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Analytic next-to-leading order electroweak corrections to Higgs boson pair production at high energies

Joshua Davies^a, Kay Schönwald^b, Matthias Steinhauser^c and Hantian Zhang^b

^a*Department of Mathematical Sciences, University of Liverpool, Liverpool, L69 3BX, U.K.*

^b*Theoretical Physics Department, CERN, 1211 Geneva 23, Switzerland*

^c*Institut für Theoretische Teilchenphysik, Karlsruhe Institute of Technology (KIT), Wolfgang-Gaede Straße 1, 76131 Karlsruhe, Germany*

E-mail: j.o.davies@liverpool.ac.uk, kay.schonwald@cern.ch, matthias.steinhauser@kit.edu, hantian.zhang@cern.ch

ABSTRACT: We compute the complete top-quark-induced next-to-leading order electroweak corrections to the form factors entering gluon-induced Higgs boson pair production. We consider the top quark contribution in the limit where the Mandelstam variables are much larger than all other scales involved in the process and compute about a hundred expansion terms in analytic form. They are used to obtain precise numerical results even for fairly low values of the transverse momentum of the Higgs boson. We show that these electroweak corrections at high energies are of the order of -10% . We also discuss the leading logarithmic corrections of the analytic expressions.

KEYWORDS: Electroweak Precision Physics, Higgs Production, Higgs Properties, Higher Order Electroweak Calculations

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1 Introduction

Higgs boson pair production is the most promising process to probe the Higgs self-coupling in the Standard Model (SM) at the Large Hadron Collider (LHC). For this process, precise theoretical predictions are indispensable to interpret the experimental data. At the LHC, the dominant contribution to the cross section comes from the loop-induced gluon-fusion process $gg \rightarrow HH$ that receives large higher-order corrections [1–4].

In this context, next-to-leading order (NLO) two-loop electroweak (EW) corrections to $gg \rightarrow HH$ at high energies are of both theoretical and phenomenological interest. The high-energy behaviour of scattering amplitudes beyond logarithmic and leading power approximations is a state-of-the-art problem in quantum field theory, whose complexity further increases when the full electroweak sector of the SM is taken into account. The high-energy expansion of the $gg \rightarrow HH$ amplitudes will be particularly useful for studying the production of boosted Higgs bosons at the LHC [5], especially in the upcoming high-luminosity phase.

In the literature, there are a number of electroweak calculations available for $gg \rightarrow HH$. The leading Yukawa corrections have been first considered in the high-energy expansion [6] and the large top-mass limit [7]. The first electroweak calculation involving top-quarks, gauge and Higgs bosons has been performed in a large top-mass expansion [8], where five expansion terms in $1/m_t^2$ are presented. The complete electroweak corrections including all SM contributions have been computed numerically [9]. In addition, the contributions including Higgs self-couplings (and Yukawa) corrections have been computed with numerical approaches [10–12] and analytic expansions [13, 14]. Exact analytic results are available for the factorizable electroweak contributions [15] and the light-quark electroweak contributions [16]. Combined results of Yukawa and light-quark contributions are also available in [17, 18]. Contributions from the quark-antiquark channel are available from ref. [19].

In the QCD sector, the high-energy amplitudes have proven to be valuable for a number of phenomenologically important processes [20–27]. For example, a deep high-energy expansion (with more than 100 expansion terms) for $gg \rightarrow HH$ [21, 22] has been successfully combined with the numerical approach [4] or forward-scattering expansions [28–31] to cover the full phase space for QCD corrections.

In this paper, we present the full analytic result for electroweak corrections in a deep high-energy expansion for $gg \rightarrow HH$ involving top quarks, gauge and Higgs bosons. We will show that our result can cover a large phase-space region, ranging from the high-energy limit to fairly low values of the Higgs boson transverse momentum. These results lead to negative corrections of the order of -10% at the partonic level.

The outline of this paper is as follows: in section 2 we present our conventions, and in section 3 we provide technical details on the computation of the Feynman diagrams and in particular on the implementation of the expansions. In section 4 we describe the renormalization, for which we use the on-shell scheme. Results for the form factors are shown in section 5 where we also discuss their high-energy structures and describe our approach to obtain Padé-improved approximations. We conclude in section 6. In the appendix we provide explicit numerical results for the one- and two-loop form factors for a few selected phase-space points.

2 Conventions

We adopt the conventions of section 2 of ref. [14]. For convenience we repeat the most important formulae in the following.

We consider the process $g(q_1)g(q_2) \rightarrow H(q_3)H(q_4)$, where all momenta are incoming. The corresponding amplitude can be decomposed as a linear combination of two Lorentz structures with scalar coefficients $\mathcal{M}_1$ and $\mathcal{M}_2$. The latter are in turn decomposed into “box” and “triangle” form factors as follows

$$\begin{aligned}\mathcal{M}_1 &= X_0 s (F_{\text{tri}1} + F_{\text{box}1}) , \\ \mathcal{M}_2 &= X_0 s F_{\text{box}2} ,\end{aligned}\tag{2.1}$$

where we have introduced

$$F_{\text{tri}1} = \frac{3m_H^2}{s - m_H^2} \left(F_{\text{tri}} + \frac{m_H^2}{s - m_H^2} \tilde{F}_{\text{tri}} \right) .\tag{2.2}$$

F_{tri} and $\tilde{F}_{\text{tri}}$ originate from Feynman diagrams with one and two cuts through s -channel Higgs boson propagators, respectively. Note that at leading order (LO) we have $\tilde{F}_{\text{tri}} = 0$. The prefactor in eq. (2.1) is given by

$$X_0 = \frac{G_F}{\sqrt{2}} \frac{\alpha_s(\mu)}{4\pi} ,\tag{2.3}$$

where μ is the renormalization scale and G_F is the Fermi constant.

For this process, the Mandelstam variables are given by

$$s = (q_1 + q_2)^2 , \quad t = (q_1 + q_3)^2 , \quad u = (q_2 + q_3)^2 ,\tag{2.4}$$

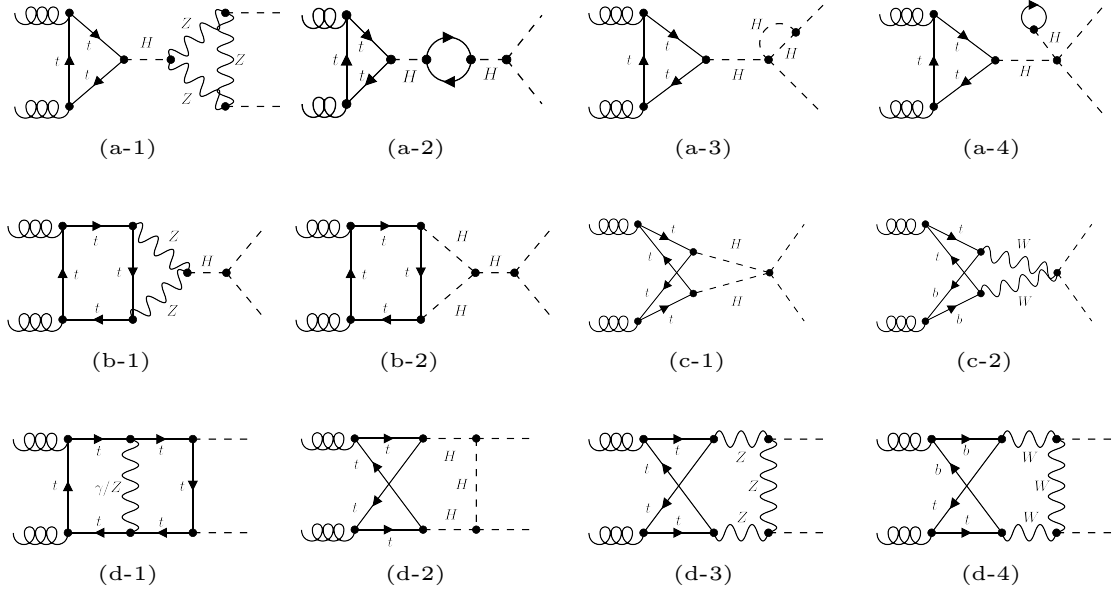


Figure 1. Sample Feynman diagrams contributing to the NLO electroweak corrections to $gg \rightarrow HH$. In the top, middle and bottom row, 1PR, triangle and box diagrams are shown. Note that (c-1) and (c-2) enter the box form factors since there is no cut through an s -channel Higgs boson propagator.

with

$$q_1^2 = q_2^2 = 0, \quad q_3^2 = q_4^2 = m_H^2, \quad s + t + u = 2m_H^2. \quad (2.5)$$

One often either uses $\sqrt{s}$ and the transverse momentum of the Higgs bosons in the center-of-mass frame, p_T , or $\sqrt{s}$ and the scattering angle θ (also in the center-of-mass frame) in order to parametrize the phase space. The relations to the Mandelstam variables are given by

$$p_T^2 = \frac{ut - m_H^4}{s}, \quad t = m_H^2 - \frac{s}{2} \left(1 - \cos \theta \sqrt{1 - \frac{4m_H^2}{s}} \right). \quad (2.6)$$

We define the perturbative expansion of the form factors as follows

$$F = F^{(0)} + \frac{\alpha_s(\mu)}{\pi} F^{(1,0)} + \frac{\alpha}{\pi} F^{(0,1)} + \dots, \quad (2.7)$$

where α_s is the strong coupling constant and α is the fine structure constant. $F^{(1,0)}$ represents the two-loop QCD corrections which are not discussed in this paper.

3 Technical details

Typical Feynman diagrams which contribute to the two-loop electroweak corrections are shown in figure 1. We want to stress that besides the one-particle irreducible (1PI) contributions to the two-loop amplitudes there are also one-particle reducible (1PR) contributions (which have been computed in ref. [15]). As a subset of the latter we also include the so-called tadpole contributions, which are necessary to obtain a finite result (see section 4).

We consider all sectors of the SM. However, since the light-quark contributions are known in analytic form from ref. [16] we concentrate on the top-quark-induced contributions. This means we consider the electroweak corrections generated by diagrams with at least one top-quark propagator (not appearing in tadpole contributions). An exploratory study of the high-energy behaviour for the diagrams with a Higgs boson exchange has been presented in ref. [6], where all fully-massive planar master integrals were computed. High-energy corrections for the subset of Feynman diagrams with non-zero top quark Yukawa and Higgs self couplings were considered in ref. [14]. In this paper, deep analytic expansions for all fully-massive non-planar master integrals have been obtained, and the results for the bare two-loop form factors have been compared to the numerical results obtained in ref. [12]. In the region in which the expansions are valid, good agreement has been found.

A calculation of the complete NLO electroweak corrections with general gauge parameters has been performed in ref. [8] in the large- m_t limit, where it was demonstrated that the gauge parameter dependence drops out for physical quantities. In the current paper we use the same set of diagrams but set all gauge parameters (γ , Z and W) to Feynman gauge. As a consequence, the neutral and charged Goldstone bosons have the same masses as the corresponding gauge bosons, Z and W . A different choice of the gauge parameters would introduce additional, but spurious, mass scales.

Our calculation involves several mass scales, which is currently too complicated to arrive at analytic results for the form factors. For this reason we perform several expansions: we perform Taylor expansions in the final-state Higgs boson mass and in the mass differences of the top quark mass m_t and the other boson masses in the loop diagrams. The resulting integrals are expanded in the high-energy limit, where the Mandelstam variables s and t are much larger than the top quark mass.

In the following we describe the computation of the 1PI two-loop diagrams. The Higgs boson mass appears in several places. Since we process them differently we distinguish them in the intermediate steps of our calculation. We denote the mass of the final-state Higgs bosons by m_H^{ext} , the one in the propagators inside loop diagrams by m_H^{int} , and the Higgs boson mass in the propagator of the triangle diagrams which is outside of the loop by m_H^{prop} . Furthermore, in the SM the Higgs boson mass appears in the expression for the Higgs boson self coupling λ . We denote it by m_H^λ thus have

$$\lambda = \frac{e^2(m_H^\lambda)^2}{8m_Z^2 c_w^2 s_w^2}, \quad (3.1)$$

where $e = \sqrt{4\pi\alpha}$, α is the fine structure constant, $c_w \equiv \cos(\theta_w)$ is the cosine of the weak mixing angle, and $s_w^2 \equiv 1 - c_w^2$.

Note that for the box diagrams, m_H^λ may have a different value to the “other” Higgs boson masses without affecting the renormalization of the amplitude. This is not the case for the triangle contributions; here we must identify $m_H^\lambda = m_H^{\text{prop}}$ for a complete renormalization. The values of m_H^{int} (which only appears at NLO) and m_H^{ext} (which is an external particle) can not interfere with the renormalization procedure. For numerical evaluation in section 5.3, we of course use the same values for m_H^{ext} , m_H^{int} , m_H^{prop} and m_H^λ , as given in eq. (5.12).

Besides the Mandelstam variables s and t , the additional scales are the top quark mass m_t and the gauge boson masses m_Z and m_W . For the latter we also distinguish the masses

in the couplings from those in the propagators inside loop diagrams, which we denote by m_Z^{int} and m_W^{int} , respectively. Note that for any given diagram there are at most two different masses present: m_t and one of the boson masses m_H^{int} , m_Z^{int} or m_W^{int} . Thus, for the generation of the form factors expressed in terms of master integrals we proceed as follows.

We use **qgraf** [32] for the generation of the amplitude, **tapir** [33] for the translation to **FORM** [34, 35] notation and the generation of auxiliary files for processing the integral topologies, and **exp** [36, 37] for the mapping of each Feynman diagram onto a minimal subset of the integral topologies from **tapir**.

The masses of the internally propagating bosons are expanded around m_t as a series in $\delta'_X = 1 - (m_X^{\text{int}}/m_t)^2$ with $X = H, Z, W$, see also refs. [6, 14]. As a result, for the diagrams containing virtual Z or Higgs bosons, all internal lines have the same mass: m_t . These integral families have already been classified in refs. [6, 14], where all relevant master integrals have been computed. For the diagrams with virtual W bosons we have both massive lines (with mass m_t) and massless lines which originate from the bottom quark propagators. The integral families correspond to those of the QCD corrections have been defined in refs. [21, 22]. Deep expansions of the master integrals have been computed in ref. [30]. Further technical details regarding calculations of master integrals can be found in refs. [38, 39]. In the final results we switch to the expansion parameter $\delta_X = 1 - m_X^{\text{int}}/m_t$, since it shows better convergence properties (as e.g. described in ref. [40]). We use the relation $\delta'_X = \delta_X (2 - \delta_X)$ and expand in δ_X up to the same order which we had for δ'_X .

We compute the amplitude using the in-house **FORM** code “**calc**”, which applies projectors to obtain the form factors, takes traces and expresses the final result in terms of scalar integrals of the integral families selected by **exp**. In this step we also perform the expansion of all boson propagators such that in the final results we have terms up to order δ_X^k . For our calculation we have $k = 3$, see also below. Note that there are no “mixed” terms in the δ_X expansion since Higgs, Z and W bosons never appear in the same Feynman diagram. At this point the scalar integrals depend on the variables $\{s, t, m_H^{\text{ext}}, m_t\}$; δ_X is only present in the integral coefficients.

The next step is to expand in the final state Higgs boson mass which is encoded in the relation of the external momenta, the Mandelstam variables and m_H^{ext} , see eqs. (2.4) and (2.5). We perform the expansion of each scalar integral at the level of the integrands with the help of **LiteRed** [41, 42]. We generate **FORM Identify** statements which are applied to the summed expressions for the form factors. The remaining scalar integrals then depend only on $\{s, t, m_t\}$. The reduction to master integrals for the fully-massive families is performed with **Kira** [43–45]. We extend the tables which have been generated for the calculation performed in ref. [14]. This part of the amplitude is expressed in terms of 168 master integrals, 28 of which are non-planar. For the QCD-like families we use the setup developed in refs. [21, 22]. In particular, we use **FIRE** [46] for the reduction and express the amplitude in terms of 161 master integrals, 30 of which are non-planar. The reduction tables are inserted into the amplitude using **FORM**. This must be programmed carefully, to avoid both generating overly-large terms and blowing up the total number of terms in the expressions.

Finally, we expand the amplitude in ϵ and for $m_t \rightarrow 0$ and insert the analytic high-energy expansion of all master integrals from refs. [6, 14, 21, 22, 30]. This final stage of the

computation is implemented efficiently in FORM by making use of Tablebases to store the ϵ and m_t expansion coefficients of the master integrals.

At this point we have obtained analytic expressions for the (bare) form factors with explicit dependence on $s, t, m_t, m_H^{\text{ext}}$ and δ_X . We have computed expansion terms up to m_t^{108} , $(m_H^{\text{ext}})^4$ and δ_X^3 , and for $(m_H^{\text{ext}})^0$ also δ_X^4 . Due to the deep expansion in m_t the expressions are quite large; they total around 6.5 GB of gzip-compressed files.

In this work, we use **Mathematica** for the numerical evaluation at 200 digits or more, which takes about half an hour (with some dependence on the phase-space point). For the calculation of hadronic cross sections, it is important to export these expressions to **Fortran** or **C** which will lead to a significant reduction of the runtime. However, this must be done carefully so that the compilation time is no more than a few hours.

The next step is to renormalize the form factors, followed by their numerical evaluation and Padé approximation procedure. We discuss the details of these parts of the calculation in the following sections.

4 Renormalization

For the NLO electroweak corrections there is no real-radiation contribution and thus all $1/\epsilon$ poles are of ultraviolet origin. They are removed after the renormalization of an independent set of quantities. In our case we choose the W , Z and Higgs boson masses, the top quark mass and the electric charge which is related to the fine structure constant via $e = \sqrt{4\pi\alpha}$. We adopt the on-shell renormalization scheme and introduce the Z factors as follows

$$\begin{aligned} e^0 &= Z_e e, \\ m_t^0 &= Z_{m_t} m_t, \\ m_W^0 &= Z_{m_W} m_W, \\ m_Z^0 &= Z_{m_Z} m_Z, \\ m_H^0 &= Z_{m_H} m_H, \end{aligned} \tag{4.1}$$

where the superscript “0” indicates bare quantities. In addition to the parameters in eq. (4.1) we have to renormalize the wave function of the Higgs boson

$$H^0 = \sqrt{Z_H} H, \tag{4.2}$$

where H is the Higgs boson field. All Z factors can be computed from two-point functions; explicit analytic expressions can, e.g., be found in refs. [47, 48]. For convenience we provide computer-readable expressions in the ancillary files of this paper [52].

Let us also stress that our NLO electroweak form factors do not have an explicit dependence on the renormalization scale since all parameters are renormalized in the on-shell scheme.

We consistently include tadpole contributions in all parts of our calculation, i.e., in the bare two-loop amplitude and the Z factors. This prescription is equivalent to the so-called *Fleischer-Jegerlehner tadpole scheme* [49]. Alternative options to treat the tadpole contributions are discussed, e.g., in ref. [50].

We renormalize the box and triangle contributions independently. In the case of the box contribution, at two-loop order there are only 1PI Feynman diagrams (see figure 1), and

contributions form diagrams where one-loop tadpoles are attached to the (virtual) top quark propagators. The identical tadpole contributions are present in the on-shell renormalization constant Z_{m_t} . Thus they exactly cancel in the on-shell scheme and finite results for F_{box1} and F_{box2} are obtained from parameter and wave function renormalization (as detailed above) by considering only 1PI contributions. This is different in the case of the triangle contribution where further types of tadpole diagrams arise, see figure 1. Furthermore, there are various 1PR diagrams with one-loop corrections to the Higgs boson propagator or one-loop vertex corrections to the final-state triple-Higgs boson vertex. We find it convenient to sum all 1PI and 1PR contributions to the two-loop amplitude and the renormalization constants and proceed with the renormalization as described above. This leads to a finite results for F_{tri1} .

In the counterterm contribution we use for all (one-loop) Z factors exact (i.e. non-expanded) results. Since the Z factors contain $1/\epsilon$ poles we need the one-loop amplitudes including linear terms in ϵ . For the triangle diagrams the (exact) results can be found in ref. [15]. They contain at most tri-logarithms. It is possible to compute the $\mathcal{O}(\epsilon)$ terms of the box contributions, however the analytic expressions are rather cumbersome. For this reason we use the deep high-energy expansion of the one-loop box amplitudes from the calculation of the QCD corrections in ref. [30] (see also ref. [14]).

An important cross check of our result is the cancellation of the $1/\epsilon$ pole between the counterterm contribution and the two-loop amplitude. If we perform a consistent expansion in m_t we observe an exact cancellation. Alternatively, if we leave the counterterm contribution unexpanded we typically observe a cancellation of more than 100 significant digits in the coefficient of the $1/\epsilon$ pole. We have further verified numerically that our renormalized form factors are independent of explicit renormalization scale choices.

In a next step we transform our results into the so-called G_μ scheme where the fine structure constant α is replaced by the Fermi constant G_F . For explicit expressions we refer, e.g., to section 5.1.1 of ref. [48]. The transformation to the G_μ scheme introduces the quantity Δr [51]. For an explicit result, see eq. (423) of ref. [48]. In our numerical evaluation we do not expand Δr but evaluate it exactly, and we evaluate the Z factors similarly. Note that $F^{(0,1)}$ in eq. (2.7) is already defined in the G_μ scheme.

5 Results

5.1 Analytic results

For convenience we provide explicit analytic results for the leading non-vanishing terms of the LO form factors. They are given by

$$F_{\text{tri1}}^{(0)} = \frac{6m_t^2 m_H^2}{s^2} \left[4 - l_{ms}^2 \right] + \mathcal{O} \left(\frac{m_t^4}{s^2}, \frac{m_H^4}{s^2} \right),$$

$$F_{\text{box1}}^{(0)} = \frac{4m_t^2}{s} \left[2 + \frac{m_H^2}{s} \left((l_{1ts} - l_{ts})^2 + \pi^2 \right) \right] + \mathcal{O} \left(\frac{m_t^4}{s^2}, \frac{m_H^4}{s^2} \right),$$

$$\begin{aligned}
 F_{\text{box}2}^{(0)} = & \frac{2m_t^2}{st(s+t)} \left[-l_{1ts}^2(s+t)^2 - l_{ts}^2 t^2 - \pi^2 (s^2 + 2st + 2t^2) + \frac{2m_H^2}{s(s+t)} \left(l_{1ts}^2 s(s+t)^2 \right. \right. \\
 & + \pi^2 s^3 + 2s^2 t (-2l_{ms} + l_{ts} + \pi^2 - 4) - st^2 (8l_{ms} + (l_{ts} - 2)l_{ts} + 16) \\
 & \left. \left. - 4(l_{ms} + 2)t^3 \right) \right] + \mathcal{O}\left(\frac{m_t^4}{s^2}, \frac{m_H^4}{s^2}\right), \quad (5.1)
 \end{aligned}$$

where

$$l_{ms} = \log\left(\frac{m_t^2}{s}\right) + i\pi, \quad l_{ts} = \log\left(-\frac{t}{s}\right) + i\pi, \quad l_{1ts} = \log\left(1 + \frac{t}{s}\right) + i\pi. \quad (5.2)$$

At two-loop order the form factors have a more complicated structure. However, the leading logarithmic contributions in $\log(m_t^2/s)$ are sufficiently compact. In the limit of vanishing external Higgs boson mass and small top-quark mass, the leading logarithmic contributions in the high-energy limit for the box form factors are given by

$$F_{\text{box}1}^{(0,1)} = \frac{m_W^4}{s^2 c_w^6 s_w^2} \left[l_{ms}^2 \frac{36c_w^6 + 32c_w^4 - 40c_w^2 + 17}{36} (l_{1ts}^2 + l_{ts}^2 - 2l_{1ts}l_{ts} + \pi^2 - 4) \right] + \dots, \quad (5.3)$$

$$\begin{aligned}
 F_{\text{box}2}^{(0,1)} = & \frac{m_W^4}{s^2 c_w^6 s_w^2} \left[l_{ms}^3 \frac{36c_w^6 + 32c_w^4 - 40c_w^2 + 17}{27} + l_{ms}^2 \left(-\frac{2}{9} \delta_Z (32c_w^4 - 40c_w^2 + 17) \right. \right. \\
 & - 8\delta_W c_w^6 + \frac{36c_w^6 + 32c_w^4 - 40c_w^2 + 17}{36t(s+t)} (l_{1ts}^2(s+t)^2 - 2l_{1ts}t(s+t) - 2l_{ts}t(s+t) \\
 & \left. \left. + l_{ts}^2 t^2 + \pi^2 (s^2 + 2st + 2t^2) \right) \right] + \dots, \quad (5.4)
 \end{aligned}$$

where the ellipsis stand for sub-leading logarithmic contributions and higher-order terms in m_t , m_H and $\delta_{W/Z} = 1 - m_{W/Z}/m_t$. We note that the counterterms do not contribute to the results shown in eqs. (5.5)–(5.4); they come exclusively from the genuine two-loop diagrams. The leading term for the box form factors come from Feynman diagrams without Yukawa couplings as shown, e.g., in diagrams (d-3) and (d-4) in figure 1. We obtain for the high energy limit of the triangle form factor

$$F_{\text{tri}1}^{(0,1)} = \frac{3m_t^2 m_W^2}{16s^2 s_w^2} l_{ms}^4 \left[-\frac{4m_t^4}{m_W^4} + \frac{1 + 2c_w^4}{c_w^4} + \frac{4s_w^2 m_H^2}{27c_w^2 m_W^2} \right] + \dots. \quad (5.5)$$

We remark that while the high energy limits of the box form factors are obtained by expanding the corresponding two-loop diagrams directly, we obtain the high energy limit of the triangle form factor by re-expanding the partially exactly known results. In the exactly known results, we do not keep track of external or internal Higgs boson masses. We therefore obtain our result by strictly expanding in the $s \rightarrow \infty$ limit, but no re-expansion in any other scale is performed. In order to shorten the expressions, we use $\log(m_X^2/s) + i\pi = l_{ms} + \mathcal{O}(\delta_X)$ and drop the terms that come from the δ expansion.

We observe that the leading term for $F_{\text{box}1}$ is a quadratic logarithm, whereas $F_{\text{box}2}$ has a cubic contribution. Note that in the LO expression shown in eq. (5.1) $F_{\text{box}1}$ has no

$\log(m_t^2/s)$ terms at $\mathcal{O}(m_t^2)$ or $\mathcal{O}(m_H^2)$ and the linear logarithms of F_{box2} are suppressed by m_H^2/s . The LO triangle form factor has a quadratic $\log(m_t^2/s)$ contribution which is suppressed at high energies by a factor of m_H^2/s due to the s -channel Higgs boson propagator. It is also interesting to remark that $F_{\text{box2}}^{(0,1)}$ has $\log^2(m_t^2/s)$ terms which are proportional to δ_W or δ_Z . We note that at NLO EW, the process $gg \rightarrow HH$ contains several coupling structures, resulting in rich structures of logarithmic and power corrections. For further analytic expressions we refer to the ancillary files [52] where we provide five expansion terms in the high-energy limit. Deeper expansions are available on request.

The generic structure of the high-energy expansion of the form factors has the form

$$F = \sum_{n=-4}^{108} \sum_{i=0}^2 \sum_{k_W=0}^4 \sum_{k_Z=0}^4 \sum_{k_H=0}^4 c_{ni}^{k_W k_Z k_H} \left(\frac{m_t}{\sqrt{s}} \right)^n \left(\frac{m_H^{\text{ext}}}{\sqrt{s}} \right)^{2i} \delta_W^{k_W} \delta_Z^{k_Z} \delta_H^{k_H}, \quad (5.6)$$

where negative powers in m_t are only present for $i > 0$. Note that for $i = 2$ we have $1/m_t^4$ terms. Furthermore, there is the additional constraint that $k_W + k_Z + k_H \leq 3$ for $i = 1, 2$ and $k_W + k_Z + k_H \leq 4$ for $i = 0$. Since there are no “mixed” contributions in the δ_X expansions at most one of the indices k_W, k_Z, k_H may be non-zero. For convenience we perform the rescaling

$$\begin{aligned} \delta_W^{k_W} &\rightarrow \delta \delta_W^{k_W}, \\ \delta_Z^{k_Z} &\rightarrow \delta \delta_Z^{k_Z}, \\ \delta_H^{k_H} &\rightarrow \delta \delta_H^{k_H}, \end{aligned} \quad (5.7)$$

and count in the following the powers of δ . The coefficients $c_{ni}^{k_W k_Z k_H}$ in eq. (5.6) depend on t/s and $\log(m_t^2/s)$.

5.2 Construction of approximations

In the following, we describe our procedure to construct approximations. First, we fix the maximal powers of δ and $(m_H^{\text{ext}})^2$ and insert the numerical values for δ_W , δ_Z , δ_H and m_H^{ext} . We then specify the phase-space point (i.e. the values of s and t). This leads to an expansion in m_t . Next we insert the numerical value for m_t if it appears in the argument of logarithms. Then we rescale odd and even power of m_t according to $m_t^{2k} \rightarrow m_t^{2k} x^k$ and $m_t^{2k-1} \rightarrow m_t^{2k-1} x^k$ and insert also here the numerical value for m_t . As a result we obtain a polynomial of degree $N = 54$ in x . Afterwards we construct a Padé approximant of the form

$$[n/m](x) = \frac{a_0 + a_1 x + \dots + a_n x^n}{1 + b_1 x + \dots + b_m x^m}, \quad (5.8)$$

where $n + m = N$. We can vary the parameters n and m and we can also choose $N < 54$. This provides the possibility to generate a large number of Padé approximants which allows us to determine a central value and uncertainty interval. This procedure is performed separately for each phase-space point. More details can be found in refs. [4, 53].

Whereas in the high-energy limit we can construct a deep expansion in m_t , we only have a few terms in the expansion in δ and the final-state mass m_H^{ext} . To improve convergence in these parameters, sum acceleration techniques can be useful. We use the so-called *Aitken*

extrapolation (see, ref. [54]) and proceed as follows: the starting point is an expansion of the form

$$A = \sum_{i=0}^{\infty} A_i y^i, \quad (5.9)$$

where in our case y tags either the expansion in δ or in $(m_H^{\text{ext}})^2$. The A_i correspond to the Padé approximations based on the specified input and expansion depths, which are described below. We then construct the partial sums

$$A^{(k)} = \sum_{i=0}^k A_i y^i, \quad k = 0, 1, \dots, k_{\text{max}}, \quad (5.10)$$

where for the expansion in $(m_H^{\text{ext}})^2$ we have $k_{\text{max}} = 3$, while we have $k_{\text{max}} = 4$ for the expansion in δ .¹ These partial sums are used to obtain the new approximation:

$$A_{\text{acc}}^{(n)} = \frac{A^{(n)} A^{(n+2)} - \left(A^{(n+1)}\right)^2}{A^{(n)} - 2A^{(n+1)} + A^{(n+2)}}. \quad (5.11)$$

It is interesting to note that $A_{\text{acc}}^{(n)}$ corresponds to the $[n+1/1]$ Padé approximant.

For the expansion in δ we can construct the accelerated results $A_{\text{acc}}^{(0)}$, $A_{\text{acc}}^{(1)}$ and $A_{\text{acc}}^{(2)}$. We can compare $A_{\text{acc}}^{(0)}$ and $A_{\text{acc}}^{(1)}$, which only include expansions up to δ^2 or δ^3 , respectively, to results which include δ^3 or δ^4 terms without any acceleration. We estimate the final uncertainty of the δ expansion using the difference between our best “raw” and accelerated results.

In principle we could also consider sum acceleration in m_H^{ext} . However, as we will see below, the expansion in m_H^{ext} converges sufficiently quickly that the application of sum acceleration is not necessary.

5.3 Numerical results

In this section we present numerical results for the form factors and the squared partonic amplitude. We use the following input values [55]

$$\begin{aligned} G_F &= 1.166378 \times 10^{-5} \text{ GeV}^{-2}, \\ m_H &= 125.2 \text{ GeV}, \\ m_Z &= 91.19 \text{ GeV}, \\ m_W &= 80.37 \text{ GeV}, \\ m_t &= 172.57 \text{ GeV}, \end{aligned} \quad (5.12)$$

and introduce the coupling (e.g. in eq. (2.7))

$$\alpha_{G_\mu} = \frac{\sqrt{2}}{\pi} G_F m_W^2 \left(1 - \frac{m_W^2}{m_Z^2}\right), \quad (5.13)$$

¹We set the expansion terms of $O(\delta^4 (m_H^{\text{ext}})^i)$, $i > 0$ to 0. The justification for this procedure is discussed in section 5.3.

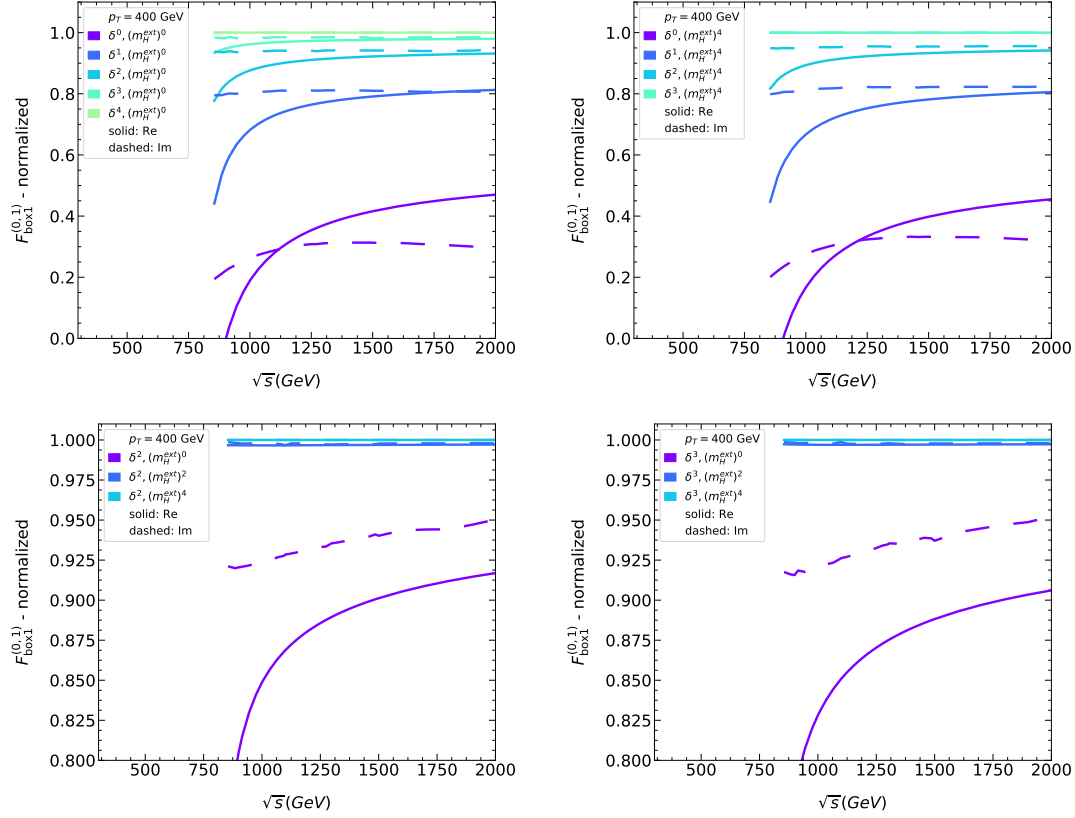


Figure 2. Ratio of various δ and m_H expansion terms normalized to the highest available approximation for F_{box1} . Results for the real and imaginary parts are shown as solid and dashed lines, respectively. The expansion depths given in the legend indicate the expansion terms which are used for the construction of the Padé approximation. Here “ $\delta^n, (m_H^{\text{ext}})^m$ ” means that all expansion terms δ^k and $(m_H^{\text{ext}})^l$ with $k \leq n$ and $l \leq m$ are included. Top row: fixed expansion depth in m_H . Bottom row: fixed expansion depth in δ .

which is our expansion parameter in the G_μ scheme. The squared matrix element can be perturbatively expanded as

$$\mathcal{U} = \frac{1}{16} (|\mathcal{M}_1|^2 + |\mathcal{M}_2|^2) = \frac{(X_0 s)^2}{16} \left(\mathcal{U}^{(0)} + \frac{\alpha_s(\mu)}{\pi} \mathcal{U}^{(1,0)} + \frac{\alpha}{\pi} \mathcal{U}^{(0,1)} + \dots \right), \quad (5.14)$$

which we use to define the ratio

$$r_{\text{EW}} = \frac{\alpha}{\pi} \frac{\mathcal{U}^{(0,1)}}{\mathcal{U}^{(0)}}, \quad (5.15)$$

which quantifies the size of the NLO corrections at the partonic level. In the results discussed below we use for the LO term in the denominator of eq. (5.15) the expanded expressions. For all phase-space points which are considered in this paper this provides an excellent approximation to the exact results.

In the following, we describe Padé results which have been obtained using expansion depths between $(m_t^2)^{46}$ and $(m_t^2)^{54}$. Furthermore, we include approximations up to $(m_H^{\text{ext}})^4$

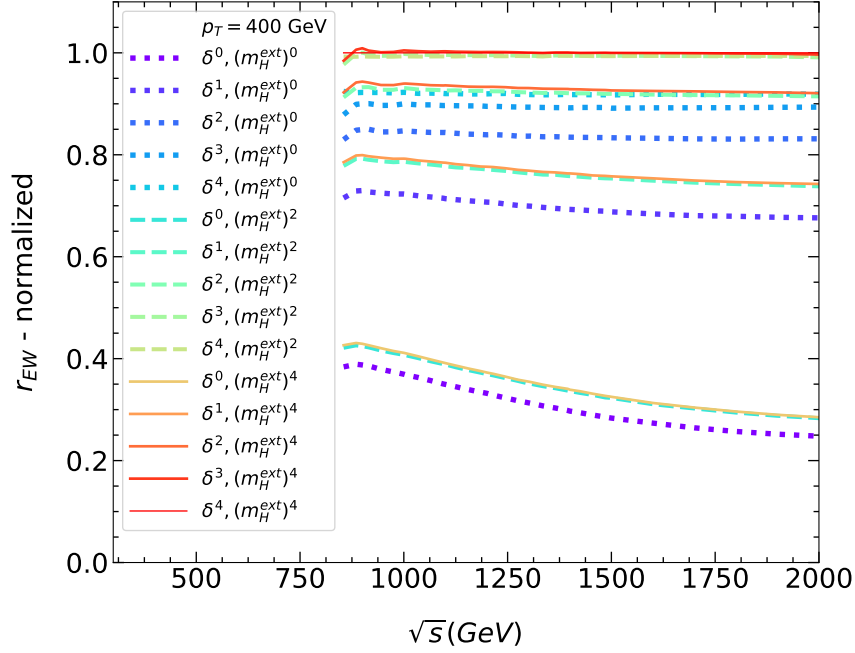


Figure 3. r_{EW} for various δ and m_H expansion terms normalized to highest available approximation for $p_T = 400$ GeV. For details concerning the meaning of the legend we refer to the caption of figure 2.

and δ^4 . Coefficients which are not available (i.e. $(m_H^{\text{ext}})^2\delta^4$ and $(m_H^{\text{ext}})^4\delta^4$) are set to zero. Their numerical effect is expected to be small since the quartic m_H^{ext} terms are, in general, of the order of 1% or smaller, see below for more details.

Our form factors are expressed in terms of the Mandelstam variables s , t and the masses for the top quark, gauge and Higgs bosons. For the numerical results discussed below, we use p_T and $\sqrt{s}$ as input and compute the value of t using

$$t = \frac{1}{2} \left(2m_H^2 - s + \sqrt{s} \sqrt{s - 4m_H^2 - 4p_T^2} \right), \quad (5.16)$$

which is then inserted in the analytic expression for the form factors. We hold one of the parameters $\sqrt{s}$ or p_T fixed and examine the dependence on the other. For a fixed transverse momentum, we choose the minimum value for $\sqrt{s}$ to be either the value dictated by the kinematic constraints or approximately $2m_t$. In the appendix we provide explicit numerical results for the one- and two-loop form factors for a few selected phase-space points.

In figures 2 and 3 we discuss the quality of the δ and m_H^{ext} expansions. In figure 2 we show results for $F_{\text{box}1}^{(0,1)}$ for $p_T = 400$ GeV. In the different panels we either vary the expansion depth in δ (top row) or m_H^{ext} (bottom row) and normalize the results in each case to the highest available approximation. We observe a good convergence of the δ expansion; in each step the distance to the next expansion order is reduced by more than a factor of two. Looking at the $(m_H^{\text{ext}})^0$ terms, where up to δ^4 terms are available, we observe a rather flat correction that induces a shift of approximately 7.5% for the δ^3 term, and approximately 3% for the δ^4 term. The missing term of order $\delta^5(m_H^{\text{ext}})^0$ can therefore be estimated to be around 1%–1.5%. A similar convergence pattern is observed for $F_{\text{box}2}^{(0,1)}$.

The convergence in m_H^{ext} is even faster, as can be seen from the panels in the bottom row of figure 2. Here we conclude that the quadratic terms in m_H^{ext} are of the order of 10% and the quartic terms are at the 1% level or below. The size of the missing terms in the m_H^{ext} expansion are therefore negligible compared to the missing terms in the δ expansion.

From the observed convergence pattern we can estimate the size of the missing m_H^{ext} terms at quartic order in δ . For the $\delta^4(m_H^{\text{ext}})^2$ [$\delta^4(m_H^{\text{ext}})^4$] terms we obtain $3\% \cdot 10\% = 0.3\%$ [$3\% \cdot 1\% = 0.03\%$] as compared to the known expansion terms. In our numerical analysis, we set these expansion terms to zero (see section 3 for details) at the point where we still use δ' (i.e. quadratic mass ratio) for the parametrization of the expansion in mass differences. Note that converting to the variable δ (i.e. linear mass ratio) introduces terms up to order δ^4 , which we keep. As we have shown above, dropping or keeping these terms only amounts to a small shift in our final numerical values. In order to account for the finite expansion orders of our result, we assign a conservative uncertainty estimate of $\pm 2\%$.

In figure 3 we test our assumption for r_{EW} . We choose again $p_T = 400$ GeV. The dotted (blueish), dashed (greenish) and solid (reddish) curves show the δ expansion in case no, quadratic, or quartic m_H^{ext} terms are included in the computation of r_{EW} . In each case we construct five approximations which differ in the expansion depth in δ . In total 15 different approximations are shown in figure 3. All results are normalized to the approximation which includes $\delta^4(m_H^{\text{ext}})^4$ terms.² We observe a similar convergence pattern as in figure 2. In particular, the dashed and solid curves, which correspond to the same δ expansion, almost lie on top of each other indicating that the quartic m_H^{ext} terms are below 1%. Furthermore, the curves which belong to $\delta^3(m_H^{\text{ext}})^2$, $\delta^4(m_H^{\text{ext}})^2$ and $\delta^3(m_H^{\text{ext}})^4$ are very close to reference line at “1”, which supports our estimate that the higher order expansion terms in δ and m_H^{ext} are below a $\pm 1\%$ correction.

We observe a very similar convergence pattern for all values of $p_T \gtrsim 300$ GeV. This suggests the assignment of a total uncertainty of $\pm 1\%$ to the squared NLO matrix element when it is used as input for the hadronic cross sections. This estimate is further backed by the sum acceleration introduced in section 5.2. We use sum acceleration for the expansion in δ , since it shows the slower overall convergence. We observe that $A_{\text{acc}}^{(0)}$, where no δ^3 terms enter, reproduces our nominal result including δ^3 terms within about 5%. Similarly, $A_{\text{acc}}^{(1)}$ reproduces the nominal result including δ^4 terms to order 1%. Furthermore, we observe that the difference between our nominal best approximation and $A_{\text{acc}}^{(2)}$ is below 1%, which again supports the conservative uncertainty estimate of 1% for r_{EW} .

Next we discuss the dependence on expansion depth in m_t . In figure 4 we show the real and imaginary parts of $F_{\text{box1}}^{(0,1)}$ for two different values of p_T , as a function of $\sqrt{s}$. In each case we show two curves and their associated uncertainty bands, which are all based on expansions up to δ^4 and m_H^2 but differ in the highest power in m_t used for the construction of the Padé-improved result, as indicated in the legend. We observe sizeable uncertainty bands for the approximations containing at most m_t^{32} terms, in particular for the imaginary part.

²Note that this approximation contains all expansion terms δ^n and $(m_H^{\text{ext}})^m$ with $n \leq 3$ and $m \leq 4$, and the $\delta^4(m_H^{\text{ext}})^0$ term. However, the (numerically small) expansion terms $\delta^4(m_H^{\text{ext}})^2$ and $\delta^4(m_H^{\text{ext}})^4$ are set to zero, as described above.

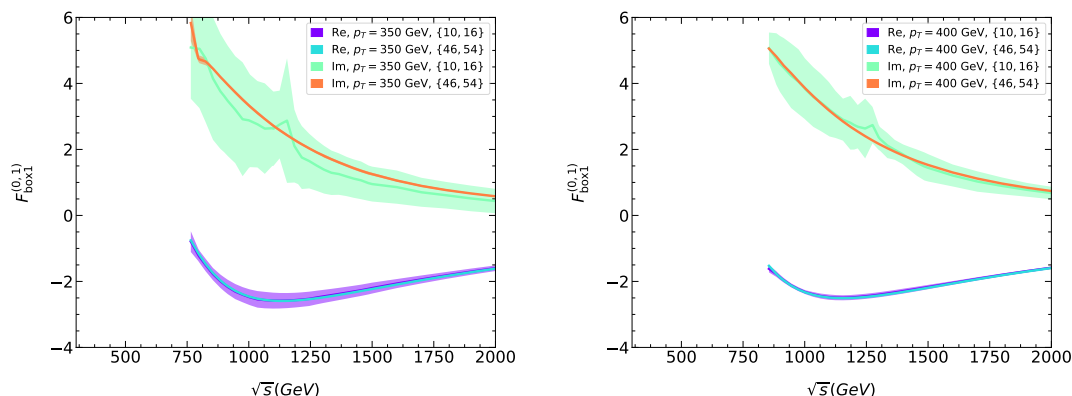


Figure 4. Real and imaginary parts of $F_{\text{box1}}^{(0,1)}$ for $p_T = 350$ GeV and $p_T = 400$ GeV as a function of $\sqrt{s}$ for different expansion depth in m_t used for the Padé approximation, as indicated in the legend: $\{n, m\}$ means that for the construction of the Padé approximation at most expansions up to $(m_t^2)^m$ and at least up to $(m_t^2)^n$ are taken into account.

The uncertainty is reduced substantially once we allow terms up to m_t^{108} terms. The latter is completely contained within the uncertainty band of the lowest approximation.

Here we note that for some phase-space points, which correspond to certain values of t , the m_t expansion does not converge in such a way that the Padé-based approximation is unstable. In these cases we use the $t \leftrightarrow u$ symmetry of the amplitude to provide an alternative, stable, evaluation. In practice, we also calculate all results with the variables t and u interchanged. For almost all generated data points, we observe perfect agreement with the results obtained without exploiting the t – u symmetry. In the remaining cases, one of the results is clearly erroneous, manifesting as a distinct, spike-like deviation from the smooth curve which comes with a huge Padé uncertainty.

In figures 5 and 6 we show NLO form factors $F_{\text{tri1}}^{(0,1)}$, $F_{\text{box1}}^{(0,1)}$ and $F_{\text{box2}}^{(0,1)}$ for fixed p_T or fixed $\sqrt{s}$, respectively. In all cases we show the highest available approximation which includes δ^4 and $(m_H^{\text{ext}})^4$ terms, together with the corresponding uncertainty band from the Padé procedure.³ This band is only visible for $p_T \lesssim 350$ GeV and only for lower values of $\sqrt{s}$ as can be seen in figure 5. It is, in general, more pronounced for the imaginary parts than for the real parts. From figure 6 we observe that for $p_T \gtrsim 350$ GeV the uncertainty induced by the Padé approximation is well below the uncertainty due to the truncated expansion in δ and m_H .

In figure 7, we show the ratio r_{EW} for fixed values of p_T as a function of $\sqrt{s}$ which we extend up to 10 TeV. For each curve we use the highest available approximation and show, besides the central value, also the uncertainty bands as obtained from the Padé approximation. These are only visible for $p_T \lesssim 300$ GeV. For $p_T \geq 350$ GeV we obtain numerically stable results. The result for r_{EW} shows a more stable behaviour than for the individual (box) form factors. This can be explained by the dominance of the triangle form factors for smaller value of $\sqrt{s}$. For larger $\sqrt{s}$ the approximations for the box form factors are stable for all p_T values shown in figure 7. The partonic quantity r_{EW} is negative and of the order of -10% , which is in agreement with the observations from ref. [9] for the hadronic invariant mass or p_T distribution.

³The estimated $\pm 2\%$ uncertainty from the truncation in δ and m_H^{ext} is not shown.

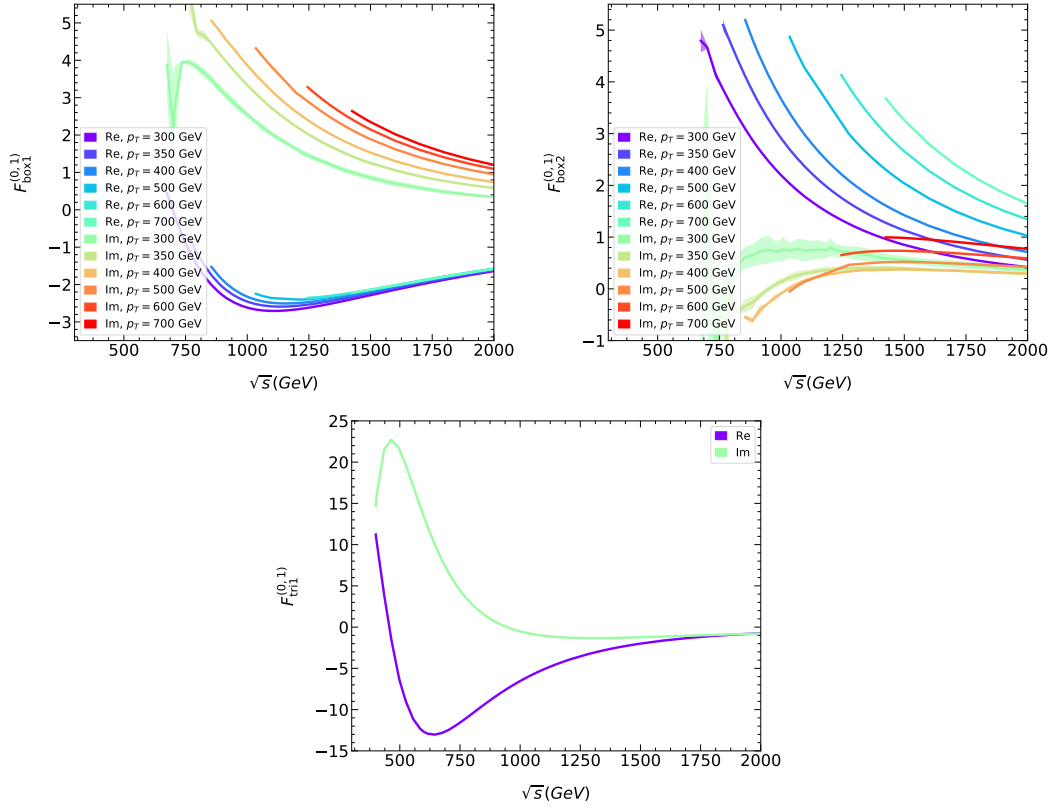


Figure 5. $F_{\text{box}1}$, $F_{\text{box}2}$ and $F_{\text{tri}1}$ as a function of $\sqrt{s}$ for fixed p_T . Note that the triangle form factor is independent of p_T .

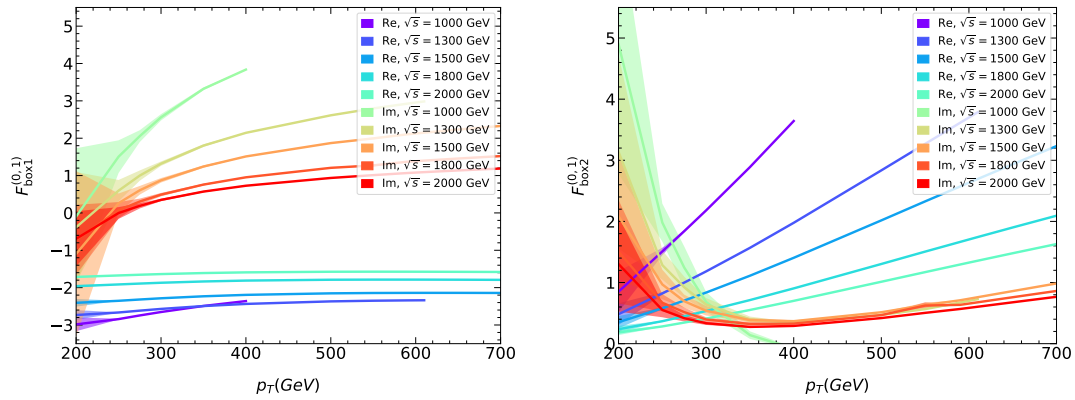


Figure 6. $F_{\text{box}1}$ and $F_{\text{box}2}$ as a function of p_T for fixed $\sqrt{s}$.

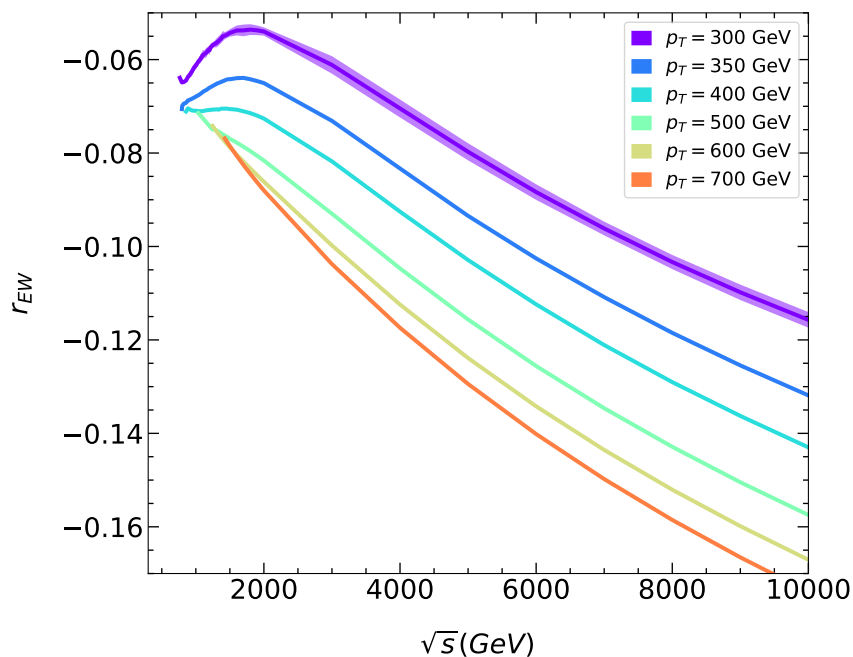


Figure 7. r_{EW} for various values of p_T as a function of $\sqrt{s}$.

6 Conclusions and outlook

In this work, we compute NLO electroweak corrections to Higgs boson pair production in gluon fusion. We consider the top quark-induced contributions in the high-energy limit. Since we include all sectors of the Standard Model, several mass scales are involved in the computation. We identify certain hierarchies that allow us to perform nested expansions in several small parameters. In particular, we assume that the mass of the Higgs boson in the final state is small as compared to the top quark mass. Furthermore, we expand in mass differences in case several distinct masses appear inside the loop diagrams. Most importantly, we perform a deep expansion with more than 100 expansion terms around the high-energy limit. The calculation of the electroweak corrections is much more challenging than in the QCD case; consequently, the intermediate and final expressions are considerably larger.

From the convergence properties of our results we estimate that the truncation uncertainty, dominated by the mass-difference expansion, is about $\pm 2\%$ of the two-loop corrections, which is well below the scale uncertainty from NLO QCD corrections. A systematic improvement is possible by computing more terms in the δ and m_H^{ext} expansion, which might be a challenging task from the computational point of view.

In the limiting case of massless external Higgs bosons and small top-quark mass, the leading high-energy expansion of the two-loop form factors starts at $(m_t^2/s)^0$, while at one loop the leading term starts at $(m_t^2/s)^1$. At NLO we observe a quadratic and cubic logarithm as the leading contribution for $F_{\text{box}1}$ and $F_{\text{box}2}$, respectively.

Our results are explicit in all input parameters, allowing for a flexible use of the form factors and straightforward numerical evaluation. The results obtained in this paper cover the phase space for $p_T \gtrsim 350 \text{ GeV}$ and lead to corrections of the order of -10% at the

partonic level in the high-energy limit. The high-energy results serve as cross check for other calculations, e.g., those based on numerical approaches (see, e.g., refs. [9, 12]). Furthermore, it is expected that in combination with the forward limit expansion, the entire phase space can be covered as in the case of QCD (see, e.g., ref. [31]).

Low-order expansions of our results are available from [52], and deep expansions are available upon request. Our results will be made public in the context of `ggxy` [31], which provides a framework to compute hadronic cross sections and distributions for gluon-induced processes.

Acknowledgments

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A Sample numerical results for the form factors

For convenience we provide in this appendix results for the form factors for a few representative phase-space points. In table 1, we present numerical results for one- and two-loop form factors, as defined in eqs. (2.1) and (2.7). We use the value for $\sqrt{s}$ and p_T given in the table together with the numerical values of input parameters are reported in eq. (5.12). We provide further detailed benchmark results by splitting the bare two-loop and counterterm contributions in the ancillary files to this paper [52].

$\sqrt{s}$ [GeV]	p_T [GeV]	$F_{\text{tri1}}^{(0)}$	$F_{\text{box1}}^{(0)}$	$F_{\text{box2}}^{(0)}$	$F_{\text{tri1}}^{(0,1)}$	$F_{\text{box1}}^{(0,1)}$	$F_{\text{box2}}^{(0,1)}$
1000	350	0.0062605 +0.054356 i	0.14270 −0.39457 i	−0.025741 −0.17797 i	−6.53488 −0.51315 i	−2.45552(60) +3.311(33) i	2.9101(27) +0.195(56) i
1000	400	0.0062605 +0.054356 i	0.15179 −0.37594 i	−0.028158 −0.21204 i	−6.53488 −0.51315 i	−2.32639(15) +3.8610(70) i	3.6713(34) −0.064(23) i
2000	350	−0.0016832 +0.0052336 i	0.056402 −0.074252 i	−0.00085560 −0.040586 i	−0.73629 −0.77771 i	−1.61342(72) +0.571(14) i	0.5623(23) +0.299(17) i
2000	400	−0.0016832 +0.0052336 i	0.057281 −0.068858 i	−0.00067097 −0.047603 i	−0.73629 −0.77771 i	−1.5945(10) +0.7359(60) i	0.71184(40) +0.3011(74) i
2000	500	−0.0016832 +0.0052336 i	0.058756 −0.059640 i	−0.000070987 −0.060279 i	−0.73629 −0.77771 i	−1.57687(5) +0.9467(13) i	1.02555(3) +0.4226(11) i
2000	600	−0.0016832 +0.0052336 i	0.059907 −0.052344 i	0.00071520 −0.071229 i	−0.73629 −0.77771 i	−1.57578 +1.09237 i	1.34029 +0.58935 i
2000	700	−0.0016832 +0.0052336 i	0.060806 −0.046603 i	0.0015758 −0.080752 i	−0.73629 −0.77771 i	−1.58221 +1.20077 i	1.64503 +0.77052 i

Table 1. Sample numerical results for one- and two-loop form factors. The results correspond to on-shell renormalization in the G_μ scheme, see also eq. (5.13). For the Padé approximation, we choose $\{N_{\text{low}}, N_{\text{high}}\} = \{45, 54\}$. The uncertainties displayed in brackets, where visible, are from the Padé procedure. We assign an additional $\pm 2\%$ uncertainty to the two-loop form factors to account for the truncation of the δ and m_H^{ext} expansions, see the discussion at the end of section 5.3.

Data Availability Statement. This article has associated data in a data repository. The data repository link is <https://www.ttp.kit.edu/preprints/2026/ttp26-006>.

Code Availability Statement. This article has no associated code or the code will not be deposited.

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