephson junction? Such an element does exist, and is called a quantum phase-slip (QPS) junction [100–104].

In Fig. 1.16, we show the equivalent circuit of a Josephson junction (Fig. 1.16.a) and its dual QPS junction (Fig. 1.16.b). To describe the Josephson junction, as seen in Section 1.1.3, we use the phase $\hat{\phi}$ and charge $\hat{n}$ conjugate variables, and write the Hamiltonian of the Josephson junction

$$\hat{\mathcal{H}}_{JJ} = E_J \cos(\hat{\phi}) = \frac{E_J}{2} \sum_n \left(|n\rangle\langle n+1| + |n+1\rangle\langle n| \right), \quad (1.71)$$

where E_J is the Josephson energy and $|n\rangle$ is the eigenstate of the charge operator such that $\hat{n}|n\rangle = n|n\rangle$, hence the state of n Cooper pairs on the island (green circle in Fig. 1.16.a).

By analogy, the QPS junction is described by the following Hamiltonian:

$$\hat{\mathcal{H}}_{\text{QPS}} = E_S \cos(2\pi\hat{q}) = \frac{E_S}{2} \sum_m \left(|m\rangle\langle m+1| + |m+1\rangle\langle m| \right), \quad (1.72)$$

where $\hat{q}$ is the quasicharge operator, $\hat{m}$ is the fluxon operator and $|m\rangle$ is the eigenstate of the fluxon operator such that $\hat{m} |m\rangle = m |m\rangle$, i.e. the state with m fluxons in the green loop in Fig. 1.16.b. $\hat{q}$ and $\hat{m}$ are conjugate variables and obey the commutation relation $[\hat{q}, \hat{m}] = i$.

When the QPS junction is realized by a Josephson junction, the phase-slip energy E_S can be written as the tunneling rate through said junction [59–61]:

$$E_S = \frac{4}{\sqrt{\pi}} (8E_J E_C)^{1/4} e^{-\sqrt{8E_J/E_C}}, \quad (1.73)$$

with E_J the Josephson energy and E_C the capacitive energy associated to the junction capacitance C_J . Note that we have already used this expression when discussing the fluxonium qubit first transition frequency at the half-flux sweet spot (Section 1.2.3). More generally, and away from the sweet spot, the fluxonium Hamiltonian can be approximated by a QPS Hamiltonian [61, 105, 106]:

$$\hat{\mathcal{H}}_{\text{QPS fluxonium}} = 2\pi^2 E_L \left(\hat{m} + \frac{\Phi_{\text{ext}}}{\Phi_0} \right)^2 + E_S \cos(2\pi\hat{q}). \quad (1.74)$$

In Ref. [61], an additional second-harmonic term $\cos(4\pi\hat{q})$ is used to capture bifluxon transitions at $\Phi_{\text{ext}} = 0$.

QPS junctions allow us to imagine and design circuits such as the chain of superconducting loops depicted in Fig. 1.16.c, inspired by Ref. [60]. Truncating to the one-fluxon manifold, the Hamiltonian of this circuit can be expressed as a tight-binding Hamiltonian:

$$\hat{\mathcal{H}}_{\text{QPS chain}} = \sum_i U_i |i\rangle\langle i| - \sum_i \frac{E_{S,i}}{2} \left(|i\rangle\langle i+1| + |i+1\rangle\langle i| \right), \quad (1.75)$$

with U_i the energy on loop i and $E_{S,i}$ the tunneling energy between loops $i-1$ and i . Depending on the values of U_i and $E_{S,i}$ (which tunes the barrier height between fluxon sites), the eigen-wavefunctions of this system are delocalized over the whole circuit or localized on a few loops. Such circuits could be used as an alternative platform for quantum simulation, where fluxons play the role of photons in usual architectures [60]. At the end of Chapter 2, in Section 2.5, we will look at such a chain of loops with periodic boundary conditions, in order to build a protected qubit from separated wavefunctions, a system we call a four-loop fluxonium.

2 Simultaneous flux-locking of gradiometric fluxoniums

This chapter aims at demonstrating the flux-locking mechanism in gradiometric fluxoniums, whereby a fluxon is trapped in the circuit during cooldown and intrinsically biases the qubit to the half-flux sweet spot. After describing the phenomenon, we show how it can solve the flux crosstalk issues encountered in quantum processors based on flux qubits and other flux-tunable elements by simultaneously locking eight fluxonium samples in a single waveguide. By compensating the inductive asymmetry between the two loops of the design, we are able to lock the effective flux-bias within $\Phi_{\text{eff}} = -3 \times 10^{-4} \Phi_0$ from the target, corresponding to only 15 % degradation in $T_{2,E}$ when operated in zero external field. Additionally, the fluxon is retained by the aluminum ring indefinitely, which is a crucial step toward fluxon-based circuits with more degrees of freedom. In particular, we will explore a four-loop circuit aimed at offering noise protection thanks to its delocalized qubit states.

2.1 Motivations

The main motivation for using a gradiometric geometry in a flux-tunable qubit is generally the suppressed susceptibility to global magnetic field. Typically, it allows experimentalists to integrate a SQUID loop as the main Josephson junction in a flux qubit, so as to make it gap-tunable [107–109]. The geometry also opens the door to hybrid systems made by coupling a flux qubit or a fluxonium to field-tunable physical systems such as spins or molecules [37, 110]. On top of the suppressed susceptibility to magnetic field, gradiometric fluxoniums also come with a flux-locking mechanism by which a fluxon trapped during cooldown intrinsically biases the qubit to the half-flux sweet spot. While the flux-locking mechanism is used in previously cited gap-tunable flux qubit implementations, there has not been, to our knowledge, any systematic study of its accuracy and reproducibility.

A previous implementation of gradiometric fluxoniums in the group has demonstrated the flux-locking mechanism and uncovered a major caveat: the granular aluminum outer ring experiences phase slips, which manifest in the qubit effective flux bias as a fluxon escape, i.e. the qubit jumps from the half-flux to the zero-flux sweet spot or vice versa. Fluxon escapes are mediated by ionizing radiation interactions with the substrate in the vicinity of the device, which are completely shielded in underground conditions [111]. Therefore, the first and most crucial goal of this chapter is to recover the expected indefinitely-long

conservation of flux in above-ground conditions. This short-term goal is also crucial to our longer-term goal, which is to realize a multi-loop fluxonium qubit with several degrees of freedom, based on Ref. [60]. In such a circuit with a trapped fluxon, qubit states are encoded in superpositions of fluxons states and can be expected to be noise-protected by topology.

Both the suppressed field susceptibility and the flux-locking mechanism make the gradiometric fluxonium a prime candidate to solve one of the specific challenges of scaling up fluxonium processors: flux crosstalk created by neighboring flux lines [112]. Indeed, characterizing and inverting the flux-crosstalk matrix is a complex task that becomes increasingly computationally demanding with the number of flux-tunable qubits, couplers and resonators on chip. It is rendered difficult by the non-linearity of the joint frequency response of these elements [113, 114] and requires frequent recalibration due to flux offset drifts and jumps over a cooldown. We argue that using gradiometric fluxoniums in a quantum processor reduces the flux crosstalk from neighboring flux lines by orders of magnitude, thanks to their sensitivity to the magnetic field gradient only. Additionally, the flux locking keeps the fluxoniums at the half-flux sweet spot with little to no static biasing, which further reduces the field seen by neighboring qubits as well as the heat load in the cryostat. This motivation is the one that lead to publishing Ref. [2], which this chapter is based on, as well as Ref. [115] from Prof. Yasunobu Nakamura's team, performed and published in parallel.

2.2 Device presentation

In this section, we present the gradiometric fluxonium in general terms and derive its Hamiltonian to prove that it is exactly equivalent to a single-loop fluxonium. Then, we turn to the fabricated samples and discuss design and fabrication techniques. Finally, we characterize the spectrum of eight samples and determine their fluxonium parameters E_J , E_C and E_L .

2.2.1 General introduction to gradiometric flux qubits

While we can see the gradiometric fluxonium as a fluxonium with two loops, it is relevant to picture it as a superconducting ring split in two halves by a wire consisting of a Josephson junction and an inductor, as in Fig. 2.1.a. Starting from 0 at the bottom, the phase acquired by traveling around is $2\pi \times m$, where m is the number of fluxons trapped in the ring. In particular, in a symmetric device, the phase directly across from the starting point is 0 or π modulo 2π depending on the parity of m . In Fig. 2.1.b we represent m versus magnetic field B_{cd} applied during cooldown. As expected from flux quantization (see Section 1.1.2 and Refs. [12, 14]), m exhibits plateaus following

$$m = \left\lfloor \frac{B_{cd}}{\Phi_0/A_{ring}} \right\rfloor, \quad (2.1)$$

with Φ_0 the magnetic flux quantum and A_{ring} the area enclosed by the outer ring of the device. In other words, it is enough to target $\Phi_0/2A_{\text{ring}} < B_{\text{cd}} < 3\Phi_0/2A_{\text{ring}}$ to lock exactly one fluxon in the device. This property of flux-locking is crucial, as it makes it both easy to apply and robust to local deviations of the magnetic field, as long as they do not exceed the width of a plateau. When m is odd (in green), the junction is π -phase biased, making the fluxonium intrinsically locked at the half-flux-quantum sweet spot. We designate it as π -locked. When m is even (in orange), including zero, the fluxonium is locked at the zero flux sweet spot and designated as 0-locked.

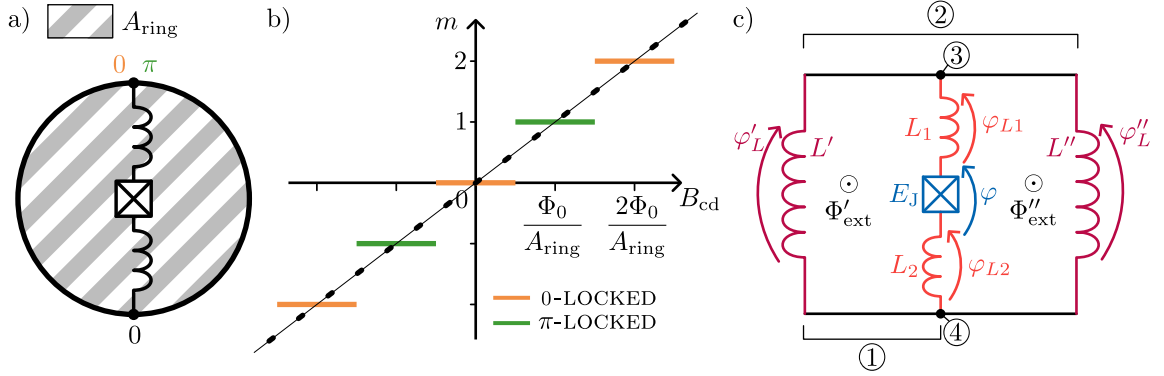


Figure 2.1: Gradiometric fluxonium. a) Simplified gradiometric fluxonium circuit, where the inductance and Josephson junctions are included on the diameter of the outer ring. The number of trapped fluxons m gives a total phase 0 or π depending on its parity. b) Number of locked fluxons m versus magnetic field during cooldown B_{cd} , for a gradiometric fluxonium with an outer ring area A_{ring} . Such quantization is exactly the one historically observed on a superconducting ring in Refs. [12, 14] and illustrated in Fig. 1.2.c. c) Full gradiometric fluxonium circuit, consisting of a Josephson junction in the middle, and two inductive loops. As an external magnetic field is threaded through the loops, each of them contributes respectively with Φ'_{ext} and Φ''_{ext} to the effective flux Φ_{eff} .

We will now derive the Hamiltonian of the general gradiometric fluxonium circuit depicted in Fig. 2.1.c, applying the same method as in Section 1.2. Let us first write the Lagrangian

$$\mathcal{L} = \frac{C}{2} \dot{\Phi}^2 - \frac{1}{2L'} \Phi_L'^2 - \frac{1}{2L_1} \Phi_{L1}^2 - \frac{1}{2L_2} \Phi_{L2}^2 - \frac{1}{2L''} \Phi_L''^2 + E_J \cos \varphi, \quad (2.2)$$

which so far has 5 degrees of freedom. The degrees of freedom are, however, not independent, and we can write a total of four relations to eliminate four of them. First are the loop relations coming from flux quantization:

$$\textcircled{1} \quad \varphi_L' - \varphi_{L1} - \varphi - \varphi_{L2} + \varphi'_{\text{ext}} = 0, \quad (2.3)$$

$$\textcircled{2} \quad \varphi_L' - \varphi_L'' + \varphi'_{\text{ext}} + \varphi''_{\text{ext}} = 2\pi m. \quad (2.4)$$

Note how in Eq. (2.4), the right-hand-side term is the total number m of fluxons trapped in the outer ring. Second are two relations coming from Kirchhoff's current law applied at points $\textcircled{3}$ and $\textcircled{4}$:

$$\textcircled{3} \quad \frac{\varphi_L'}{L'} + \frac{\varphi_{L1}}{L_1} + \frac{\varphi_L''}{L''} = 0, \quad (2.5)$$

$$\textcircled{4} \quad \frac{\varphi_L'}{L'} + \frac{\varphi_{L2}}{L_2} + \frac{\varphi_L''}{L''} = 0. \quad (2.6)$$

From Eqs. (2.5) and (2.6), we get

$$\varphi_{L1} = -L_1 \left(\frac{\varphi'_L}{L'} + \frac{\varphi''_L}{L''} \right), \quad (2.7)$$

$$\varphi_{L2} = -L_2 \left(\frac{\varphi'_L}{L'} + \frac{\varphi''_L}{L''} \right), \quad (2.8)$$

and already the Lagrangian becomes

$$\mathcal{L} = \frac{C}{2} \dot{\Phi}^2 - \frac{L' + L_1 + L_2}{2L'^2} \Phi_L'^2 - \frac{L'' + L_1 + L_2}{2L''^2} \Phi_L''^2 - \frac{L_1 + L_2}{L'L''} \Phi_L' \Phi_L'' + E_J \cos \varphi. \quad (2.9)$$

With Eqs. (2.3) and (2.4) we get:

$$\begin{pmatrix} \frac{L' + L_1 + L_2}{L'} & \frac{L_1 + L_2}{L''} \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \varphi'_L \\ \varphi''_L \end{pmatrix} = \begin{pmatrix} \varphi - \varphi'_{\text{ext}} \\ 2\pi m - \varphi'_{\text{ext}} - \varphi''_{\text{ext}} \end{pmatrix}. \quad (2.10)$$

Hence,

$$\varphi'_L = \Lambda \left(\varphi - \varphi'_{\text{ext}} + \frac{L_1 + L_2}{L''} (2\pi m - \varphi'_{\text{ext}} - \varphi''_{\text{ext}}) \right), \quad (2.11)$$

$$\varphi''_L = \Lambda \left(\varphi - 2\pi m + \varphi''_{\text{ext}} - \frac{L_1 + L_2}{L'} (2\pi m - \varphi'_{\text{ext}} - \varphi''_{\text{ext}}) \right), \quad (2.12)$$

with

$$\Lambda = \frac{L'L''}{L'L'' + (L_1 + L_2)(L' + L'')} \quad (2.13)$$

Finally, we get the gradiometric fluxonium Hamiltonian, which is exactly equivalent to the fluxonium Hamiltonian (see Section 1.2.3),

$$\hat{\mathcal{H}} = 4E_C \hat{n}^2 + \frac{E_L}{2} \left(\hat{\varphi} + \varphi_{\text{eff}} + 2\pi m \frac{1 + \alpha}{2} \right)^2 - E_J \cos \hat{\varphi}, \quad (2.14)$$

where m is the number of trapped fluxons, $\alpha = (L'' - L')/(L'' + L')$ is the inductive asymmetry between the loops, and the effective flux bias is given by

$$\Phi_{\text{eff}} = \varphi_{\text{eff}} \frac{\Phi_0}{2\pi} = \frac{\Phi'_{\text{ext}} - \Phi''_{\text{ext}}}{2} + \alpha \frac{\Phi'_{\text{ext}} + \Phi''_{\text{ext}}}{2}, \quad (2.15)$$

where Φ'_{ext} and Φ''_{ext} are the external fluxes created by B_{ext} in each loop. Note that the effective flux Φ_{eff} only designates the flux created by the applied magnetic field B_{ext} , which we explicitly distinguish from the flux arising from the trapped fluxons and given by $m(1 + \alpha)/2$.

2.2.2 Chip design and fabrication

The design is based on the circuit shown in Fig. 2.1.c, for which we derived the Hamiltonian in Eq. (2.14). In order to mitigate the flux escapes observed in Ref. [111], we replace the granular aluminum in the outer ring by pure aluminum, whose orders-of-magnitude higher superfluid stiffness ($\rho_{s,\text{Al}}/\rho_{s,\text{grAl}} \sim 100$) ensures a suppressed susceptibility to quasiparticle-burst-activated phase slips. As a result, the majority of the inductance in our circuit stems from the inductive wires in the center.

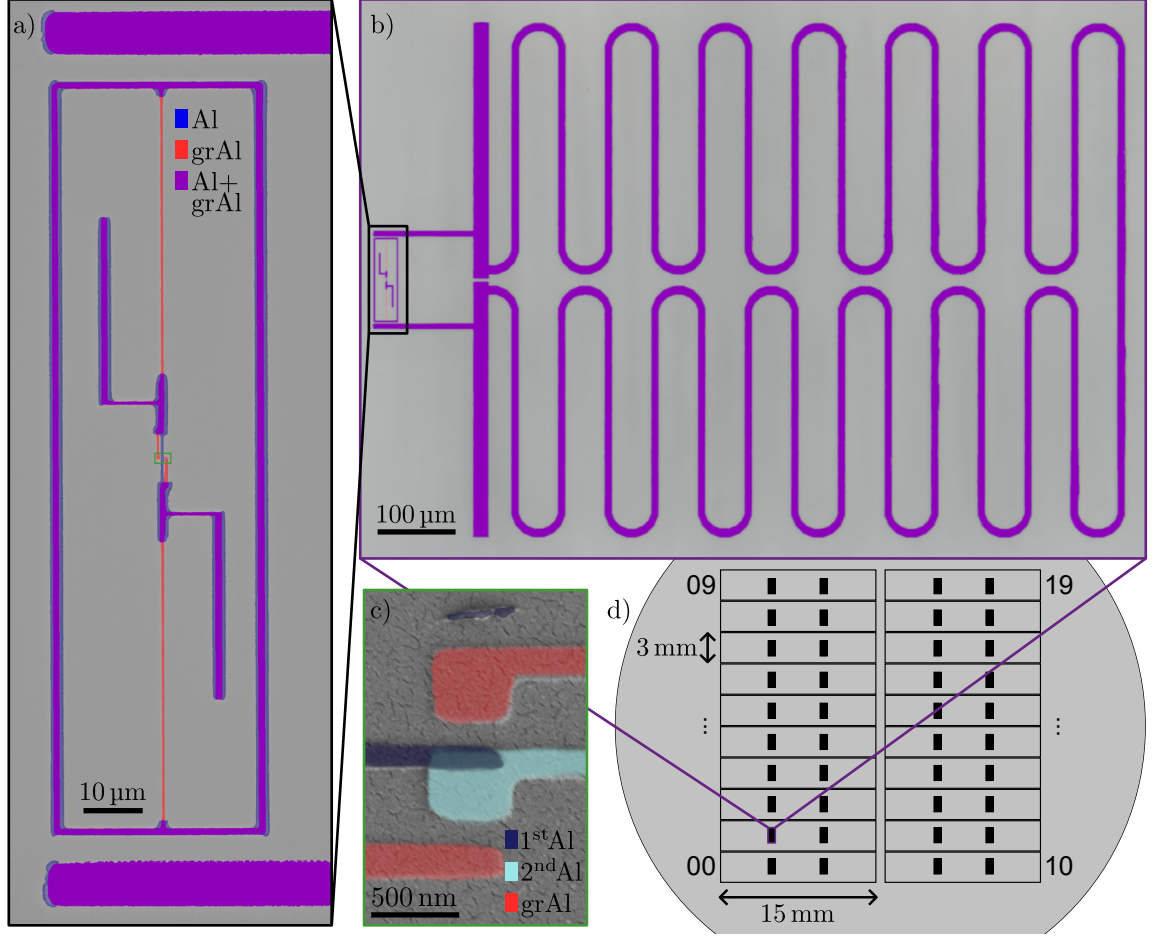


Figure 2.2: Sample pictures. **a, b)** Optical microscope pictures of the qubit (a) and the aluminum LC resonator (b) for sample nicknamed 01L. The samples are fabricated in aluminum (Al) and granular aluminum (grAl). Each sample's resonator is designed with a different number of meanders in order to spread the frequencies between 5.5 GHz and 9 GHz. **c)** Scanning electron microscope picture of the Josephson junction at the center of the sample, showing how the first two layers of aluminum overlap, while the third layer of grAl is not connected. **d)** Schematics of the twenty 3 mm × 15 mm chips fabricated on a single 2-inch sapphire wafer. Each chip is labeled by its column and row indices. Each qubit is additionally labeled L (left) or R (right). For redundancy, the columns are identical.

Figure 2.2.a shows a false-colored optical microscope picture of one such sample, where the aluminum ring encloses the constitutive elements of the fluxonium: inductors (made of grAl, in red), capacitive pads (in purple) and Josephson junction (in blue). A zoom-in on

the Josephson junction, taken in scanning electron microscopy, is provided in Fig. 2.2.c and depicts the overlap between the first two aluminum layers, forming a typical SIS junction. The grAl pads visible in Fig. 2.2.c come from the third step of evaporation and do not participate in the junction. Each fluxonium sample is capacitively coupled to an aluminum LC resonator for readout (see Fig. 2.2.b). The resonators are designed with a different number of meanders and central distance in order to spread the frequencies between 5.5 GHz and 9 GHz and linewidths between 100 kHz and 20 MHz. These patterns allow us to fabricate a total of forty qubit-and-resonator samples on a single 2-inch sapphire wafer, as schematically represented in Fig. 2.2.d, and measure up to twelve simultaneously. Each $3\text{ mm} \times 15\text{ mm}$ chip hosts two qubits with different resonator frequencies and is labeled according to its row and column indices. Columns are identical in order to have redundancy.

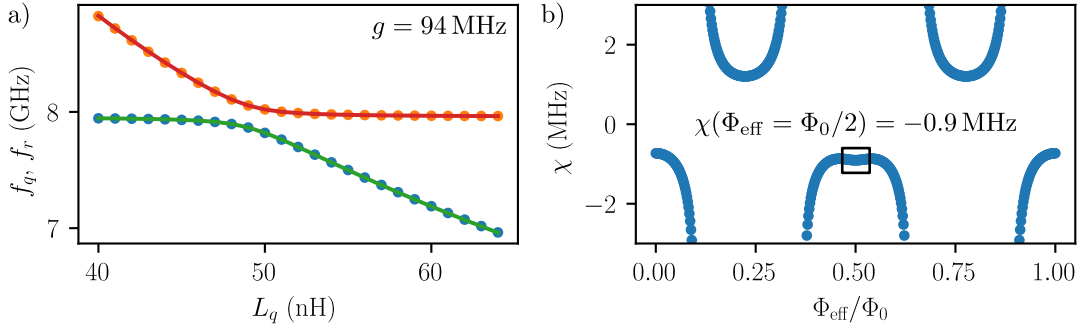


Figure 2.3: Simulation of qubit-resonator coupling. **a)** Finite-element simulation of the qubit and resonator frequencies (dot markers). In these classical simulations, both the qubit and the resonator are considered harmonic oscillators. The qubit’s inductance L_q is swept in order to control the qubit’s frequency. Coupling g is fitted at the anticrossing (solid lines). **b)** Computation of the dispersive shift for a fluxonium qubit with the mean parameters of Table 2.1, a resonator at frequency $f_r = 7\text{ GHz}$ and qubit-resonator coupling $g = 94\text{ MHz}$ obtained from electromagnetic simulations. At the half-flux sweet spot, $\chi = -0.9\text{ MHz}$. The discontinuities in χ are due to higher-order contributions from the $1-2$ transition crossing the resonator.

The resonators and their frequencies were designed with finite-element simulations. Additionally, we simulate the qubit-resonator coupling g (see Fig. 2.3.a) and compute the expected dispersive shift χ (see Fig. 2.3.b). At the half-flux sweet spot, we expect $\chi \approx -1\text{ MHz}$. In general, it is recommended to target $\chi \approx \kappa$, in order to balance the effects of a high χ/κ (increased photon-shot noise, i.e. resonator photons dephase the qubit and thus drastically reduce T_2) and a low χ/κ (more readout photons needed, because each of them carries little information). In our case, since we use granular aluminum fluxoniums that can be read with large photon numbers without quantum demolition [116] and dimer Josephson amplifiers (DJA) that are low-noise, near-quantum-limited amplifiers [117], we can tolerate going to the low χ/κ regime and benefit from the increased T_2 .

The fabrication recipe is similar to Ref. [36]. All samples shown in this chapter were fabricated on a single double-side polished c-plane $330\text{-}\mu\text{m}$ -thick sapphire wafer. A double stack of MMA(8.5)MAA EL13 and 950 PMMA A4 resists was spincoated on the wafer and patterned using a 50 keV e-beam lithography tool. After developing in a IPA/H₂O 3:1 solu-

tion for 2 min 30 s, aluminum was deposited in a three-angle shadow evaporation process in a Plassys e-beam evaporation system. The thin film deposition recipe includes plasma cleaning and Ti gettering to clean the sample and the chamber, -31° -tilted aluminum evaporation at 1 nm/s for 20 s, 6-min oxidation at 11.54 mbar, 31° -tilted aluminum evaporation at 1 nm/s for 30 s, argon milling to ensure contact, and granular aluminum deposition consisting of a 0° aluminum evaporation at 1 nm/s for 36 s under a constant oxygen flow at 5.0 sccm. After lift-off in hot acetone (50°C , 13 h) and hot NEP (90°C , 27 min), the wafer was cleaned in acetone and IPA. Finally, the wafer was diced in $3\text{ mm} \times 15\text{ mm}$ chips after coating it with a protective S1818 G2 photo-resist, which is removed in hot DMSO (80°C , 10 min).

2.2.3 Device characterization

Six chips, each hosting two qubit-resonator samples, are mounted in the same 3D copper waveguide, equipped with a global magnetic field coil, similarly to Refs. [118, 119] (see also the experimental setup in Appendix Section A). We perform standard dispersive microwave reflection measurements at 30 mK. In order to achieve single-shot readout, we employ a dimer Josephson amplifier (DJA) [117] operated in reflection. The spectra of eight working samples a–h, measured with two-tone spectroscopy in the same waveguide over several cooldowns, are shown in Fig. 2.4. The transition frequencies were extracted by fitting a Lorentzian function at each field, and are represented as different markers depending on the cooldown. The spectra are fitted to the (gradiometric) fluxonium Hamiltonian (see Eq. (2.14)) using the scqubits Python package [55]. All measured transitions, including multi-photon transitions (e.g. $0 - 2/2$ on samples d, e, f and g) and transitions with the resonator levels (e.g. $0 - 1 + f_{\text{res}}$ in samples a, b, c, d and f) are taken into account in the fitting routine. Together with thermal resets under different magnetic fields B_{cd} , it enables a robust characterization of fluxonium parameters, including in cases where the low flux susceptibility prevented us from measuring a full spectrum in one cooldown. Spectra are shown as a function of $\Phi_{\text{tot}} = \Phi_{\text{eff}} + m(1 + \alpha)/2$.

Sample	E_J/h (GHz)	E_C/h (GHz)	E_L/h (GHz)
• a	9.21 ± 0.16	3.97 ± 0.07	1.95 ± 0.05
• b	9.19 ± 0.16	4.05 ± 0.04	1.59 ± 0.04
• c	7.50 ± 0.13	4.48 ± 0.07	1.96 ± 0.03
• d	8.87 ± 0.23	3.88 ± 0.03	1.87 ± 0.03
• e	9.06 ± 0.15	4.09 ± 0.06	1.84 ± 0.05
• f	8.45 ± 0.15	4.16 ± 0.05	2.05 ± 0.05
• g	8.90 ± 0.14	4.09 ± 0.08	1.58 ± 0.04
• h	9.22 ± 0.15	3.97 ± 0.07	2.01 ± 0.06

Table 2.1: Gradiometric fluxonium parameters for all eight samples a–h.

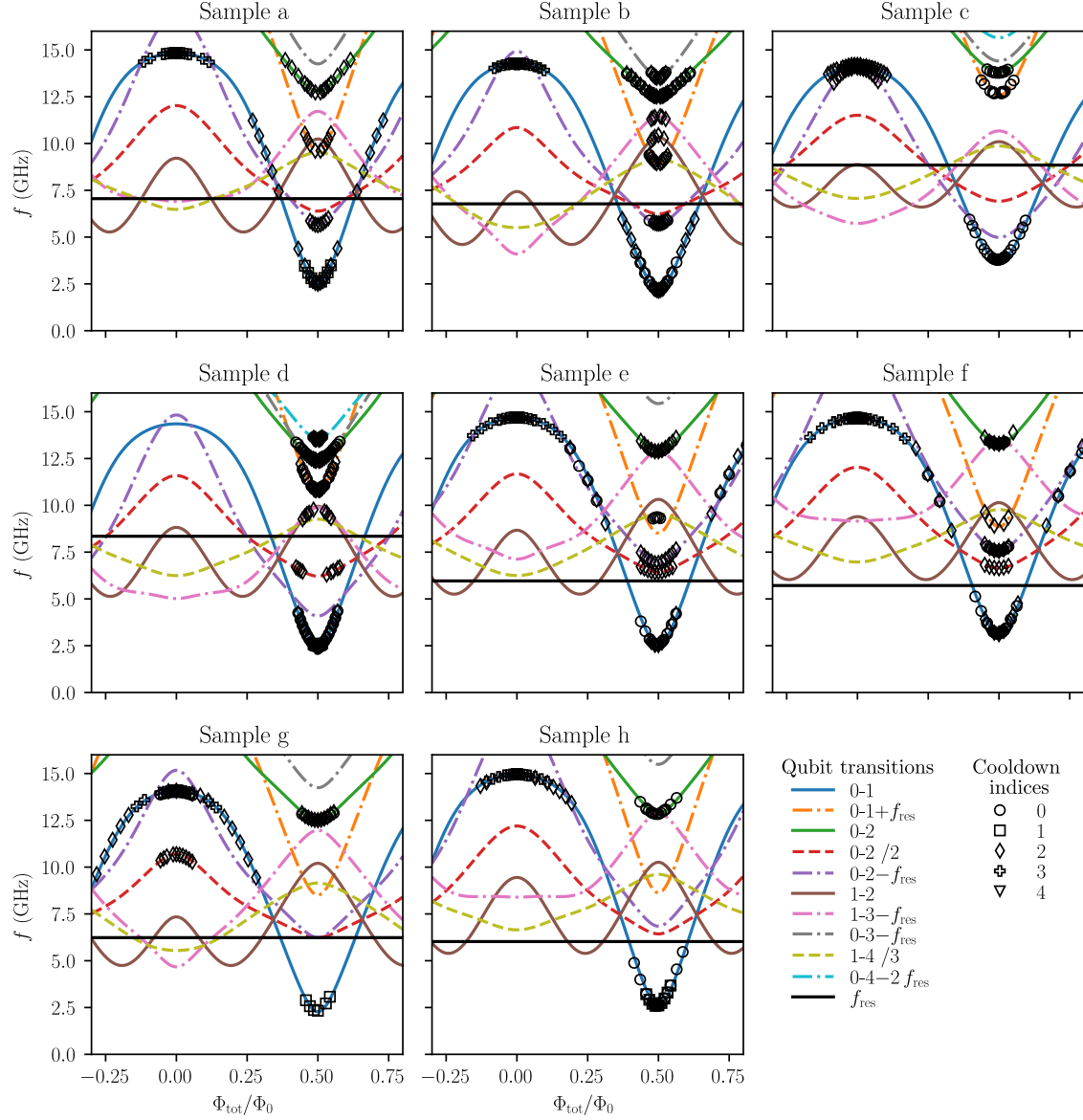


Figure 2.4: Spectra of all eight gradiometric fluxonium samples. Spectra with fits of all eight samples measured in this chapter. Markers depict the transition frequencies extracted from a two-tone spectroscopy, with marker type indicating different cooldowns. Colored solid lines represent the main transition frequencies, dashed lines represent multiple-photon transitions and dash-dotted lines represent transitions mediated by resonator photons. Black solid lines represent the readout resonator frequency f_r of each sample.

All samples have similar fluxonium parameters (see Table 2.1), with mean values

$$\bar{E}_J/h = 8.80 \text{ GHz}, \quad \bar{E}_C/h = 4.09 \text{ GHz}, \quad \bar{E}_L/h = 1.86 \text{ GHz}, \quad (2.16)$$

or in other words, $\bar{C} = 4.7 \text{ fF}$ and $\bar{L} = 88 \text{ nH}$, corresponding to a mean sheet inductance $\bar{L}_{\square, \text{grAl}} = 0.22 \text{ nH}/\square$ for the granular aluminum. Because all of these samples were fabri-

cated in one evaporation, it is interesting to look at the relative standard deviation of their parameters:

$$\frac{\sigma(E_J)}{\bar{E}_J} = 6.2\%, \quad \frac{\sigma(E_C)}{\bar{E}_C} = 4.2\%, \quad \frac{\sigma(E_L)}{\bar{E}_L} = 9.1\%.$$

We note that the dispersion is overall below 10 %. We also note that, unsurprisingly, the most stable parameter is the capacitance, as it is mostly set by geometry, while the least stable is the inductance set by the resistivity of grAl.

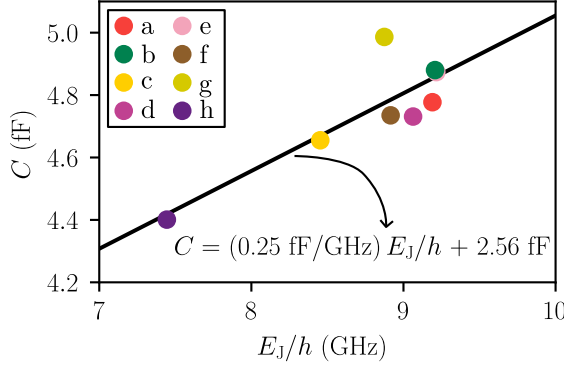


Figure 2.5: E_J and C scaling. Gradiometric fluxonium total capacitance C versus Josephson energy E_J for all eight samples. C follows a linear trend consistent with the fact that both C_J and E_J are proportional to the overlap area of the junction. The remaining capacitance is the geometric capacitance $C_{\text{geo}} = 2.56$ fF.

In Fig. 2.5 we plot the total capacitance C for these samples as a function of their Josephson energy E_J . For a Josephson junction of overlap area A_{overlap} , we expect $E_J \propto A_{\text{overlap}}$ (which intuitively can be understood as adding inductors in parallel, which brings the inductance down, hence E_J up) and the intrinsic Josephson capacitance $C_J \propto A_{\text{overlap}}$ (which corresponds to a parallel plate capacitor). As expected, the total capacitance of our samples follows a linear trend with E_J . The vertical offset capacitance is the geometric capacitance set by the shape of the capacitive pads next to the junction in our design, $C_{\text{geo}} = 2.56$ fF. This value corresponds to the value estimated from finite-element simulations.

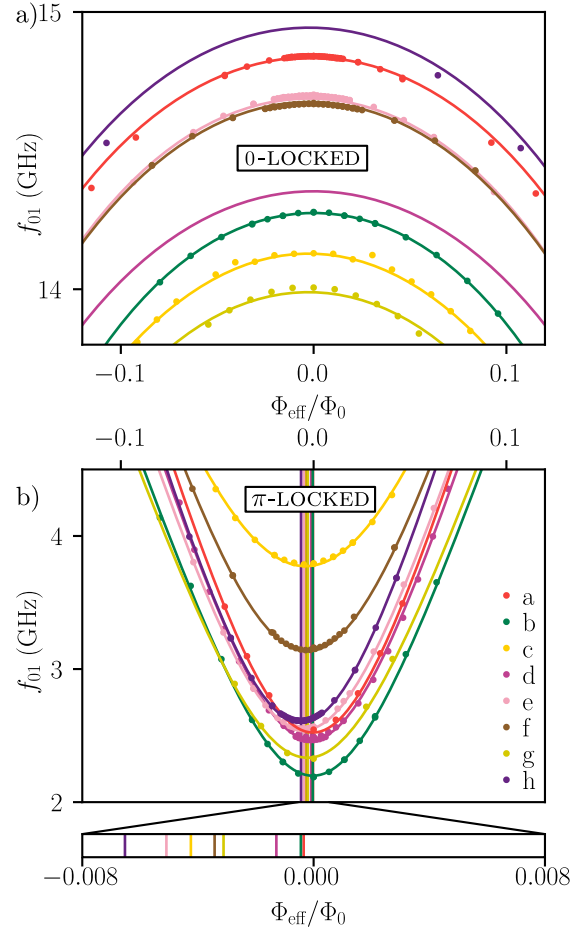
2.3 Demonstration of flux-locking

2.3.1 Simultaneous locking of several samples

We demonstrate the flux-locking mechanism in Fig. 2.6 by cooling down in a magnetic field B_{cd} corresponding to either $m = 0$ (Fig. 2.6.a) or $m = 1$ (Fig. 2.6.b) fluxons trapped in the ring. Specifically, $m = 1$ is obtained for $B_{\text{cd}} \approx 0.66 \mu\text{T}$, corresponding to $1 \Phi_0$ in the ring area of $A_{\text{ring}} = 107 \times 29.4 \mu\text{m}^2$. As previously shown in Fig. 2.4, all measured transition frequencies are fitted to the fluxonium Hamiltonian. We note that, in Fig. 2.6.b, the frequency minima do not align exactly with $\Phi_{\text{eff}} = 0$. These flux offsets from perfect sweet-spot locking result from a deliberate sweep of the loop width asymmetry $(w' - w'')/2$ and will be explored in Section 2.3.2.

The previous implementation in the group, with a granular aluminum ring, has shown flux escape at base temperature. In Fig. 2.7.a, we show the resonator spectroscopy of one

Figure 2.6: Flux-locking demonstration. a) Spectra of eight gradiometric fluxonium samples measured in the vicinity of $\Phi_{\text{eff}} = 0$, after cooling down either in zero field or b) in a field $\Phi_0/2A_{\text{ring}} < B_{\text{cd}} < 3\Phi_0/2A_{\text{ring}}$, corresponding to $1\Phi_0$ trapped in the aluminum ring of the device. Markers depict the frequency of the $0 - 1$ transition extracted from a two-tone spectroscopy and lines represent a fit to the fluxonium Hamiltonian in Eq. (2.14). Vertical lines indicate the position of the minimal frequency and are shown in detail in the bottom inset.



such sample, adapted from Ref. [111]. In the left panel, the fluxonium is π -locked. As its half-flux-quantum frequency is around 4 GHz, it sits below the resonator, which is confirmed by the anticrossings at $\Phi_{\text{eff}} = 0.25\Phi_0$. In the right panel however, measured a few hours later, the qubit frequency now sits above the resonator at 10 GHz, because the qubit is 0-locked. In the meantime, a trapped fluxon has escaped from the ring, as shown in the middle panel, after about eighty minutes. These flux escapes are mediated by ionizing radiation interactions with the substrate in the vicinity of the device and are completely suppressed in underground conditions. Above ground, they happen on typical timescale of hours. In contrast, we do not observe any flux escape in the aluminum ring gradiometric fluxoniums after locking at base temperature. Figure 2.7.b shows a two-tone spectroscopy trace, taken on an additional sample (from a previous fabrication batch) biased at the half-flux sweet spot, for over 50 hours. The qubit does not undergo any frequency jump over this period, and none were observed over tens of samples, including during several-month-long cooldowns.

However, we typically observe simultaneous π -locking for only 80% of the qubits, i.e. in each cooldown one or two devices lock to even values of m instead of odd. This behavior is unexpected and could be due to ionizing radiation activated flux escapes happening during the cooldown, when the superfluid stiffness $\rho_{s,\text{Al}}$ is suppressed close to the superconducting

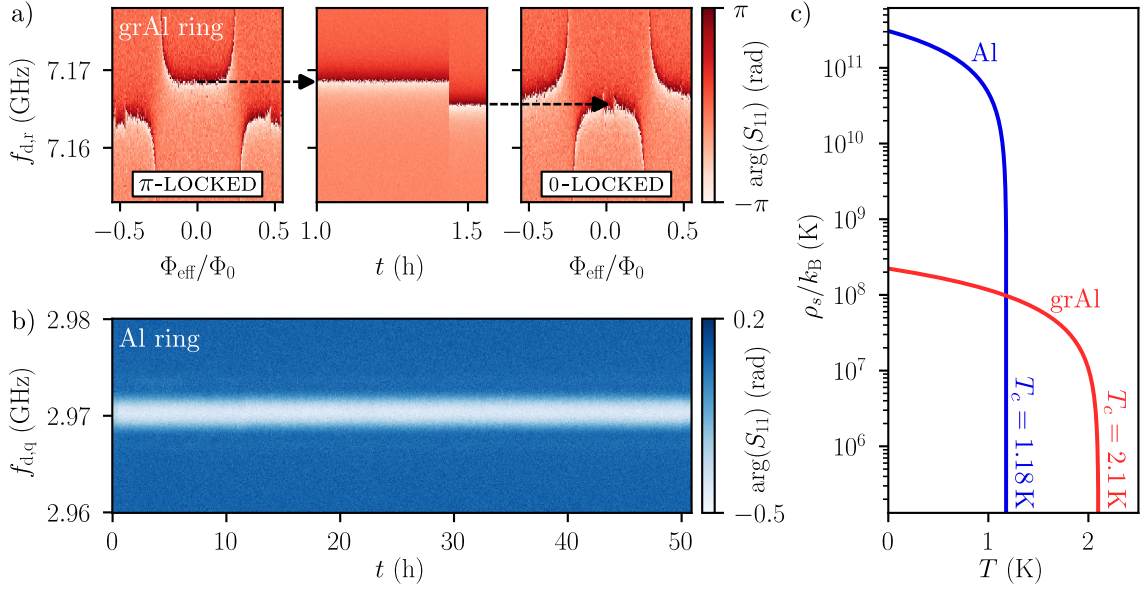


Figure 2.7: Flux locking preservation. **a)** Resonator spectroscopy of a gradiometric fluxonium with a granular aluminum outer ring. Colors show the phase of the reflection coefficient S_{11} of the readout resonator versus drive frequency $f_{d,r}$ and effective flux Φ_{eff} . In the left panel, the qubit is π -locked, while in the right panel it is 0-locked. In the meantime, a trapped fluxon has escaped, which is shown in the middle panel as a sudden frequency jump. Figure adapted from Ref. [111]. **b)** Two-tone spectroscopy of a gradiometric fluxonium with an aluminum outer ring, biased at the half-flux sweet spot. Colors show the phase of the reflection coefficient S_{11} of the readout resonator versus qubit drive frequency $f_{d,q}$ and time. No flux escape is observed over 50 h of measurement and no flux escape was observed in any aluminum sample over month-long cooldowns. **c)** Superfluid phase stiffness ρ_s of aluminum (blue) and granular aluminum (grAl) versus temperature T . At $T = 0$, the superfluid stiffness of granular aluminum is three orders of magnitude lower than that of aluminum, which is why the qubit in panel a undergoes flux escapes contrary to the qubit in panel b. Additionally, during cooldown, the superfluid stiffness of aluminum stays lower or on the order of the superfluid stiffness of grAl $\rho_{s,\text{grAl}}(T=0)/k_B = 2.2 \times 10^8 \text{ K}$ over a small range of temperatures $T \lesssim T_c$. This could explain why in each cooldown around 20% of the samples do not lock at the target m .

critical temperature, rendering the ring vulnerable to phase slips. In Fig. 2.7.c, we plot the superfluid stiffness of aluminum and granular aluminum versus temperature, showing how on a narrow range of temperatures $T \lesssim T_c$, $\rho_{s,\text{Al}}$ is orders of magnitude lower than $\rho_{s,\text{grAl}}$. We note that the observed yield is similar to the one found in Ref. [115], despite their using a wider TiN outer ring. Potential mitigation strategies align with those used to address quasiparticle poisoning in superconducting circuits, such as employing quasiparticle traps adjacent to the sample [120] or shielding from cosmic radiation underground [111]. An additional strategy could involve accelerating the cooldown or enhancing sample thermalization to minimize the time spent near the transition.

2.3.2 Fine tuning the locking precision

As we saw in Fig. 2.6.b, different samples lock to the half-flux sweet spot with different offsets. This results from a purposeful sweep of the loop width asymmetry $(w' - w'')/2$, over

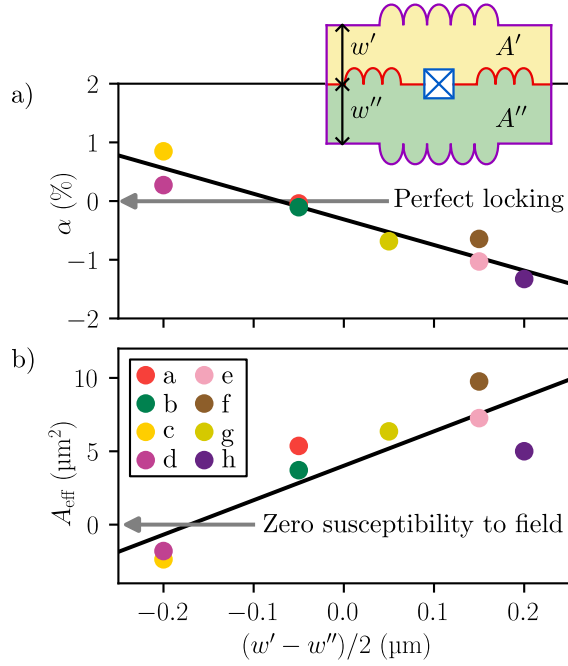


Figure 2.8: Asymmetry fine tuning. **a)** Inset: Circuit diagram of the gradiometric fluxonium, showing the Josephson junction (blue), the granular aluminum inductances (red) in the middle, and the low-value inductances (purple) of the aluminum ring. The difference in loop widths $w' - w''$ is tuned while keeping the total size constant. Inductive asymmetry $\alpha = (L'' - L')/(L'' + L')$ is obtained from the offset of the spectrum when the fluxonium is π -locked (see Fig. 2.6.b), such that $\Phi_{\text{offset}}/\Phi_0 = \alpha/2$. Perfect sweet-spot locking is realized when $\alpha = 0$. **b)** Effective area A_{eff} vs. loop width asymmetry. A_{eff} is extracted from fitting the spectra together with fluxonium parameters E_J , E_C and E_L . Zero susceptibility to homogeneous magnetic field is achieved when $A_{\text{eff}} = 0$.

the range -200 nm to 200 nm , in order to target fully symmetric samples more accurately. More specifically, the loop width asymmetry contributes to two quantities:

$$\text{Inductive asymmetry } \alpha = \frac{L'' - L'}{L'' + L'}, \quad (2.17)$$

$$\text{Effective area } A_{\text{eff}} = \frac{A' - A''}{2} + \alpha \frac{A' + A''}{2} = \frac{\Phi_{\text{eff}}}{B_{\text{ext}}}. \quad (2.18)$$

The inductive asymmetry α is extracted from the flux offset from perfect sweet-spot locking when the samples are π -locked, represented as vertical lines in Fig. 2.6.b and its inset. As the total applied phase in the Hamiltonian is $\varphi_{\text{tot}} = \varphi_{\text{eff}} + 2\pi m(1 + \alpha)/2$, at $B_{\text{ext}} = 0$ and $m = 1$, we get α from the residual phase offset $2\pi(1 + \alpha)/2$. The effective area A_{eff} is fitted alongside the fluxonium parameters in Section 2.2.3. Note that, from Eq. (2.18), A_{eff} has both a geometric (from the area difference $A' - A''$) and an inductive (from α) contribution.

In Fig. 2.8, we plot these two quantities versus the loop asymmetry $(w' - w'')/2$. As expected, we observe a linear dependence of α (Fig. 2.8.a) and A_{eff} (Fig. 2.8.b) on the asymmetry, as follows:

$$\alpha = (-4.4 \text{ \%/}\mu\text{m}) \frac{w' - w''}{2} - 0.31 \text{ \%,} \quad (2.19)$$

$$A_{\text{eff}} = (23.6 \mu\text{m}^2/\mu\text{m}) \frac{w' - w''}{2} + 4.0 \mu\text{m}^2. \quad (2.20)$$

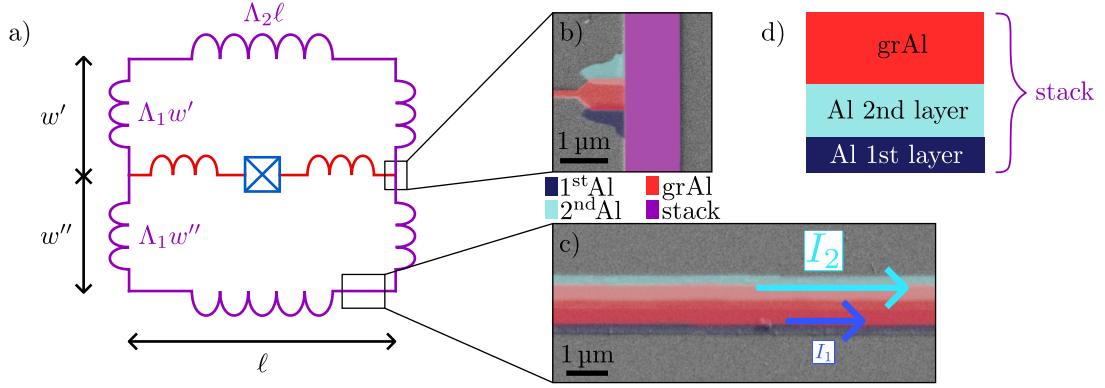


Figure 2.9: Asymmetry in the gradiometric fluxonium circuit. **a)** Circuit diagram showing the proposed parametrization for the inductance of the aluminum ring. **b), c)** Scanning-electron microscopy of the outer ring at the connection with the grAl wire (b) and in the lower-loop long arm (c). **d)** Sketch of the Al-grAl stack.

Let's parametrize the inductance in the different arms by writing the inductance of the aluminum loops, following the circuit diagram in Fig. 2.9.a, as

$$L' = \Lambda_2' \ell + 2\Lambda_1 w', \quad (2.21)$$

$$L'' = \Lambda_2'' \ell + 2\Lambda_1 w'', \quad (2.22)$$

where Λ_1 is the linear inductance in the short arms and Λ_2 in the long arms. This gives

$$\alpha = \frac{(\Lambda_2'' - \Lambda_2')\ell + 2\Lambda_1(w'' - w')}{(\Lambda_2'' + \Lambda_2')\ell + 2\Lambda_1 w} \quad (2.23)$$

$$= \frac{-2}{\frac{\Lambda_2' + \Lambda_2''}{2\Lambda_1}\ell + w} \left(\frac{w' - w''}{2} \right) + \frac{\frac{\Lambda_2'' - \Lambda_2'}{2\Lambda_1}\ell}{\frac{\Lambda_2' + \Lambda_2''}{2\Lambda_1}\ell + w}. \quad (2.24)$$

From the linear fit in Eq. (2.19) and the dimensions of the circuit $\ell = 107 \mu\text{m}$ and $w = 29.4 \mu\text{m}$, we extract

$$\frac{(\Lambda_2' + \Lambda_2'')/2}{\Lambda_1} = 0.15, \quad (2.25)$$

$$\frac{\Lambda_2' - \Lambda_2''}{(\Lambda_2' + \Lambda_2'')/2} = 0.018. \quad (2.26)$$

In short, the relative difference of inductance between the long arms of the two loops is 2 %, while the aluminum stack in the short arms is 7 times more inductive than in the long arms. Qualitatively, it is not surprising that the inductance is different in the long and short arms, as the stacking of aluminum (and grAl) layers is very different. As depicted in Fig. 2.9.b, the two aluminum layers in the short arms are stacked right above each other, while in the long arms, they are separated with little overlap (Fig. 2.9.c). However, the amount by which they differ is very large, which suggests a contribution from geometric inductance that is not taken into account in this parametrization. The origin of the small

(2%) inductive asymmetry between the inductance of the long arms in each loop probably lies in the shadow evaporation technique, employed to fabricate the junction. In order to fabricate the SIS Josephson junction in the middle of the qubit, we evaporate two layers of aluminum, with tilt angles -31° and 31° . This shadow evaporation creates a stack of 20-nm-thick and 30-nm-thick aluminum layers, respectively, as depicted in Fig. 2.9.d. As a result, the supercurrent paths in the upper and lower loop are different and bring slightly different inductive contributions.

The different paths taken by the supercurrent also result in an asymmetry in $A' - A''$. From Eqs. (2.19) and (2.20) we extract the area difference

$$\frac{A' - A''}{2} = (92.7 \mu\text{m}^2/\mu\text{m}) \frac{w' - w''}{2} + 8.9 \mu\text{m}^2. \quad (2.27)$$

The slope of the area difference aligns with the device's $107 \mu\text{m}$ length. The remaining offset corresponds to a 100 nm geometric offset, explained by the persistent current path through the two aluminum layers with different kinetic inductances. From the dependence of the penetration depth on the film thickness measured in Ref. [121], we estimate the sheet inductance of each layer to be $2.1 \text{ pH}/\square$ and $0.9 \text{ pH}/\square$, respectively. The current is thus twice as large in the 30-nm-thick layer as in the 20-nm-thick layer: $I_2/I_1 = 2$. The aluminum layers are 600 nm apart, with 400 nm overlap, i.e. the middle of the thicker layer is $\delta = 300 \text{ nm}$ away from the middle of the bilayer stack. Correspondingly, the effective offset for the persistent current path is $\delta (1 - I_1/I_2)/(1 + I_1/I_2) = 100 \text{ nm}$, close to the value we find experimentally.

Overall, this means that the zero crossings of α and A_{eff} occur at different values of $(w' - w'')/2$, shifted away from zero (which is also reflected in the non-zero constant contribution in Eq. (2.27)). This observation implies that adjusting $w' - w''$ alone cannot offer a device that simultaneously locks perfectly to the sweet spots and remains insensitive to magnetic fields. From a practical point of view, it actually ensures that the samples with the lowest α , which are the ones locked closest to zero, are still tunable with a global magnetic field and hence easier to manipulate in our setup. More generally, a finer understanding of the supercurrent pathways within the Al-grAl stack, as well as a better control over it during fabrication, might be necessary to make symmetric samples with improved accuracy.

2.4 Coherence

In order to characterize the inductive asymmetry α in our most symmetric samples a and b more accurately, we turn to coherence measurements. In particular, the goal of this section is to measure the decoherence time T_2 as a function of external flux to determine the locking accuracy.

Characterizing the coherence of qubits necessitates a readout method able to distinguish their states. Thanks to our low-noise parametric amplifiers [117], we can achieve single-shot QND readout in a standard dispersive coupling. In Fig. 2.10.b, we show the 2D

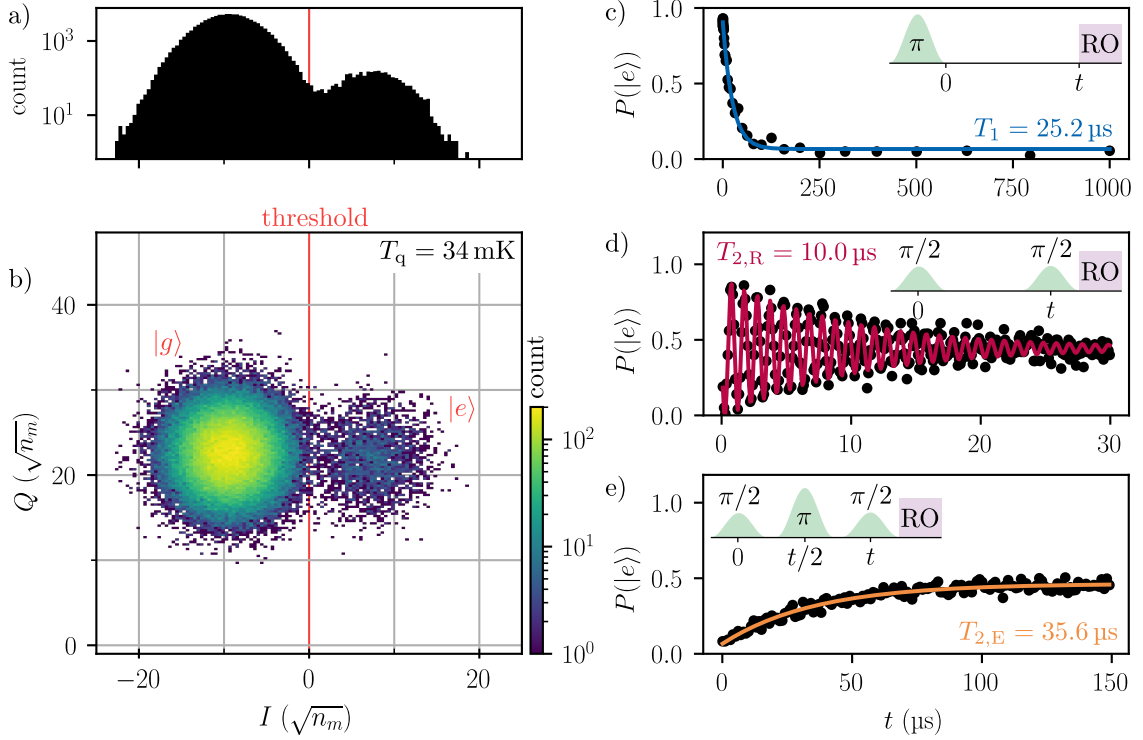


Figure 2.10: Time-domain qubit characterization. **a, b)** IQ quadrature measurement of sample **c**, **b)** in measurement photons $n_m = \sqrt{\bar{n}_r \tau \kappa_r}$, with $\bar{n}_r$ the mean number of photons in the resonator, τ the readout duration and κ_r the resonator linewidth. The red line in both plots indicates the threshold used to discriminate between states $|g\rangle$ and $|e\rangle$. **c, d, e)** Measurements of the energy relaxation time T_1 (c), the Ramsey decoherence time $T_{2,R}$ (d) and the Hahn-echo decoherence time $T_{2,E}$ (e) at the sweet spot on sample **c**. For each measurement, an inset shows the pulse sequence.

histogram of IQ quadratures obtained by measuring sample **b**, biased at the sweet spot, 100,000 times. The plot shows two well-separated clouds, corresponding to the ground $|g\rangle$ and excited $|e\rangle$ states of the qubit. The states are discriminated with a threshold at $I = 0$ (red line). By measuring every 100 μ s, we ensure that the state of the qubit before measurement is the thermal state and estimate the qubit temperature to be $T_q = 34$ mK, close to the base temperature of the cryostat at the mixing chamber stage $T = 20$ mK.

After calibrating the position of the ground and excited states in the IQ plane, we calibrated a π -pulse and a $\pi/2$ -pulse from Rabi oscillations when driving the qubit. These calibrated pulses allow us to measure the relaxation and decoherence times T_1 , $T_{2,R}$ and $T_{2,E}$, defined as follows:

- The energy relaxation time T_1 is obtained by applying a π -pulse and measuring the relaxation of the qubit, which is fitted to an exponential decay e^{-t/T_1} (see Fig. 2.10.c),
- the decoherence time $T_{2,R}$ is obtained from a Ramsey sequence consisting of two $\pi/2$ -pulses separated by a time t , and the population is fitted to a cosine with an exponential envelope $\cos(2\pi\Delta f t)e^{-t/T_{2,R}}$, with Δf the detuning between qubit and drive frequencies (see Fig. 2.10.d),

- the Hahn-echo decoherence time $T_{2,E}$ is obtained from a Hahn-echo sequence consisting of two $\pi/2$ -pulses with a π -pulse in between, and fitted to an exponential decay $e^{-t/T_{2,E}}$ at the sweet spot (see Fig. 2.10.e), or with an additional Gaussian decay $e^{-\Gamma_{2,E}^0 t} e^{-(\Gamma_{2,E}^\Phi t)^2}$ away from the sweet spot to take flux noise into account. Following Ref. [69], $T_{2,E}$ is then calculated as

$$T_{2,E} = \frac{\sqrt{(\Gamma_{2,E}^0)^2 + 4(\Gamma_{2,E}^\Phi)^2} - \Gamma_{2,E}^0}{2(\Gamma_{2,E}^\Phi)^2}. \quad (2.28)$$

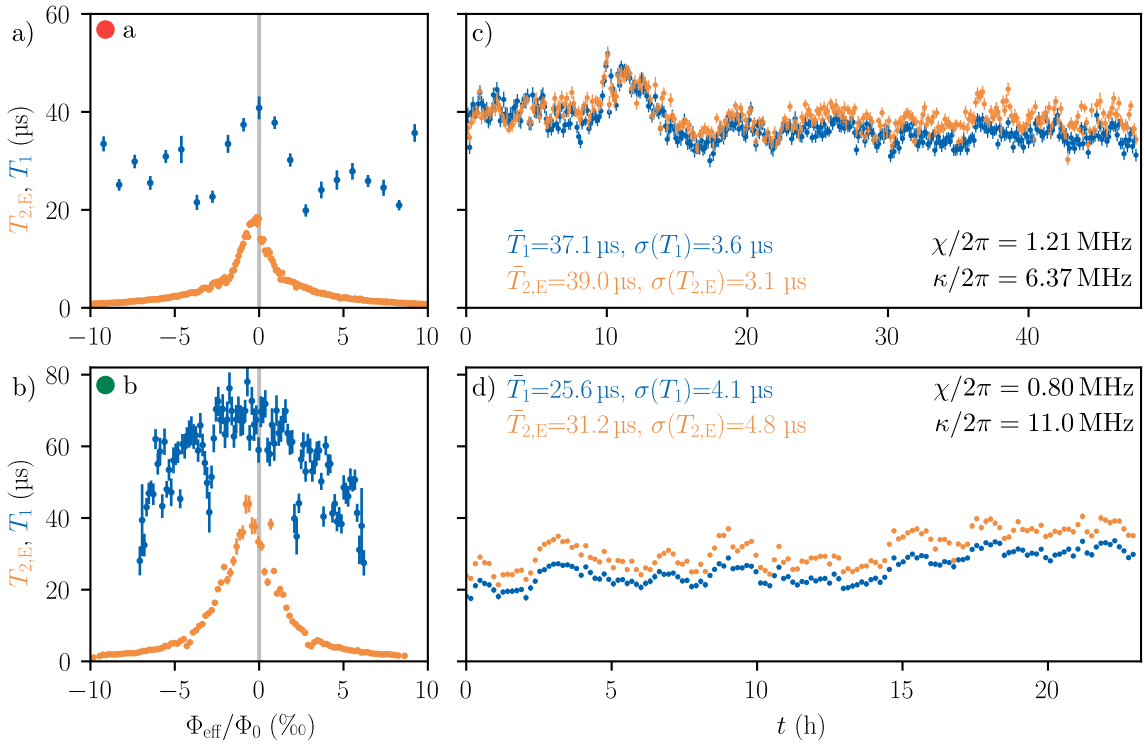


Figure 2.11: Gradiometric fluxonium coherence. **a, b)** T_1 (in blue) and $T_{2,E}$ (in orange) coherence times for samples $\bullet$ a (a) and $\bullet$ b (b) with $m = 1$ fluxon trapped vs. effective flux Φ_{eff} around zero field, illustrating the precision of the locking mechanism. Markers show the measured values with uncertainty. The light gray vertical line indicates zero field. **c, d)** T_1 (in blue) and $T_{2,E}$ (in orange) coherence times for the same samples in a different cooldown, at the sweet spot, over several hours.

To investigate the locking accuracy in more detail, we measure $T_{2,E}$ around zero field for samples a and b, which are the closest to zero in Fig. 2.8.a. In Fig. 2.11, we report $T_{2,E}$ values of respectively $20 \mu\text{s}$ (a) and $44 \mu\text{s}$ (b) at $\Phi_{\text{eff}} = -3 \times 10^{-4} \Phi_0$ and $\Phi_{\text{eff}} = -5 \times 10^{-4} \Phi_0$, similar to values reported for granular aluminum fluxoniums in Ref. [36]. At zero field, they correspond to degradations of 15 % and 32 % respectively. For both samples, $T_{2,E}$ at the sweet spot follows an exponential decay, not limited by T_1 , which could be explained, for example, by critical current noise or photon shot noise with $n_{\text{res}} \approx 0.01$ residual photons in the resonator. Additionally, Fig. 2.11.c and d reveal significant correlated fluctuations in T_1

and $T_{2,E}$ over several hours, as commonly observed in the literature [122, 123], though their origin remains unknown. These measurements over time were performed interleaved, i.e. each few-minute-long measurement alternated between a T_1 sequence and a Hahn-echo sequence. We report the following relative standard deviation:

$$\text{Sample a: } \frac{\sigma(T_1)}{\bar{T}_1} = 9.7 \%, \quad \frac{\sigma(T_{2,E})}{\bar{T}_{2,E}} = 7.9 \%, \quad (2.29)$$

$$\text{Sample b: } \frac{\sigma(T_1)}{\bar{T}_1} = 16 \%, \quad \frac{\sigma(T_{2,E})}{\bar{T}_{2,E}} = 15 \%. \quad (2.30)$$

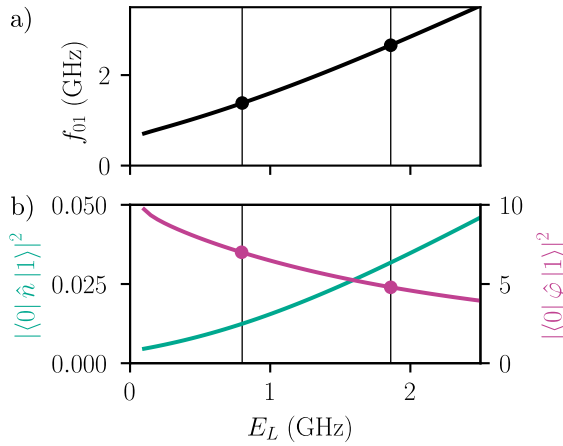


Figure 2.12: Dielectric and inductive noise for low-frequency granular aluminum fluxoniums. **a,b)** Numerically computed 0 – 1 transition frequency (a) and matrix elements $|\langle 0 | \hat{n} | 1 \rangle|^2$ and $|\langle 0 | \hat{\phi} | 1 \rangle|^2$ (b) versus inductive energy E_L for the average fluxonium with parameters $\bar{E}_J$ and $\bar{E}_C$ from Eq. (2.16). The vertical lines indicate the mean value $\bar{E}_L = 1.86$ GHz for the samples discussed here and for a different fabrication batch with $E_L = 0.80$ GHz.

The samples presented so far in this chapter are not quite in the fluxonium regime of high inductance $E_L \ll E_J$, as $\bar{E}_J/\bar{E}_L = 4.7$. The reason why the fluxonium (or even, for low E_C , the heavy fluxonium) regime is usually preferred for quantum information is that by decreasing E_L , we are tuning the zero-point fluctuation of the charge operator $n_{\text{ZPF}} = \sqrt[4]{E_L/32E_C}$ down. This method is very efficient at driving T_1 up, because in regular superconducting qubits dielectric noise is the largest source of noise, orders of magnitude above inductive noise. With granular aluminum inductors however, inductive noise is generally larger and is often the limiting factor for coherence, as shown in Ref. [124]. When using high inductance, the qubit coherence now sees an increased contribution from the phase zero-point fluctuation $\varphi_{\text{ZPF}} = \sqrt[4]{2E_C/E_L}$. As an illustration, Fig. 2.12 shows the dependence of the qubit frequency (a) and the charge $\hat{n}$ and phase $\hat{\phi}$ matrix elements (b) on the inductive energy E_L , for the average parameters $\bar{E}_J$ and $\bar{E}_C$ taken from Table 2.1. The right-most vertical line denotes the average $\bar{E}_L$ for the samples discussed in this chapter, while the left-most vertical line corresponds to a sample from a previous fabrication batch, with similar E_C , but different E_L and E_J . That sample, with $E_L = 0.80$ GHz, has a typical $T_1 = 3 \mu\text{s}$ at half-flux ($f_q = 0.5$ GHz), about one order of magnitude below samples a and b. While the T_1 reduction does not match quantitatively with the increase in $T_1 \propto |\langle 0 | \hat{\phi} | 1 \rangle|^2$ suggested by Fig. 2.12.b, it is consistent with a inductive-loss-limited qubit.

In Table 2.2, we report the qubit temperature T_q , relaxation time T_1 , Ramsey decoherence time $T_{2,R}$ and Hahn-echo decoherence time $T_{2,E}$ for four samples out of the eight discussed so far. These quantities were measured with different cryogenic setups, from using only a

Sample	Perf.	dir.c. (20 dB)	dir.c.+10 dB (30 dB)	30 dB	50 dB, 10 dB, IR filters
● a	T_q (mK)				34
	T_1 (μ s)				43
	$T_{2,R}$ (μ s)				12
	$T_{2,E}$ (μ s)				20
● b	T_q (mK)	200		91	35
	T_1 (μ s)	20.6		53.6	67
	$T_{2,R}$ (μ s)	0.3		10	14
	$T_{2,E}$ (μ s)	0.6			50
● d	T_q (mK)			85	
	T_1 (μ s)			41.4	
	$T_{2,R}$ (μ s)			1.5	
	$T_{2,E}$ (μ s)			2	
● h	T_q (mK)	197	95	91	
	T_1 (μ s)	11.9	13	21	
	$T_{2,R}$ (μ s)	0.3	0.1	0.5	
	$T_{2,E}$ (μ s)	0.4	0.1	1.8	

Table 2.2: Qubit temperature and coherence in different cryogenic setups. In the first three columns, attenuation is only for the readout line. *dir.c.* stands for directional coupler. In the last column, 10 dB corresponds to the qubit drive line, and IR filters are added on both lines.

directional coupler giving 20 dB of attenuation on the readout line, to using circulators and 50 dB attenuation on the readout line, 10 dB on the qubit drive line, and IR filters on all lines. Improving the setup in such ways reduced the qubit temperature from 200 mK to 35 mK and increased both T_1 and T_2 . In particular, sample b saw its T_1 at half-flux triple and its $T_{2,E}$ was multiplied by 10. Results from this table highlight the importance of a good cryogenic setup when reporting qubit performance and coherence.

2.5 Four-loop fluxonium-like qubit

Thanks to the previous sections, we know that we can trap a fluxon indefinitely in a superconducting ring and get qubit states as superpositions of fluxon states in two loops. Now, we turn to a more complex system comprising four loops and four Josephson junction. While the circuit resembles the gradiometric fluxonium circuit, its additional degrees of freedom given by each junction make it more complex but potentially also more robust to noise. In this section, we present the work done toward this system. Because of its complexity, we will focus on building intuition and exploring realized samples and encountered experimental challenges.

2.5.1 Concept, circuit and intuition

Superconducting circuits with several degrees of freedom such as the four-loop fluxonium offer the possibility of hosting qubit states whose wavefunctions do not overlap, offering protection against bit flip [125]. It is the core idea behind protected qubits discussed in Section 1.2.5 such as $0 - \pi$ qubits [77], fluxonium molecules [91], $\cos(2\varphi)$ qubits [87] or bifluxon qubits [126]. In our circuit, the qubit states are superpositions of fluxon states and therefore delocalized over the four loops. At specific bias points, the qubit states are degenerate.

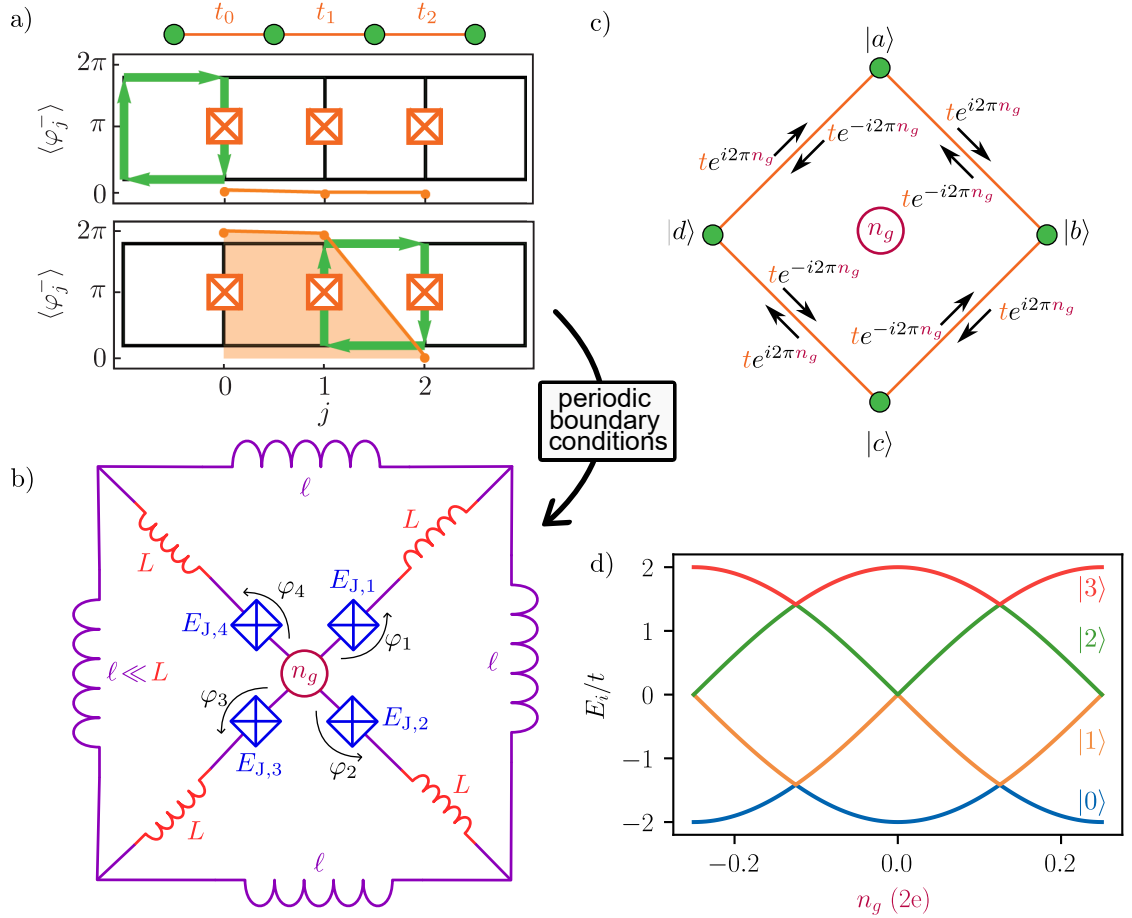


Figure 2.13: Four-loop fluxonium: a concept, a circuit, an intuition. **a)** Circuit of a 1D loop chain with fluxon states represented as green arrows, as well as the average phase across the junction $\langle \varphi_j^- \rangle$ plotted versus junction index j . Figure adapted from Ref. [60]. **b)** Circuit diagram of the four-loop fluxonium, built from an aluminum outer ring in which we incorporate four junctions and four inductors to realize a periodic loop chain. **c)** Fluxon tight-binding model corresponding to the four-loop fluxonium circuit with $m = 1$ trapped fluxon (or equivalent applied external field), where the complex tunneling rates are due to the acquired phase when circulating around a charge, also known as the Aharonov-Casher effect. **d)** Eigenenergies E_i/t versus offset charge n_g in the tight-binding paradigm.

The idea for this specific circuit originates in Ref. [60]. In Fig. 2.13.a, we reproduce a figure showing a chain of four loops separated by a total of three junctions indexed $j = 0, 1, 2$.

The green arrows depict the current associated with a fluxon state in the first loop (top) or the third loop (bottom), when there is a total of $m = 1$ fluxon in the system. These fluxon states correspond to a kink in the phase $\langle \varphi_j^- \rangle$ across junction j , as shown by the orange markers. By treating the Josephson junctions as quantum phase slip (QPS) elements, we can understand this system as a 1D tight-binding model, where the interaction at junction j is set by the tunneling rate [59–61]

$$t_j = \frac{4}{\sqrt{\pi}} (8E_{J,j}^3 E_{C,j})^{1/4} e^{-\sqrt{8E_{J,j}/E_{C,j}}}, \quad (2.31)$$

as schematically represented at the top of the figure. The eigenstates of this circuit are given by superpositions of the fluxon states and are therefore delocalized over the circuit.

Here, we propose to realize this system with periodic boundary conditions. In other words, we make the chain a closed loop by connecting the first and last loops with a Josephson junction, as represented in the circuit in Fig. 2.13.b. In order to preserve the total number of fluxons m in the circuit, we incorporate the necessary inductances in the junction arms, and keep only low-inductance regular aluminum in the outer ring. From the circuit, we derive the four-loop fluxonium Hamiltonian

$$\hat{\mathcal{H}}_{4LF} = \sum_{i=1}^4 4E_{C,i} (\hat{n}_i - n_g)^2 + \frac{1}{2} E_L \sum_{i=1}^4 (\hat{\phi}_i + \tilde{\Phi}_i)^2 - \frac{1}{8} E_L \left(\sum_{k=1}^4 \hat{\phi}_k \right)^2 - \sum_{i=1}^4 E_{J,i} \cos(\hat{\phi}_i), \quad (2.32)$$

with the effective fluxes

$$\tilde{\Phi}_i = \frac{3}{8} (\Phi_i - \Phi_{i+1}) - \frac{1}{8} (\Phi_{i+2} - \Phi_{i+3}), \quad (2.33)$$

where the indices have to be understood modulo 4, i.e. $i \equiv i + 4$.

Several observations can be made from this Hamiltonian. First, notice that $\hat{\mathcal{H}}_{4LF}$ is charge sensitive. As the central node is connected to the rest of the circuit only by junctions and capacitors, it forms a superconducting island with an offset charge n_g that tunes the spectrum. Second, it consists of four fluxonium Hamiltonians with effective external flux $\tilde{\Phi}_i$ coupled together through term $-\frac{1}{8} E_L (\sum_{k=1}^4 \hat{\phi}_k)^2$. This strong and non-perturbative coupling makes computing the eigenstates numerically demanding, and out of the scope of this thesis. We can however build intuition from the tight-model introduced previously.

In Fig. 2.13.c, we represent the equivalent tight-binding model of our circuit, with periodic boundary conditions and charge-dependent complex tunneling rates $te^{i2\pi n_g}$ due to Aharonov-Casher effect¹. This model can be written as a matrix:

$$H_{\text{tight-binding}} = \begin{pmatrix} U_a & -te^{i2\pi n_g} & 0 & -te^{-i2\pi n_g} \\ -te^{-i2\pi n_g} & U_b & -te^{i2\pi n_g} & 0 \\ 0 & -te^{-i2\pi n_g} & U_c & -te^{i2\pi n_g} \\ -te^{i2\pi n_g} & 0 & -te^{-i2\pi n_g} & U_d \end{pmatrix}, \quad (2.34)$$

¹ As a personal sidenote, I enjoy the conceptual similarity of this system with the polarized exciton-polaritons in a honeycomb lattice (also called photonic graphene) I studied in my Master1 internship.

and leads to four charge-dependent eigenstates plotted in Fig. 2.13.d. Interestingly, some of these states are degenerate for specific values of n_g . From such eigenstates, we can hope to realize a low-frequency superconducting qubit with enhanced coherence thanks to its delocalized wavefunctions.

2.5.2 Design and fabrication

We fabricated two sets of samples, with two different capacitive coupling schemes, in order to couple to different modes of the circuit. Type A (see Fig. 2.14.a) couples the resonator to the terminals of two junctions, while type B (see Fig. 2.14.b) couples the resonator to the terminals of only one junction. Building upon the work done in the previous sections, the samples are realized with an aluminum outer ring, ensuring the conservation of trapped fluxons. The inductors are made from granular aluminum and the junctions as typical SIS overlap junctions, exactly like the gradiometric fluxoniums.

Similarly to the gradiometric fluxoniums in Section 2.2.2, we fabricate twenty samples across twenty chips on a 2-inch wafer, as depicted in Fig. 2.14.c. We can measure up to four samples in a single waveguide because we planned four different resonator frequencies. On the wafer lie test structures alongside the samples. These structures, drawn schematically in Fig. 2.14.d, are intended to test the feasibility and reproducibility of evaporating four nearly identical junctions, in the same conditions and for the same design and relative position as in the samples. We note however that exposing the surrounding structures, especially the capacitive pads in both type A (see Fig. 2.14.a) and type B (see Fig. 2.14.b), modifies the effective electron-beam dose at the junctions by proximity effect.

In panels e, f and g of Fig. 2.14, we plot the Josephson energy E_J for the test structure junctions, estimated from room-temperature resistance R_n with the Ambegaokar-Baratoff formula [127]:

$$E_J = \frac{\Phi_0 \times 1.76 k_B T_c}{4eR_n}, \quad (2.35)$$

where $T_c = 1.2$ K is the superconducting critical temperature of aluminum. In each panel, corresponding to a column of test structures (left, middle, right), we plot E_J for each junction, in a color that represents the number of working junctions in a given structure (see legend as an inset of Fig. 2.14.e). The black horizontal line depicts the mean E_J and the gray area is the standard deviation. We note that only a handful of structures, all in the middle, host four working junctions, illustrating the difficulty in fabricating samples with four working junctions. Additionally, the relative standard deviation is

$$\frac{\sigma(E_{J,\text{left}})}{\bar{E}_{J,\text{left}}} = 27 \%, \quad \frac{\sigma(E_{J,\text{middle}})}{\bar{E}_{J,\text{middle}}} = 18 \%, \quad \frac{\sigma(E_{J,\text{right}})}{\bar{E}_{J,\text{right}}} = 38 \%, \quad (2.36)$$

which shows the difficulty in fabricating such samples with reproducible parameters. However, considering only the structures with four working junctions (green markers), the relative standard deviation within a structure is 4 % on average, with only 1.5 % in the best case and up to 9 % in the worst case, which gives confidence in getting sufficiently symmetric samples from this fabrication process.

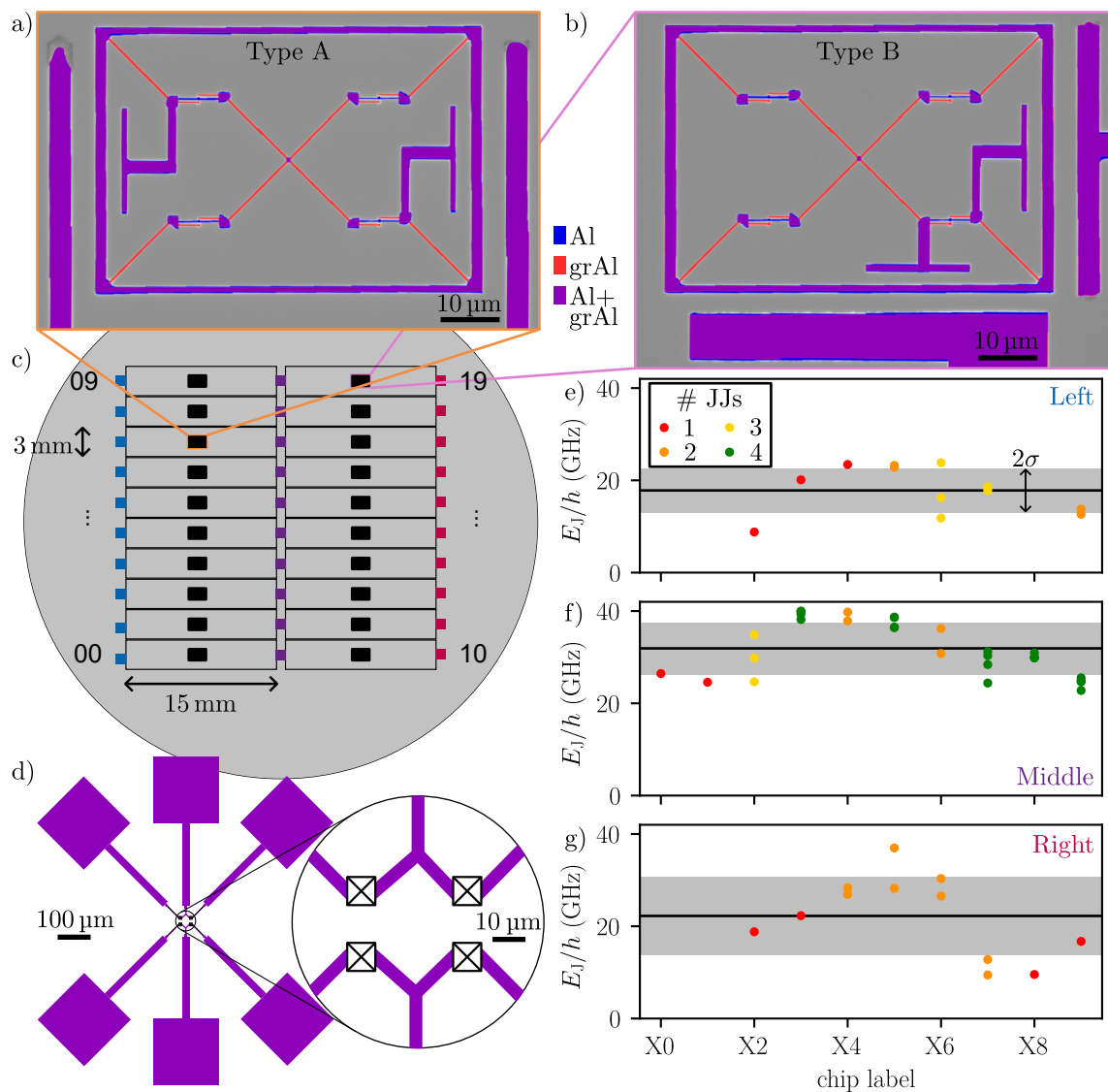


Figure 2.14: Four-loop fluxonium design and fabrication challenges. **a, b)** Optical microscope pictures, in false colors, of samples 07 (**a**) and 19 (**b**). These samples are examples of two different types of capacitive coupling of the circuit to the resonator (coupling to two junctions or one junction). **c)** Schematic diagram of a wafer, showing twenty chips with different resonators and three columns of test structures intended to measure the room-temperature resistance of four junctions, identical to the samples in their design and relative position. **d)** Schematic of the four-junction test structures. **e, f, g)** Estimated Josephson energy E_J of the test-structure junctions for each column. Marker color depicts the number of working junctions in each 4-junction structure. The number of working junctions and the spread of E_J illustrate the difficulty of making a truly symmetric working sample.

2.5.3 Results and challenges

We measure eight samples from two fabrication batches across multiple cooldowns. The samples are designed with both coupling types shown in Fig. 2.14, with varied junction and inductance parameters due to fabrication.

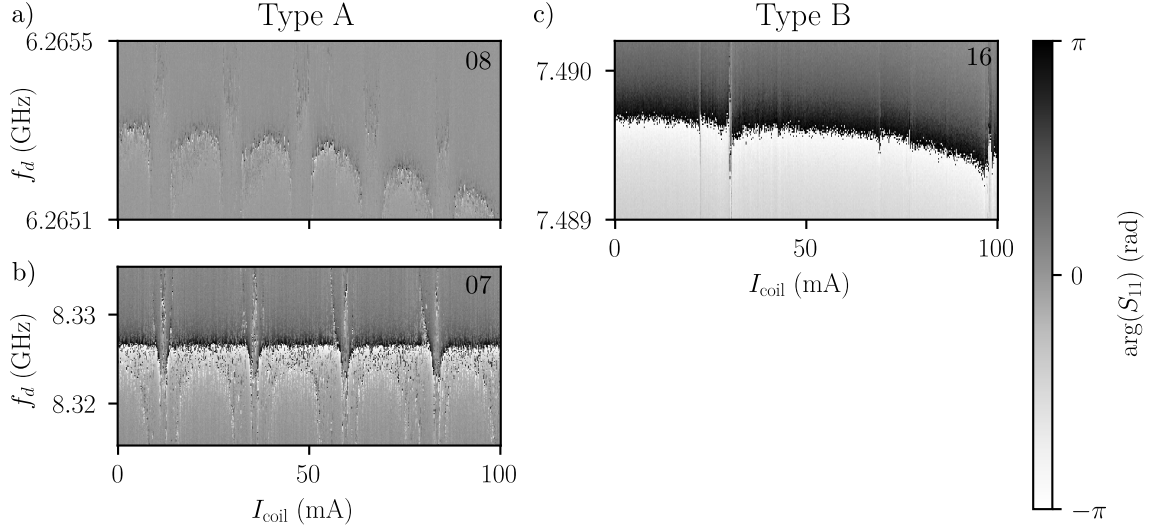


Figure 2.15: Portrait gallery. **a, b, c)** Resonator spectroscopy of samples 08 (a), 07 (b) and 16 (c) measured in reflection, versus current I_{coil} in the coil wound around the waveguide. For each panel, the phase of the reflection coefficient S_{11} is shown according to the colorbar on the right-hand side.

As a first step, let us look at the magnetic field response of the readout resonator. Figure 2.15 shows the resonator spectroscopy of three different samples when sweeping the current I_{coil} flowing in the coil wound around the waveguide. As the current creates a perpendicular magnetic field, it tunes the flux in each of the sample's loops. Just like in a gradiometric fluxonium, the asymmetry due to the different area and inductance of each loop means each loop is tuned slightly differently. Indeed, we can see that, for both sample 08 and sample 07 (Fig. 2.15.a and Fig. 2.15.b), the field-dependent resonator frequency is not exactly periodic. While Fig. 2.15.a exhibits a global decreasing trend corresponding to suppression of the superconducting gap by the perpendicular magnetic field, which tunes the resonator inductance, both samples show anticrossings appearing quasi-periodically. The behavior between anticrossings is different from period to period, confirming that we are tuning the flux in the loops at different speeds. Sample 16, which is coupled differently to the resonator, shows a different behavior in Fig. 2.15.c. Over the same current range 0 to 100 mA, one can only discern a handful of anticrossings and no periodicity. These observations likely come from the capacitive pads creating a different loop asymmetry in that sample compared to the other two.

The spectroscopies shown in Fig. 2.15 are plagued by noise, visible especially in panels a and b where the resonators often jump between several frequencies while biased at a fixed current I_{coil} . In the following, we show how this noise comes, in part, from charge drifts and jumps. In Fig. 2.16, we plot the resonator spectroscopy of sample 02 versus I_{coil} and charge offset on the central island n_g . Figure 2.16.a shows a flux sweep similar to the ones observed in Fig. 2.15 for type-A coupling, as expected. We note that thanks to the gradiometric nature of those samples, the spectrum is symmetric and well centered around $I_{\text{coil}} = 0$ mA. It shows several anticrossings over five quasi-periods. In each period, two regions of particular interest are the region around 0 mA that shows high levels of jitter,

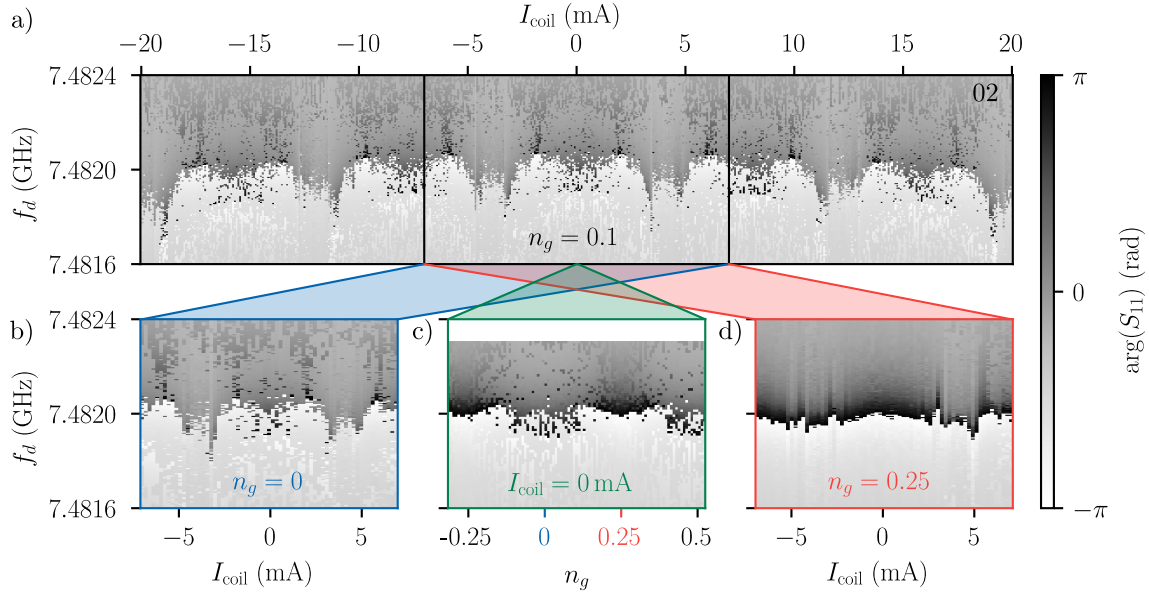


Figure 2.16: Flux and charge sweeps. **a)** Reflection spectroscopy of sample 02, at $n_g \approx 0.1$, showing 5 quasi-periods in I_{coil} . **b, d)** Spectroscopies versus I_{coil} at $n_g = 0$ and $n_g = 0.25$. The values of n_g were calibrated with panel c. **c)** Spectroscopy versus charge offset in the center island n_g , at zero field $I_{\text{coil}} = 0$ mA.

most probably due to parity switches on the central island, and the region around ± 4 mA where the anticrossings suggest it is similar to the half-flux sweet spot in a fluxonium, where the qubit frequency sits at its minimum below the resonator. Focusing on the first region, we measure the same resonator response at 0 mA versus DC voltage V_g , applied on a pin close to the samples via the drive port of the waveguide (see Fig. 2.16.c). We use the periodicity to calibrate the charge offset in the central island n_g . Specifically, we attribute $n_g = 0$ to the point with high level of jitter, because it is the point where parity switches (i.e. n_g jumping from 0 to 0.5 due to a broken Cooper pair tunneling through a junction) have the most impact on the qubit frequency, as seen from the energies calculated with the tight-binding model in Fig. 2.13.d. In panels b and d, we provide a zoom-in over one period, around 0 mA, at $n_g = 0$ (b) and $n_g = 0.25$ (d). The first major difference between these two plots is the level of jitter. Because of the periodicity in charge offset, $n_g = 0.25$ and $n_g = 0.75$ are equivalent and a parity switch has no impact on the qubit frequency and hence on the resonator frequency. Therefore, panel d looks flatter and less noisy. We also note that the resonator frequency tunes over a 200 kHz range when $n_g = 0$ and only 100 kHz when $n_g = 0.25$, with less visible anticrossings. The dispersive shift can be estimated to be on the order of 100 kHz.

In order to get a better understanding of the flux and charge tunability of the system, we measure the IQ quadratures of the S_{11} readout signal at a single frequency. Owing to our low-noise parametric amplifier in the readout chain, we obtain a good signal-to-noise ratio despite the relatively low dispersive shift (around 100 kHz). This is visible in the separation of the clouds in a 2D histogram (see Fig. 2.17.a). In particular, we can distinguish two pairs of clouds, corresponding to an even and an odd number of electrons on the island. The odd pair of clouds is not well separated, indicating that the dispersive shift is lower than for

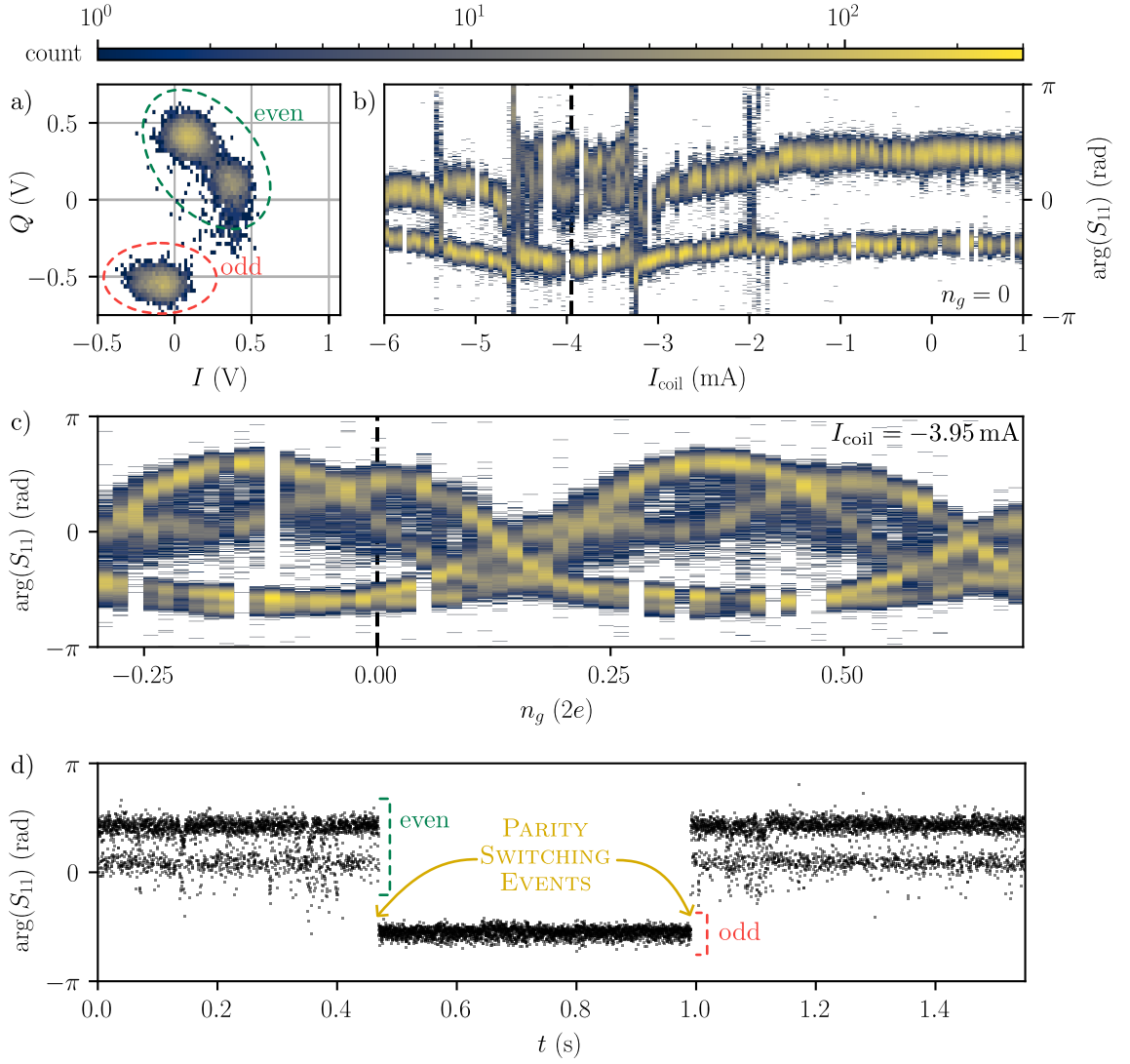


Figure 2.17: IQ clouds, bias sweeps and parity switching. **a)** Histogram of the IQ quadrature of the demodulated readout signal S_{11} at $I_{\text{coil}} = -3.95$ mA and $n_g = 0$ (see vertical dashed lines in panels b and c). At these values, the signal exhibits at least four clouds, two of them corresponding to an even ($n_g = 0$) or odd ($n_g = 0.5$) number of quasiparticles in the island. Lack of control on the charge prevents us from measuring the AC-Stark shift effect to calibrate the axes in number of measurement photons. **b), c)** Histogram of the phase of the readout signal $\arg(S_{11}) = \arg(I + iQ)$ versus coil current I_{coil} (b) or charge offset n_g (c). Both plots illustrate that the circuit frequencies depend heavily on the charge, hence on the parity. So, a good control of the charge is necessary in order to probe and characterize such a sample. **d)** $\arg(S_{11})$ versus time at the same bias parameters as panel a. The plot reveals two parity switching events happening after 470 ms and 990 ms, i.e. the parity switching time is at least a few hundred milliseconds.

the even pair. This proves that for a given n_g , we have a quantum system with two states, that we are able to measure in single shot. In panels b and c, we show how these clouds evolve with flux bias (I_{coil}) and charge bias (n_g) by plotting histograms of the reflection coefficient's phase, $\arg(S_{11})$. At $n_g = 0$, the flux sweep reveals again two groups of clouds for $n_g = 0$ and $n_g = 0.5$. Interestingly, the only bias parameters showing state separation are around the sweet spot at -4 mA for $n_g = 0$. Of course, without a full circuit spectrum,

it is difficult to attribute this increase in dispersive shift to an increase of the main qubit frequency (via the naive approach $\chi \propto g^2/(f_q - f_r)$) or a decrease of the qubit frequencies that would bring higher transitions close to resonance with the resonator. The labeling of even and odd pairs of clouds is therefore arbitrary. At the sweet spot $I_{\text{coil}} = -3.95$ mA, we measure again such IQ histograms when sweeping the offset charge n_g (Fig. 2.17.c). It reveals a complex intertwining of four clouds moving with charge. The dependence is periodic and clearly shows even and odd number of charges. Unexpectedly, and contrary to most charge-dependent superconducting circuits, the angular position of the clouds is not symmetric. While we do not know the exact reason for this behavior, we suspect it is due to uncontrolled parameters in the fabrication, giving asymmetry between the loops or even creating stray junctions in the wires.

Finally, we note that in panels b and c, some measurement points show only odd or even clouds. This is due to the relatively small measurement time, that allows the parity to stay constant over 10,000 measurement points. In Fig. 2.17.d, we show the time trace of the same IQ quadrature as in panel a. It exhibits only two parity switching events over 1.5 s. This indicates a parity switching time of at least a few hundred milliseconds, higher than many measured parity switching times in the literature, especially without specific engineering or microwave filtering. It might be due to the nature of the superconducting island, that has a small total volume and is mostly made of granular aluminum, whose higher superconducting gap prevents some tunneling of broken Cooper pairs.

2.6 Conclusion

In this chapter, we have demonstrated simultaneous and stable sweet-spot flux locking of eight gradiometric fluxoniums equipped with an aluminum ring. The devices exhibit no flux escapes, unlike gradiometric fluxoniums with granular aluminum rings. Furthermore, we are able to reduce the field susceptibility and asymmetry of the samples down to an effective area of a few square micrometers and an inductive asymmetry less than 1‰, which implies that fluxonium qubits can be locked within $\Phi_{\text{eff}} = -3 \times 10^{-4} \Phi_0$ from the half-flux-quantum sweet spot. At zero field, $T_{2,E}$ is reduced by only 15 % from its maximum. While further research would be necessary to pinpoint and mitigate the reasons why the locking mechanism fails about 20 % of times, we consider that this scheme is beneficial for large scale integration in any quantum processor that includes flux-tunable elements. The stability of the flux locking enables the implementation of more complex systems of trapped fluxons in superconducting circuits such as the proposed four-loop fluxonium.

We have shown that we are able to fabricate four-loop fluxonium circuits with four working junctions, with above 90% reproducibility in Josephson energy. We have identified challenges in fabrication and measurement. In particular, while most measured samples have shown a similar behavior, the signatures in resonator spectroscopy are different enough to suspect vastly different parameters across samples. Nonetheless, the biggest challenge remains in measurement, where the lack of control on the charge offset prevents us from measuring the mode frequencies in standard two-tone spectroscopy, calibrating

pulses, etc. Going further, several avenues could be explored. Regarding charge-offset control, we can apply methods widely employed in charge-tunable superconducting qubits, in particular in transmons. Such strategies include increasing the capacitance in order to flatten the charge dependence, engineering the asymmetry of superconducting gap between junction electrodes, filtering infrared radiations in the microwave lines, etc. In Chapter 3, we explore these strategies on the minimal example of a charge-tunable fluxonium qubit. Another idea is to shunt the central island with normal metal in order to equilibrate the charge with the outer ring. This approach is being tested in different academic groups for Cooper-pair boxes and similar qubits (see for instance Ref. [128]) and seems to necessitate large resistance and capacitance for such flux qubits.

3 Charge-tunable fluxonium qubits

3.1 Motivations

At the core of the field of superconducting circuits sits the duality between conjugate variables charge and flux. Each of them is associated with an experimental parameter, linked to a geometrical property of the circuit under study: charge offset on a superconducting island and external flux in a superconducting loop. Each parameter is also associated with noise. However, the duality is not symmetric. In particular, charge noise is typically about two orders of magnitude larger than flux noise in superconducting qubits [125]. Charge offset also undergoes drifts, jumps and parity switching events due to quasiparticle tunneling and thus requires frequent recalibration, in contrast with the external flux that, with appropriate magnetic shielding, is a generally stable parameter. For these reasons, offset charge tunability is typically designed to be exponentially suppressed by an additional shunt capacitance, which is the transmon strategy [56], or completely removed by a shunt inductance, which is the strategy employed in fluxoniums [129]. In parallel, a number of design, fabrication and filtering tricks can be played to attenuate noise as well as parity switching events, drifts and jumps.

The transmon and the fluxonium strategies can unfortunately not be used on any system. In particular, circuits aimed at realizing protected qubits often come with a superconducting island separated from the rest of the circuit. In Section 2.5, we presented a four-loop fluxonium design that includes such an island. Since adding an inductive shunt would break the necessary periodic boundary condition and large capacitances are difficult to reach in the design, we want to apply tricks to improve control of the charge offset. In order to test and prove different strategies, we focus on the minimal working example of a charge-tunable fluxonium qubit, where the main phase-slip junction is replaced by two junctions in series, forming a superconducting island in between. The offset charge on the island can be controlled by an applied DC voltage and tunes the qubit spectrum with a charge dispersion set by the Josephson energies of both junctions. This system, sometimes referred to as a bifluxon qubit, is interesting on more than one account. On top of its proximity to the four-loop circuit, it provides bit-flip protection when biased at $n_g = 0.5$ for identical junctions thanks to the disjoint support of the first two eigen-wavefunctions. The bifluxon qubit presented in Ref. [126] is also promising because parity switching events happen very rarely, on timescales of tens of minutes to hours.

Last but not least, as their coherence has improved year after year, superconducting qubits are becoming valuable as low-noise highly-sensitive quantum sensors. In the proposed charge-tunable fluxonium circuit, charge offset acts as a secondary tuning knob that,

additionally to the external flux, tunes the spectrum and matrix elements of the qubit. We argue that it can be a useful tool to uncover environment effects on qubits, unknown decoherence channels, weakly coupled two-level systems (TLS), etc. Specifically, we will present a method, specific to charge-tunable fluxoniums, to measure the type of coupling (i.e. the qubit operator that mediates the coupling) between qubits and long-lived TLSs.

3.2 Device presentation

In this section, we first derive the Hamiltonian of a charge-tunable fluxonium qubit and explore its spectrum. We then discuss design ideas, fabrication processes and sample characterization.

3.2.1 Theory of a flux- and charge-tunable qubit

We derive the Hamiltonian of two similar circuit designs for charge-tunable fluxoniums. Both designs, shown in Fig. 3.1.a (type 1) and Fig. 3.1.b (type 2), have two Josephson junctions (in blue) forming a superconducting island, represented as a black dot, on which the charge offset n_g is set by a DC voltage V_g through capacitance C_g . The junctions are shunted by an inductance L (in red) to create a fluxonium loop. To keep it general, we denote the effective flux through the loop Φ_{eff} . In the measured samples, it corresponds to either a real external flux through one loop or an effective flux through gradiometric loops (see Section 2.2.1), which does not change the physics in either sample type. The difference between the designs in Fig. 3.1 instead lies in the large additional capacitance C . In the first design, it shunts only the first junction, while in the second design it shunts both junctions at once.

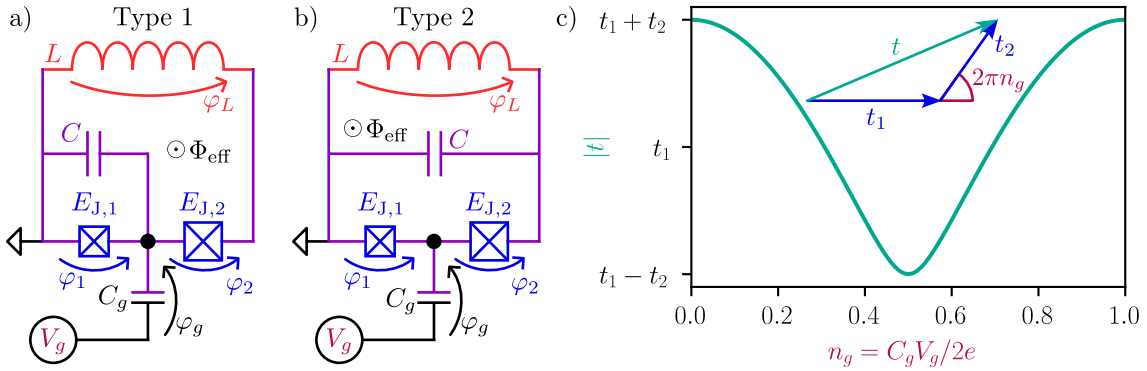


Figure 3.1: Charge-tunable fluxonium qubit. **a, b)** Circuit diagram of the two charge-tunable fluxonium implementations studied hereafter. In both diagrams, the qubit consists of two Josephson junctions (in blue, with Josephson energies $E_{J,1}$ and $E_{J,2}$), an inductor (in red, with inductance L) and a capacitor (in purple, with capacitance C). A DC source of voltage V_g is capacitively coupled to the island between the two junctions with capacitance C_g . In panel a, the capacitor is in parallel with the first junction while in panel b it is in parallel with both junctions. **c)** Total tunneling rate $|t| = |t_1 + t_2 e^{i2\pi n_g}|$ versus charge offset $n_g = C_g V_g / 2e$ in the quantum phase slip paradigm for two junctions separated by a superconducting island.

Type 1

Let us derive the Hamiltonian in the first design. We first write the Lagrangian

$$\mathcal{L} = \frac{C_{J,1} + C}{2} \dot{\Phi}_1^2 + \frac{C_{J,2}}{2} \dot{\Phi}_2^2 + \frac{C_g}{2} \dot{\Phi}_g^2 - \left(\frac{1}{2L} \Phi_L^2 - E_{J,1} \cos \varphi_1 - E_{J,2} \cos \varphi_2 \right), \quad (3.1)$$

and two loop relations:

$$\varphi_1 + \varphi_2 - \varphi_L + \varphi_{\text{eff}} = 0, \quad (3.2)$$

$$\Phi_1 - \Phi_g - V_g t = 0, \quad (3.3)$$

where the second relation corresponds to an open loop and takes the applied DC voltage into account (here, t is time). It can be written equivalently $\dot{\Phi}_1 - \dot{\Phi}_g = V_g$. Hence, the Lagrangian becomes

$$\mathcal{L} = \frac{C_{J,1} + C}{2} \dot{\Phi}_1^2 + \frac{C_{J,2}}{2} \dot{\Phi}_2^2 + \frac{C_g}{2} (\dot{\Phi}_1 - V_g)^2 - \left(\frac{1}{2L} (\Phi_1 + \Phi_2 + \Phi_{\text{eff}})^2 - E_{J,1} \cos \varphi_1 - E_{J,2} \cos \varphi_2 \right). \quad (3.4)$$

We determine the conjugate charge variables to the flux variables Φ_1, Φ_2 as:

$$Q_1 = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}_1} = (C_{J,1} + C + C_g) \dot{\Phi}_1 - C_g V_g, \quad (3.5)$$

$$Q_2 = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}_2} = C_{J,2} \dot{\Phi}_2, \quad (3.6)$$

which allows us to write the Hamiltonian

$$\mathcal{H} = \dot{\Phi}_1 Q_1 + \dot{\Phi}_2 Q_2 - \mathcal{L} \quad (3.7)$$

$$= \frac{1}{2C_\Sigma} (Q_1 + C_g V_g)^2 + \frac{1}{2C_2} Q_2^2 + \frac{1}{2L} (\Phi_1 + \Phi_2 + \Phi_{\text{eff}})^2 - E_{J,1} \cos \varphi_1 - E_{J,2} \cos \varphi_2, \quad (3.8)$$

where we removed constant terms in the second line.

Finally, writing the reduced charges $n_i = Q_i/2e$ and the offset $n_g = C_g V_g/2e$, as well as the phases $\varphi_i = 2\pi \Phi_i/\Phi_0$, we can write the charge-tunable fluxonium Hamiltonian:

$$\hat{\mathcal{H}}_{CTF1} = 4E_{C_\Sigma} (\hat{n}_1 + n_g)^2 + 4E_{C_2} \hat{n}_2^2 + \frac{E_L}{2} (\hat{\varphi}_1 + \hat{\varphi}_2 + \varphi_{\text{eff}})^2 - E_{J,1} \cos \hat{\varphi}_1 - E_{J,2} \cos \hat{\varphi}_2. \quad (3.9)$$

Type 2

For the second design, in Fig. 3.1.b, we apply the same method, with Lagrangian

$$\mathcal{L} = \frac{C_{J,1}}{2} \dot{\Phi}_1^2 + \frac{C_{J,2}}{2} \dot{\Phi}_2^2 + \frac{C_g}{2} \dot{\Phi}_g^2 + \frac{C}{2} \dot{\Phi}_L^2 - \left(\frac{1}{2L} \Phi_L^2 - E_{J,1} \cos \varphi_1 - E_{J,2} \cos \varphi_2 \right), \quad (3.10)$$

and the same loop equations

$$\varphi_1 + \varphi_2 - \varphi_L + \varphi_{\text{eff}} = 0, \quad (3.11)$$

$$\Phi_1 - \Phi_g - V_g t = 0, \quad (3.12)$$

that yield

$$\mathcal{L} = \frac{C_{J,1}}{2} \dot{\Phi}_1^2 + \frac{C_{J,2}}{2} \dot{\Phi}_2^2 + \frac{C_g}{2} (\dot{\Phi}_1 - V_g)^2 + \frac{C}{2} (\dot{\Phi}_1 + \dot{\Phi}_2)^2 - \left(\frac{1}{2L} \Phi_L^2 - E_{J,1} \cos \varphi_1 - E_{J,2} \cos \varphi_2 \right). \quad (3.13)$$

Therefore, the conjugate variables Q_1 and Q_2 are

$$Q_1 = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}_1} = (C_{J,1} + C + C_g) \dot{\Phi}_1 + C \dot{\Phi}_2 - C_g V_g, \quad (3.14)$$

$$Q_2 = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}_2} = C \dot{\Phi}_1 + (C + C_{J,2}) \dot{\Phi}_2, \quad (3.15)$$

which can be written in matrix form as

$$\begin{pmatrix} Q_1 + C_g V_g \\ Q_2 \end{pmatrix} = \begin{pmatrix} C_{J,1} + C + C_g & C \\ C & C + C_{J,2} \end{pmatrix} \begin{pmatrix} \dot{\Phi}_1 \\ \dot{\Phi}_2 \end{pmatrix}. \quad (3.16)$$

We define $C_\Delta^2 = (C_{J,1} + C + C_g)(C + C_{J,2}) - C^2$ the matrix determinant and re-organize the terms in the Hamiltonian $\mathcal{H}_{CTF2} = \dot{\Phi}_1 Q_1 + \dot{\Phi}_2 Q_2 - \mathcal{L}$, so that:

$$\begin{aligned} \mathcal{H}_{CTF2} &= \frac{C + C_{J,2}}{2C_\Delta^2} (Q_1 + C_g V_g)^2 + \frac{C_{J,1} + C + C_g}{2C_\Delta^2} Q_2^2 - \frac{C}{C_\Delta^2} (Q_1 + C_g V_g) Q_2 \\ &\quad + \frac{1}{2L} (\Phi_1 + \Phi_2 + \Phi_{\text{eff}})^2 - E_{J,1} \cos \varphi_1 - E_{J,2} \cos \varphi_2. \end{aligned} \quad (3.17)$$

Finally, with reduced charges $n_i = Q_i/2e$ and charge offset $n_g = C_g V_g/2e$, we reach

$$\begin{aligned} \hat{\mathcal{H}}_{CTF2} &= 4E_{C_1} (\hat{n}_1 + n_g)^2 + 4E_{C_2} \hat{n}_2^2 - 4E_{C_\Delta^2/2C} (\hat{n}_1 + n_g) \hat{n}_2 \\ &\quad + \frac{E_L}{2} (\hat{\varphi}_1 + \hat{\varphi}_2 + \varphi_{\text{eff}})^2 - E_{J,1} \cos \hat{\varphi}_1 - E_{J,2} \cos \hat{\varphi}_2, \end{aligned} \quad (3.18)$$

with $C_1 = C_\Delta^2/(C + C_{J,2})$ and $C_2 = C_\Delta^2/(C + C_{J,1} + C_g)$.

Comparison

Both Hamiltonians in Eq. (3.9) and Eq. (3.18) look similar, and we can prove that they are equivalent with a simple change of variables. Let us define:

$$\tilde{\varphi}_1 = \varphi_1 + \varphi_2, \quad (3.19)$$

$$\tilde{\varphi}_2 = \varphi_1 - \varphi_2, \quad (3.20)$$

which, enforcing the commutation relations $[\tilde{\varphi}_i, \tilde{n}_j] = i\delta_{ij}$, entails:

$$\tilde{n}_1 = \frac{n_1 + n_g + n_2}{2}, \quad (3.21)$$

$$\tilde{n}_2 = \frac{n_1 + n_g - n_2}{2}. \quad (3.22)$$

In both Hamiltonians, the potential terms (functions of φ) are already equal. Focusing on the kinetic terms $\mathcal{T}(n_1, n_2)$, we write:

$$\mathcal{T}_{\text{CTF},1} = 4E_{C_\Sigma}(\tilde{n}_1 + \tilde{n}_2)^2 + 4E_{C_2}(\tilde{n}_1 - \tilde{n}_2)^2 \quad (3.23)$$

$$= 4(E_{C_\Sigma} + E_{C_2})\tilde{n}_1^2 + 4(E_{C_\Sigma} + E_{C_2})\tilde{n}_2^2 + 4(E_{C_\Sigma} - E_{C_2})2\tilde{n}_1\tilde{n}_2, \quad (3.24)$$

and

$$\mathcal{T}_{\text{CTF},2} = 4E_{C_1}(\tilde{n}_1 + \tilde{n}_2)^2 + 4E_{C_2}(\tilde{n}_1 - \tilde{n}_2)^2 - 4E_{C_{\Delta^2/2C}}(\tilde{n}_1 + \tilde{n}_2)(\tilde{n}_1 - \tilde{n}_2) \quad (3.25)$$

$$= 4(E_{C_1} + E_{C_2} - E_{C_{\Delta^2/2C}})\tilde{n}_1^2 + 4(E_{C_1} + E_{C_2} + E_{C_{\Delta^2/2C}})\tilde{n}_2^2 + 4(E_{C_1} - E_{C_2})2\tilde{n}_1\tilde{n}_2. \quad (3.26)$$

In short, the Hamiltonians are equivalent, up to a parameter renormalization, and will be treated as such in the following.

In both cases, we can intuitively understand the charge tunability by treating the junctions as quantum phase slip (QPS) elements. In this paradigm, that we introduced in Section 1.2.7 and used for the four-loop fluxonium in Section 2.5.1, we are interested in the tunneling amplitudes [59–61]

$$t_j = \frac{4}{\sqrt{\pi}}(8E_{J,j}^3 E_{C,j})^{1/4} e^{-\sqrt{8E_{J,j}/E_{C,j}}}. \quad (3.27)$$

In Fig. 3.1.c we show as an inset their vector representation, where the angle between the vectors is set by the charge offset n_g because of Aharonov-Casher interference. Therefore, the 0 – 1 transition frequency at the half-flux-quantum sweet spot is

$$f_{01}(\Phi_{\text{eff}} = \Phi_0/2, n_g) = |t_1 + t_2 e^{i2\pi n_g}| \quad (3.28)$$

$$= \left| (t_1 + t_2) \cos\left(2\pi \frac{n_g}{2}\right) - i(t_1 - t_2) \sin\left(2\pi \frac{n_g}{2}\right) \right| \quad (3.29)$$

$$= \sqrt{(t_1 + t_2)^2 \cos^2\left(2\pi \frac{n_g}{2}\right) + (t_1 - t_2)^2 \sin^2\left(2\pi \frac{n_g}{2}\right)}. \quad (3.30)$$

The qubit frequency oscillates between $t_1 + t_2$ and $t_1 - t_2$, which in the following will be called the charge dispersion Δf . For identical junctions, the formula reduces to $f_{01} = (t_1 + t_2)|\cos 2\pi n_g/2|$ and the frequency reaches zero at $n_g = 0.5$ (a regime known as bifluxon qubit). For very different junctions on the other hand, where $t_2 \ll t_1$, f_{01} becomes $|t_1 + t_2 \cos(2\pi n_g)|$, i.e. a fluxonium with a very small charge tunability.

The circuits presented here showcase a large variety of behaviors, from a barely-charge-sensitive fluxonium¹ to a fully-symmetric bifluxon where fluxons tunnel by pairs at

¹ Mitchell once suggested to shorten it to "BCS fluxonium".

$n_g = 0.5$, regimes that only differ by the relative values of $E_{J,1}$ and $E_{J,2}$. In the remaining of this chapter, we will focus on relatively small charge tunabilities, but the fabrication, measurement and noise mitigation strategies presented in the following can be adapted to symmetric bifluxon devices.

3.2.2 Sample design and fabrication

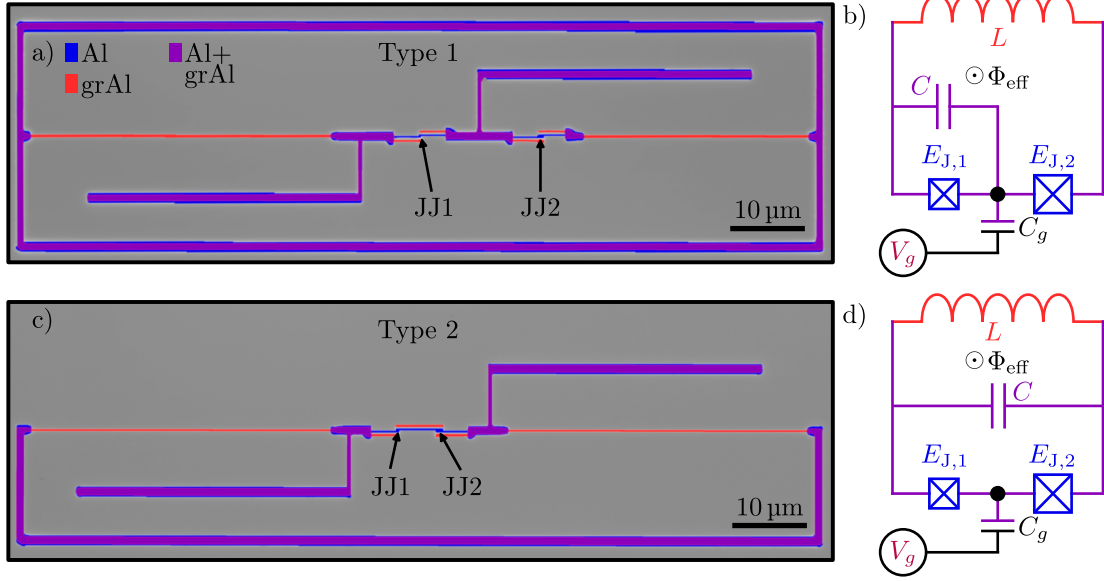


Figure 3.2: Charge-tunable fluxoniums sample pictures. **a, c)** False-colored optical microscope picture of a type-1 (a) or type-2 (c) sample. In type 1, the geometric capacitance from the pad shunts only the first junction. This sample is made to resemble previous gradiometric samples with a spurious second junction. In type 2, the capacitance shunts both junctions and the island is made purposefully small. Both gradiometric and non-gradiometric samples are fabricated for both types. **b, d)** Equivalent circuit of type-1 and type-2 samples, from which we derived the Hamiltonians in Section 3.2.1.

Building upon the gradiometric fluxoniums we designed, fabricated and measured in Chapter 2, we adapt the design to include two junctions in each sample. The second junction is added in two ways. In the type-1 design (see Fig. 3.2.a), the second junction is placed to the right of the capacitive pad, so that the capacitance shunts only the first junction. This corresponds to the circuit in Fig. 3.1.a, repeated in Fig. 3.2.b. This design is meant to test an hypothesis for the unwanted charge dispersion observed in some gradiometric fluxonium samples, namely that a spurious junction can form at the contact between aluminum and granular aluminum, at the position of JJ2 in Fig. 3.2.a. In the type-2 design (see Fig. 3.2.c), we instead fabricate the junctions close to each other in order to reduce the volume of the central superconducting island. The capacitance now shunts both junctions, in accordance with the circuit in Fig. 3.1.b and Fig. 3.2.d. Note that we chose to fabricate both gradiometric and single-loop samples for each type, which has no impact on the equivalent circuit or the qubit spectrum.

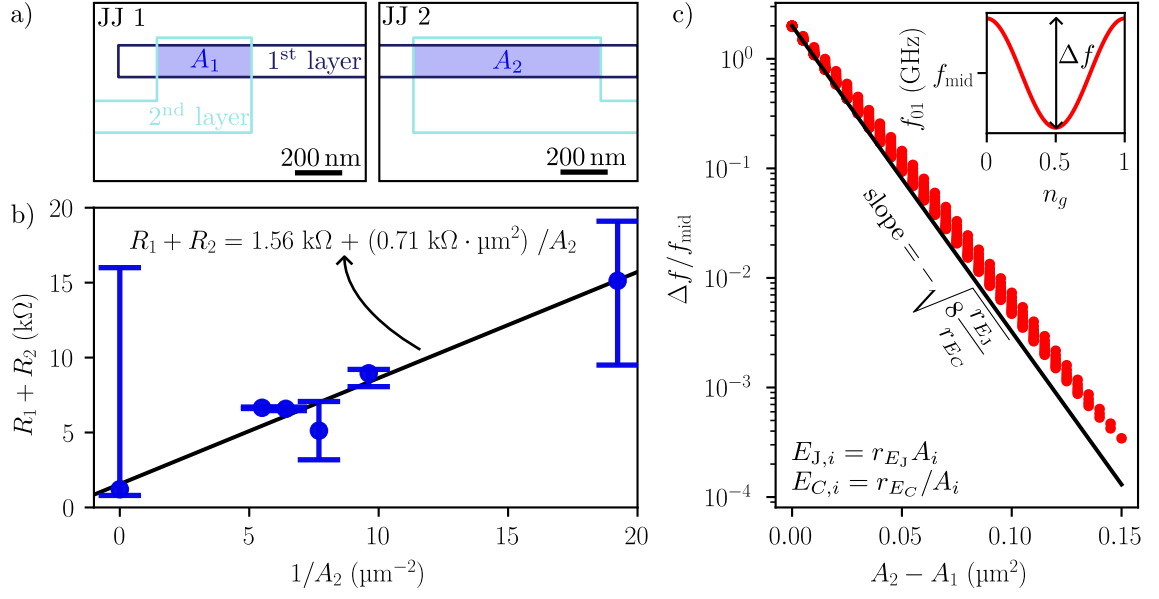


Figure 3.3: Josephson junction design and charge dispersion. **a)** Schematic of the two junctions in a sample, representing the first and second aluminum layers overlapping on area A_i for junction i . The overlap area A_i controls the Josephson energy $E_{J,i} \propto A_i$ and the junction capacitance $C_{J,i} \propto A_i$. **b)** Measured room-temperature resistance $R_1 + R_2$ of test structures with various second-junction area A_2 . Points at $1/A_2 = 0 \mu\text{m}^{-2}$ correspond to test structures with only one junction. Markers represent the median value over all working test junctions, while errorbars represent the minimum and maximum values. A linear dependence to $1/A_2$ is fitted. **c)** Computed relative charge dispersion $\Delta f/f_{\text{mid}}$ (see inset) versus difference in junction area $A_2 - A_1$ (red markers). The coefficients used to compute E_J and E_C are $r_{E_J} = h \times 200 \text{ GHz} \cdot \mu\text{m}^{-2}$ and $r_{E_C} = h \times 0.39 \text{ GHz} \cdot \mu\text{m}^2$ (equivalently, $r_{C_J} = 50 \text{ fF} \cdot \mu\text{m}^{-2}$). Black line represents the scaling expected from tunneling rates.

In order to have access to different regimes, from bifluxon qubits with equal junctions to barely-charge-sensitive fluxoniums, we vary the second junction's Josephson energy $E_{J,2}$ and intrinsic capacitance $C_{J,2}$. Both are tuned by the overlap area A_2 , represented schematically in Fig. 3.3.a, via

$$E_{J,2} \propto A_2, \quad C_{J,2} \propto A_2. \quad (3.31)$$

We estimate the Josephson energy from room-temperature resistance measurements on test structures composed of two junctions in series, where only the second junction's area is swept. As $E_{J,2} \propto A_2$, we expect the total resistance $R_{2\text{JJ}s} = R_1 + R_2$ of such test structures to be

$$R_{2\text{JJ}s} = R_1 + r/A_2, \quad (3.32)$$

with r a coefficient of proportionality. In Fig. 3.3.b, we report the resistance values measured in one of the fabricated wafers, versus the inverse area $1/A_2$. At $1/A_2 = 0 \mu\text{m}^{-2}$, the points correspond to test structures with only the first junction. $R_{2\text{JJ}s}$ follows a linear trend

$$R_{2\text{JJ}s} = 1.56 \text{ k}\Omega + (0.71 \text{ k}\Omega \cdot \mu\text{m}^2)/A_2, \quad (3.33)$$

from which we attribute $R_1 = 1.56 \text{ k}\Omega$, corresponding to $E_{J,1}/h = 77 \text{ GHz}$. This value is much higher than targeted, which was solved in the next fabrication iterations. We

also extract the proportionality coefficient $r = 0.71 \text{ k}\Omega \cdot \mu\text{m}^2$, which we can relate to the coefficient for intrinsic capacitance² $r_{C_J} = 50 \text{ fF} \cdot \mu\text{m}^{-2}$ [131–133] and the proportionality found in Fig. 2.5 from Section 2.2.3, $C_J = (0.25 \text{ fF/GHz}) \times E_J/h$. Indeed,

$$r_{E_J} = \frac{E_J}{A} = \frac{E_J}{C_J} \times \frac{C_J}{A} = h \times (4 \text{ GHz/fF}) \times (50 \text{ fF} \cdot \mu\text{m}^{-2}) \quad (3.34)$$

$$= h \times 200 \text{ GHz} \cdot \mu\text{m}^{-2}. \quad (3.35)$$

From the resistance measurements, using the Ambegaokar-Baratoff formula [134], we find:

$$r_{E_J} = \frac{\Phi_0 \times 1.76 k_B T_c}{4e} \times \frac{1}{r} = h \times 197 \text{ GHz} \cdot \mu\text{m}^{-2}. \quad (3.36)$$

Thus, the scaling of E_J with the junction overlap area is quantitatively consistent with the literature.

Such a control in fabrication over the Josephson energy gives access to a range of charge dispersion values, from gigahertz to kilohertz, as illustrated from numerically computed frequencies in Fig. 3.3.c. In the inset, we define the charge dispersion $\Delta f = f_{01}(n_g = 0) - f_{01}(n_g = 0.5)$ as the amplitude of charge tunability over charge offset n_g at the half-flux sweet spot $\Phi_{\text{eff}} = \Phi_0/2$. We also define the middle frequency $f_{\text{mid}} = (f_{01}(n_g = 0) + f_{01}(n_g = 0.5))/2$. The relative charge dispersion $\Delta f/f_{\text{mid}}$ follows an exponential trend downward when sweeping the area difference $\Delta A = A_2 - A_1$. Indeed, applying the proportionality relations in Eq. (3.31) to the tunneling rate in Eq. (3.27), we get

$$t_j = \frac{4}{\sqrt{\pi}} (8r_{E_J}r_{E_C})^{1/4} e^{-A_j \sqrt{8r_{E_J}/r_{E_C}}}, \quad (3.37)$$

where r_{E_C} is the proportionality coefficient for E_C in a junction, therefore given by $r_{E_C} = e^2/2r_{C_J}$. Considering that the qubit frequency is well approximated by the tunneling rate $|t_1 + t_2 e^{i2\pi n_g}|$ in the fluxonium regime (see Section 1.2.3 and Section 1.2.7), we come to

$$\Delta f = f_{01}(n_g = 0) - f_{01}(n_g = 0.5) = (t_1 + t_2) - (t_1 - t_2) \quad (3.38)$$

$$= 2t_2, \quad (3.39)$$

$$f_{\text{mid}} = (f_{01}(n_g = 0) + f_{01}(n_g = 0.5)) = ((t_1 + t_2) + (t_1 - t_2))/2 \quad (3.40)$$

$$= t_1, \quad (3.41)$$

where we consider that the second junction is the largest, thus $t_2 < t_1$. The relative charge dispersion is therefore:

$$\frac{\Delta f}{f_{\text{mid}}} = 2e^{-(A_2 - A_1) \sqrt{8r_{E_J}/r_{E_C}}}. \quad (3.42)$$

In Fig. 3.3.c, we plot the corresponding exponential dependence, with the coefficients $r_{E_J} = 200 \text{ GHz} \cdot \mu\text{m}^{-2}$ and $r_{E_C} = 0.39 \text{ GHz} \cdot \mu\text{m}^2$, also used in the numerical computation of the frequencies by diagonalizing the full Hamiltonian. Both methods align, while the

² This value corresponds to a parallel plate capacitor with the AlO_x relative permittivity $\epsilon_r = 4$ [130] and oxide thickness $d = 0.7 \text{ nm}$.

slope of the exponential trend differs slightly, most probably a signature of the fact that the tunneling rates fail to capture all aspects of the fluxonium spectrum.

Thanks to the tuning of the Josephson energy with area, we are able to fabricate many different samples in different regimes on a single wafer and with only one evaporation step. This allows us to obtain the desired charge dispersion as well as redundancy in the measurements.

3.2.3 Characterization

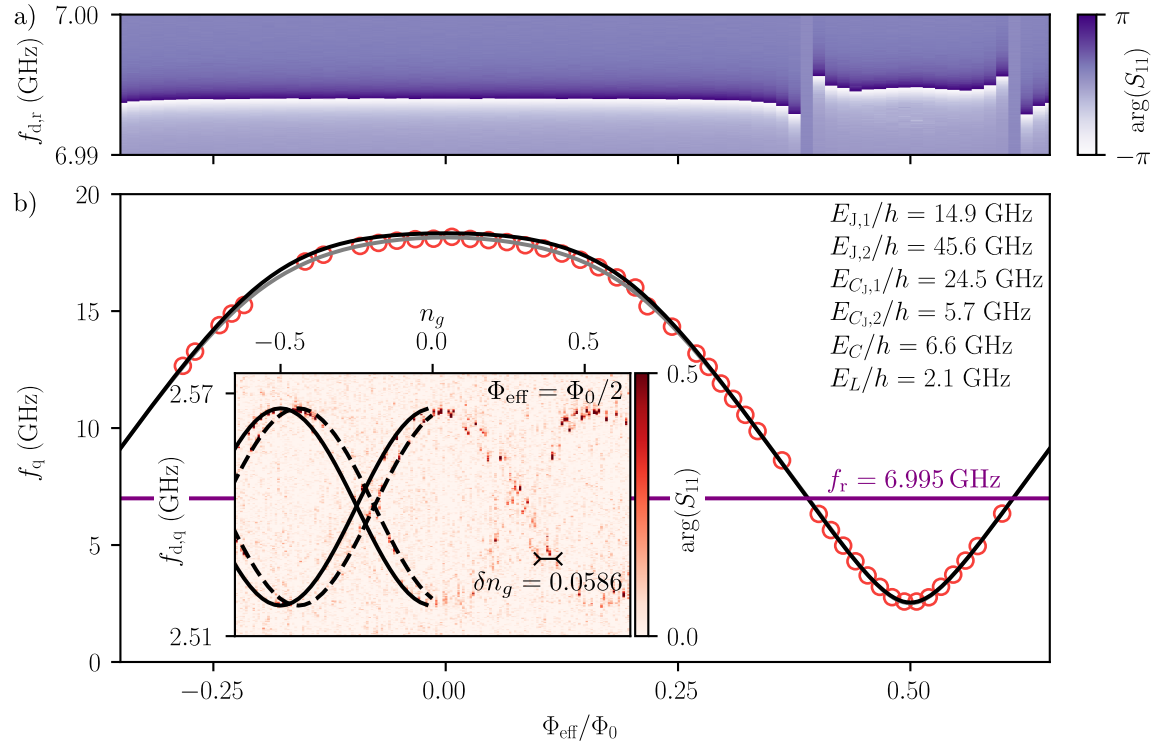


Figure 3.4: Sample characterization. **a)** Resonator spectroscopy versus external flux Φ_{eff} showing two anticrossings as expected for a fluxonium qubit. **b)** Qubit spectrum versus external flux, extracted from two-tone spectroscopy. The spectrum fits to a two-junction fluxonium Hamiltonian with $E_{J,1}/h = 14.93$ GHz, $E_{J,2}/h = 45.60$ GHz, $E_{C,1}/h = 24.47$ GHz, $E_{C,2}/h = 5.72$ GHz, $E_C/h = 6.56$ GHz, $E_L/h = 2.10$ GHz (black line). For comparison, we show the best single-junction fluxonium fit (gray line). Inset: two-tone spectroscopy versus charge offset n_g at $\Phi_{\text{eff}} = \Phi_0/2$. We observe two distinct charge offset jump amplitudes. One is ± 0.5 and corresponds to parity switches. The other is ± 0.0586 (dashed line) and its origin is unknown.

We characterize a sample by measuring its frequency f_q in a standard two-tone spectroscopy. The measurement is performed versus both the external flux and the charge offset parameters. In Fig. 3.4.a we show the resonator spectroscopy corresponding to sample 02R. The colorplot represents the phase of the resonator's reflection coefficient $\arg(S_{11})$. As expected for a fluxonium, even a charge-tunable one, the spectrum reveals two anticrossings at $\Phi_{\text{eff}}/\Phi_0 = 0.39$ and $\Phi_{\text{eff}}/\Phi_0 = 0.61$, allowing to calibrate the position

of the half-flux-quantum sweet spot. Additionally, the periodicity allows to calibrate the flux from the current in the biasing coil.

We then measure the qubit frequency f_q in standard two-tone spectroscopy, fitting to a Lorentzian function at each flux bias point. In Fig. 3.4.b we represent the extracted qubit frequency as red circles and the resonator frequency $f_r = 6.995$ GHz as a purple line. The spectrum can be fitted to a fluxonium spectrum (gray line) with parameters

$$E_J^{(1)}/h = 11.93 \text{ GHz}, \quad E_C^{(1)}/h = 4.87 \text{ GHz}, \quad E_L^{(1)}/h = 2.09 \text{ GHz}, \quad (3.43)$$

where the exponent ⁽¹⁾ symbolizes that the model includes only one Josephson junction. This device is in a similar regime as the gradiometric fluxoniums in Chapter 2, with $E_J^{(1)}/E_L^{(1)} = 5.7$. Contrary to a fluxonium spectrum however, the device is tunable in charge offset. The inset of Fig. 3.4.b displays the two-tone spectroscopy of the device at the sweet spot $\Phi_{\text{eff}} = \Phi_0/2$, versus charge offset n_g and qubit drive frequency $f_{d,q}$. This measurement is repeated around the sweet spot, from $\Phi_{\text{eff}}/\Phi_0 = 0.474$ to $\Phi_{\text{eff}}/\Phi_0 = 0.526$. The qubit frequency versus charge, together with the spectrum versus flux, is fitted to the charge-tunable fluxonium Hamiltonian (Eq. (3.18), plotted as a black line), which yields:

$$E_{J,1}^{(2)}/h = 14.93 \text{ GHz}, \quad E_{J,2}^{(2)}/h = 45.60 \text{ GHz}, \quad (3.44)$$

$$E_{C,1}^{(2)}/h = 24.47 \text{ GHz}, \quad E_{C,2}^{(2)}/h = 5.72 \text{ GHz}, \quad (3.45)$$

$$E_C^{(2)}/h = 6.56 \text{ GHz}, \quad E_L^{(2)}/h = 2.10 \text{ GHz}. \quad (3.46)$$

Both fits follow the frequency in flux well and actually yield similar results. First, notice how E_L takes the same value, corresponding to a total inductance $L = 78$ nH, i.e. the sheet kinetic inductance of granular aluminum is $L_\square = 0.13$ nH/ $\square$. Second, the Josephson junction inductances and capacitances are consistent in both cases. Regarding the inductances, $L_J^{(1)} = 13.7$ nH and $L_{J,1}^{(2)} + L_{J,2}^{(2)} = 14.5$ nH, consistent with two junctions in series. Regarding the capacitances, $C^{(1)} = 3.98$ fF is consistent with $C^{(2)} + 1/(1/C_{J,1}^{(2)} + 1/C_{J,2}^{(2)}) = 3.59$ fF.

In the two-tone spectroscopy versus charge offset (inset of Fig. 3.4.b), the qubit exhibits at least four different charge offsets. While it is expected to observe parity switches that bring n_g to $n_g \pm 0.5$, the origin of the second offset jump is unknown. We can attribute different charge offsets to these branches, so that

$$n_g = C_g V_g + n_{g,0}, \quad (3.47)$$

$$n'_g = C_g V_g + n'_{g,0}. \quad (3.48)$$

In the figure, Eq. (3.47) corresponds to the solid black lines and Eq. (3.48) to the dashed lines. From the fitting routine, we also extract $\delta n_{g,0} = n'_{g,0} - n_{g,0} = -0.05864$. The fact that the charge offset jumps between those two values suggests that the additional offset is due to a charged element toggling between two values, for example a two-level system similar to the one observed in a weakly-charge-sensitive transmon in Ref. [135]. Overall, this illustrates the difficulty of controlling the charge offset on the island, from parity switches to additional drifts and jumps.

3.3 Of the control of charge on a superconducting island

Charge offset is a parameter that is notoriously difficult to control and charge noise is detrimental to the coherence of all charge-tunable qubits with an amplitude about two orders of magnitude larger than flux noise [125]. In this section, we describe different types of charge offset noise, from parity switching events to small jumps, and highlight measurement as well as mitigation methods. We also explore additional phenomena linked to the charge offset, namely parity driving and Coulomb blockade on a small island.

3.3.1 Parity switching: concept and measurement methods

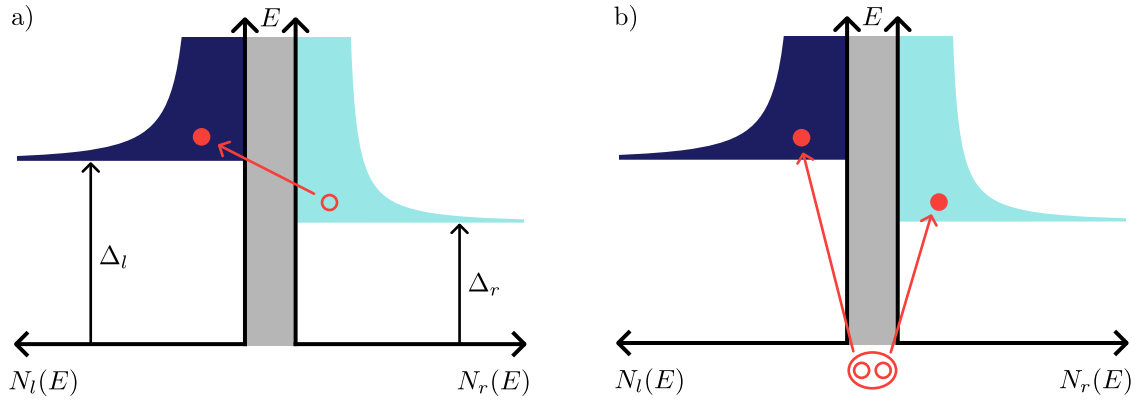


Figure 3.5: Parity switching processes. Density of states $N_l(E)$ (respectively, $N_r(E)$) in the left (respectively, right) superconducting electrode of a Josephson junction versus energy E . The gray rectangle represents the insulating barrier separating the electrodes. The right electrode has a lower superconducting gap than the left electrode, $\Delta_r < \Delta_l$. **a)** Process by which an existing quasiparticle tunnels through the junction, conserving the total number of quasiparticles in the device (NUPS). **b)** Process by which a quasiparticle is created in each junction electrode by the breaking of a Cooper pair near the insulating barrier, in either electrode (PAPS).

Parity switching is the phenomenon by which a quasiparticle (i.e. a broken Cooper pair) is added or removed on the superconducting island, effectively switching the charge offset n_g by ± 0.5 . Parity switching happens through two mechanisms, often referred to as number-conserving parity switching (NUPS) and photon-assisted parity switching (PAPS) in the literature [136, 137], respectively illustrated in Fig. 3.5.a and Fig. 3.5.b. For both mechanisms, we consider a Josephson junction with leads of different superconducting gaps $\Delta_l > \Delta_r$ and represent the density of states $N(E)$ on each side. In Fig. 3.5.a, a quasiparticle tunnels from the right to the left. Due to the gap asymmetry, the parity switching rate Γ_p is proportional to $e^{-\delta\Delta/k_B T}$ with $\delta\Delta = \Delta_l - \Delta_r$. Nonetheless, the energy stored in the qubit can also help the quasiparticle tunnel and trigger a relaxation of the qubit, in which case $\Gamma_p \propto e^{(hf_{01} - \delta\Delta)/k_B T}$. To summarize, we can write

$$\Gamma_p^{(a)} \propto p_q e^{(hf_{01} - \delta\Delta)/k_B T} + (1 - p_q) e^{-\delta\Delta/k_B T}, \quad (3.49)$$

with p_q the qubit population in the excited state.

In Fig. 3.5.a, a quasiparticle is created on both sides by the breaking of a Cooper pair, for example by high-energy photons (hence the name photon-assisted parity switching) [138]. The parity switching rate then depends on the total gap $\Delta_l + \Delta_r$, which needs to be surpassed to create quasiparticles, and on the flux of high-energy photons $E_{\text{photon}} > \Delta_l + \Delta_r$ reaching the sample. We can write the rate as

$$\Gamma_p^{(b)} \propto e^{-(\Delta_l + \Delta_r)/k_B T}. \quad (3.50)$$

In the following, we go through four different methods to measure the parity switching time $T_p = 1/\Gamma_p$. Each method falls into one of two categories: direct methods, which involve measuring the qubit frequency, and state-mapping methods, which map the frequency on the qubit states before measuring them.

Direct measurement methods

In Fig. 3.6, we present two methods to measure the parity switching time in two different samples. These methods are called *direct* because they involve measuring the switching of qubit frequency, either by a two-tone spectroscopy (panels a, b, c) or a repeated measurement of resonator frequency (panels d, e, f). In both methods, the goal is to construct a measurement of the parity $p(t) = -1$ or 1 depending on the parity (odd or even) of the number of quasiparticles on the island. We then calculate the power spectral density $S_{\text{parity}}(f)$ of $p(t)$ from its Fourier transform:

$$S_{\text{parity}}(f) = \lim_{T \rightarrow +\infty} \frac{1}{T} |\mathcal{F}(p(t))|^2 \quad (3.51)$$

$$= \lim_{T \rightarrow +\infty} \frac{1}{T} \left| \int_0^{+\infty} p(t') e^{-i2\pi f t'} dt' \right|^2. \quad (3.52)$$

In the following, S_{parity} is computed by the periodogram method. We expect the parity to follow a random telegraphic noise model, such that

$$S_{\text{parity}}(f) = \frac{A}{1 + (\pi f T_p)^2} + B, \quad (3.53)$$

where T_p is the parity switching time and A, B are constants.

In the first method, the frequency of the qubit is monitored by stroboscopic two-tone spectroscopies. In Fig. 3.6.a, we show the average qubit population P_e obtained from such a measurement on a gradiometric type-1 sample (see Fig. 3.2.a). The two-tone spectroscopy displays two peaks, indicating that the qubit toggles between two frequencies because of quasiparticle tunneling. As the measurements were taken with short waiting times in order to measure with a high sampling frequency, the qubit does not relax to the ground state between measurements and the peaks are not Lorentzian. This detail does not prevent us from attributing a parity p to each measurement time. By averaging over only 20 single measurements at a time, we can measure the qubit frequency every 2.5 ms as in Fig. 3.6.b.

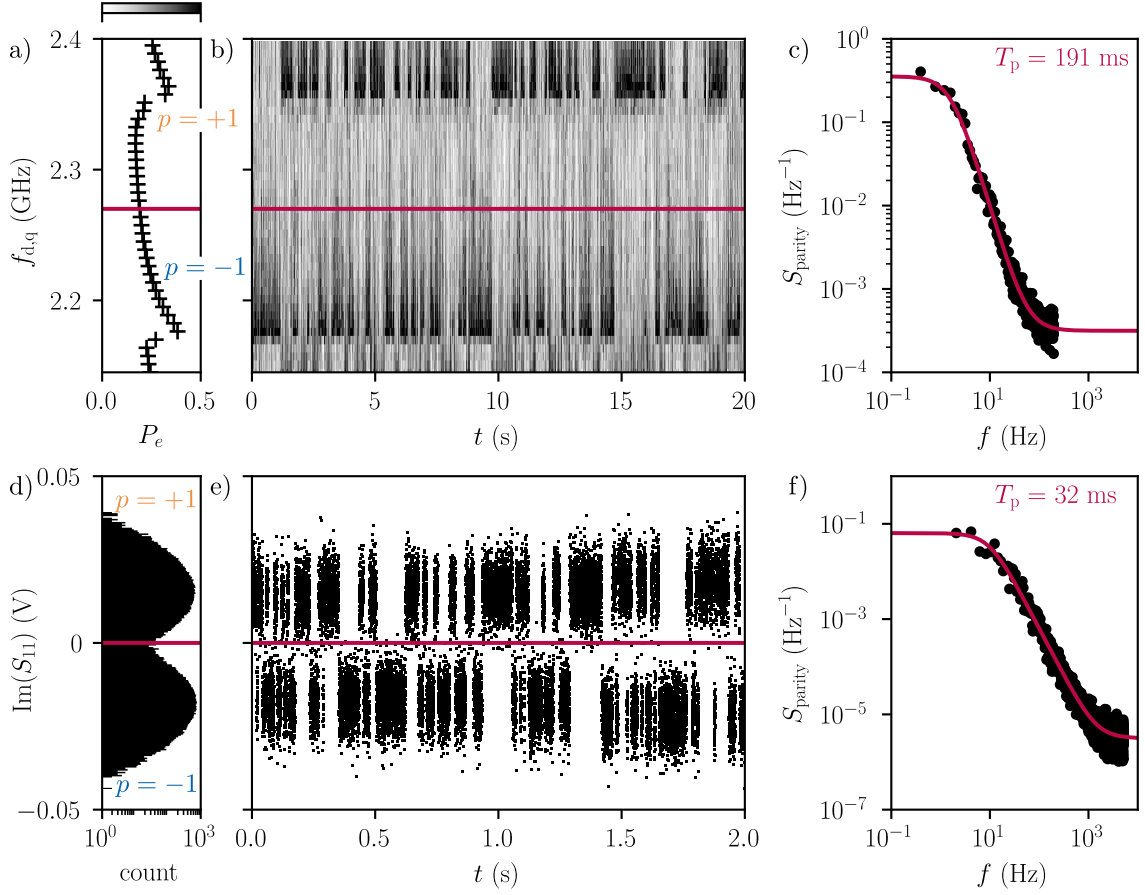


Figure 3.6: Parity switching time measurement, direct methods. **a, b, c)** Measurement of parity with two-tone spectroscopy. **a)** Excited-state population P_e , averaged over 64 s, versus qubit drive frequency $f_{d,q}$. The two-tone spectroscopy shows two peaks, each corresponding to a parity. **b)** Time trace of the same two-tone spectroscopy over twenty seconds, with colors representing P_e according to the axis of panel a. Each column is averaged over 20 measurements (2.5 ms). The parity is attributed at each time according to whether the qubit frequency is above or below the threshold $f_{thr} = 2.27$ GHz (carmine line). **c)** Power spectral density S_{parity} of the parity $p(t)$. S_{parity} fits well to a random telegraphic noise model with parity switching time $T_p = 191$ ms. **d, e, f)** Measurement of parity in direct dispersive measurement. **d)** Histogram of the imaginary part of the resonator reflection coefficient $\text{Im}(S_{11})$, showing two well-separated Gaussians, each corresponding to a parity. **e)** Time trace of the same measurement, where each point is averaged for 97 μ s. Threshold at 0 (carmine line) gives the parity at each time. **f)** Power spectral density S_{parity} of the parity $p(t)$. Fitting to a random telegraphic noise model gives $T_p = 32$ ms.

Based on the threshold at $f_{thr} = 2.27$ GHz, a parity value $p(t)$ is attributed at each time t . As explained, the power spectral density S_{parity} of $p(t)$ carries the information of the parity switching time T_p , which is uncovered by fitting to a random telegraphic noise model (see Fig. 3.6.c). In the case of this sample, $T_p = 191$ ms, which is a relatively large value, thanks to the mitigation strategies that will be detailed in Section 3.3.2. As we see from the time trace and the power spectral density, this method can only be used for large parity switching times because a two-tone spectroscopy with enough frequency sampling and averages can already take several milliseconds. It is however a valuable alternative to

a stroboscopic Ramsey measurement for devices with a charge dispersion larger than the shortest available $\pi/2$ -pulse in frequency domain.

The second direct method relies only on measuring the readout resonator response, without probing the qubit. For devices with a large charge dispersion, such that the resonator frequency depends heavily on the charge offset parity, measuring a time trace of S_{11} reveals two clouds in the complex plane, exactly as it does for qubit states in the IQ plane. In Fig. 3.6.d, the histogram of the imaginary part of the reflection coefficient $\text{Im}(S_{11})$, measured with a Vector Network Analyzer (VNA) on a single-loop type-1 sample (see Fig. 3.2.a), exhibits as expected two symmetrical Gaussians. From the time trace in Fig. 3.6.e, we compute the power spectral density and fit it with parity switching time $T_p = 32$ ms (Fig. 3.6.f). In this sample, T_p is relatively large, which is why it can be measured with such a method. While this method is not available for every sample, because it requires a large charge dispersion, it comes with the advantage of not pushing the qubit away from the thermal state, making the switching time more suitable for comparison across samples. It is also a method that can be applied with only a VNA (provided T_p is larger than the measurement time), making it easy to apply, with no calibration needed.

State-mapping measurement methods

Another technique to measure parity is to map it onto the qubit states. In Fig. 3.7, we present two methods based on this idea. Both methods involve precise manipulation of the qubit but allow to measure short parity times, including in devices with low charge dispersion. The first method is taken from Ref. [139] and is often used in the offset-charge-sensitive transmon literature. It consists in the modified Ramsey sequence shown in Fig. 3.7.a, where two orthogonal $\pi/2$ -pulses (along the X and Y axes) are played at drive frequency f_d in the middle between the two qubit frequencies. Because the sign of the detuning between the actual qubit frequency and f_d depends on the charge parity, the parity is mapped onto the qubit state at the end of the sequence. In the schematical Bloch spheres, the two parity values $p = -1$ or $p = +1$ are represented respectively in blue and orange. We calibrate the drive frequency f_d by performing a Ramsey experiment. The Fourier transform of the qubit population P_e at the end of the sequence is shown in Fig. 3.7.b. In that experiment, the data was post-selected on the parity value obtained from applying the state-mapping sequence right before each single Ramsey measurement. We thus prove that this method is able to accurately select the qubit frequency. Repeating the state-mapping sequence many times gives a parity time trace from which we compute the power spectral density in the same way as the direct methods, and fit it to a random telegraphic noise model. In Fig. 3.7.c, we show the data obtained from applying this method on a gradiometric fluxonium, designed with a single junction, that exhibited a charge dispersion due the presence of a spurious junction, presumably resulting from a poor contact between aluminum and granular aluminum and that happened to have a large parity switching time $T_p = 366$ ms. This method is especially suited for devices with low charge dispersion (typically, Δf around 1 MHz) and makes it possible to measure short parity times.

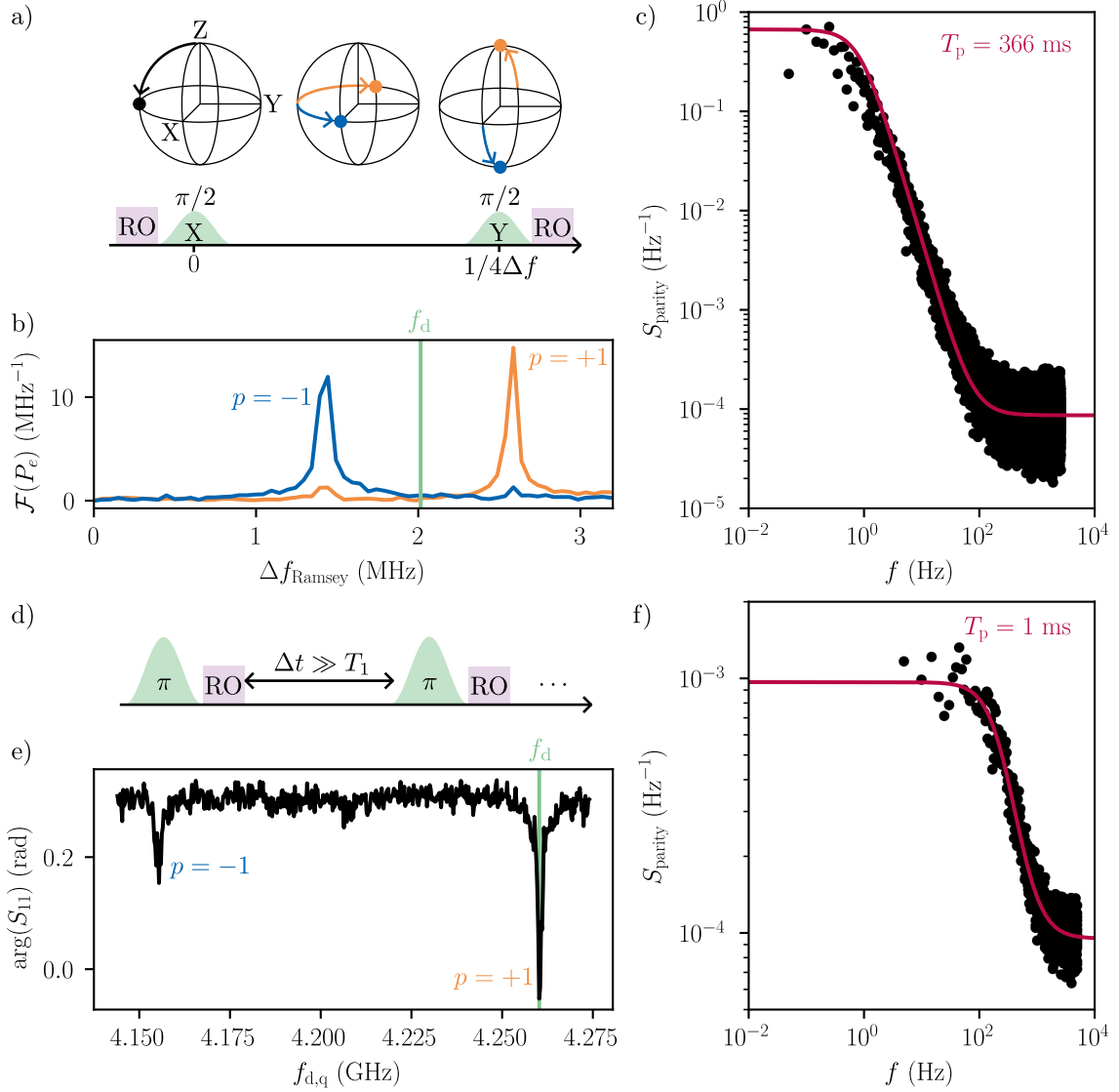


Figure 3.7: Parity switching time measurement, state mapping methods. **a, b, c)** Measurement of parity by mapping to the qubit states with a modified Ramsey sequence. **a)** Pulse sequence and representation of the corresponding qubit state on Bloch spheres. The pulse sequence is based on a Ramsey sequence, with the second $\pi/2$ -pulse orthogonal to the first one. By waiting $1/4\Delta f$ between the pulses, the parity is mapped onto the qubit state at the end of the sequence. **b)** Fourier transform of the qubit population from a (regular) Ramsey sequence applied directly after measuring parity p with the sequence from panel a. In blue (respectively, orange), the data was post-selected on parity $p = -1$ (respectively, $p = +1$). This measurement also calibrates the drive frequency f_d . **c)** Power spectral density of a parity time trace obtained from applying the modified Ramsey sequence, which fits to a telegraphic noise model. **d, e, f)** Measurement of parity by mapping on the qubit state with a selective π -pulse. **d)** Pulse sequence, consisting of a π -pulse at one of the qubit frequencies, repeated regularly after relaxation time. **e)** Reflection coefficient phase $\arg(S_{11})$ from a two-tone spectroscopy used to calibrate the qubit frequencies. In particular, the higher frequency is the drive frequency f_d in the sequence. **f)** Power spectral density of a parity time trace obtained from applying the π -pulse sequence, which fits to a telegraphic noise model.

The second method involving qubit state mapping uses a selective π -pulse instead of $\pi/2$ -pulses. As shown in the pulse sequence in Fig. 3.7.d, applying a π -pulse at drive frequency f_d corresponding to one of the qubit frequencies maps the parity on the qubit states, which is measured by a readout pulse right after. The sequence is repeated after the qubit has time to decay into the thermal state (close to the ground state). In Fig. 3.7.e, we show how f_d is chosen from a two-tone spectroscopy measured previously on the device. The device used in these panels is a charge-tunable gradiometric fluxonium such as the one depicted in Fig. 3.2.a. As for every method described here, we fit the power spectral density of the parity time trace in Fig. 3.2.f, which gives $T_p = 1$ ms, among the smallest measured in all the samples fabricated in this thesis. Note that measuring such a small parity switching time would have been impossible with the direct methods, even though the charge dispersion Δf is relatively large ($\Delta f = 105$ MHz). Moreover, the other state-mapping method could also not be applied, because it requires a $\pi/2$ -pulse whose width in frequency domain embeds both qubit frequencies, contrary to the present method where the selective π -pulse is at one of the two qubit frequencies.

Note that all the parity switching times T_p shown in this section are larger or equal to one millisecond, i.e. T_p is orders of magnitude larger than the relaxation time of the qubits, typically in the tens of microseconds. This means that qubit dynamics can easily be resolved at given n_g values. In the literature, these measurement methods are used to uncover the mechanisms that impact T_p and review mitigation strategies. In the next section, we apply two of such mitigation strategies to our samples.

3.3.2 Mitigation strategies: gap engineering and infrared filtering

Mitigation strategies can be sorted in two categories: first, reducing the quasiparticle density by shielding, filtering, trapping high-energy particles away from the circuit and, second, suppressing the quasiparticle tunneling rate at the junction. The latter consists in engineering the superconducting gap Δ of the layers that make up the circuit such that the quasiparticle tunneling rate is reduced. Typically, this is done by increasing the gap difference $\delta\Delta$ between the junction electrodes, in order to create a barrier that exponentially suppresses quasiparticle tunneling. In particular, $\delta\Delta/h$ should be higher than the qubit frequency. This approach has proved beneficial in transmons, increasing the parity switching time up to 7 s [140]. Another gap engineering strategy is to make both junction electrodes with a higher gap than the rest of the circuit. In Ref. [141], this has proved effective to fabricate transmons with $T_p = 2.7$ s. Yet another gap engineering strategy, combining both ideas, seems to completely suppress parity switching, with $T_p > 1 \times 10^3$ s in transmons [142]. Reducing the quasiparticle density x_{QP} is also effective at increasing T_p , and additionally improves the relaxation time T_1 of the qubit. In Ref. [143], phonons traps realized as normal metal reservoirs on the chip's back side bring T_p to 44 s.

In charge-tunable fluxoniums, also known as bifluxon qubits, and other circuits combining Cooper-pair boxes and inductances, these strategies have shown parity switching times above 20 minutes [86, 126, 144]. Such results open the door to controlling the charge

offset as accurately as the external flux, making both quantities useful tuning knobs for many experiments. In the following, we apply the typical gap engineering strategy on our samples and explore how infrared filtering, by removing radiations above 20 GHz, greatly improves the parity switching time.

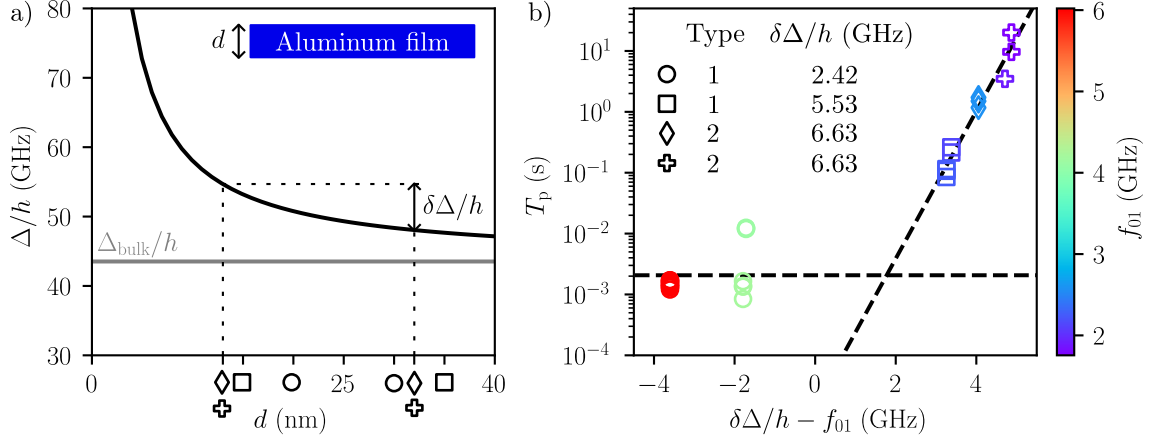


Figure 3.8: Gap engineering applied to charge-tunable fluxoniums. a) Superconducting gap of aluminum versus film thickness d . Adapted from Ref. [145]. On the x-axis, the thicknesses of the aluminum layers in the fabricated samples are reported, with the corresponding markers in panel b. b) Parity switching time T_p versus $\delta\Delta/h - f_{01}$, where $\delta\Delta$ is the superconducting gap difference across the junctions and f_{01} is the qubit frequency. $\delta\Delta$ is estimated from the nominal thickness calibrated in the metallic film evaporator and the scaling in panel a.

Figure 3.8 illustrates the improvements in T_p in charge-tunable fluxoniums, obtained by tuning the gap difference $\delta\Delta$ at the fabrication step. This strategy relies on the dependence of the gap Δ in thin-film superconductors on the film thickness d . In Fig. 3.8.a, we represent that dependence for aluminum, as exhibited in Ref. [145] from experimental data in Refs. [146, 147]:

$$\Delta/h = \Delta_{\text{bulk}}/h + \frac{a}{d}, \quad (3.54)$$

where $\Delta_{\text{bulk}}/h = 43.52$ GHz and the scaling factor $a = 145$ GHz · nm. By fabricating the junction from films of different thicknesses, we tune the gap difference $\delta\Delta$ between the junction electrodes. For example, samples marked as $\diamond$ and $\oplus$ are fabricated with thicknesses $d_1 = 13$ nm and $d_2 = 32$ nm, resulting in $\delta\Delta = 6.63$ GHz. In Fig. 3.8.b, we show the parity switching time T_p measured on such gap-engineered samples. When the gap difference is smaller than the qubit frequency f_{01} (depicted as the marker color), the energy stored in the qubit participates in the tunneling of quasiparticles through the junctions and the parity rate is increased. As long as $\delta\Delta/h - f_{01} < 0$, the parity rate is constant (flat dashed line). Gap engineering is therefore crucial to make $\delta\Delta/h - f_{01}$ positive, and as large as possible, which exponentially suppresses this process and increases T_p . We extract from these experimental results the following exponential scaling:

$$T_p = B e^{A(\delta\Delta/h - f_{01})}, \quad (3.55)$$

with $A = 2.79$ GHz⁻¹ and $B = 1.47 \times 10^{-5}$ s. While we expect $A = h/k_B T$ for the qubit-relaxation-induced parity switching time T_p^{01} , the present data was measured with different

methods, yielding a different mean qubit population. Thus, T_p follows an exponential trend as expected but we are not able to quantitatively assess the scaling. The highest parity switching time, $T_p = 20$ s, is achieved for $\delta\Delta/h - f_{01} = 4.87$ GHz. However, we note that while these values can be compared as orders of magnitude, they should not be used to draw conclusions from small changes, as they were measured in different setups and with different methods.

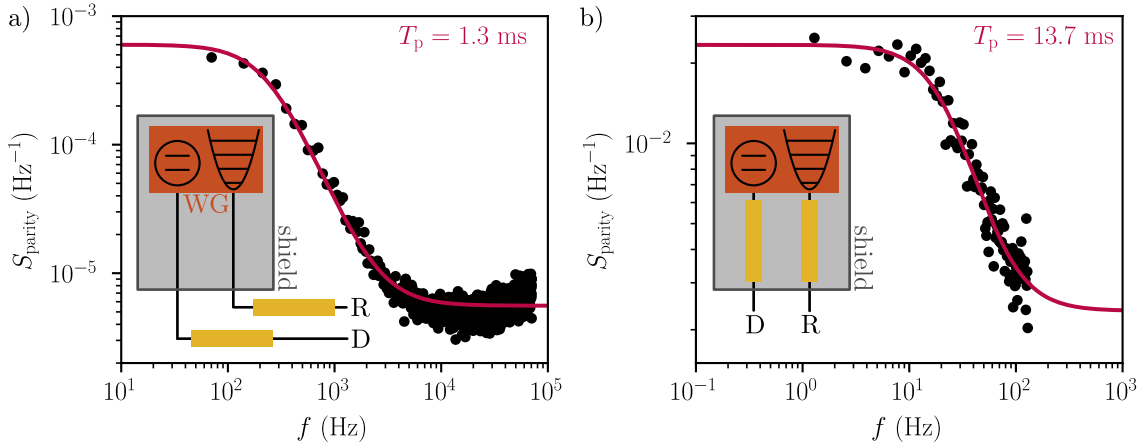


Figure 3.9: Infrared filtering for parity switches. **a, b)** Power spectral density S_{parity} of the parity of charge offset ($n_g = 0$ or $n_g = 0.5$) on the central island on sample 02R. Insets represent the measurement setup where the qubit-resonator system is placed in a copper waveguide, shielded from magnetic fields and radiations by copper-aluminum and μ -metal shields. The infrared filters in yellow are positioned outside (a) or inside (b) the shield. R and D represent the readout and qubit drive lines, respectively. Both measurements fit well to the random telegraphic noise model, with a typical parity switching time improved tenfold with the filters inside the shield.

All the parity switching times T_p quoted in this chapter were measured with some infrared filtering on both the readout and the qubit drive lines. However, the absolute attenuation above 20 GHz, which depends on the type of IR filter, as well as the position of the filters relative to the waveguide, also impacts T_p . In Fig. 3.9, we measure T_p on the same sample in two different cooldowns, with different positions for the infrared filters³. In Fig. 3.9.a, the filters were placed outside the shield, right before the feedthroughs. Parity was measured over time with the π -pulse-based state-mapping method described in Fig. 3.7.d. The power spectral density fits to a telegraphic noise model with $T_p = 1.3$ ms. When moving the same filters inside the shield, as close as possible to the waveguide hosting the sample, the parity switching time obtained from a two-tone-spectroscopy time trace (see Fig. 3.6.a) was increased to $T_p = 13.7$ ms, a tenfold improvement. This emphasizes the sensitivity of superconducting circuits to high-frequency radiations channeled by microwave cables and shows that SMA connectors are leaky at infrared frequencies.

³ HERD-2 filter from Sweden Quantum, <https://www.swedenquantum.com/products/herd-2>

3.3.3 Parity driving

These mitigation strategies efficiently increase the parity switching time T_p . They do not allow, however, the operator to prepare parity $p = -1$ or $p = +1$. In this section, we show a measurement where the parity is driven by an additional tone sent to the qubit, such that $p = -1$ or $p = +1$ is favored.

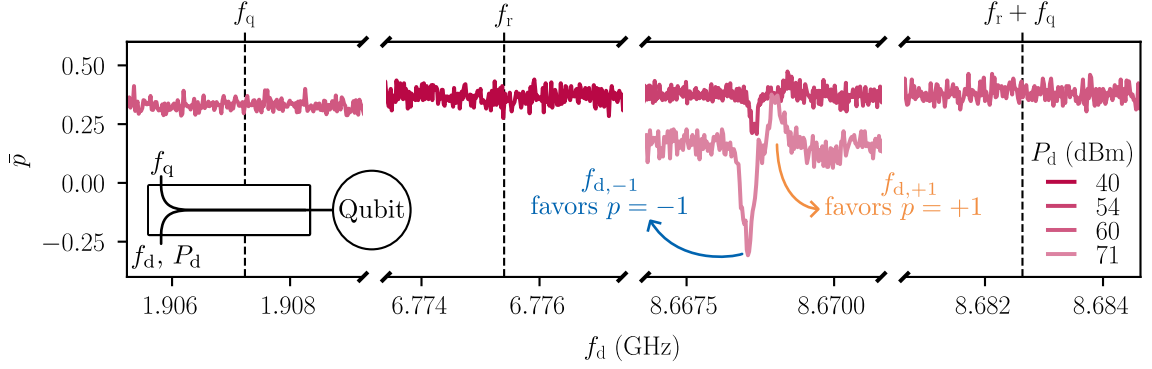


Figure 3.10: Parity driving spectroscopy. Mean parity $\bar{p}$ measured with the state-mapping method described in Fig. 3.7.a versus frequency f_d of an additional long pulse applied to the qubit with power P_d (see inset in the bottom left corner). Parity is driven only at $f_{d,-1} = 8.66855$ GHz and $f_{d,+1} = 8.669$ GHz, 14 MHz below $f_r + f_q$.

First, we performed a three-tone spectroscopy, where the qubit parity is measured with the method shown in Fig. 3.7.a after driving at frequency f_d . Figure 3.10 shows the mean parity $\bar{p}$ obtained from this measurement. Mean parity $\bar{p}$ only deviates from its equilibrium value $\bar{p}_{eq} = 0.36$ at two values of f_d . In particular, we observe no effect when driving at the qubit frequency f_q , the resonator frequency f_r or $f_r + f_q$. We observe an effect at $f_{d,-1} = 8.66855$ GHz (favoring $p = -1$) and $f_{d,+1} = 8.669$ GHz (favoring $p = +1$), 14 MHz below $f_r + f_q$. We note that increasing the drive power P_d to 71 dBm (at the input of the cryostat) not only increases the height of the peak and the dip but also decreases the mean parity baseline $\bar{p}_{eq}$. This is likely due to the additional noise provided by the added room-temperature amplifier on the drive line.

As $f_{d,-1}$ and $f_{d,+1}$ do not straightforwardly correspond to a combination of transition frequencies of the system, we can hypothesize that they involve the gap difference at one of the junctions. The sample this measurement was performed on was actually designed and fabricated as a regular gradiometric fluxonium, comprising only one junction. The charge dispersion $\Delta f \approx 1$ MHz indicates the presence of a spurious second junction, probably formed at the contact between aluminum and granular aluminum. The gap difference at the spurious junction is therefore the difference of superconducting gap between thin-film aluminum and granular aluminum. For resistivity $\rho \approx 2 \times 10^3 \mu\Omega \cdot \text{cm}$, granular aluminum has a critical temperature $T_c = 1.9$ K [50] (see Section 1.1.4), which corresponds to a gap $\Delta_{\text{grAl}} = 1.76 \times k_B T_c = h \times 69.7$ GHz. Thin film aluminum, on the other hand, has a gap $\Delta_{\text{Al}} = h \times 50.8$ GHz (thickness $d = 20$ nm) and $\Delta_{\text{Al}} = h \times 48.4$ GHz

($d = 30$ nm) [145]. Therefore, the peaks at $f_{d,\pm 1}$ seem to match with a multi-photon process, involving two photons of the drive and one photon from the qubit, such that

$$f_{d,\pm 1} = \frac{\delta\Delta/h - f_{q,\pm 1}}{2}, \quad (3.56)$$

with

$$f_{q,+1} = 1.9077 \text{ GHz}, \quad f_{q,-1} = 1.9068 \text{ GHz}, \quad \delta\Delta/h = 19.2 \text{ GHz}, \quad (3.57)$$

consistent with the gaps of 20-nanometer-thick aluminum and granular aluminum.

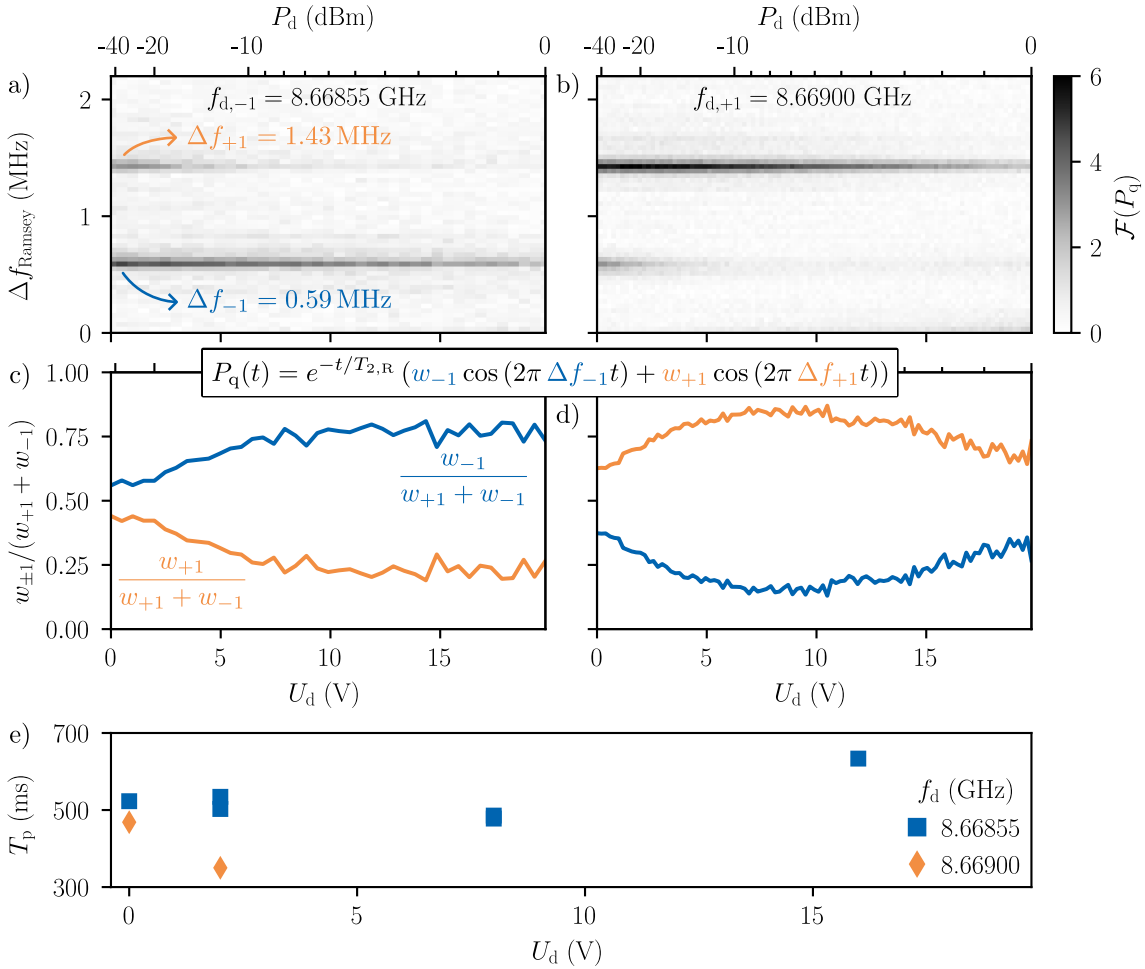


Figure 3.11: Parity driving versus power. Ramsey experiment performed right after driving at f_d for $50 \mu\text{s}$ at amplitude U_d corresponding to power P_d at the qubit. **a, b)** Fourier transform of the qubit population P_q measured after driving at $f_d = 8.66855$ GHz (a) or $f_d = 8.669$ GHz (b) and performing a Ramsey sequence. Both panels show two peaks at Δf_{+1} and Δf_{-1} , reflecting the existence of two qubit frequencies. **c, d)** Relative weights of parity $+1$ and -1 obtained from fitting P_q to a beating Ramsey (see Eq. (3.58)). **e)** Parity switching time T_p measured after driving at f_d at a few amplitude values U_d .

Now that the phenomenon of parity driving is established and we have identified which frequency corresponds to decreasing mean parity $\bar{p}$, and which to increasing $\bar{p}$, we are interested in the fidelity of preparing the parity degree of freedom in $p = -1$ or $p = +1$.

Figure 3.11 presents the results of performing a Ramsey sequence after driving for $50 \mu\text{s}$ at $f_{d,-1}$ (panels a and c) or $f_{d,+1}$ (panels b and d), at increasingly large powers. Drive voltage U_d , also expressed as a drive power P_d in dBm, is calibrated from Rabi measurements and therefore quoted at the qubit in the waveguide. As expected from the previous measurement in Fig. 3.10, the qubit frequencies remain globally unchanged when increasing P_d . Their relative amplitudes, however, given by the Fourier transform of the qubit population $\mathcal{F}(P_q)$, shown in Fig. 3.11.a and Fig. 3.11.b, clearly favor one or the other qubit frequency $f_{q,\pm}$. We also observe that both amplitudes decrease at higher powers, which we attribute to small Stark shifts, induced by the additional drive, that degrade the fidelity of the $\pi/2$ -pulse in the Ramsey sequence.

In order to measure quantitatively the weight of each parity, we fit the data obtained from these Ramsey sequences with the following model, comprising two Ramsey frequencies Δf_{-1} and Δf_{+1} :

$$P_q(t) = e^{-t/T_{2,R}} (w_{-1} \cos(2\pi \Delta f_{-1} t) + w_{+1} \cos(2\pi \Delta f_{+1} t)). \quad (3.58)$$

In Fig. 3.11.c and Fig. 3.11.d, we show the relative weights extracted from fits to Eq. (3.58). In both cases, we are able to saturate the parity to be preferably in $p = -1$ or $p = +1$ with above 80% fidelity. We note however that the effect seems to saturate at high power when driving at $f_{d,-1}$, while it instead decreases for $f_{d,+1}$. The origin of this discrepancy is unknown. We also note that the measurements exhibit different weights at $U_d = 0 \text{ V}$, which is due to short wait times between high and low power points.

Nonetheless, such a preparation would be useless if it resulted in an increased parity switching rate. In Fig. 3.11.e, we show the parity switching time T_p measured separately at different powers, mostly with drive frequency $f_{d,-1}$ (square markers). For these points, T_p stays globally constant, around 500 ms. This result highlights the potential of parity driving and could motivate exploring this method further. In particular, further experiments could resolve the gap difference of both junctions, identify whether the effect saturates differently for different frequencies and confirm or infirm the hypothesis stated above, so far mostly based on frequency matches.

3.3.4 Jumps and drifts

On top of parity switching, the charge offset in charge-sensitive qubits also undergoes drifts and jumps [139, 148, 149]. Ref. [149] identifies the signature of charge noise in power spectral density to a power law $S(f) = A/f^\alpha$ with $S(1 \text{ Hz}) = 2.9 \times 10^{-4} e^2/\text{Hz}$ and $\alpha = 1.93$. In our samples, we observe both drifts and jumps. In particular, some samples exhibit a toggling between charge offsets, jumping back and forth every few seconds.

In Fig. 3.12, we show two-tone spectroscopies performed on a type-1 (Fig. 3.12.a) and a type-2 (Fig. 3.12.b) sample versus charge offset n_g (see sample pictures in Fig. 3.2). At each

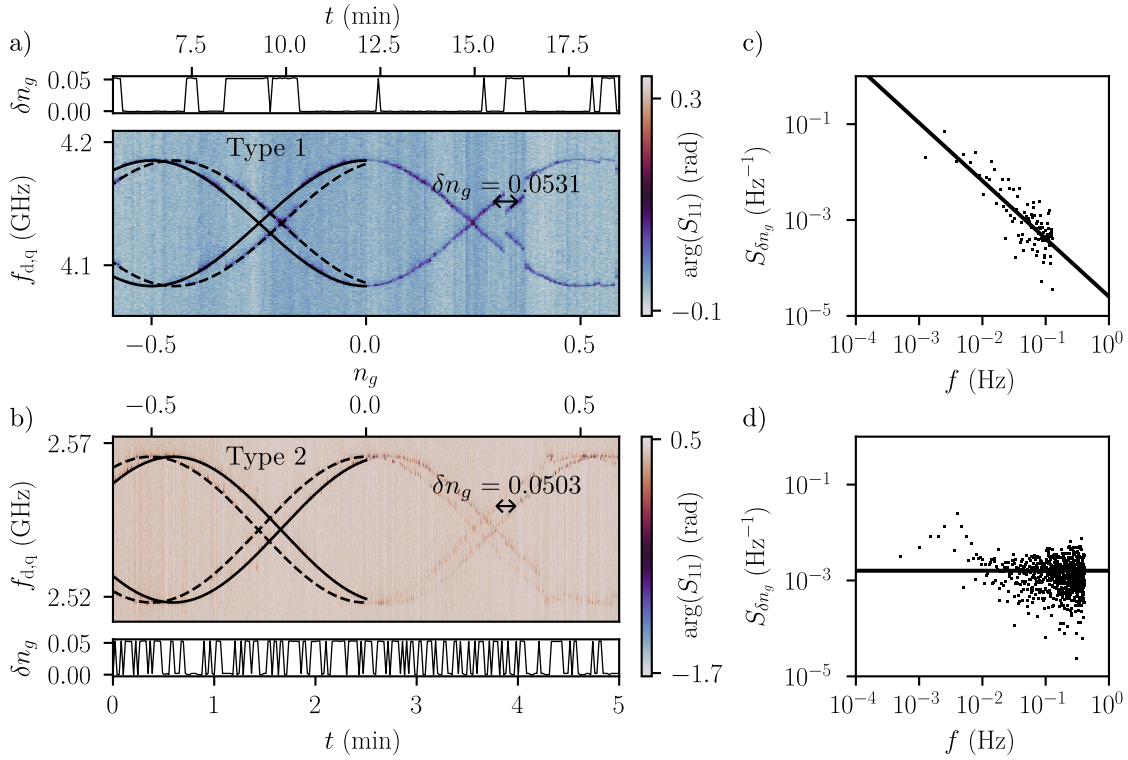


Figure 3.12: Charge-offset jumps below $1e$. **a, b)** Two-tone spectroscopies of a type-1 (a) and a type-2 (b) sample versus charge offset n_g . In color, the phase of the resonator reflection coefficient shows dips or peaks at the qubit frequency. Despite the different size and layer composition of the island and the different parity rate in the samples, both exhibit the same behavior. Additionally to the parity switches, the offset charge also jumps by $\delta n_g = \pm 0.05$, back and forth over a voltage sweep corresponding to one period. In the insets, δn_g versus time shows the temporal occurrences of the jumps. In a), the time interval corresponds to the charge offset interval of the two-tone spectroscopy, while in b) it is only plotted over 5 minutes. **c, d)** Power spectral density $S_{\delta n_g}$ of $\delta n_g(t)$ the additional charge offset extracted from the two-tone spectroscopies. In c), $S_{\delta n_g}$ fits to a power law A/f^α with $\alpha = 1.208$ and $S_{\delta n_g}(1 \text{ Hz}) = 2.53 \times 10^{-5} \text{ Hz}^{-1}$. In d), $S_{\delta n_g}$ is constant over the interval $f = 0.5 \text{ mHz}$ to $f = 0.4 \text{ Hz}$, $S_{\delta n_g} = 1.60 \times 10^{-3} \text{ Hz}^{-1}$.

applied voltage, the qubit frequency is extracted and all points are fitted to four cosine models, each with a different offset:

$$f_{q,i}(V_g) = f_{q,\text{mid}} + \Delta f \cos\left(2\pi \frac{V_g - V_i}{V_{\text{period}}}\right), \quad (3.59)$$

with $f_{q,\text{mid}}$ the qubit frequency at $n_g = 0.25$, Δf the charge dispersion, V_g the applied voltage, V_{period} the period such that $n_g = V_g/V_{\text{period}}$ and V_i the offset associated with branch i . This fit allows for a robust calibration of n_g and gives the additional offset associated with both parity switches and below- $1e$ jumps. For visualization, we plot on both panels a and b $f_{q,i}$ according to their associated relative offset Δn_g , as a solid line for $\Delta n_g = 0$ and 0.5 , as a dashed line for the additional offset δn_g . From the fits, we are able to attribute $\delta n_g(t)$ at each measurement time. As insets, we plot the time traces of only the additional charge offset δn_g obtained from the same two-tone spectroscopies. For the type-1 sample in panel a, the qubit frequency jumps 9 times in 15 minutes (excluding the single-point

jumps that correspond to attribution errors), while the rate for the type-2 sample in panel b is one or two orders of magnitude higher.

Interestingly, despite both samples exhibiting the same $\delta n_g = 0.05$, their noise spectrum is very different. In Fig. 3.12.c and Fig. 3.12.d, we show the power spectral density $S_{\delta n_g}$ calculated from the time traces of $\delta n_g(t)$. The type-1 sample follows a power law A/f^α with $\alpha = 1.208$ and $S_{\delta n_g}(1 \text{ Hz}) = 2.53 \times 10^{-5} \text{ Hz}^{-1}$. While the level of noise is similar to Ref. [149], the exponent on the frequency f is almost half, this sample being closer to $1/f$ noise, while theirs is close to $1/f^2$. The type-2 sample, however, does not even follow a power law on the accessible interval $f = 0.5 \text{ mHz}$ to $f = 0.4 \text{ Hz}$. Instead, the charge noise corresponds to a white noise, with a constant power spectral density at $S_{\delta n_g} = 1.60 \times 10^{-3} \text{ Hz}^{-1}$.

The microscopic mechanism for these jumps is unknown, and we note that making the island smaller in type-2 samples, while it decreased the parity switching rate, also resulted in more frequent small charge jumps. An interpretation could be that on larger islands, small changes in the electrical environment are screened or averaged out over the spatial extension of the island. While removing such jumps entirely would greatly improve our ability to control the qubit, jumps with a typical timescale of seconds can, in many applications, be circumvented by dynamical recalibration or monitoring of the qubit frequency.

3.3.5 Blockade

In one of the measured type-1 samples, we observed periodic steps of charge offset that we attribute to a Coulomb blockade phenomenon. While we could not identify all the components of this story, it is clear from the measurements that the blockade is made possible by the existence of an additional, spurious Josephson junction, creating a secondary island where the charge is quantized. The existence of a spurious junction in those samples is not entirely unexpected, as it was already the case in some single-loop and gradiometric fluxonium samples fabricated with a similar design and the same recipe. In those samples, the signature of the spurious junction is a charge tunability exactly like in the two-junction fluxoniums we have been studying in this chapter. Similarly, the third junction in the present sample adds a charge modulation, on top of the expected charge dispersion.

In Fig. 3.13.a, we present the two-tone spectroscopy versus charge offset n_g of the type-1 sample exhibiting a blockade, biased at the half-flux-quantum sweet spot. The spectrum is cut at the mean qubit frequency $f_{q,\text{mid}}$, showing only the top half of the two qubit frequencies given by the parity switches. The qubit frequency deviates from the expected cosine model in a subtle way, only visible by measuring over several periods. The maximum qubit frequency modulates, which is equivalent to having a periodic instead of constant $\Delta f(V_g)$ in Eq. (3.59). In Fig. 3.13.b, the same two-tone spectroscopy data is plotted, choosing only the points corresponding to the maximum qubit frequency over each period. This reveals a modulation of Δf of about 4 MHz over one period $n_{g,\text{period}} = 9.6$. This is the

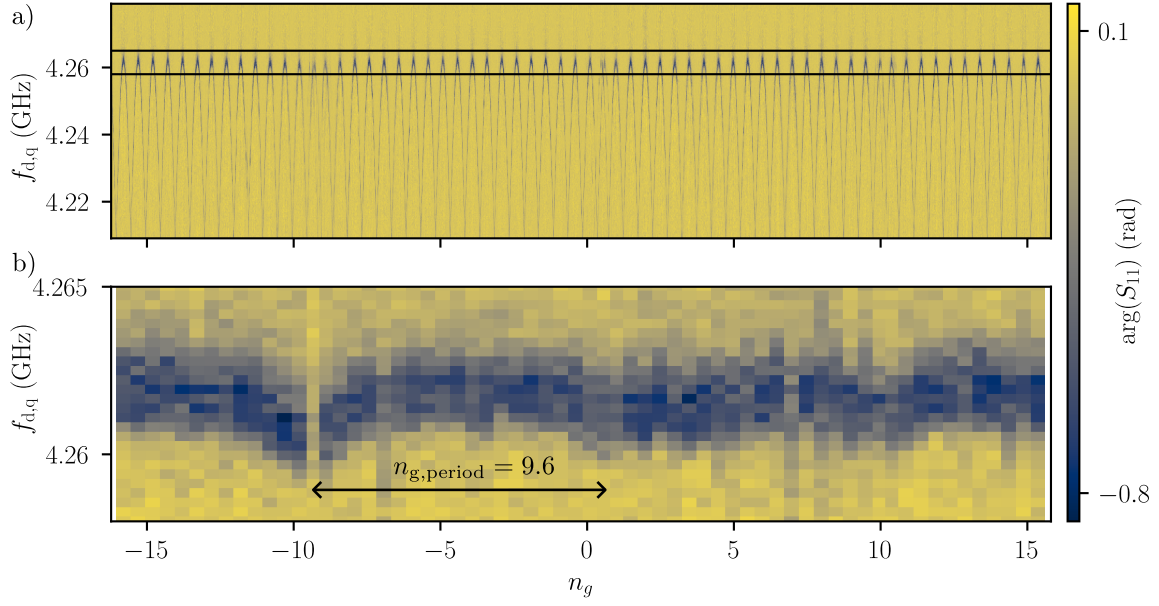


Figure 3.13: Evidence of a spurious third junction in two-tone spectroscopy. **a)** Two-tone spectroscopy of a type-1 sample versus charge offset n_g tuned by applying DC voltage V_g at the driving pin next to the qubit chip. Qubit frequency shows a periodic modulation on top of the cosine model and periodic charge offset jumps. Black rectangle indicates the frequency span displayed in panel b. **b)** Zoom-in displaying only the points corresponding to the maximum frequency of each period. The additional modulation is the signature of a third junction.

signature of a spurious junction, probably resulting from the contact between aluminum and granular aluminum in our design.

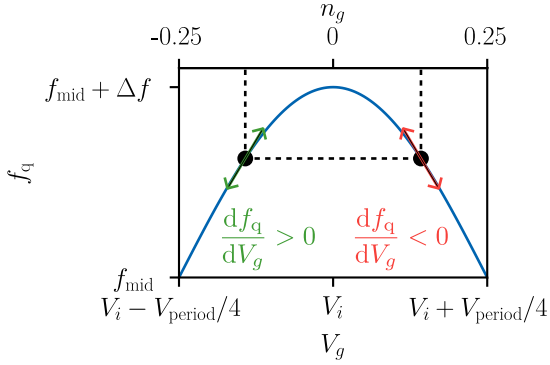


Figure 3.14: Charge offset measurement from qubit frequency. Qubit frequency f_q versus applied DC voltage V_g and charge offset n_g over half a period. Using both f_q and the sign of its derivative $\frac{df_q}{dV_g}$, we calibrate n_g from measuring the qubit frequency.

Qubit frequency f_q is extracted at each charge offset from the two-tone spectroscopy in Fig. 3.13. Thanks to the periodic dependence of f_q , n_g is calibrated independently from the applied DC voltage V_g , as illustrated in Fig. 3.14. Because of the spontaneous parity switches, we can only measure n_g modulo 0.5. Additionally, the symmetry of f_q around $n_g = 0$ means that we need to use the sign of its first order derivative $\frac{df_q}{dV_g}$ to distinguish between positive and negative values. This method allows to attribute a measured charge

offset $n_g^{(\text{meas})}$ to each applied charge offset $n_g = C_g V_g$ and define the additional charge offset δn_g as

$$\delta n_g = n_g^{(\text{meas})} - n_g [0.5]. \quad (3.60)$$

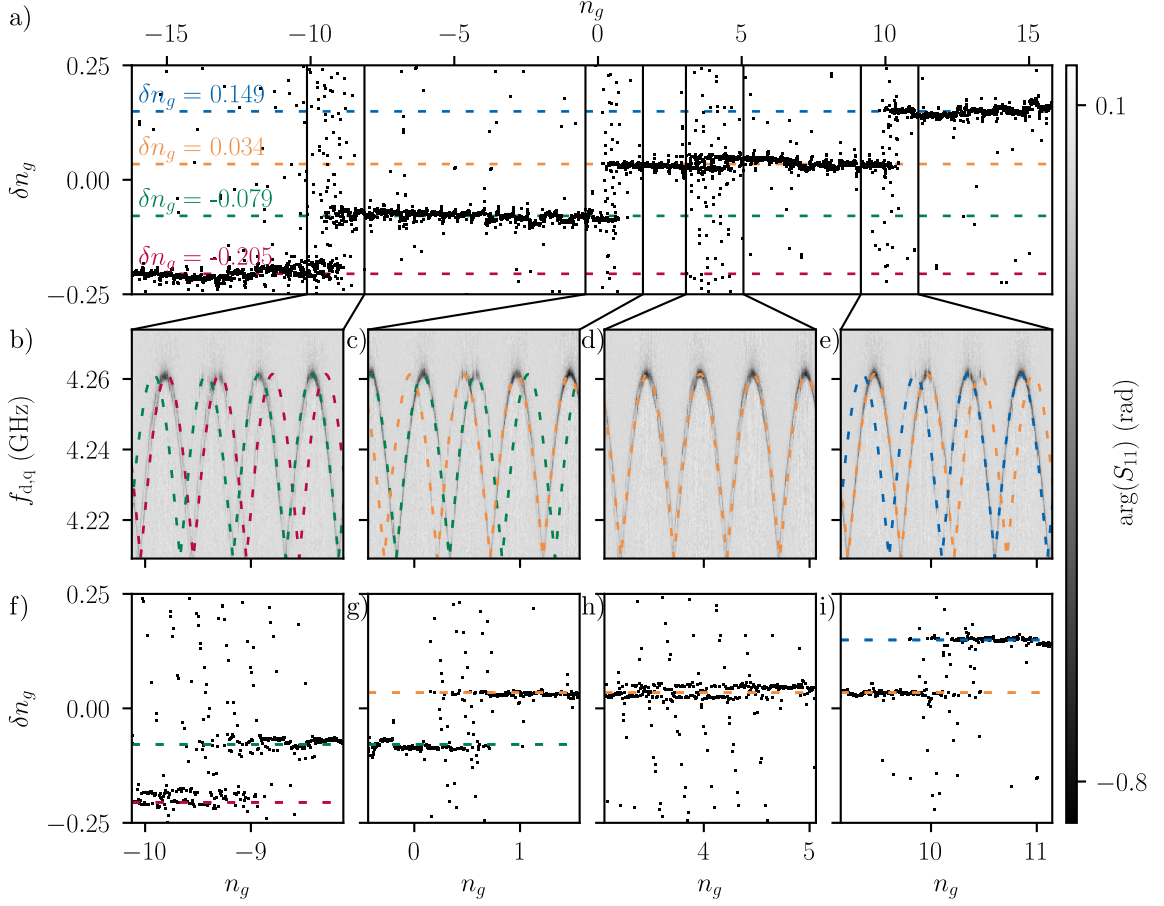


Figure 3.15: Coulomb blockade evidenced by steps of charge offset. **a)** Additional charge offset δn_g extracted from the two-tone spectroscopy in Fig. 3.13, revealing a quantization and evenly-spaced charge jumps of $\Delta(\delta n_g) \approx +0.12$. The periodicity of these jumps matches the periodicity of the additional modulation identified in Fig. 3.13.b. Horizontal dashed lines represent the four δn_g given by fitting the whole charge spectrum with four constrained cosine models with the same charge dispersion and period. **b, c, d, e)** Two-tone spectroscopy versus n_g around the defects where δn_g jumps by $+0.12$ (b, c, e) and around $n_g = 4$ where a smaller jump $\Delta(\delta n_g) = +0.02$ is visible (d). **f, g, h, i)** Corresponding additional charge offset δn_g for the same n_g intervals as panels b to e.

The additional charge offset δn_g is represented in Fig. 3.15.a. Because of parity switches, δn_g can only be defined on the $[-0.25, 0.25[$ interval. For a regular sample with no other element than intended in the design, in particular with only two junctions, δn_g is expected to be overall constant, albeit with drifts and random jumps as commonly observed in charge-tunable circuits measured over long times (here, the measurement takes three and a half hours). While δn_g is indeed constant over ranges of n_g , it forms four evenly-spaced steps of nearly-equal height $\Delta(\delta n_g) \approx 0.12$ over the studied range of applied voltage V_g . The step lengths match with the additional frequency modulation identified in Fig. 3.13.a, with

a period $n_{g,\text{period}} = 9.63$, linking the two phenomena and showing that δn_g corresponds to the (quantized) charge on the additional island formed by the spurious junction.

In parallel to this analysis and in order to extract periodicity V_{period} , mean frequency $f_{q,\text{mid}}$ and charge dispersion Δf versus charge offset, we apply the same fitting routine as in Section 3.3.4. We fit all the frequency dependence to four cosine models (see Eq. (3.59)), with the same V_{period} , $f_{q,\text{mid}}$, Δf and four voltage offsets V_i . Translated in δn_g , these offsets are shown as horizontal dashed lines in Fig. 3.15.a, which match with the identified steps of δn_g . To show how the fit performs, we plot in panels b to e the same two-tone spectroscopy as in Fig. 3.13, with the fitted model as dashed lines, for n_g around the position of the step edges (b, c, e) and an additional smaller charge jump (d). Focusing on the step edges, we see from these spectroscopies that the qubit switches from one branch to the other (i.e. from a δn_g value to another) in less than two periods. From panels f, g and i, showing a zoom-in of δn_g on these ranges, we see that the two values of δn_g overlap for about half a period. Noticeably, in Fig. 3.15.f, the main step jump happens next to a smaller step jump of height $\Delta(\delta n_g) = 0.02$.

The additional charge offset δn_g exhibits additional features on top of the evenly-spaced steps. Indeed, it undergoes drifts and random jumps illustrated for example by the global downward trend between $n_g = -9$ and $n_g = 0$ and the standard deviation $\sigma(\delta n_g) = 0.01$ on each step. We also observe smaller steps in δn_g , such as between $n_g = 3$ and $n_g = 5$ where the step height is on the order of the standard deviation $\Delta(\delta n_g) = 0.02$, similar to the small step height in Fig. 3.15.f. In Fig. 3.15.d, we show a zoom-in of the corresponding two-tone spectroscopy on that range. While the orange dashed line represents only one fitted frequency with offset $\delta n_g = 0.034$, one can distinguish a toggling between two very close frequencies. This toggling in frequency corresponds to a toggling in δn_g , $\Delta(\delta n_g) = 0.02$, as can be seen in Fig. 3.15.h. Finally, as the measurement method uses the derivative to map $n_g^{(\text{meas})}$, all these jump features cause misattributions of δn_g , which appear as artifacts. The artifacts are particularly visible around the position of said features, in the zoom-in provided in panels f to i.

In order to tune the charge offset on each island, we introduce a second DC voltage bias V'_g . Both V_g and V'_g are applied at pins pointing in the waveguide, their relative position to the sample giving different capacitances (see waveguide setup in Fig. A.2 in the appendix). Due to the different nature and shape of the two islands, their respective charge offsets $n_{g,1}$ and $n_{g,2}$ are set by linear combinations of the applied voltages and the capacitances from each pin to each island, as illustrated by the circuit in Fig. 3.16.a:

$$\begin{pmatrix} n_{g,1} \\ n_{g,2} \end{pmatrix} = \begin{pmatrix} C_{g,1} & C'_{g,1} \\ C_{g,2} & C'_{g,2} \end{pmatrix} \begin{pmatrix} V_g \\ V'_g \end{pmatrix}. \quad (3.61)$$

The measurements thus aim first at determining $n_{g,1}$ and $n_{g,2}$ from n_g and n'_g . The latter quantities are simply the applied voltages V_g and V'_g divided by the apparent period.

We perform the same experiment as previously, namely a two-tone spectroscopy, this time versus both V_g and V'_g and plot the qubit frequency f_q in Fig. 3.16.b. Because of the low parity switching time, only the top half of the charge dispersion, from $f_q = f_{\text{mid}} = 4.134$ GHz to

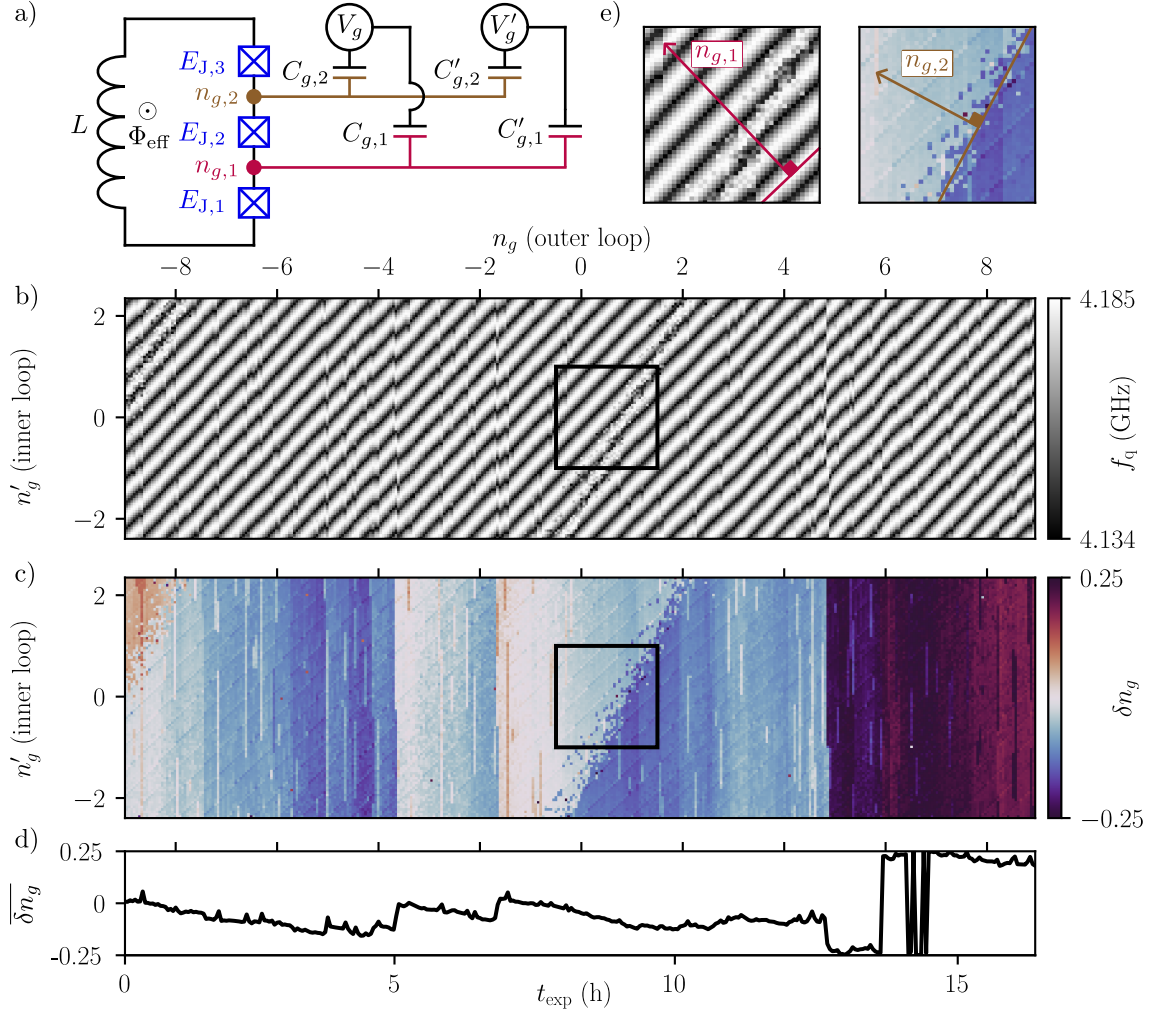


Figure 3.16: Coulomb blockade with two tuning knobs. **a)** Charge-tunable fluxonium circuit with a spurious third junction. V_g and V'_g are DC voltage biases applied at two different pins close to the sample in a waveguide, therefore coupling with different capacitances to the two independent islands. **b)** Qubit frequency f_q versus DC biases n_g and n'_g , defined from the periodicity in V_g and V'_g . In the measurement, n_g is the outer loop (slow parameter), while n'_g is the inner loop. **c)** Additional charge offset δn_g measured from f_q with the method described in Fig. 3.14. On top of drifts and small jumps appearing as vertical lines, two large steps identical to the ones revealed by Fig. 3.15 (in the top left corner and in the center) tune with n'_g . **d)** Mean additional charge offset $\overline{\delta n_g}$ over n'_g values versus n_g and experiment time t_{exp} . Drifts and jumps are apparent for the total 16 hours of data acquisition. **e)** Zoom-ins of panels b and c. Axes $n_{g,1}$ and $n_{g,2}$ are defined from the periodicity of f_q and δn_g .

$f_q = f_{\text{mid}} + \Delta f = 4.185$ GHz, is considered. Note that, despite using the same device as in Fig. 3.13 and Fig. 3.15, these measurements were taken in a subsequent cooldown three months later, during which the qubit frequency decreased by 75 MHz. From the rotated periodic pattern, it appears that both n_g and n'_g tune the charge offset between the two main junctions, $n_{g,1}$. In Fig. 3.16.e (left), we reproduce the portion of Fig. 3.16.b indicated by the black square, to show how $n_{g,1}$ is determined. $n_{g,1}$ is defined as the axis orthogonal to the diagonal lines of constant frequency (in this case, at the maximum $f_q = 4.185$ GHz).

Following the analysis performed before (see Fig. 3.14), we determine the additional charge offset δn_g from f_q , which is represented in Fig. 3.16.c. Several features appear. The most prominent are the two lines corresponding to the step jumps seen in Fig. 3.15, visible in the top left corner and in the center. These lines are parallel to each other but are not orthogonal to the $n_{g,1}$ axis defined in the last paragraph. As these steps of constant δn_g correspond to the additional island, we use these lines to define the $n_{g,2}$ axis in Fig. 3.16.e (right). The angle between $n_{g,1}$ and $n_{g,2}$ is small, indicating that the coupling capacitances C_g have similar values for the two islands.

On top of the step jumps, δn_g also reveals random jumps and drifts over time. In Fig. 3.16.b and Fig. 3.16.c, they appear as vertical lines because the measurements were performed with n'_g as the inner loop (fast parameter) and n_g as the outer loop (slow parameter). In Fig. 3.16.d, we report the average additional charge offset $\overline{\delta n_g}$ at every n_g value over the experiment time t_{exp} (17 hours in total). It reveals a global mostly-downward trend with occasional jumps, sometimes as large as $\Delta(\delta n_g) = 0.17$ (at $t_{\text{exp}} = 12.5$ h), consistent with random drifts and jumps observed in other charge-sensitive circuits (see Section 3.3.4).

With these experiments, we revealed the existence of an additional island in the circuit, created by a spurious third junction. The charge on this island appears to be quantized, giving step jumps in the total apparent charge offset of the device. Contrary to typical Coulomb blockade experiments, we do not observe any diamond shape in the data, because charge offset is kept periodic by tunneling of Cooper pairs through the junctions. Such a system, giving access to quantized charges in a coherent environment, opens the door to fundamental experiments. Such experiments could probe, for example, the electronic temperature on the island in order to compare it to the cryostat and qubit temperatures.

3.4 Unveiling coupling between qubits and two-level systems

Two-level systems (TLSs) are part of the history of superconducting qubits since the dawn of the field [150]. As design and fabrication techniques improved, especially on Josephson junctions, strongly-coupled TLSs disappeared from qubit spectra. Yet, two-level systems had not vanished! With low couplings, they are not visible in two-tone spectroscopies but have a huge impact on qubit coherence. Specifically, some of them have been shown to have lifetimes far exceeding those of the qubits, thus providing a memory effect in the environment [151–153]. In Ref. [153], we have shown that we are able to distinguish the qubit relaxation to a Markovian environment from its relaxation into a long-lived two-level system (TLS) from quantum jump measurements. Nevertheless, and despite numerous experiments performed on TLSs (long-lived or not), their physical nature is still unknown. In this section, we propose to take advantage of the two tuning knobs offered by charge-tunable fluxoniums (flux and charge) to change the matrix elements of the qubit in order to tune, in situ, the qubit-TLS coupling rate Γ_{qt} , possibly giving additional information on their physical nature.

3.4.1 Concept

In this section, we argue in favor of using charge-tunable fluxoniums to tune the qubit-TLS coupling rate Γ_{qt} . Computing the qubit matrix elements reveals different behaviors at a constant frequency when tuning flux and charge, depending on the type of coupling. Namely, we explore basic coupling schemes via the current (proportional to $\hat{\phi}$ or $\sin \hat{\phi}$) or the voltage ($\hat{n}$). Of course, one could imagine exotic couplings involving combinations of the two.

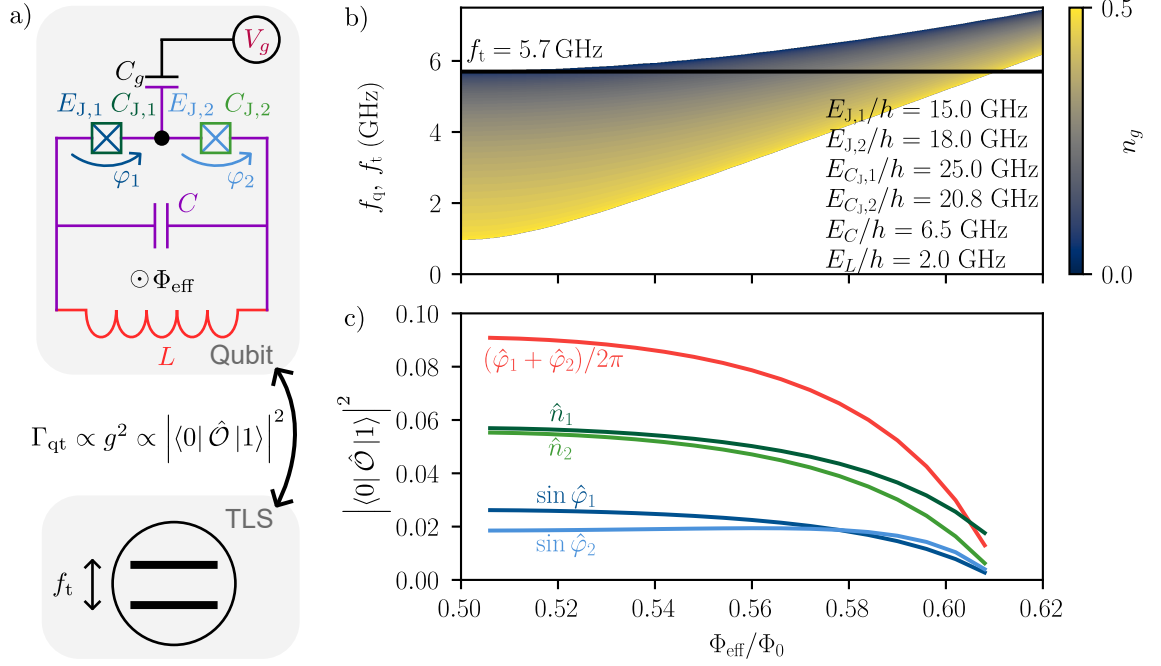


Figure 3.17: Qubit-TLS coupling in charge-tunable fluxonium. **a)** Schematic of a charge-tunable fluxonium qubit coupled with coupling rate Γ_{qt} to a two-level system (TLS) of frequency f_t , following the model in Ref. [153]. **b)** Qubit frequency f_q versus effective flux Φ_{eff} (x-axis) and charge offset $n_g = C_g V_g$ (colorbar), for similar parameters to the sample characterized in Section 3.2.3. To illustrate the concept of tuning the qubit-TLS coupling with the charge-tunable fluxonium, a TLS with constant frequency f_t is considered (black horizontal line). **c)** Matrix elements $|\langle 0 | \sin \hat{\phi}_i | 1 \rangle|^2$ and $|\langle 0 | \hat{n}_i | 1 \rangle|^2$ versus flux, for each junction i , and $|\langle 0 | (\hat{\phi}_1 + \hat{\phi}_2) | 1 \rangle|^2$. Qubit frequency f_q is kept at f_t by changing the charge offset.

The model followed here is the same as in Ref. [153], i.e. a qubit coupled to a single long-lived TLS. In Fig. 3.17.a, we represent the charge-tunable fluxonium qubit circuit, in the type-2 design defined in Fig. 3.1.b. This qubit is coupled to a single TLS of frequency f_t , with rate Γ_{qt} defined in Refs. [151, 153] as:

$$\Gamma_{\text{qt}} = \frac{2g^2\Gamma_2}{\Gamma_2^2 + (f_q - f_t)^2}, \quad (3.62)$$

where $\Gamma_2 = \Gamma_{2,q} + \Gamma_{2,t}$ is the total decoherence rate, f_q (respectively, f_t) is the qubit (respectively, TLS) frequency and g is the transverse coupling strength. Considering a general operator $\hat{O}$, mediating the coupling, g is therefore proportional to the matrix element $|\langle 0 | \hat{O} | 1 \rangle|$.

In order to illustrate the idea, let us take a sample with reasonable parameters

$$E_{J,1}/h = 15 \text{ GHz}, \quad E_{J,2}/h = 18 \text{ GHz}, \quad (3.63)$$

$$E_{C,1}/h = 25 \text{ GHz}, \quad E_{C,2}/h = 20.8 \text{ GHz}, \quad (3.64)$$

$$E_C/h = 6.5 \text{ GHz}, \quad E_L/h = 2.0 \text{ GHz}, \quad (3.65)$$

close to the real sample characterized in Section 3.2.3, except the second junction is only 1.2 times larger than the first junction (instead of 3 times). In Fig. 3.17.b, we represent the qubit frequency f_q computed numerically versus external flux Φ_{eff} and charge offset n_g (in color). Because of the quasi-symmetry in the junctions, the charge dispersion at $\Phi_{\text{eff}} = \Phi_0/2$ is $\Delta f = 4.71 \text{ GHz}$, larger than most samples shown in this chapter.

We consider now a TLS at a constant frequency $f_t = 5.7 \text{ GHz}$ and compute the matrix elements squared $|\langle 0 | \sin \hat{\phi}_i | 1 \rangle|^2$ and $|\langle 0 | \hat{n}_i | 1 \rangle|^2$, corresponding respectively to the TLS coupling to the current (proportional to $\sin \hat{\phi}_i$) or the charge ($\hat{n}_i$) at junction i . The chosen matrix elements are shown in Fig. 3.17.c versus Φ_{eff} . Because of the symmetry between the junctions, the operators take similar values for each of junction. Their dependence to Φ_{eff} is also different between $\sin \hat{\phi}$ and $\hat{n}$. Note in particular the non-monotonous dependence of $|\langle 0 | \sin \hat{\phi}_2 | 1 \rangle|^2$, which gives confidence in this method being able to distinguish between $\hat{\phi}$ and $\hat{n}$ coupling mechanisms. Additionally, we show the dependence of operator $\hat{\phi}_1 + \hat{\phi}_2$, corresponding to coupling to the current in the granular aluminum inductor. Going further, comparing the variation in Γ_{qt} in TLSs with different susceptibilities to, for instance, electric field would allow us to calibrate their electric dipole, giving additional information on their nature and position in the qubit circuit.

3.4.2 Preliminary experimental results

The method is applied to the sample characterized in Section 3.2.3, despite its relatively low charge dispersion at $\Phi_{\text{eff}} = \Phi_0/2$ ($\Delta f = 49 \text{ MHz}$), because it exhibits a large parity switching time $T_p = 1 \text{ s}$, thanks to gap engineering and infrared filtering (see Section 3.3.2). The goal of this section is to show how the qubit-TLS coupling can be measured in practice, including in charge-sensitive devices, and gauge the next steps in terms of sample design and measurement techniques.

Measuring the qubit-TLS coupling rate Γ_{qt} is done by measuring quantum jump traces of the qubit and applying the so-called correlation spectroscopy, which disentangles the Markovian relaxation of the qubit (Γ_q) from the coupling to one long-lived TLS (Γ_{qt}). As the measurement and its analysis themselves are out of the scope of this thesis, the reader is referred to Ref. [153] for details on the technique. In order to account for charge offset drifts, jumps and parity switches, we interleave a two-tone spectroscopy with quantum jump traces, as illustrated in the pulse sequence in Fig. 3.18.a. The two-tone sequence sweeps only once through all $N_{f_{d,q}}$ values of qubit drive frequency $f_{d,q}$, before the quantum jump sequence records $N_{\text{QJ}} = 1000$ qubit state readout results ($\text{RO}^{(2)}$). The measurements are then averaged N_{slice} times. As an example, we show in Fig. 3.18.b the result of such a two-tone spectroscopy, averaged for 13 seconds. At $\Phi_{\text{eff}} = \Phi_0/2$ and $n_g = 0.18$, the qubit

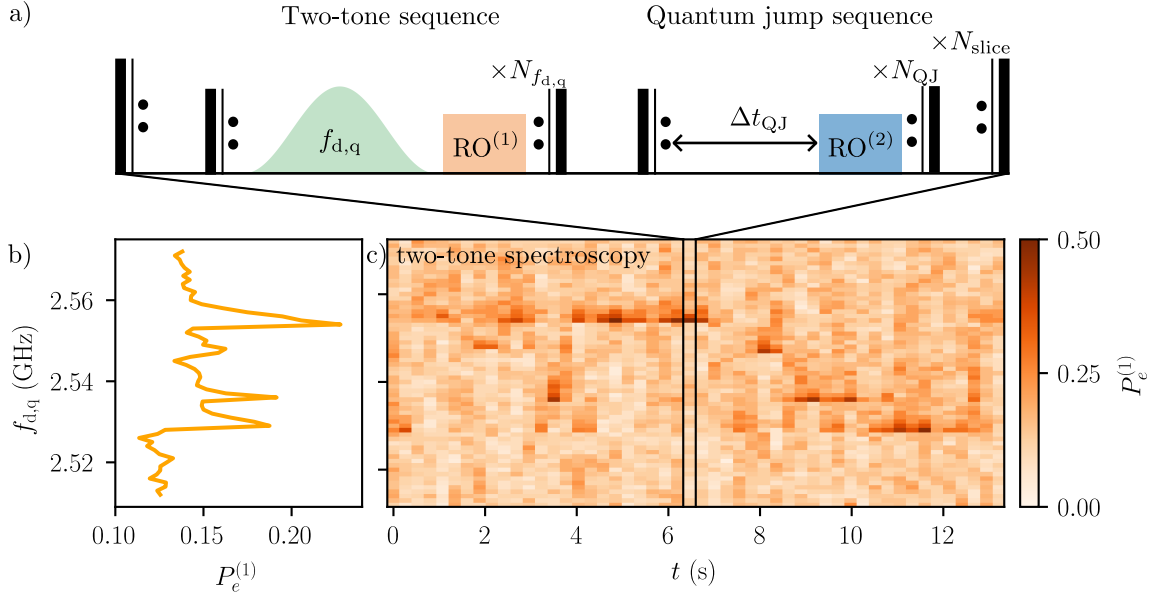


Figure 3.18: Interleaved two-tone spectroscopy and quantum jumps. **a)** Pulse sequence including a two-tone spectroscopy measured by RO⁽¹⁾ (sweeping through all $N_{f_{d,q}}$ values of qubit drive frequency $f_{d,q}$) and a quantum jump sequence measured by RO⁽²⁾ (in the following, $N_{QJ} = 1000$ and $\Delta t_{QJ} = 3 \mu s$). The sequence is repeated over 13.2 seconds and cut into 270-millisecond-long slices (with N_{slice} sequences per slice). **b, c)** Two-tone spectroscopy obtained by applying this sequence to a charge-tunable fluxonium and extracting the excited-state population $P_e^{(1)}$ from RO⁽¹⁾. On average, $P_e^{(1)}$ exhibits four peaks versus $f_{d,q}$ (b), but slicing the measurement resolves each peak separately in time (c), ensuring that the qubit stays at the measured frequency during the quantum jump sequence.

excited-state population shows four peaks in $f_{d,q}$. These peaks are resolved in time in Fig. 3.18.c, showing how the qubit stays at constant frequencies for up to a few seconds. Interleaved this way, these measurements allow us to associate the actual qubit frequency with each quantum jump trace.

From the quantum jump traces at each frequency, we build the following proxies to detect and discern long-lived two-level systems in the environment: the excited state population right after measuring the qubit in the ground state ($P_q^{(g)}$, see Fig. 3.19.a, b), or right after measuring a jump-down event ($P_q^{(eg)}$, see Fig. 3.19.c, d). Diagrams in insets of Fig. 3.19.a and Fig. 3.19.c represent the described post-selection. Both quantities are shown versus the tuning parameters of the device, external flux Φ_{eff} (a, c) and charge offset n_g (b, d). While Φ_{eff} is simply calibrated from the applied magnetic field and the qubit spectrum periodicity in flux, n_g is back-calculated from applied Φ_{eff} and measured qubit frequency. The n_g axis is therefore a combination of voltage bias V_g , applied on purpose, and charge drifts, jumps and parity jumps that are not controlled by the operator.

We observe features in the same positions with both proxies, in flux and charge, though the signal is stronger in $P_q^{(eg)}$ (note the different color axes on the right). These features are identified to be long-lived two-level systems, associated with a memory effect whose signature is visible in $P_q^{(eg)}$. We name three of them A, B and C (see Fig. 3.19.c, d) to

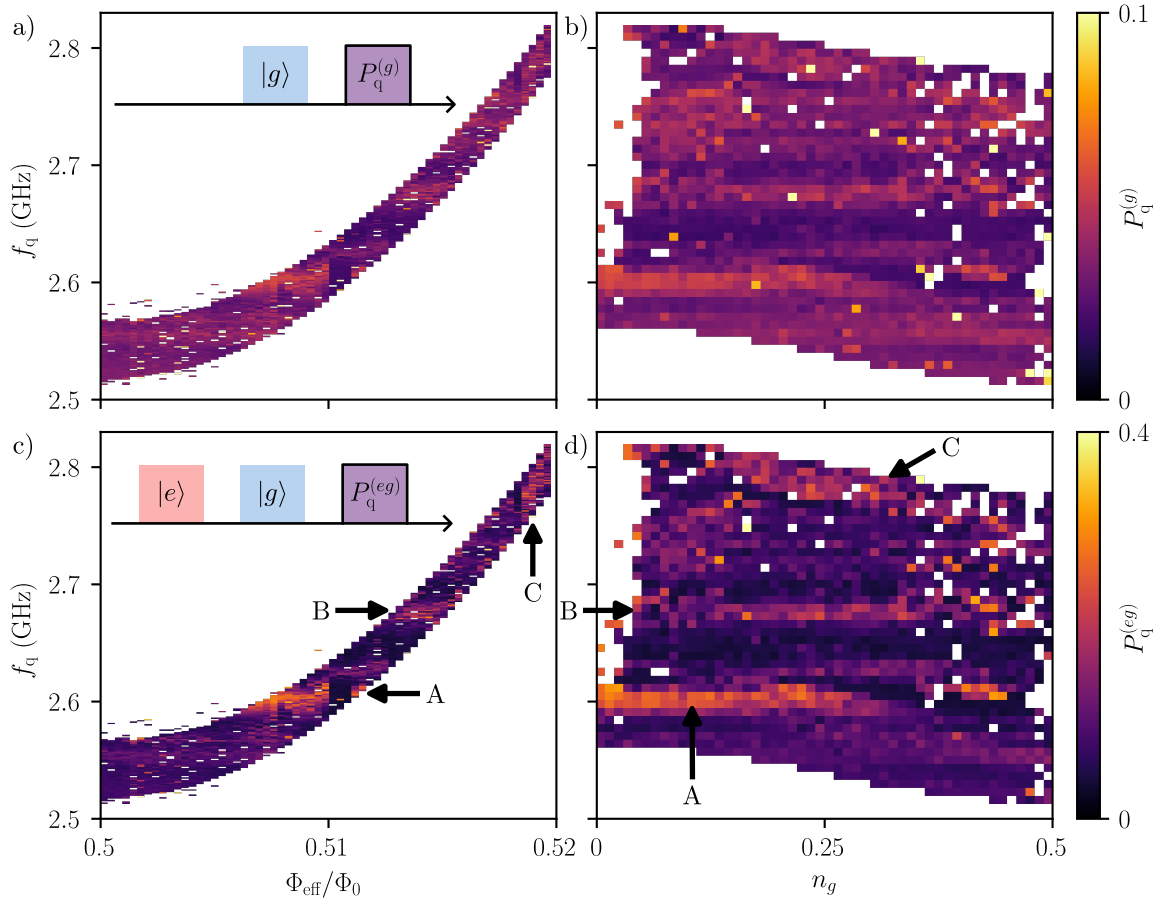


Figure 3.19: Correlation spectroscopy in a charge-tunable fluxonium. **a, b)** Qubit excited-state population right after measuring the qubit in the ground state $|g\rangle$ (see inset in panel a), as a proxy for T_1 drop due to the presence of a TLS. $P_q^{(g)}$ is plotted versus qubit frequency f_q and external flux Φ_{eff} (a) or charge offset n_g (b). Note that f_q is set by both parameters Φ_{eff} and n_g . **c, d)** Qubit excited-state population right after measuring a jump-down event, $|e\rangle$ followed by $|g\rangle$ (see inset in panel c). This is a proxy for memory effects due to a long-lived TLS in the qubit environment. Note the different color axis. $P_q^{(eg)}$ is also plotted versus f_q and Φ_{eff} (c) or n_g (d). Three features identified to long-lived TLSs are named A, B and C.

be discussed further. We observe that TLSs A and B have constant frequencies in both flux and charge, although the frequency of TLS A jumped at least three times during the measurement (at $\Phi_{\text{eff}}/\Phi_0 = 0.5095, 0.51$ and 0.511). In particular, it jumps out of frame for a few measurement points, which should not be confused with a variation of $P_q^{(eg)}$ versus n_g . TLS C, on the other hand, appears as a collection of noisy points rather than a line in Fig. 3.19.c, but seems to have a charge-offset-dependent frequency in Fig. 3.19.d.

Overall, we do not observe any variation of the qubit-TLS coupling in these measurements, when sweeping external flux and charge offset. Considering the low charge dispersion of this particular device, the matrix elements would only slightly vary on this parameter range, not drastically enough to induce a measurable change in Γ_{qt} or $P_q^{(eg)}$. It can be pointed out that the apparent linewidth of these TLSs is large compared to other reported long-lived TLSs in fluxonium qubits [152, 153], which comes from the lower decoherence

time T_2 of the device, due to the charge sensitivity. This is an intrinsic limitation to measuring the qubit-TLS coupling rate Γ_{qt} in relation with qubit matrix elements, as Γ_{qt} also depends on the decoherence of the qubit-TLS system. Nonetheless, T_2 has two sweet spots at $n_g = 0$ and $n_g = 0.5$, which are the points with the most drastic difference in matrix elements, allowing a fair comparison between at least two points, should the coherence be otherwise too low.

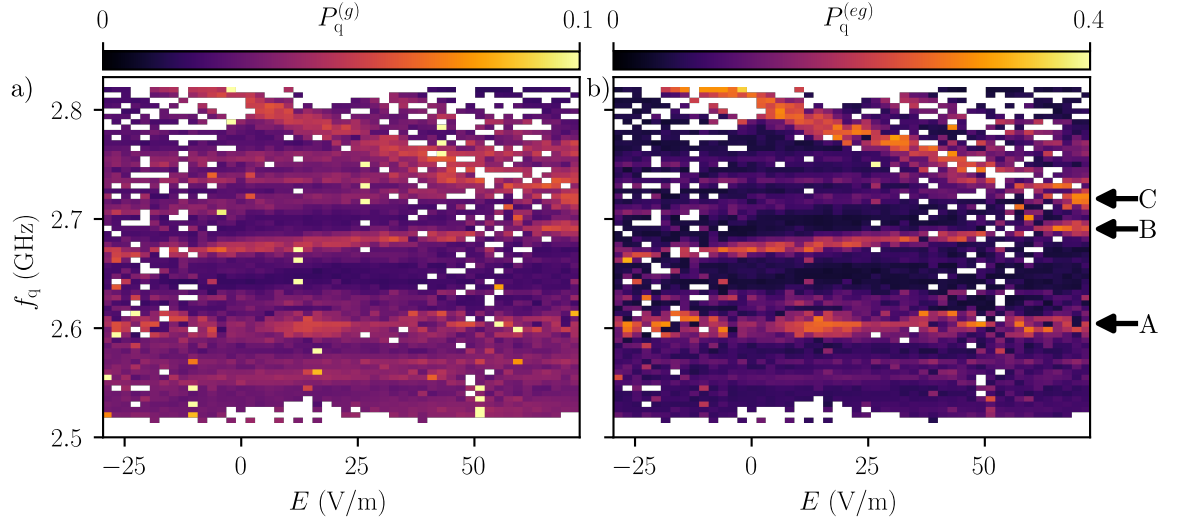


Figure 3.20: Electric-field tunable TLSs. a, b) Qubit excited-state population after the same post-selection as in Fig. 3.19, $P_q^{(g)}$ (a) and $P_q^{(eg)}$ (b), versus electric field E and qubit frequency f_q . Charge offset n_g , that tunes f_q alongside Φ_{eff} , is set by both the global electric field E and local charge drifts, jumps or parity switches.

Charge drifts and jumps give the opportunity to untangle the actual charge offset n_g from the applied electric field E . In Fig. 3.20, we represent the same two proxies $P_q^{(g)}$ (a) and $P_q^{(eg)}$ (b) from the same measurements versus E , estimated from finite-element simulations. We identify the same three TLSs A, B and C. While the frequency of TLS A stays constant over the studied range $E = -30$ V/m to 73 V/m, TLSs B and C each show a linear dependence on E . This is consistent with our recent findings of electric-field-tunable long-lived TLSs in fluxonium qubits [153]. The large tunability of TLS C explains why it did not appear as a single line versus flux in Fig. 3.19.c. On the other hand, $P_q^{(eg)}$ shows a beating versus E at the frequency of TLS A, which is unexpected and does not appear when plotted versus the actual charge offset n_g . Finally, the better resolution in Fig. 3.20.b reveals more TLSs, whose coupling to the qubit is probably smaller than the main three we identified. They exhibit different susceptibilities to electric field.

In conclusion, a sample with more matrix-element tunability, i.e. with higher charge dispersion, is needed to carry out the analysis described in Section 3.4.1 and propose a microscopic model for the coupling of TLSs to fluxonium-like qubits. This information, combined with the electric-field susceptibility, would help the community understand the origin of (long-lived) two-level systems and propose mitigation strategies.

3.5 Conclusion

In this chapter, we have designed, fabricated and characterized charge-tunable qubits based on the now standard fluxonium architecture. By incorporating a second junction in series with the main junction, we introduced a superconducting island whose charge can be tuned, offering an additional control knob on top of the external flux. Doing so, we also introduced a source of noise. In particular, charge is known to be an experimental challenge, due to local phenomena triggering charge drifts and jumps. The exact sources of these phenomena are not known and mitigation strategies therefore not straightforward. Charge offset also undergoes parity switches because of quasiparticles (broken Cooper pairs) tunneling through the junctions. Applying conventional techniques to mitigate quasiparticle tunneling (gap engineering) and quasiparticle creation (infrared filtering), we have successfully increased the parity switching time by four orders of magnitude, reaching up to 20 seconds. We also reviewed other phenomena observed in our samples, in particular parity driving and the signature of Coulomb blockade, the latter observed in a device exhibiting a spurious third junction. By tuning the charge offset on each island, we identify that it is quantified on the second island formed by said spurious junction, though the exact mechanism is not resolved.

Finally, we propose a measurement scheme based on correlation spectroscopy in quantum jump traces to uncover how ubiquitous long-lived two-level systems couple to fluxonium-like qubits. Using the two control knobs offered by our devices, flux and charge, we argue that a constant-frequency TLS would couple more or less strongly to the qubit depending on the current and voltage in the circuit. We demonstrate this idea by computing the matrix elements for reasonable circuit parameters, at different values of external flux and charge offset. Going further, we apply said measurements to one of our devices with long parity switching time and uncover a set of two-level systems by monitoring the qubit frequency in real time, therefore mitigating the impact of charge drifts and jumps. We find that the TLSs are electric-field tunable, as were the ones we reported in regular fluxonium qubits [153]. Nonetheless, the device did not offer enough matrix-element tunability to access measurable shifts in the qubit-TLS coupling.

4 Conclusions & Outlook

This chapter concludes an adventure in which we explored different fluxonium devices, on the way to a proposed four-loop fluxonium circuit. In this work, we identified two major challenges to tackle in order to control the proposed qubit, namely flux-locking and offset charge sensitivity. For each challenge, we engineered specific devices to study the phenomena at play and solved or mitigated them with state-of-the-art techniques.

Summary

In Chapter 2, we demonstrated the simultaneous sweet-spot locking of several gradiometric fluxonium qubits in a single waveguide, paving the way to fluxonium processors with reduced crosstalk from flux lines to next-neighbor flux-tunable qubits, couplers or resonators. By sweeping the perimeter and area across our samples, we were able to uncover the consequences of fabrication asymmetries on both locking accuracy and magnetic field susceptibility. This understanding allowed us to propose hypotheses on the origins of these asymmetries, relate them to material properties and compensate them. As a result, the two best compensated samples lock respectively at $\Phi_{\text{eff}} = -3 \times 10^{-4} \Phi_0$ and $\Phi_{\text{eff}} = -5 \times 10^{-4} \Phi_0$, which means that the coherence peak overlaps with zero field bias. In other words, the coherence time $T_{2,E}$ at zero magnetic field is reduced by only 15 % and 32 %, respectively. These results were published in Ref. [2]. The demonstration of flux locking, without any flux escape on a month timescale, opens the door to studying more complex systems of tunneling fluxons in a multi-loop device. In particular, we fabricated a proposed four-loop fluxonium circuit aimed at realizing a protected qubit from wavefunctions delocalized over several loops. We were able to track the state-dependent resonator frequency versus both magnetic field and charge offset on the central island and identified that the system is plagued by charge noise on the island, which manifests in measurements as charge-offset jumps, drifts and parity switching events. Therefore, better control, monitoring and mitigation of the charge parameter are needed to pursue this endeavor, which is the goal of Chapter 3.

In Chapter 3, we focused on the minimal working example of a charge-tunable fluxonium, a fluxonium design in which the main junction is replaced by two junctions in series, creating a superconducting island with a charge offset. Across different samples, we uncovered both expected and unexpected phenomena linked to charge-offset noise. On the expected side, we observed charge jumps and drifts that are consistent with the literature on charge-tunable qubits, as well as parity switches. We showed that, thanks to both infrared filtering close to the waveguide and gap engineering at the junctions, we were

able to increase the parity switching time up to twenty seconds, larger than most published charge-tunable qubits. Unexpected, however, was the observation of parity driving, by which a subharmonic drive, combined with a photon from the qubit, triggers a quasiparticle tunneling event, saturating the charge offset parity with a fidelity above 80%. Finally, an unexpected blockade was witnessed in a sample exhibiting plateaus of charge offset, probably due to a spurious third junction forming an additional small island on which the charge is quantized. Overall, we gained a better control over charge offset in charge-tunable fluxoniums, especially on parity switches, which makes these samples suitable for probing experiments. In particular, we proposed a scheme to unveil the coupling mechanism of long-lived two-level systems to fluxonium qubits and proved its feasibility. Preliminary results, while not conclusive regarding the coupling, revealed the presence of electric-field-tunable long-lived TLSs in the environment, which led to the findings in Ref. [153].

Outlook on four-loop fluxoniums

The work on the four-loop fluxonium should be carried on two fronts simultaneously. On the theory side, because diagonalizing the Hamiltonian directly is computationally demanding, methods such as tensor networks could be employed [154]. This would allow us to predict the eigen-frequencies of the device, the coupling of each state to the resonator and compare at least qualitatively with the behavior of the fabricated samples. It would also allow us to target specific sets of parameters and estimate the effect of fabrication asymmetry of the junctions and inductances.

On the experimental side, applying the knowledge acquired in Chapter 3 would allow us to increase the parity switching time and help distinguish the even and odd parity states better. Starting from the state measurements with well-resolved IQ clouds, we can perform two-tone spectroscopies and postselect on parity to resolve the flux- and charge-dependent spectrum of the device. We would then be able to characterize its Hamiltonian parameters. In parallel, using the IQ cloud as quantum jump traces, we could estimate the relaxation time T_1 at different bias points and assess whether we observe protection at specific values of charge and flux.

Additionally, one could imagine reading out the fluxon states in each loop. Drawing inspiration from persistent-current readout in historical flux qubit experiments [155], we propose to couple a SQUID to a specific loop, via mutual or shared inductance. The frequency of the SQUID therefore depends on the amount of the persistent current in the loop, with maximal sensitivity at $\Phi_{\text{ext}} = \Phi_0/4$. In a standard fluxonium qubit, this scheme realizes a $\hat{\sigma}_q^x \hat{\sigma}_r^z$ coupling, as the readout SQUID $\hat{\sigma}_r^z$ operator couples to the current $I_c \sin \hat{\varphi} \propto \hat{\sigma}_q^x$ in the qubit. In that case, the coupling is purely longitudinal only at $\Phi_{\text{ext}} = \Phi_0/4$. In both cases, this proposal opens the door to preparing state superpositions from measurement alone, and potentially probe elements in the environment (for instance, TLSs) that couple longitudinally to the qubit.

Outlook on charge-tunable fluxoniums

Several improvements on the devices and directions for the project are conceivable. Regarding charge control, focusing on mitigating small jumps and drifts would be meaningful for other charge-tunable qubit applications such as the four-loop fluxonium or other devices in the literature. On this topic, several groups have considered freezing the charge offset in transmons with a resistive shunt, as (unexpectedly) observed in Ref. [128]. There are lower and upper bounds to the value of resistance R that should be added. Indeed, the time to discharge the island into the other electrode or to ground is RC , with C the capacitance across the shunt. Therefore, RC should be smaller than the parity switching time, so that charges can equilibrate before quasiparticles can tunnel through the junction, but larger than T_1 , otherwise the losses in the resistor dominate. In a sample such as the ones shown in Chapter 3, where C is a few femtofarads, this results in a large R , for which one would have to find the appropriate material to deposit, that can be contacted with aluminum or granular aluminum on the devices.

Regarding the proposed TLS probing experiment, the obvious next step is to apply it on a device with a larger charge dispersion. This comes with experimental challenges, as a fast monitoring of the qubit frequency with two-tone spectroscopy is not possible anymore. A finer calibration, for example involving the determination of the parity from IQ clouds, might be needed. The second challenge lies in dephasing due to charge noise, which reduces coherence drastically and makes comparisons between charge offset biases difficult. Two encouraging signs, however, are that we are able to resolve TLSs away from the flux sweet spot in fluxoniums and that the coupling at the two charge sweet spots $n_g = 0$ and $n_g = 0.5$, where coherence is not limited by the charge-noise dephasing, should already differ for different coupling mechanisms. Another strategy would be to rely on the electric-field tunability exhibited by some long-lived TLSs. By tuning the frequency of a regular fluxonium, with external flux, to the TLS frequency at each electric field value, the qubit-TLS decay rate Γ_{qt} should also depend on the corresponding operator matrix element of the qubit. It would nonetheless also depend on the TLS operator element, which can only be accurately estimated with an appropriate model for their electric dipole. In short, this strategy trades a level of complexity in the data acquisition for one in the analysis.

Regarding the devices themselves, one path forward could be to build a charge-tunable fluxonium with equal junctions, making it a so-called bifluxon, a protected qubit where the charge matrix element is heavily suppressed [126]. On top of the obvious positives of having a protected qubit in the lab, such a device, with low dielectric losses thanks to its low charge matrix element, could be used as a testbed for inductive losses in granular aluminum or other materials.

Appendix

A Experimental setup

A.1 Waveguide and barrel

All samples were measured in a waveguide, as detailed in Ref. [156]. The waveguide is mounted in a copper-aluminum barrel, as shown in Fig. A.1.a. Two RF lines are fed through the barrel cap (bottom), infrared-filtered and plugged to either the readout port (which couples to the waveguide modes) or the qubit drive port (which ends in a pin close to the chips). In total, six chips hosting up to two qubit-resonator systems can be measured in a single waveguide, positioned next to each other as shown in the montage.

The barrel is further shielded with a mu-metal magnetic shield, and placed on the mixing chamber stage of a cryostat. In Fig. A.1.b, we show the setup in Sionludi XL cryostat Phoenix where, contrary to most dilution cryostats, the lowest temperature stages are up. The black-and-white background corresponds to other experimental setups. Three RF lines are used in this experiment: readout, qubit drive and DJA pump. DC is used for flux biasing the qubits (coil in Fig. A.1.a) and the DJA, as well as charge or electric-field biasing via the bias-tee on the RF qubit drive line. Attenuation and filtering is according to the best setup identified in Table 2.2 (Section 2.4) and represented in Fig. A.3.a. On the picture, the output chain from the DJA is not visible. Behind the barrels are ports that lead down to the HEMT amplifiers at the 4 K stage. Input and output to and from the cryostat are also outside the frame, at the bottom.

Finally, the experiments in Section 3.3.5 (Fig. 3.16) are performed in a waveguide equipped with a four-DC-pin lid, as shown in Fig. A.2. By using both a bias-tee on the qubit drive line and one of these DC pins, we are able to tune the charge offset on two different islands. These DC lines are filtered by the low-pass filters on the picture, with a cut-off frequency around 1 MHz (3 dB attenuation).

A.2 Cryostat wiring and measurement setup

Figure A.3 shows the measurement setup, inside and outside of the cryostat. In particular, we highlight the best cryogenic setup for coherence in Fig. A.3.a, as demonstrated in Table 2.2 of Section 2.4.

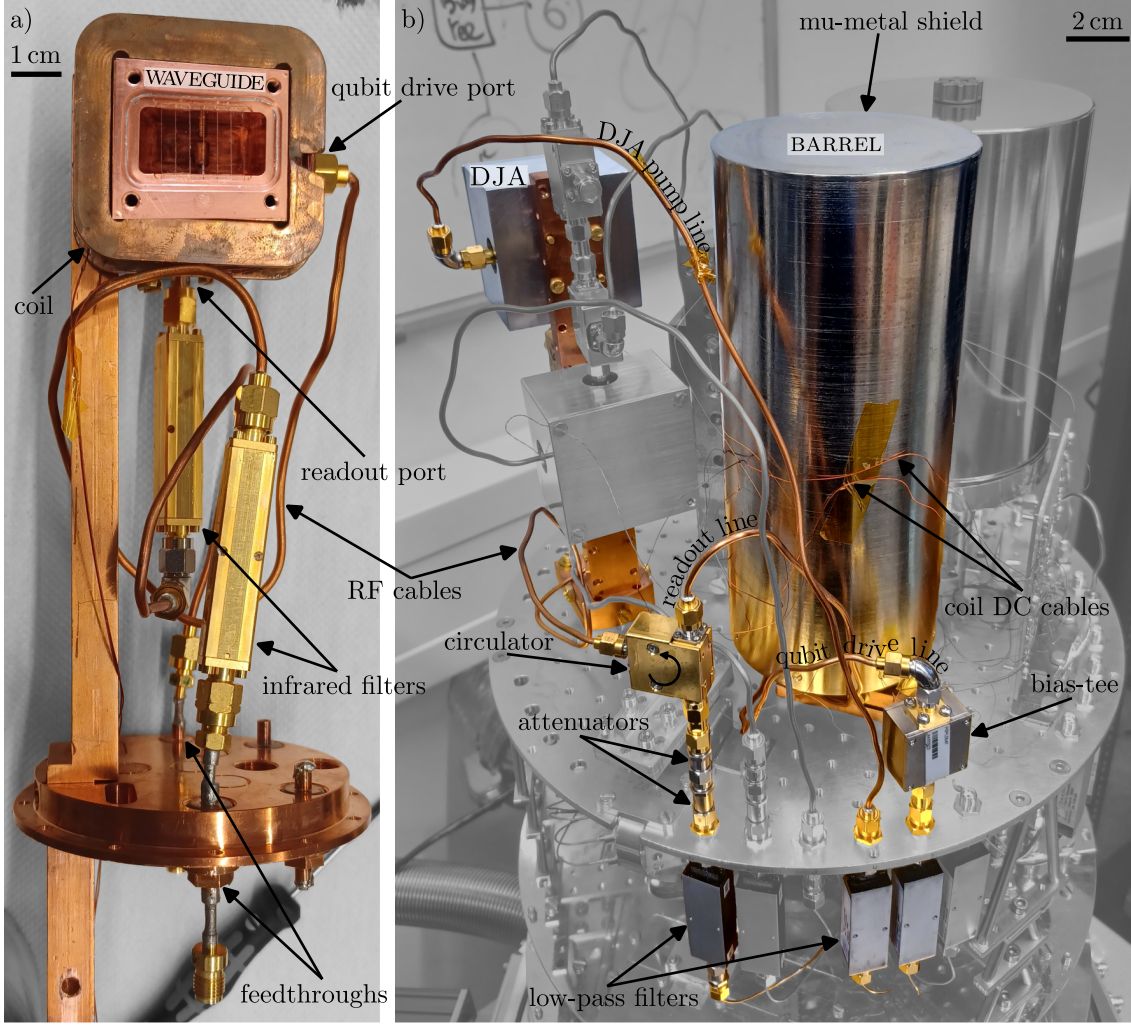


Figure A.1: Waveguide and barrel shield. **a)** Montage of waveguide mounted in open barrel, with readout and qubit drive lines visible, both infrared filtered. The open waveguide is taken from a different picture to show the positioning of up to six $3\text{ mm} \times 15\text{ mm}$ chips. After mounting the waveguide, the barrel is closed by a copper-aluminum shield and a mu-metal shield. **b)** Barrel mounted in Sionludi XL cryostat Phoenix, with elements belonging to other experimental setups in the faded black-and-white background.

The experiments in this thesis were mostly performed in a Qinu Sionludi XL cryostat, nicknamed Phoenix, at a base temperature of 20 to 30 mK. Frequency-domain measurements, namely resonator spectroscopy and two-tone spectroscopy, are used to characterize the device parameters. They are performed with the setup represented in Fig. A.3.b, using a Vector Network Analyzer (VNA) and a microwave source. Time-domain measurements, in particular coherence measurements, are performed with the setup represented in Fig. A.3.c, using either an OPX from Quantum Machines or a Presto from Intermodulation Products. From these measurements, we improved the cryogenic setup and converged on the filtering and attenuation shown in Fig. A.3.a (see Table 2.2 in Section 2.4). In particular, the total attenuation on the readout line was increased to 80 dB in order to reduce the photon shot noise.

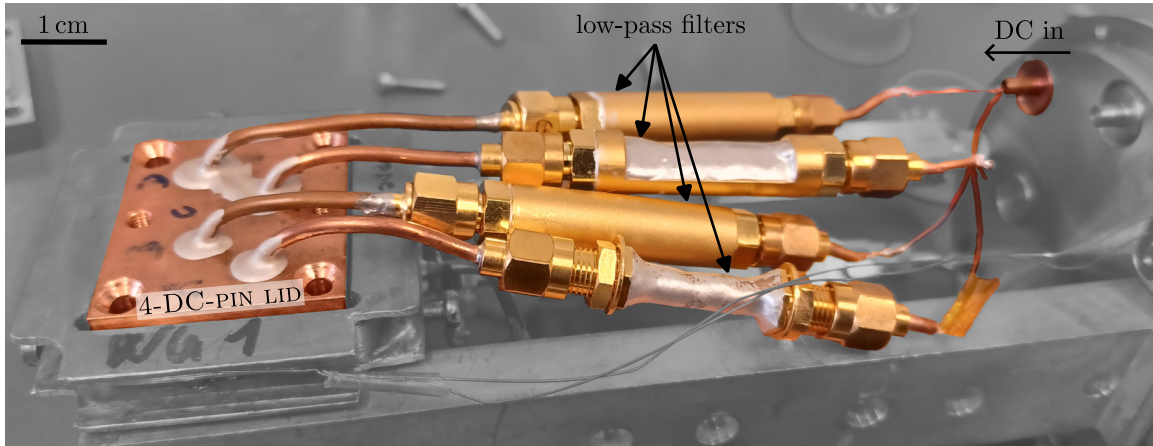


Figure A.2: Four-DC-pin waveguide lid. Waveguide mounted in open barrel with lid adapted with four DC pins in order to tune the charge offset on several islands for the experiments in Section 3.3.5. The low-pass filters have a very low cutoff frequency so that only DC signal is not attenuated.

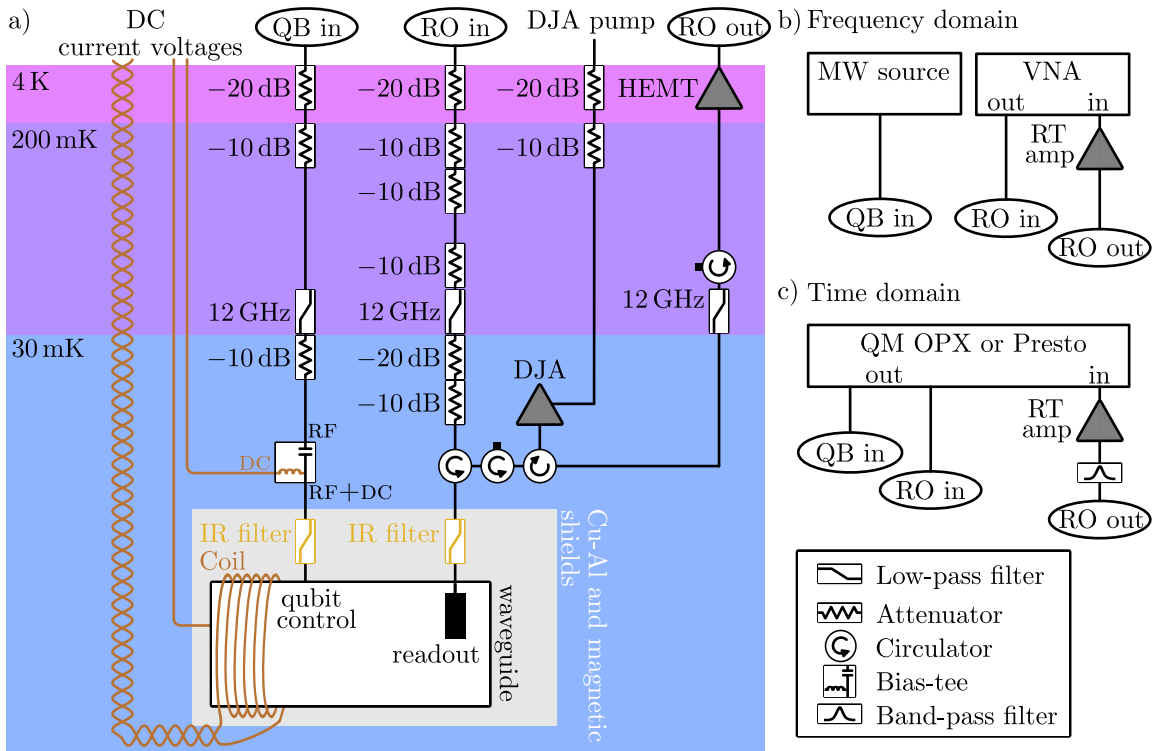


Figure A.3: Cryostat wiring and measurement setup. **a)** Cryostat complete setup, including RF lines for qubit control and readout, dimer Josephson amplifier (DJA), as well as DC lines for flux (DC current) and charge offset (DC voltages) biasing. Filtering and attenuation correspond to the last column of Table 2.2 in Section 2.4. **b, c)** Room temperature measurement setup, for frequency domain measurements, i.e. resonator spectroscopy, two-tone spectroscopy, etc (b) and time domain measurements, i.e. pulsed two-tone spectroscopy, coherence measurements, etc (c).

B Additional sample characterizations

Wafer	Sample name	f_r (GHz)	Q_i	Q_c	$\chi/2\pi$ (MHz)	g (GHz)	f_{01} (GHz)	E_J/h (GHz)	E_C/h (GHz)	E_L/h (GHz)	T_1 (μ s)	$T_{2,R}$ (μ s)	$T_{2,E}$ (μ s)
DB001	fluxonium	8.368				0.21	1.08	12.9	4.4	1.12			
DB003	-	9.005	2.8×10^3	4.3×10^2	-2.5		2.97	11.96	3.846	3.353	30.6	5.5	
DB006	15R	6.33					0.32	17.25	2.28	2.96	7.8	0.3	
DB017	07L	6.57					3.96	12.35	7.22	1.91			
DB018	00R	8.337					1.28	9.99	4.24	0.85			
	01R	8.386					1.35	12.08	4.51	1.2			
	00L	8.833					1.01	10.86	3.94	0.90			
	01L	8.888					0.93	12.81	4.46	0.83			
DB019	04L						1.45	12.29	2.3	3.64			
DB020	18R	5.811					0.513				2.8	3	
	19L	6.078	1.1×10^6	8.9×10^3			0.57	10.57	3.19	0.71	3		
	07L	6.594					0.522				2.9	2.2	4.5
	12L	7.958					0.600				2.7	1	
DB021	08R (● f)	5.715					3.128	8.45	4.16	2.05			
	08L (● e)	5.961					2.538	9.06	4.09	1.84			
	19L (● h)	6.024	5.0×10^6	2.5×10^4	-0.3		2.619	9.22	3.97	2.01	22	0.5	1.8
	16R (● g)	6.234	5.4×10^4	2.5×10^4			2.319	8.9	4.09	1.58	20		
	17L	6.491	1.3×10^4	1.5×10^3	-0.4		2.999	8.32	3.68	2.25	19	0.2	0.8
	14R (● b)	6.776	5.7×10^3	8.8×10^2	-0.8		2.16	9.19	4.05	1.59	25.6 ± 4.1	11	31.2 ± 4.8
	14L (● a)	7.056			-1.2		2.41	9.21	3.97	1.95	37.1 ± 3.6	12	39.0 ± 3.1
	01R (● d)	8.345	5.8×10^3	1.1×10^3			2.497	8.87	3.88	1.87	41	1.5	2
	01L (● c)	8.848					3.720	7.50	4.48	1.96			

Table B.1: Characterized gradiometric fluxonium samples. When possible, T_1 and $T_{2,E}$ are given as mean and standard deviation of time fluctuations.

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