

A novel approach to the equivalence principle paradox

Does gravity make a charged body radiate?

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Abstract

The longstanding question of whether an electrically charged particle emits electromagnetic radiation when accelerated by a gravitational field has led to what is often called the equivalence principle paradox. While it is commonly assumed that the answer must be binary—either radiation occurs in accordance with the Larmor formula or there is no radiation at all—we show that a more nuanced view is advisable. By analyzing the energy and momentum exchange between a charged body and its associated electromagnetic field in both homogeneous and inhomogeneous gravitational fields, we show that radiation is not an all-or-nothing phenomenon. The Larmor formula gives the upper bound of the radiated energy flux, which is realized only when the body alone receives the full energy-momentum input. In a perfectly homogeneous gravitational field, both the body and its electromagnetic field are in free fall and the radiation disappears completely. If the gravitational field is not homogeneous, energy and momentum transports take place within the electromagnetic field due to the tidal effect. The stronger these are, the more radiation is emitted. Our conclusions are extended from point charges to arbitrary rigid configurations of charged or magnetized bodies. Our analysis relies only on classical field theory, Einstein’s mass-energy equivalence, and basic properties of tidal forces, offering a conceptually clear resolution of the paradox.

Keywords: equivalence principle, tidal force, paradox, Larmor formula

1. INTRODUCTION

Our readers may already be familiar with the paradox we address in this article. Nevertheless, we would like to briefly recall it. The question is whether or not a gravitationally accelerated, electrically charged particle emits electromagnetic radiation (Figure 1).

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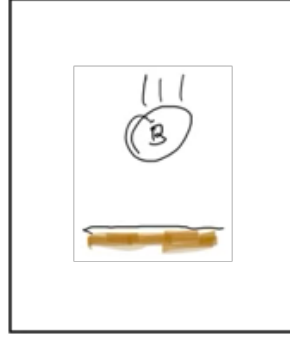


FIGURE 1. B is in free fall in Earth's gravitational field. In B's own reference frame, the gravitational field strength is zero.

The energy flow (power) P of the radiation emitted by an accelerated, electrically charged particle is given by the Larmor formula [1]

$$P = k \cdot Q^2 \cdot a^2. \quad (1)$$

Here, a is the acceleration of the particle as measured in an inertial frame, and Q is its electric charge. We have abbreviated

$$k = \frac{1}{6\pi\epsilon_0 c^3} \quad (2)$$

where ϵ_0 is the electric constant and c is the speed of light.

Well-known examples of such radiation include bremsstrahlung at the anode of an X-ray tube and synchrotron radiation in a particle accelerator. The paradox arises in the case where the acceleration is caused by a gravitational field.

Since the internal structure of the accelerated object will play a role in what follows, we prefer to speak not of a particle, but of a body, which we will refer to as B.

The paradox emerges when we describe the situation in two mutually accelerating reference frames.

In the Earth's frame, the gravitational force acts on B, causing it to accelerate. It is well established that accelerated electrically charged particles or bodies emit radiation—so B should radiate. In B's own frame, however—which is accelerating relative to Earth—the situation looks different. According to the equivalence principle, the gravitational field strength in this frame is $\mathbf{g} = 0$, B is at rest and should not radiate.

The question of whether an accelerated, charged particle emits radiation was already discussed in the years following the development of Maxwellian electrodynamics; see for instance [2–5]. As early as 1897, Larmor [2] showed that a charged particle accelerated by an electric field should emit radiation and derived the energy flux formula (1), now named after him. Schott [3] likewise studied electrically charged particles accelerated by an electric field. He also concluded that the particle emits radiation. Born [4] and later Pauli [5] approached the problem differently, discussing a hyperbolic motion of a body—i.e., motion with constant proper acceleration. Both concluded that the particle does not emit radiation.

Initially, no paradox was recognized. However, over time, the existence of a problem became the subject of further studies [6 - 11, 13].

Fulton and Rohrlich [7] provide an extensive treatment of a charged particle in a gravitational field. They show that the particle does not radiate if the accelerating field is mathematically homogeneous, but emphasize that a perfectly homogeneous field does not physically exist.

There are also proposals for resolving the paradox in which the radiation appears in one reference frame but not in another—suggesting that it could be transformed away or hidden behind an observer's event horizon [8, 10, 11].

Another idea is found in the paper by Harpaz and Soker [9], who argue that the key factor is not simply the acceleration of the charged particle, but rather the relative acceleration between the particle and its electromagnetic field.

Most of these works involve considerable mathematical complexity, particularly [7, 8, 10, 11, 13], employing the metric tensor and the energy-momentum tensor from general relativity, as well as Maxwell’s theory in accelerated frames. One gets the impression that the paradoxical situation is easy to describe, but that resolving the paradox is difficult.

In all publications known to us, the conclusion is either that the particle emits radiation according to the Larmor formula or that it does not emit radiation at all. Nowhere is the possibility considered that an accelerated body—if it does radiate—might emit less radiation than predicted by the Larmor formula.

We aim to show that it is possible to resolve the paradox using only elementary tools, assuming only knowledge of Einstein’s mass-energy equivalence and the classical description of tidal forces. We generalize our result to inhomogeneous gravitational fields and to extended objects beyond point particles—specifically, to any rigid configuration of electrically charged or magnetized bodies.

We will demonstrate that, in addition to the two extreme cases (the body emits radiation as predicted by the Larmor formula or does not radiate at all), there exists a continuum of intermediate possibilities. In other words, the Larmor energy flux represents the maximum possible radiated energy flux. In the following, we will investigate the factors determining the actual radiation emitted by the body.

Before we begin, a comment on the suggestion that radiation can be eliminated by changing the reference frame [8,10,11], which we mentioned earlier. The emission of radiation is an irreversible process in the thermodynamic sense, and as such, it must be frame-independent. Otherwise, it would violate the second law of thermodynamics.

And finally, a brief remark on the nature of the question at hand. For the setup shown in Figure 1, the radiation calculated using the Larmor formula would be extremely small. This becomes clear when comparing Earth’s gravitational acceleration (9.8 m/s^2) with the accelerations occurring in the generation of bremsstrahlung in an X-ray tube (on the order of 10^{18} m/s^2) or synchrotron radiation ($10^{14} - 10^{15} \text{ m/s}^2$). Thus, our aim is not to explain an observable effect, but rather to resolve an apparent inconsistency in the interplay between gravitation and electromagnetism.

2. Who Is Being Accelerated?

The problem we are dealing with is typically discussed in the context of a point charge. In nearly all studies known to us, we have noticed an inconsistency that may seem minor at first glance but will prove to be crucial later on. It is generally stated that a “charge” is being accelerated. However, electric charge is a physical quantity—i.e., a mathematical entity. It makes little sense to speak of the acceleration of a mathematical object. What is actually accelerated is a particle or a body, in other words, a real-world physical object. The particle (or body) is characterized by various physical quantities, of which charge is only one.

When we talk about the acceleration of a particle in a gravitational field—that is, a transfer of momentum to the particle—its mass is of primary importance. In contrast, radiation is governed by its electric charge. But the spatial distribution of mass is not the same as that of charge.

Let us briefly recall that for every component of a physical system, whether matter or field, the following relation holds

$$E = mc^2. \quad (3)$$

Since the terms *energy* and *mass*, as well as the symbols E and m , are not always used consistently, we wish to clarify that in Equation (3), the energy and mass refer to those quantities whose densities appear in the energy-momentum tensor of Einstein’s field equations—that is, the quantities that describe the inertia and gravitational behavior of the system in question. The equation does not distinguish between what we typically call *matter* and what we call a *field*. In other words, mass and energy are quantities of matter and of the field.

To make it clear that the distribution of mass is not the same as that of charge, let us examine a spatially extended, electrically charged body “under a microscope.” The body consists of particles—some electrically charged, some neutral—as well as fields, which are also electrically neutral. All constituents have mass, but only some carry electric

charge. In a gravitational field, *every* component of the body is accelerated—more precisely, each receives momentum through the gravitational field—but *not every component contributes to radiation*.

It is important to emphasize that this statement applies not only to the interior of the body but also to its external electromagnetic field—i.e., the electric field surrounding it due to its charge, and the magnetic field generated by its motion.

3. What Is Meant by Free Fall?

What do we mean when we speak of free fall or free float? The usual short answer is: a body is in free fall when it is acted upon only by the force of gravity and no other forces.

However, defining free fall in this way neglects an important aspect of the phenomenon—one that we will examine more closely below and which must be taken into account.

Figure 2 shows an extended, rigid body B in an inhomogeneous gravitational field.

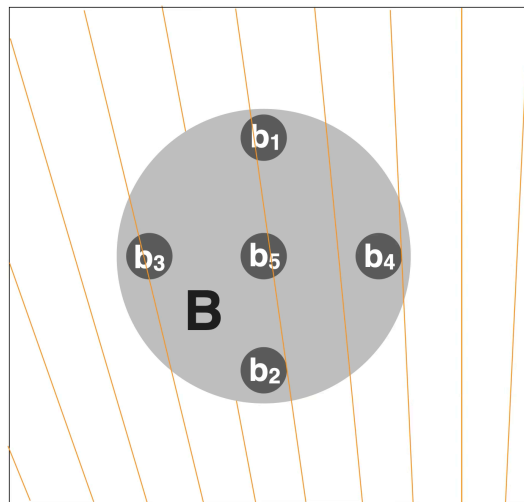


FIGURE 2. Extended body B in an inhomogeneous gravitational field. We treat the segments b1, b2, b3, b4 and b5 as individual bodies. Only b5 is in (nearly) free fall. b1 receives momentum from the rest of B; b2 gives momentum away. Likewise, b3 and b4 exchange momentum with the rest of B. If B is in motion, such momentum exchanges are accompanied by energy exchanges.

Let us consider a segment b₁ of body B, and treat this segment as a separate body in its own right. Suppose b₁ is located in a region where the gravitational field strength is weaker than at the center of mass of B. According to a refined definition of free fall, b₁ is *not* in free fall, because it is subject not only to the gravitational force, but also to a tensile force exerted by the rest of body B. In other words, b₁ receives momentum and energy not only via the gravitational field, but also *mechanically* from the rest of B. The same applies to the other subregions b₂, b₃ and b₄. For example, b₂ transfers part of the momentum and the energy it receives from the gravitational field to its immediate material surroundings. Such internal momentum transfers are commonly referred to as *tidal forces*.

Thus, the energy increase of any subregion is not determined solely by the contribution coming directly from the gravitational field. Energy also enters or exits through the subregion's material neighborhood, recognizable by the fact that the body is under mechanical stress. The only part of B that is approximately in free fall is b₅, which lies near the center of mass—provided that there is any matter located there.

Why this detour? Because in our context, it is important to keep in mind that these considerations apply not only to the body B, but also to the electromagnetic field surrounding B, assuming B is electrically charged. Since we will often be referring to the body together with its external electromagnetic field, we assign this combined system a dedicated name: BF.

We can likewise mentally divide the external electromagnetic field—now part of BF—into smaller regions. Each portion of the field stores energy and, in accordance with Eq. (3), thereby also carries mass and, as a consequence,

acquires momentum from any gravitational field. These subregions are, in general, *not* in free fall either, namely when they “experience” gravitational fields of differing strength. As a result, they receive momentum and energy not only directly via the gravitational field, but also through exchanges with their local surroundings.

We now proceed to analyze extreme cases of acceleration—that is, different modes of supplying the system BF with momentum and energy.

4. Homogeneous and Inhomogeneous Gravitational Fields

4.1. Homogeneous Gravitational Field

Our system BF (body + electromagnetic field) is located in the Earth’s gravitational field, which we assume to be sufficiently homogeneous for our purposes. Not only is BF as a whole in free fall, but so is every subregion X of it, such as bf_1 or bf_2 in Figure 3.

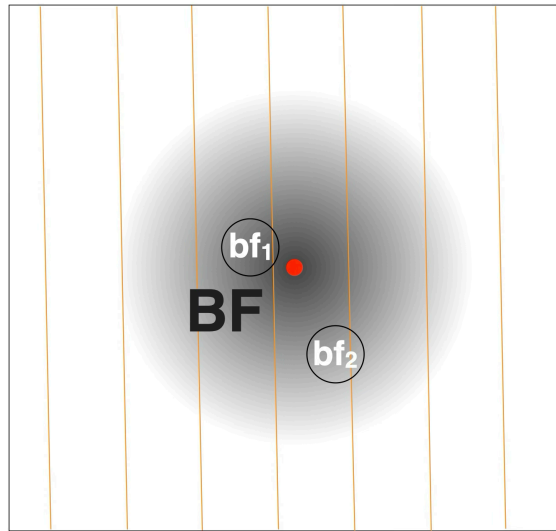


FIGURE 3. Electrically charged body with its electromagnetic field in a homogeneous gravitational field.

The momentum and energy of each subregion increase, and the momentum and energy required for this increase are supplied to X directly via the gravitational field. In particular, the electromagnetic field obtains its energy not from the body or from the adjacent electromagnetic field. In this respect, X behaves as a freely falling body.

It follows that we have no energy transport within the electromagnetic field that is subject to retardation, and thus we have no radiation that moves away from the accelerated body with its field. In other words, in the situation described, the charged body does not radiate.

If the same process is described in a freely falling reference frame, the body and its field, which is now a purely electrostatic field, are at rest and BF does not radiate.

Thus, the statement by Born [4] and Pauli [5]—that a charged particle undergoing hyperbolic motion does not radiate—is correct, provided the momentum per unit mass is identical for each part of the particle’s electromagnetic field and the body itself.

We therefore come to the conclusion that the charged body does not radiate, regardless of whether we describe the process in the reference frame of the earth or in that of the falling body. The paradox is resolved.

However, in which cases is it correct to say that an accelerated, charged body radiates?

4.2. Non-Gravitational acceleration

This second extreme case arises when only the body B (including its internal fields) receives momentum from an external source, while the field F receives none. This can be achieved in various ways—for example, by transferring

momentum to B through direct mechanical contact. Alternatively, B could be accelerated by an electric or magnetic field, similar to the mechanisms responsible for X-ray bremsstrahlung or synchrotron radiation.

Since the accelerated body must “drag along” its field, no matter how it is done, the greater part of the momentum is stored in B, but another part is transferred to the external electromagnetic field. This ensures that the momentum transfer per unit mass is the same for all regions of the field, just as for the body. Since momentum transfer is accompanied by energy transfer, a small fraction of the energy supplied to the body is passed to the field. This energy-momentum transfer within the electromagnetic field follows the laws of electrodynamics and is thus retarded. When the acceleration is reversed, the same energy and momentum do not return to the body.

That means, that radiation is generated and the process is irreversible. This form of acceleration corresponds to the known mechanisms described by the Larmor formula.

4.3. Inhomogeneous Gravitational Field

Let us now consider the situation between the two extreme cases discussed above.

We again allow BF to fall in a gravitational field—but one that is no longer homogeneous over the region occupied by the body and its electromagnetic field (see Figure 4).

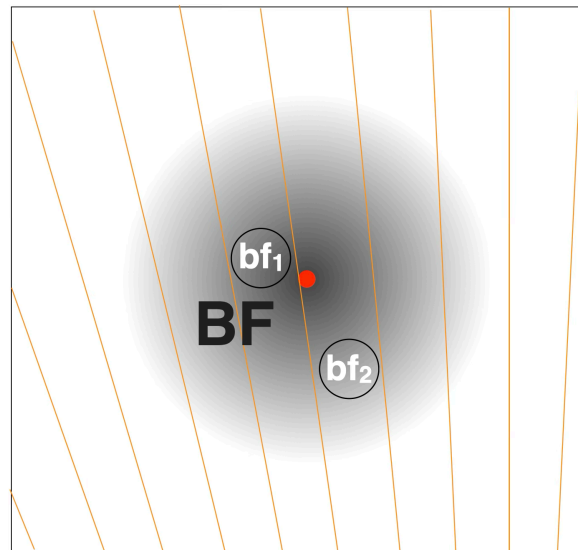


FIGURE 4. Electrically charged body with its electromagnetic field in an inhomogeneous gravitational field. Neither bf_1 nor bf_2 is in free fall.

Now, although each small region of the electromagnetic field receives momentum from the gravitational field, the momentum per unit mass is no longer the same in all regions. These differences are balanced by internal momentum flows within the electromagnetic field, similar to the case of the extended body in Figure 2. This leads to energy-momentum transport within BF, which is retarded and results in radiation. The stronger the inhomogeneity of the gravitational field, the greater this transport becomes. If the field were perfectly homogeneous, this internal transport and hence the radiation would vanish.

We thus see that radiation according to the Larmor formula and no radiation at all are two extreme cases—and all intermediate situations are, in principle, possible.

4.4. What Homogeneity means in practice

A perfectly homogeneous gravitational field does not exist, but it is both reasonable and useful to ask over what region the field must be approximately homogeneous so that the body does not radiate in practice. From the above considerations, it follows that it is not enough for the gravitational field to be homogeneous over the volume of the

body (excluding its external field); it must also be homogeneous over the region occupied by the body's surrounding electromagnetic field.

However, this external field extends to infinity. Does this mean radiation-free fall is impossible? That conclusion would go too far. What matters is not the field strength of the electrical field, but the *energy density*, since it is energy (or mass) that determines how much momentum is transferred from the gravitational field to the electromagnetic field. The energy density of the electric field of a charged sphere of radius r falls off with the fourth power of r . The same holds for the energy density of the magnetic field. About 99% of the field energy of a charged sphere is located within a distance of roughly 100 radii.

We can thus state: if the gravitational field is homogeneous within a region about 100 times the radius of the accelerated body, then the body will practically not radiate.

Let us try to get a sense of the conditions under which radiation from a freely falling, electrically charged spherical body of radius r could be expected.

Several factors make the effect exceedingly small:

1. The acceleration available in experimentally accessible gravitational fields is extremely low—especially when compared to the accelerations causing X-ray bremsstrahlung or synchrotron radiation.
2. To generate a strong gravitational field, one would require a body of enormous size. Such a body would also have a large extension, resulting in an extremely homogeneous field over the $100\text{-}r$ region.

Thus, if we restrict ourselves to realistic scenarios, we may conclude:

- A charged particle accelerated by non-gravitational means radiates in accordance with the Larmor formula.
- A charged particle accelerated in a (realistic) gravitational field emits virtually no radiation.

5. Generalization

We had mentioned that it is more insightful to consider an extended body rather than a point particle. We can now generalize our results accordingly. Everything we have concluded applies not only to small “particles” but to any rigid configuration of electrically charged or magnetized bodies. Such a configuration does not radiate when accelerated in a homogeneous gravitational field. The extent to which it does radiate depends on how inhomogeneous the gravitational field is in the region of its electromagnetic field. The size of this region is defined by the energy density distribution of the field.

6. Conclusion

The so-called equivalence principle paradox concerns the question of whether an electrically charged particle accelerated in a gravitational field emits radiation. It is commonly assumed that the answer must be either yes or no. If it does radiate, then the energy flux should be calculable using the Larmor formula.

We have shown that these are not the only possible answers. The particle may radiate more or less. The maximum possible radiated energy flux is given by the Larmor formula. This maximum is reached when all energy and momentum transfer occur exclusively via the accelerated body itself. The more the surrounding electromagnetic field of the body also falls freely, the weaker the radiation becomes. If the gravitational field is perfectly homogeneous, then each part of the electromagnetic field receives the momentum and energy it needs to follow the body directly from the gravitational field.

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