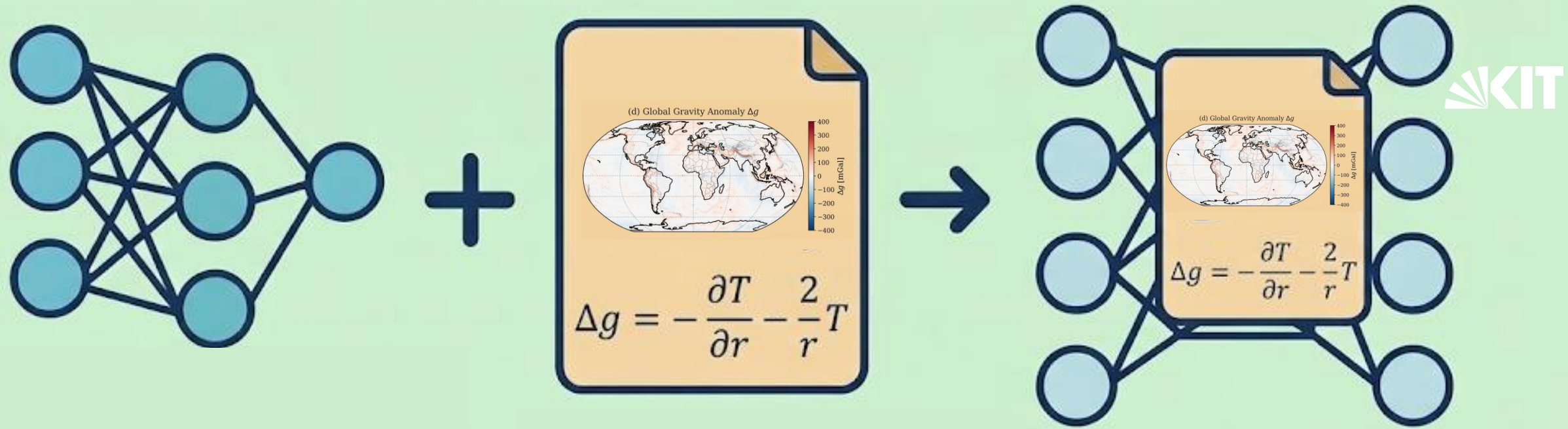




(Karmarkar, 2025)

Physics-Informed Neural Networks for Regional Gravity Field Modelling: A Case Study Based on Synthetic Data

Marie Freund, Charlotte Gschwind, Thomas Grombein, Hansjörg Kutterer

Geodetic Institute Karlsruhe

Problem Statement and Motivation

Trade-offs in Gravity Field Modelling

- **Physical Reference Surface:** Essential for physical height determination ($H = h - N$)
- **Modern Geoid Requirements:** Demands seamless spatial continuity, high resolution and strict physical field consistency

Classic Geoid Determination

- Strictly satisfies fundamental potential field equations
- Sensitive to irregularly distributed, noisy and incomplete data
- Reaches computational limits with very large datasets

(Torge et al., 2012)

Pure Data-Driven Machine Learning (MLPs)

- Capable of highly flexible function approximation and capturing complex spatial patterns
- Operates as a black box without incorporating prior physical constraints
- Prone to overfitting and generating physically implausible artefacts in data gaps

(Goodfellow et al., 2016)

Challenge: How can we leverage the enormous flexibility of neural networks without sacrificing the physical integrity of potential theory?

Methodology and PINN Framework

Concept of Physics-Informed Neural Networks

Paradigm Shift in Deep Learning

- Classical ML: purely fitting the measurement points → Black Box

$$\mathcal{L}_{\text{total}} = \mathcal{L}_T = \frac{1}{N} \sum_{i=1}^N (\hat{T}^{(i)} - T^{(i)})^2$$

- PINN Framework: Integration of prior physical knowledge directly into the learning process (*Raissi et al., 2019*)

How It Works

- The network functions as a continuous function approximator for the disturbing potential $\hat{T}(\mathbf{x})$ with $\mathbf{x} = (\lambda, \phi, h)^T$
- No restriction to fixed computational grids through the use of Automatic Differentiation (AD) (*Baydin et al., 2018*)

$$\mathcal{L}_{\text{total}} = w_T \mathcal{L}_T + w_{\Delta g} \mathcal{L}_{\Delta g}$$

Linearized boundary condition in spherical approximation:

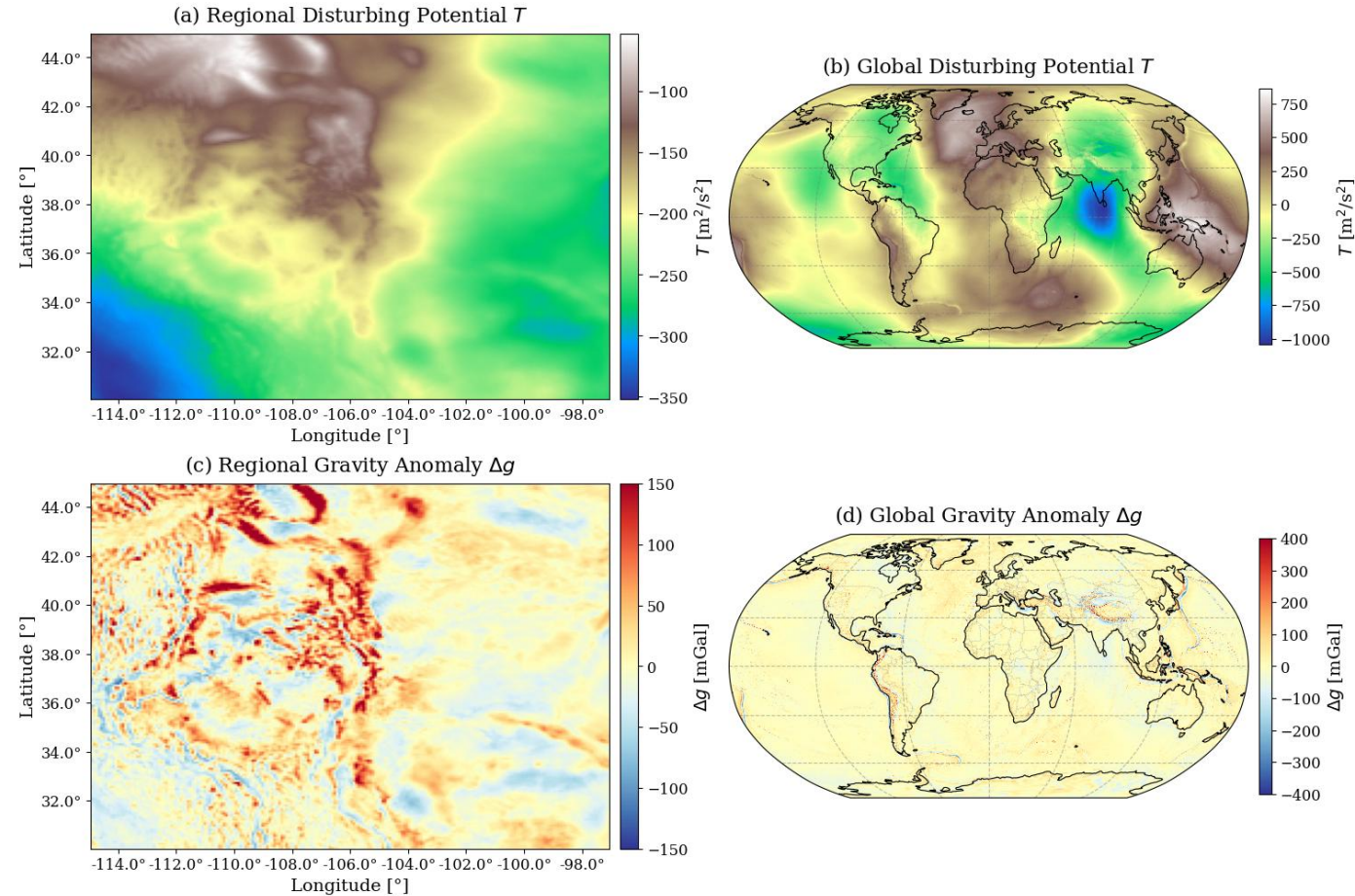
$$\Delta g = -\frac{\partial T}{\partial r} - \frac{2}{r}T \quad \Rightarrow \quad \mathcal{L}_{\Delta g} = \frac{1}{N_d} \sum_{i=1}^{N_d} \left| \Delta g_i - \left(-\frac{\partial \hat{T}}{\partial r} - \frac{2}{r} \hat{T} \right)_i \right|^2$$

Methodology and PINN Framework

Data Basis and Experimental Setup

Data and Study Area

- Regional coverage: Synthetic grid values of Δg and T based on the EIGEN-6C4 model with $N_{max} = 2190$
- Input Features: Three-dimensional geocentric spatial coordinates (λ, θ, h) with normalized height $h = r - R$ and Δg
- Output: Disturbing potential $\hat{T}$
- Area: Colorado Field, wide range of terrain characteristics, high-density dataset available for the transition to real-world data

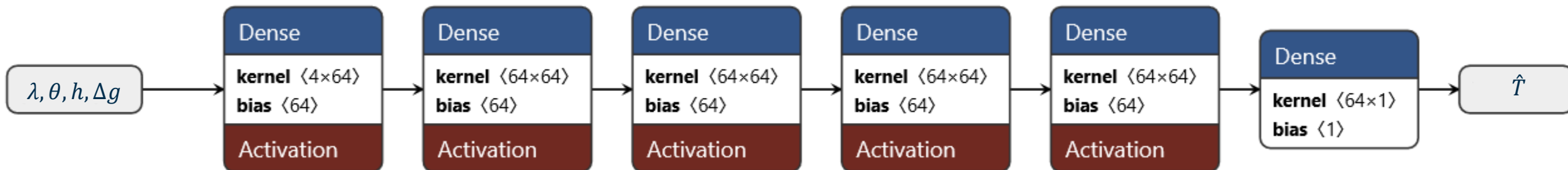


Methodology and PINN Framework

Data Basis and Experimental Setup

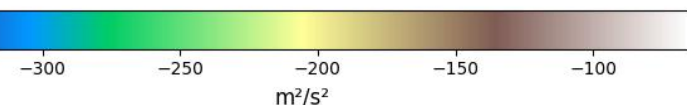
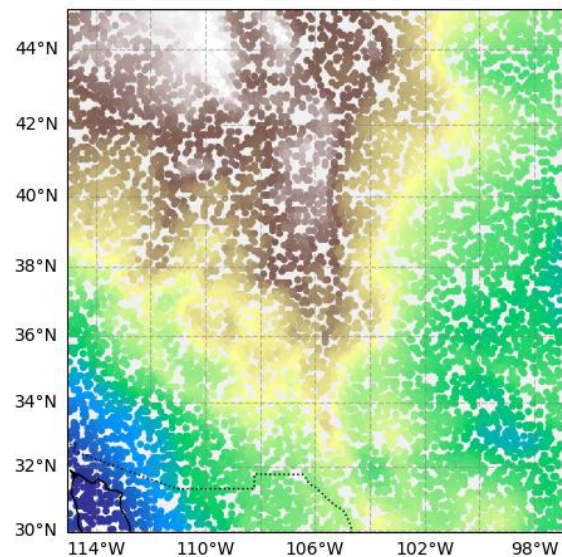
Network Architecture and Training Strategy

- Architecture: Multilayer Perceptron (MLP)
- Input and Output: 4 input features and 1 target variable
- Five Hidden Layers with 64 neurons each
- Activation function: continuously differentiable tanh function applied after every hidden layer

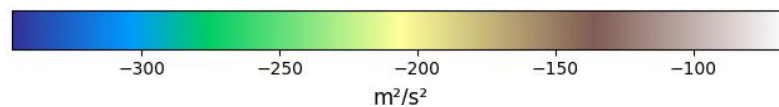
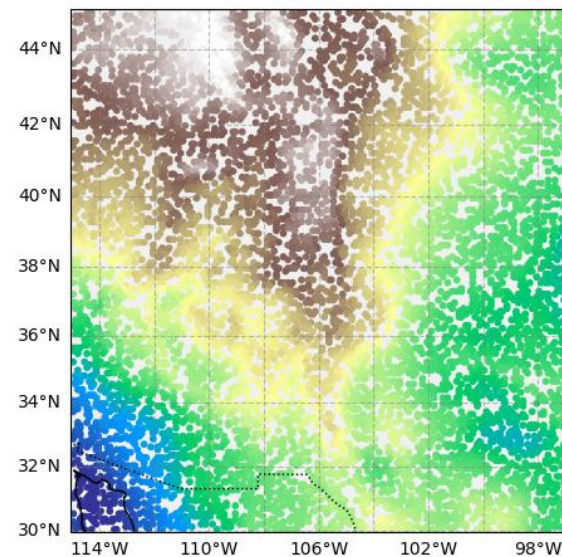
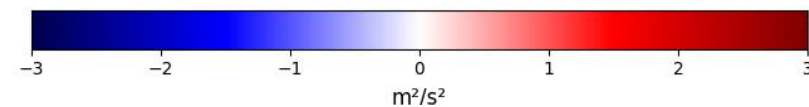
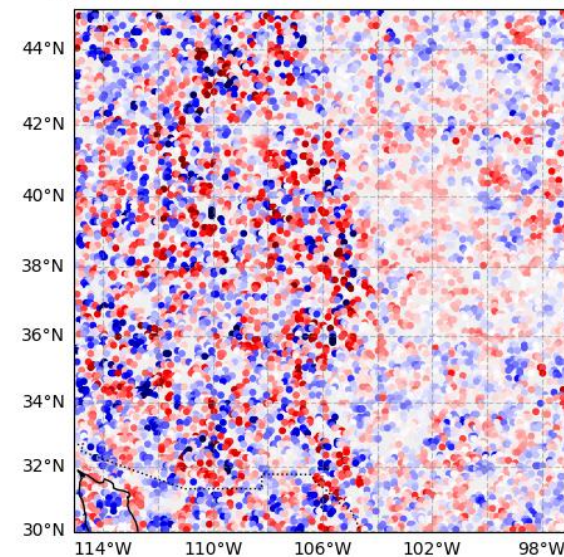


MLP vs. PINN

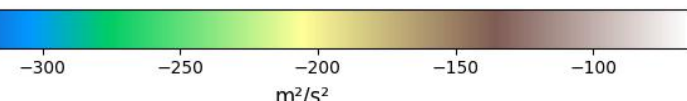
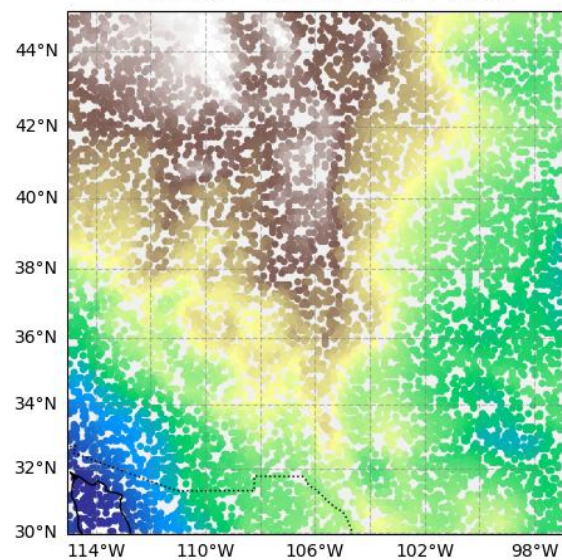
Colorado - Ground Truth T (Test)



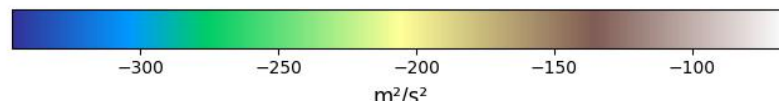
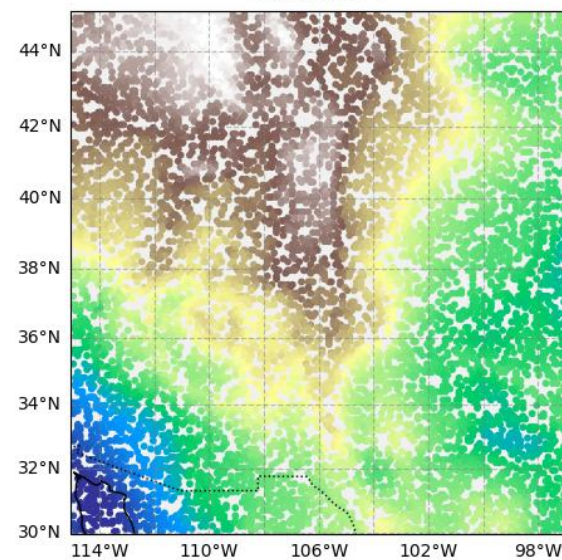
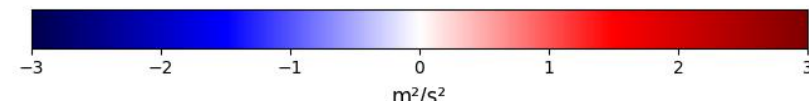
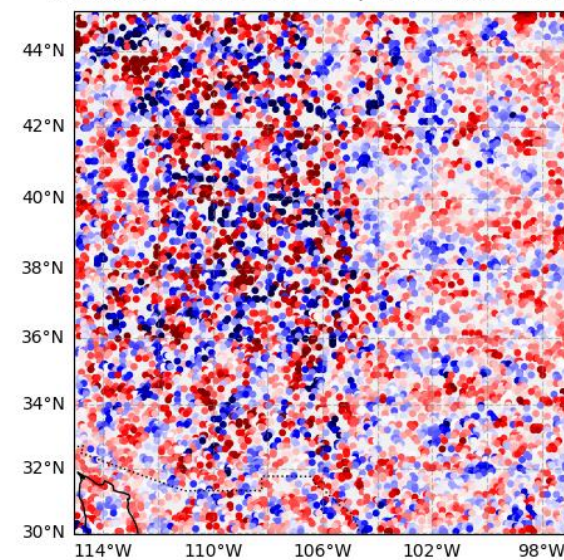
PINN Pred T

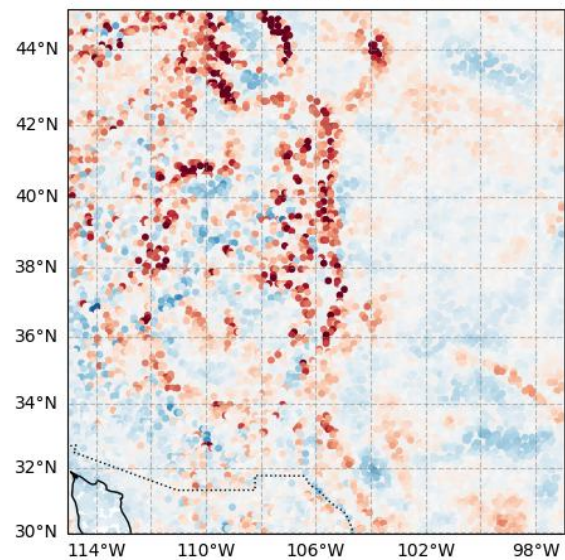
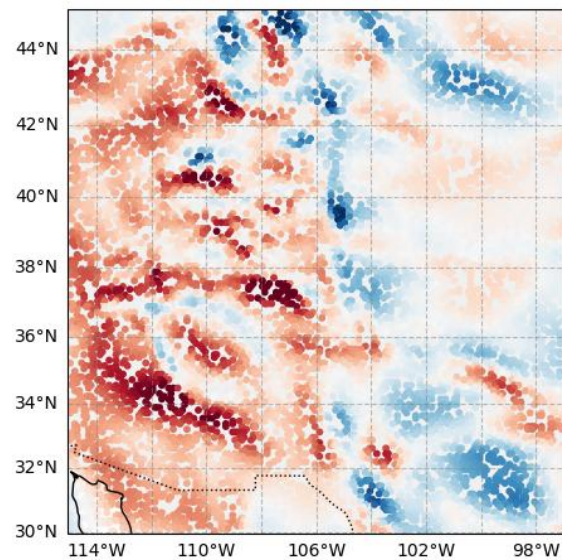
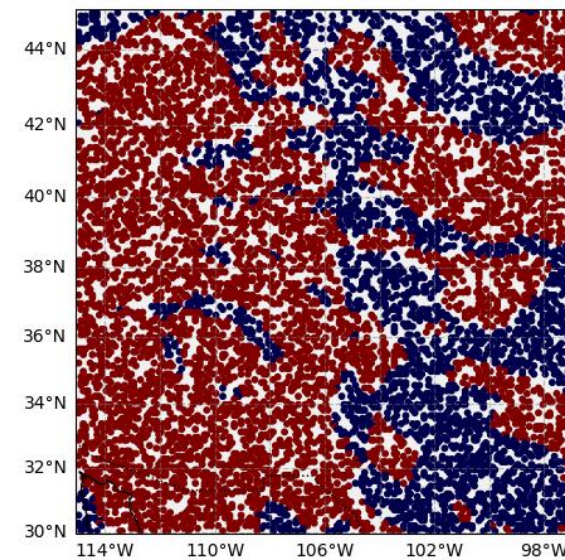
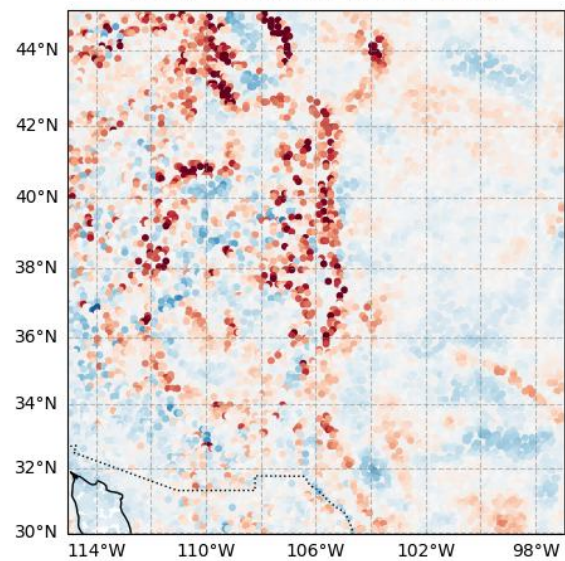
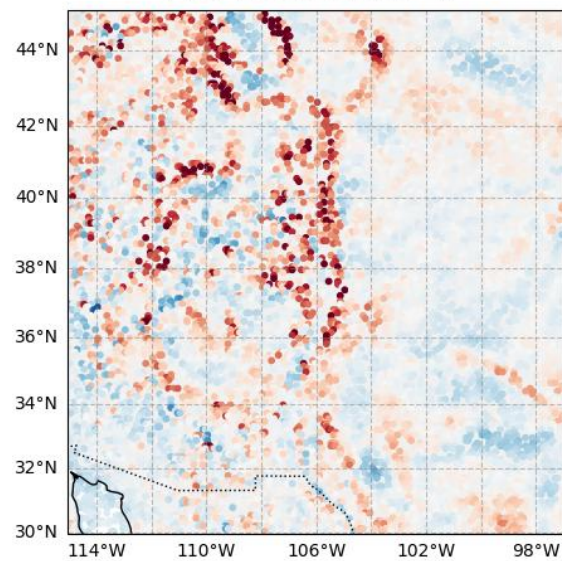
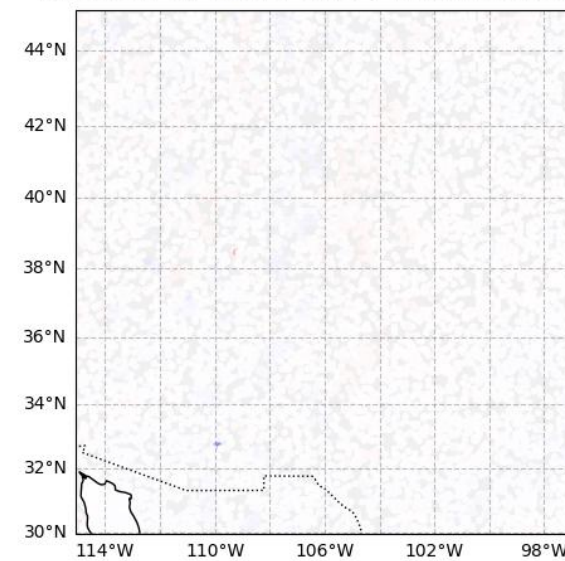
Residuals T (Scale: ± 3.0 | Test-RMSE: 0.867)

Colorado - Ground Truth T (Test)



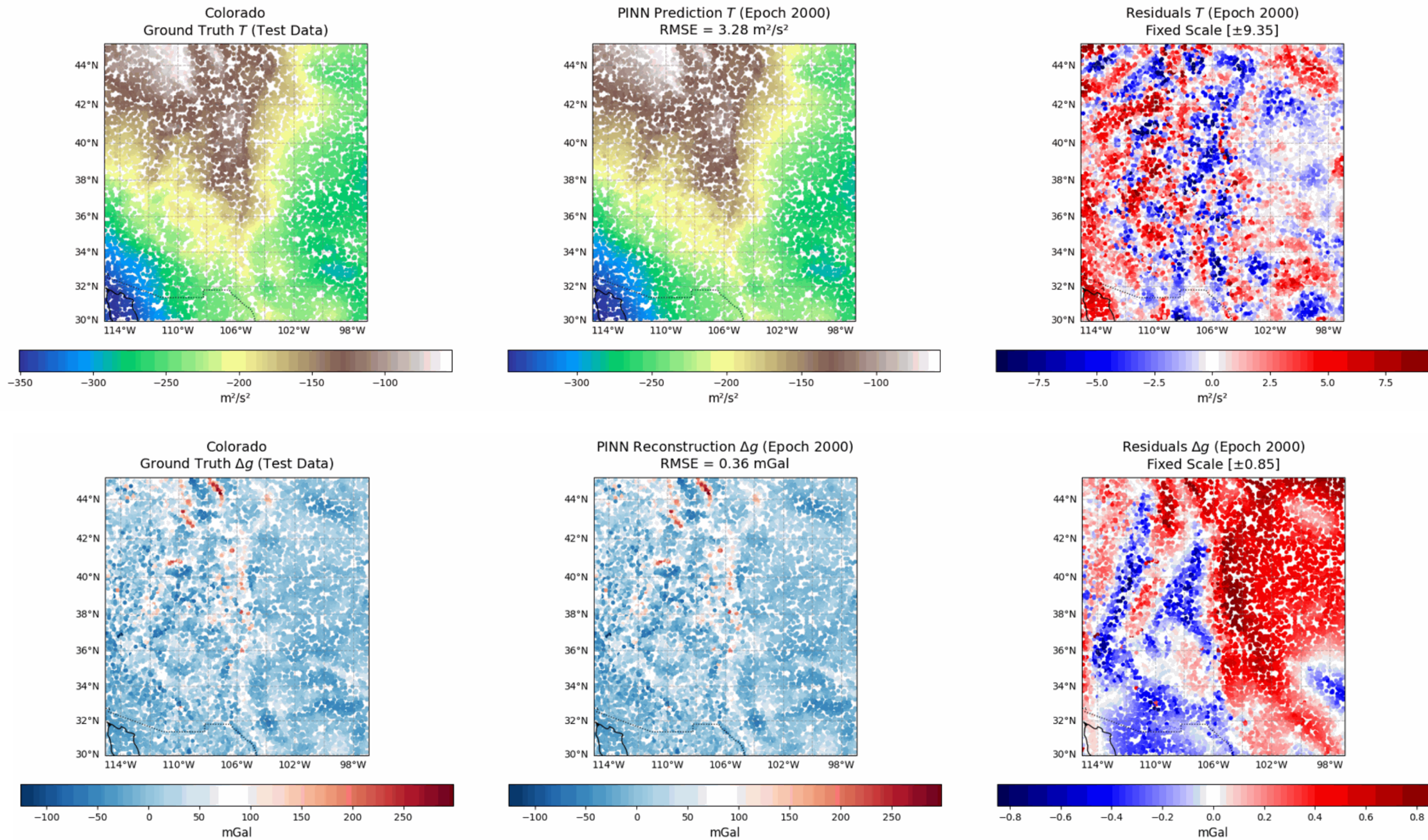
PINN Pred T

Residuals T (Scale: ± 3.0 | Test-RMSE: 1.441)

Noisy Test-Input Δg (0.0 mGal)PINN Reconstruction Δg Residuals Δg (Scale: ± 5.0 | Test-RMSE: 6340.945)Noisy Test-Input Δg (0.0 mGal)PINN Reconstruction Δg Residuals Δg (Scale: ± 5.0 | Test-RMSE: 0.035)

Evolution of Training Over Epochs

Result



Results and Evaluation

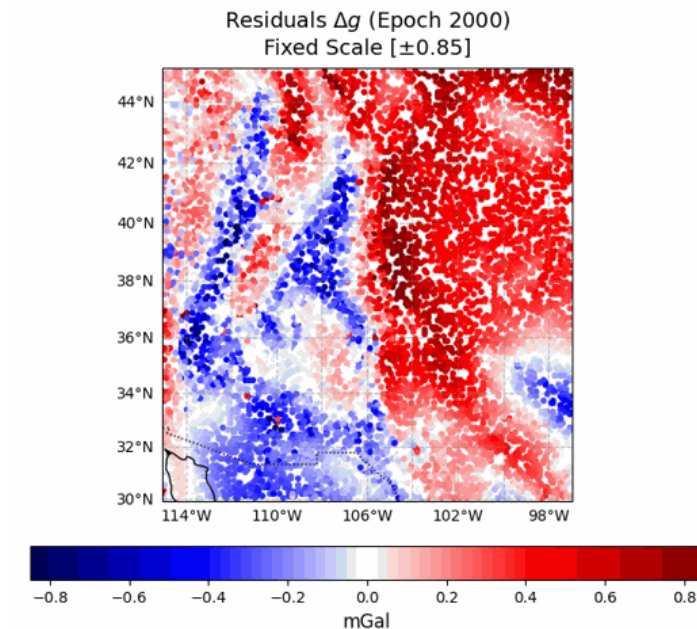
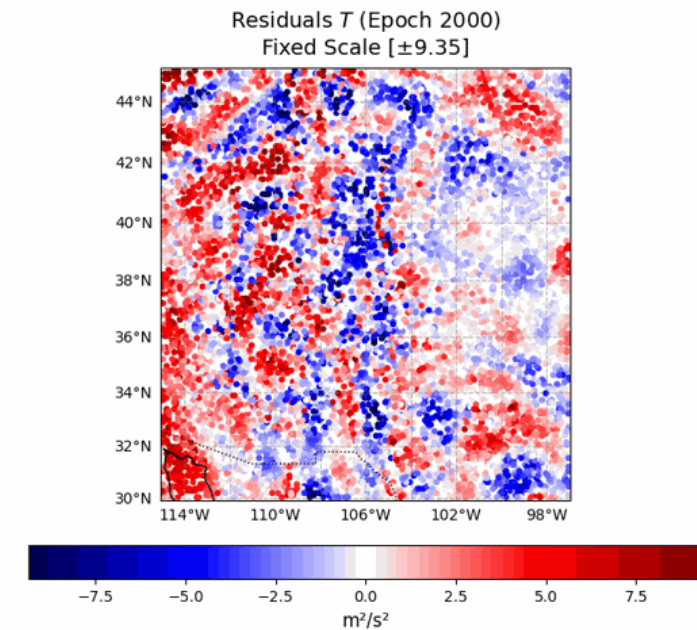
Evolution of Training Over Epochs

Coarse-to-Fine Learning Behavior

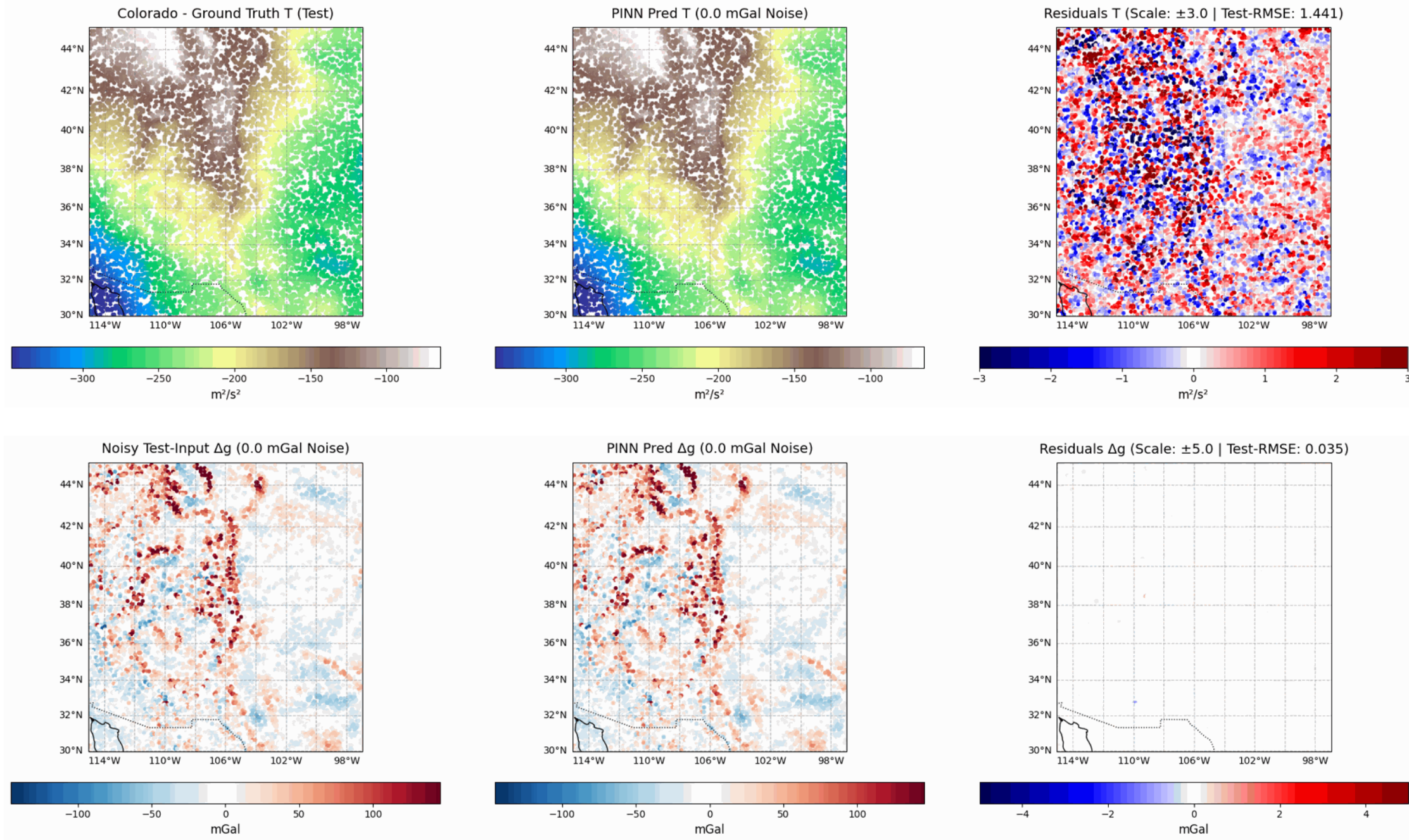
- Early Stage: Captures large-scale, long-wavelength structures, smoothed predictions
- Intermediate Stage: Sharpens gradients and resolves high-frequency topographic details
- Final Convergence: Homogeneous, unstructured residual noise centered around zero

RMSE Trajectory:

- Disturbance Potential $\hat{T}$: $3.26 \rightarrow 1.40 \text{ m}^2/\text{s}^2$
- Gravity Anomaly Δg : $0.36 \rightarrow 0.03 \text{ mGal}$



Robustness Analysis Under Noisy Data



Results and Evaluation

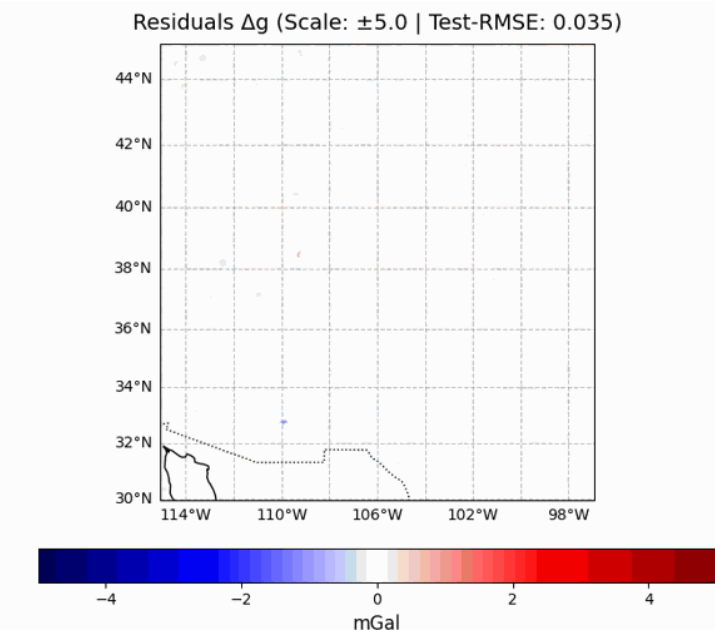
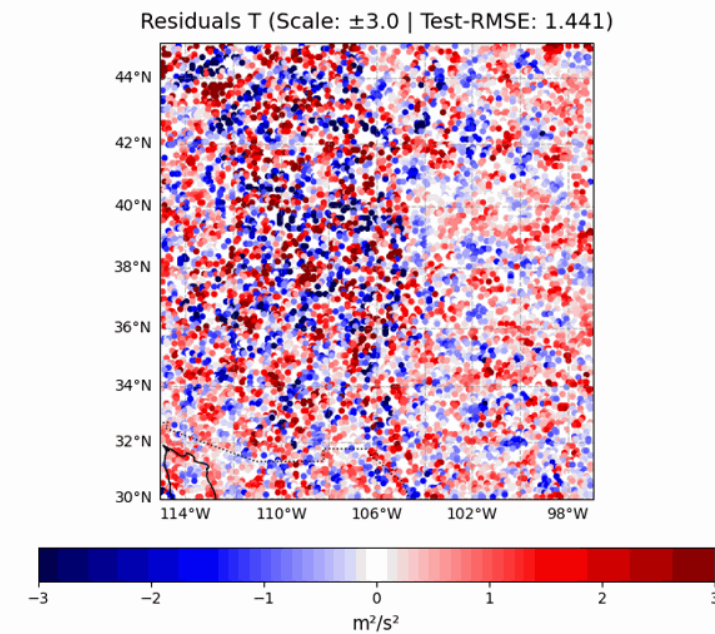
Robustness Analysis Under Noisy Data

Disturbing Potential ($\hat{T}$)

- High Stability: Consistently robust reconstruction across all noise levels with minor RMSE increase
- Protected by the integrating nature of potential theory and physical constraints

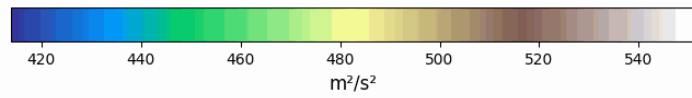
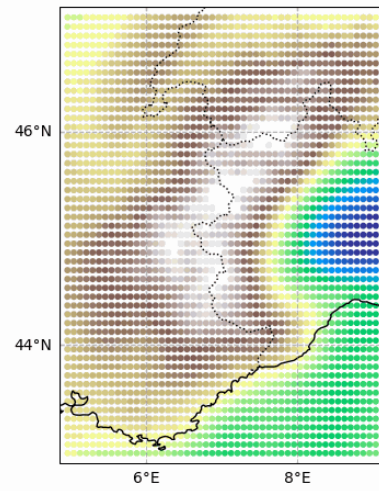
Gravity Anomaly ($\Delta\hat{g}$)

- Linear Error Growth
- High-frequency noise degrades derivative accuracy more strongly than integrated quantities
- **Physical regularization provides strong noise immunity for $\hat{T}$, while spatial derivatives (Δg) remain sensitive to input noise levels.**

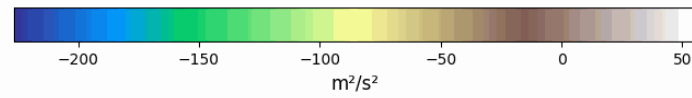
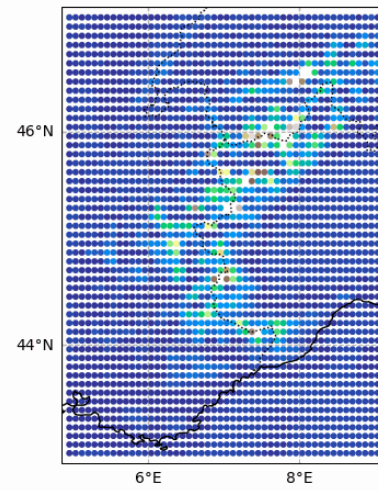


Performance Evaluation Outside the Training Area

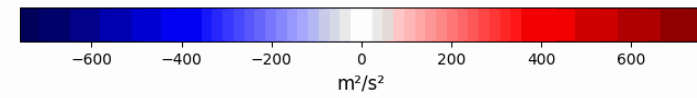
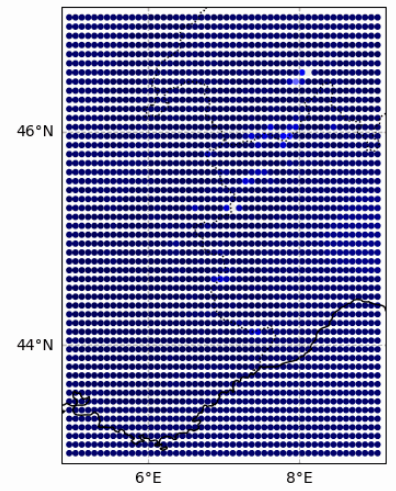
Alpine Region (Extreme Mountain Terrain) - Ground Truth T



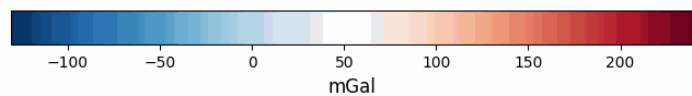
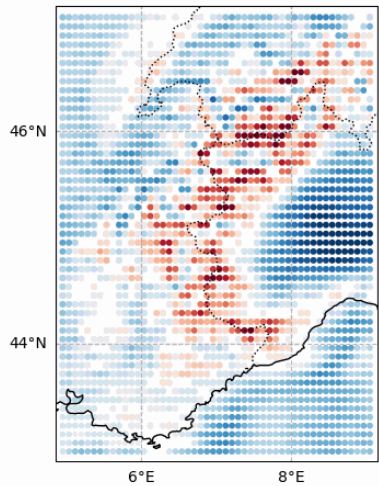
PINN Prediction T



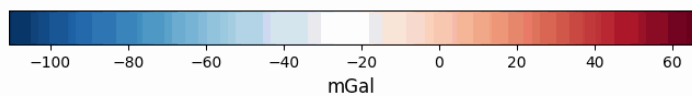
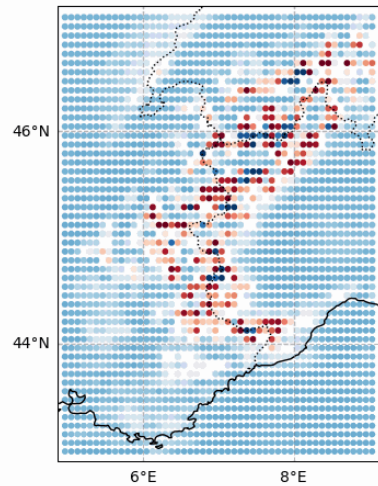
Residuals (Pred - True)



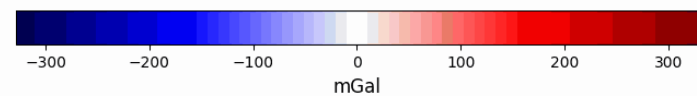
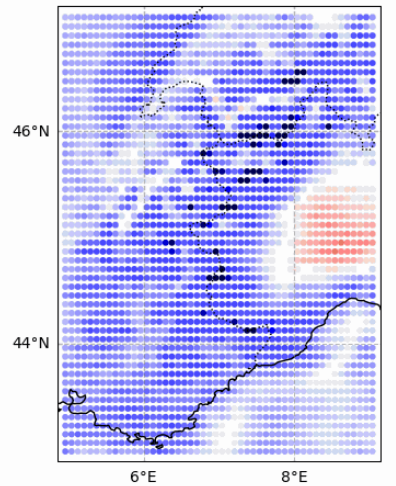
Alpine Region (Extreme Mountain Terrain) - Ground Truth Δg



PINN Derived Δg



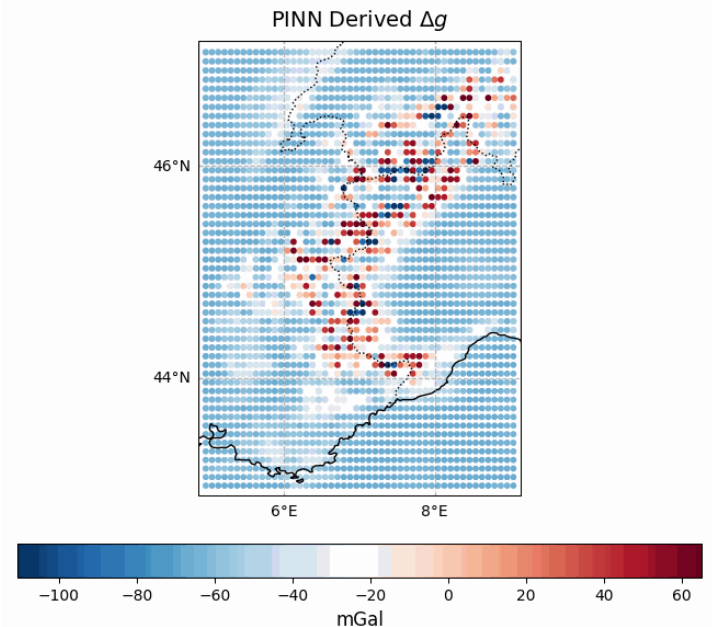
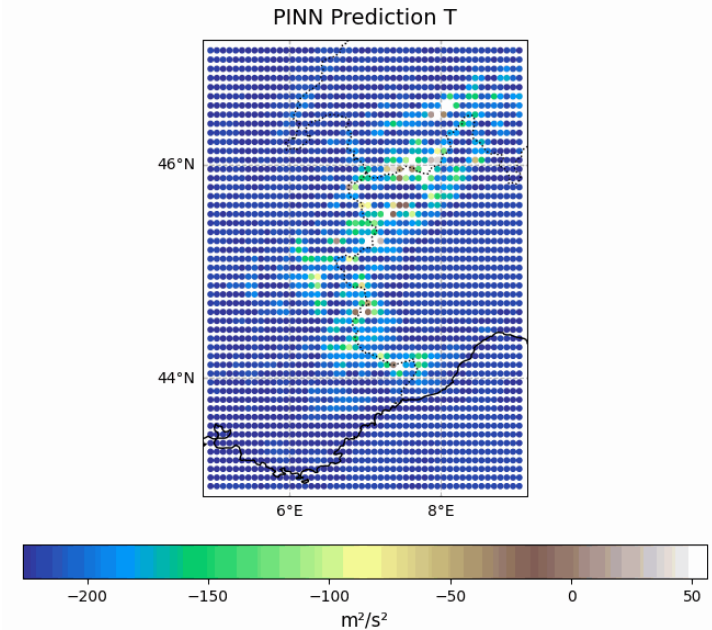
Residuals (Pred - True)



Results and Evaluation

Performance Evaluation Outside the Training Area

- **Setup & Objective:**
 - Evaluation outside Colorado using synthetic data
 - Comparison across 4 diverse geological regimes (Flatland, Extreme Terrain, Mid-Mountains, Ocean)
- **Disturbance Potential ($\hat{T}$):** Reconstruction fails in external regions due to strong coordinate reliance and severe spatial overfitting to Colorado
- **Gravity Anomaly ($\Delta\hat{g}$):** Automatic differentiation ($\partial T/\partial h$) amplifies coordinate artifacts, resulting in extreme, unphysical residual gradients
- **Core Limitation:** Strongly bound to the training regime reliable global extrapolation requires a broader representation of geological settings



Conclusion and Outlook

Conclusion and Discussion

- **Physical Consolidation:** Long training phase (10k epochs) required to balance data fitting and physical PDE constraints
- **Coordinate Dependency:** Strong reliance on absolute spatial coordinates, that leads to regional overfitting and limiting generalizability
- **Model Limitations:** High-frequency topographic details still leave minor residuals, showing the boundaries of the 5-layer/64-neuron architecture
- **Higher Computational Cost:** The PINN training takes roughly three times as long as the MLP training

Conclusion and Outlook

Outlook and Future Work

- **Real-World Application:** Transitioning from synthetic data to noisy, real-world measurements
- **Spatial Generalization:** Implementing relative coordinate representations and multi-region training to eliminate absolute spatial dependency.
- **Input Expansion:** Incorporating additional physical features (e.g., Digital Elevation Model/DEM), to better capture complex topographic effects
- **Architectural Optimization:** Investigating multi-scale PINNs or adaptive activation functions to accelerate convergence and increase resolution

Literature

- Baydin, Atilim Gunes, Barak A. Pearlmutter, Alexey Andreyevich Radul und Jeffrey Mark Siskind (5th February 2018). *Automatic differentiation in machine learning: a survey*. doi: 10.48550/arXiv.1502.05767. arXiv: 1502.05767[cs.SC]. url: <http://arxiv.org/abs/1502.05767> (accessed on 17 July 2026)
- Farea, Amer, Olli Yli-Harja und Frank Emmert-Streib (29th August 2024). “Understanding Physics-Informed Neural Networks: Techniques, Applications, Trends, and Challenges”. In: AI 5.3, S. 1534–1557. issn: 2673-2688. doi: 10.3390/ai5030074. url: <https://www.mdpi.com/2673-2688/5/3/74> (accessed on 23 February 2026)
- Goodfellow, Ian, Yoshua Bengio und Aaron Courville (2016). *Deep Learning*. MIT Press. url: <https://www.deeplearningbook.org/> (accessed on 05 May 2026)
- Jiang, Tao, Zejie Tu und Jiancheng Li (17th January 2026). “Physics-informed neural networks for geoid modeling”. In: *Journal of Geodesy* 100.1, S. 4. issn: 1432-1394. doi: 10.1007/s00190-025-02017-6. url: <https://doi.org/10.1007/s00190-025-02017-6> (accessed on 26 January 2026)
- Karmarkar, Vivek (7th July 2025). A Short Introduction to Physics-informed Neural Networks (PINNs). Medium. url: <https://medium.com/@vivek-karmarkar/a-short-introduction-to-physics-informed-neural-networks-pinns-cd342f5a3c5e> (accessed on 13 September 2026).
- Karniadakis, George Em, Yannis Kevrekidis, Lu Lu und Paris Perdikaris (May 2021). “Physics-informed machine learning”. In: *Nature Review Physics*. doi: 10.1038/s42254-021-00314-5. url: https://www.researchgate.net/publication/351814752_Physics-informed_machine_learning (accessed on 14 July 2026).
- Raissi, Maziar, Paris Perdikaris und George Em Karniadakis (1st February 2019). “Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations”. In: *Journal of Computational Physics* 378, S. 686–707. issn: 0021-9991. doi: 10.1016/j.jcp.2018.10.045. url: <https://www.sciencedirect.com/science/article/pii/S0021999118307125> (accessed on 27 January 2026).
- Torge, Wolfgang, Jürgen Müller und Roland Pail (2023). *Geodesy*. Berlin, Boston: De Gruyter Oldenbourg. isbn: 978-3-11-072330-4. doi: 10.1515/9783110723304. url: <https://doi.org/10.1515/9783110723304> (accessed on 26 January 2026)

Thank you for your attention!



Marie Freund

marie.freund@student.kit.edu