

Analytical solution and numerical validation of long-term evolution of pore pressure in anisotropic porous media with special emphasis on boundary effects

Tatia Sharia ^a,* , Thomas Niederhuber ^b, Birgit Müller ^b, Andreas Rietbrock ^a

^a Geophysical Institute (GPI), Karlsruhe Institute of Technology (KIT), Hertzstraße 16, Karlsruhe, 76187, Germany

^b Institute of Applied Geosciences, Karlsruhe Institute of Technology (KIT), Adenauerring 20b, Karlsruhe, 76131, Germany

ARTICLE INFO

Editors-in-Chief:

Professor Lyesse Laloui and Professor Tomasz Hueckel

Keywords:

Long-term fluid injection
Pore pressure evolution
Infinite elements
3D FEM modelling
Analytical solution
Hydraulic anisotropy

ABSTRACT

Accurate prediction of pore pressure evolution during fluid injection is essential for assessing reservoir performance and associated effective stress changes that may promote induced seismicity. Analytical solutions for pressure diffusion commonly assume isotropic permeability, whereas long-term numerical simulations often require large computational domains to reduce boundary effects. We investigate injection-induced pressure evolution over timescales comparable to geothermal project lifespans under hydraulic anisotropy, using analytical and numerical methods with finite and infinite elements. We derive a closed-form three-dimensional analytical solution for continuous point-source injection in a medium with permeability anisotropy. Our closed-form analytical solution shows that the permeability component along a given axis governs the temporal pressure evolution, while the perpendicular components control the pressure amplitude. To identify limitations of numerical modelling for long-term scenarios, we compare analytical and numerical results for isotropic permeability. Boundary effects in conventional finite element models manifest earlier in materials with higher permeability. For an 8 km cubic Berea sandstone model, deviations increase from below 1.4% after 0.1 years to more than 40% after 10 years. Infinite elements reduce the maximum deviation below 0.56% over 10 years while also lowering computational runtime. Finally, we modelled a geothermal doublet to demonstrate how doublet orientation and permeability anisotropy jointly control pore pressure evolution. The combined analytical and numerical framework provides rapid pressure estimates and practical guidance for long-term modelling of geothermal and other subsurface fluid-injection systems.

1. Introduction

Anthropogenic activities have been shown to be related to induced seismicity,^{1–4} which became a detrimental factor in the acceptance of the use of the subsurface. Such activities include fluid injection and extraction during hydrocarbon production and gas storage,^{5,6} mining operations,⁷ geothermal energy exploitation,^{8,9} or wastewater disposal.^{3,10} Induced seismicity in fluid injection systems is commonly attributed to increases in pore pressure that reduce the effective normal stress acting on critically stressed faults.¹¹ Previous studies demonstrated that even minor stress or pressure perturbations, often well below 1 MPa, may be sufficient to induce seismic events.^{12–14} Thus, the accurate characterization of the spatial and temporal evolution of pore pressure changes is essential for assessing induced seismicity during subsurface fluid injection. Two key challenges arise in this context: first, matrix permeability anisotropy (and hereafter “permeability

anisotropy”) strongly influences fluid flow pathways and pressure distribution, making its correct representation critical; second, numerical models must be designed to minimize model boundary effects that can distort or bias the physical behaviour, especially for long-term injection. Addressing these challenges is fundamental for improving both analytical and numerical approaches for reliable prediction of the evolution of pore pressure.

The theoretical framework for the evolution of pore pressure in porous and permeable media was first developed by Biot.^{15,16} Biot’s theory was the basis for the analytical solutions of fundamental problems for applications in subsurface fluid flow, derived by Rice and Cleary,¹⁷ Cleary,¹⁸ Rudnicki.¹⁹ They typically approximate fluid injection as either instantaneous or continuous from a point-like or axisymmetric source within an unbounded medium. The analytical solutions

* Corresponding author.

E-mail address: tatia.sharia@kit.edu (T. Sharia).

<https://doi.org/10.1016/j.gete.2026.100913>

Received 25 July 2026; Received in revised form 27 September 2026; Accepted 4 October 2026

Available online 14 October 2026

2352-3808/© 2026 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).

Open access funding enabled and organized by Projekt DEAL.

have been extensively used to evaluate the spatio-temporal evolution of pore pressure during fluid injection and to relate operational parameters of subsurface systems to induced seismicity.^{20–22}

Pore pressure changes due to fluid injection are strongly influenced by geological properties of the reservoir. Among these properties, permeability is one of the key parameters. Permeability can vary significantly between different subsurface layers, and it often has a directional dependence.²³ Permeability anisotropy plays a critical role in governing subsurface fluid flow, with direct implications for gas migration in underground storage applications, including hydrogen (H_2), methane (CH_4), and carbon dioxide (CO_2) sequestration in depleted reservoirs or aquifers. Field observations, such as those by Heffer and Lean²⁴ who analysed flooding directionality in oil fields, indicate permeability anisotropy resulting from preferential opening or closing of microcracks as a consequence of differential horizontal stress. Kirkby and Massiot²⁵ use 2D numerical models (80 m $\times$ 80 m) to characterize permeability anisotropy of a faulted reservoir. The influence of permeability on fluid flow is also reflected in induced seismicity: Shapiro et al.²⁶ assigned the migration of the microseismic triggering front to the permeability tensor of the seismically active volume. Similarly, the ellipsoidal shape of seismicity clouds has been attributed to permeability anisotropy, with the largest permeability component aligning with the maximum principal stress direction.^{27,28}

Permeability anisotropy is often not included in numerical and analytical models used to assess pore pressure and stress changes associated with fluid injection. Rothert and Shapiro²⁹ proposed an analytical point-source solution for pressure diffusion in an anisotropic medium. Additionally, they evaluated the critical pressure required to induce seismic events based on the estimated permeability tensor derived from seismicity clouds. In contrast to the approach presented in this study, their pressure magnitudes are normalized by the arithmetic mean of the principal components of the permeability tensor. Carcione and Gei³⁰ provided a normalized form for spatio-temporal pressure changes for a uniform injection rate. Fokker et al.³¹ presented analytical models of pore pressure evolution for different well and reservoir configurations for the harmonic pulse testing of wells, including cases with vertical anisotropy. Li et al.³² incorporated permeability anisotropy into a two-dimensional axisymmetric analytical model of pressure evolution and coupled it with numerical displacement calculations to reproduce the elliptical uplift patterns observed in geodetically measured surface deformation rates.

While analytical approaches provide computationally fast solutions, they are limited to simple geometries. Realistic subsurface systems, such as geothermal reservoirs, hydrocarbon formations, or CO_2 storage sites, typically require numerical models to capture lateral boundaries, complex geological settings and well configurations.³³ To evaluate the performance of a reservoir during all aspects of reservoir lifetime and to plan the optimized exploitation scenario, simulation tools for pore pressure are widely used.³⁴ However, relatively few numerical studies explicitly investigate injection and extraction processes in formations with permeability anisotropy.^{35–37} Related directional effects are sometimes captured indirectly either by modelling stratified media as isotropic layers with varying permeabilities, which may upscale to an effective anisotropy,^{38,39} or from structural heterogeneity associated with permeable faults.^{40,41} Panja et al.⁴² investigated the effects of permeability anisotropy and layering on the pressure and temperature evolution during cyclic injection for a reservoir with 400 m lateral extent by 3D numerical simulations.

Even for isotropic permeability, numerical models are subject to inherent limitations due to their finite spatial extent when lateral model boundaries are intended to represent far-field or unbounded conditions. To minimize model boundary effects, it is common practice to extend the computational domain far beyond the region of interest.^{43,44} The influence of the lateral boundaries depends on the injection duration and the hydraulic diffusivity of the medium.⁴⁵ For typical geothermal systems with operational lifetimes exceeding 10–20 years, the

achievement of sufficient model accuracy remains computationally demanding.

We provide a systematic comparison of modelling assumptions for fluid injection that control pore pressure evolution. These include permeability anisotropy, model boundary condition treatment, injection interval representation, and numerical discretization. In particular, we address two critical challenges in modelling pore pressure evolution during fluid injection: (i) the absence of an exact analytical solution for point-source continuous injection in a medium with permeability anisotropy, and (ii) mitigation of the influence of model boundaries in numerical models, which often require computationally expensive domain enlargement, especially for long prediction times (>0.5 year). To overcome these limitations, we extend the general theoretical approach by our closed-form solution for pore pressure evolution in a three-dimensional anisotropic medium which we validate against finite element simulations. Furthermore, we adopt an efficient, but so far relatively uncommon, numerical approach based on virtual infinite domains⁴⁶ to minimize model boundary effects. While infinite elements are commonly used in other modelling contexts,^{47,48} their application to pore pressure evolution in 3D permeable media in the context of subsurface energy applications remains limited.⁴⁹ The setup is benchmarked in the isotropic case against the well-known solution of Rudnicki.¹⁹ Our results show that they can substantially improve simulations intended to represent fluid injection in an unbounded medium. Finally, our simulations of pore pressure diffusion in a geothermal doublet system with anisotropic permeability highlight the importance of accounting for hydraulic anisotropy when predicting pore pressure evolution and the associated effective stress changes relevant to induced seismicity. This study therefore provides both an analytical framework for efficient prediction of pressure evolution in the planning phase of geothermal projects and practical recommendations for the development of robust numerical models necessary for long-term simulations.

2. Methodology

2.1. Material properties used for analytical and numerical calculations

The spatial and temporal evolution of pore pressure is strongly influenced by the hydraulic properties of the porous medium. In particular, permeability and hydraulic diffusivity govern the rate and extent of pressure diffusion.¹⁹

In this study, we used the material properties listed in Table 1, based on Wang.⁵⁰ The fluid properties are assumed to be a dynamic viscosity of 1 mPa·s and a bulk modulus of 2.5 GPa. For the analytical approach, corresponding undrained moduli were derived using the equations of Gassmann⁵¹ together with the relations provided in Wang.⁵⁰, Tab. B.1 The diffusivity values shown in the last row of Table 1 were computed according to relations given in Section 2.2.

We used Berea sandstone in each model configuration. In addition, for the isotropic point-source model, we also used the material properties of Ohio, Ruhr and Weber sandstones, to assess the difference between numerical and analytical solutions for various material property combinations. Nelson and Kibler⁵² compiled data from oil and gas wells in different sandstone reservoirs. They obtained an average porosity of $24 \pm 7.8\%$. For Berea sandstone, we have chosen a permeability of 190 mD and a porosity of 19% as those values are in the typical range for many applications. In comparison, porosity in carbonate reservoirs ranges from 1%–35% and for limestones average values of 12% are reported.⁵³ According to Lucia,⁵⁴ Fig. 1.7 carbonate permeability ranges from 0.1 to 1000 mD. The data used in this study are typical for conventional reservoirs and not for tight gas reservoirs or enhanced geothermal systems (EGS), where lower permeability and smaller porosity are common.

Table 1
Material properties.

	Variable	Berea	Ohio	Ruhr	Weber
Material parameters of sandstones adopted from Wang, ⁵⁰ Tab. C.1					
Shear modulus (GPa)	G	5.6	6.8	13	12
Poisson's ratio	ν	0.17	0.18	0.12	0.15
Biot-Willis coefficient	α	0.77	0.74	0.65	0.64
Porosity	ϵ	0.19	0.19	0.02	0.06
Permeability (mD)	k	190	5.6	0.2	1
Fluid properties					
Dynamic viscosity (mPa s)	γ	1			
Bulk modulus (GPa)	K_f	2.5			
Derived material parameters					
Young's modulus (GPa)	E	13.10	16.05	29.12	27.60
Lamé modulus for drained response (GPa)	λ	2.88	3.83	4.11	5.14
Lamé modulus for undrained response (GPa)	λ_u	9.05	9.71	20.83	15.41
Specific storage (GPa ⁻¹)	S	0.38	0.12	0.04	0.05
Diffusivity (m ² /s)	c	1.357	0.044	0.005	0.018

2.2. Analytical approach

Fluid injection into a porous elastic medium produces bidirectionally coupled excessive pore pressure and poroelastic stress fields. The governing equations (A.1), (A.2), (A.4), (A.5), (A.6) of poroelasticity are generally challenging to solve, but can be simplified analytically by making assumptions regarding the characteristics of the source and the medium. Wang⁵⁰ provided a comprehensive overview of these equations and identified specific conditions that facilitate such simplifications. Among these, point-like fluid injection into an infinite medium generates an irrotational displacement field ($\vec{\nabla} \times \vec{u} = 0$), leading to decoupling of pore pressure from the mechanical term.

Here, we consider point-source continuous injection with an injection rate q_v and assume the medium to be an infinite, homogeneous, elastic and fully saturated porous material, with a permeability matrix given by:

$$\mathbf{k} = \begin{pmatrix} k_{xx} & 0 & 0 \\ 0 & k_{yy} & 0 \\ 0 & 0 & k_{zz} \end{pmatrix} \quad (1)$$

The corresponding uncoupled diffusion equation, in the absence of a body force and a medium with directional permeability, is given by:

$$\frac{\partial p}{\partial t} = \frac{1}{\gamma S} \left(k_{xx} \frac{\partial^2}{\partial x^2} + k_{yy} \frac{\partial^2}{\partial y^2} + k_{zz} \frac{\partial^2}{\partial z^2} \right) p + \frac{Q(x, y, z, t)}{S} \quad (2)$$

where p is an excessive pore pressure and $Q(x, y, z, t)$ represents the volumetric source term at point $\vec{r} = (x, y, z)$ and time t . The origin of the coordinate system coincides with the injection point. The poroelastic material parameters and corresponding nomenclature are given in Table 1. S is the specific storage coefficient obtained after eliminating volumetric strain from the coupled poroelastic equations. Its relationship to the constant strain poroelastic storage coefficient, S_p , is given in Appendix A. It is related to other parameters by:

$$S = \frac{\alpha^2(\lambda_u + 2\mu)}{(\lambda_u - \lambda)(\lambda + 2\mu)} \quad (3)$$

For hydraulically isotropic material, the solution for the pore pressure is well-known.^{19,50,55} The pressure increase resulting from the instantaneous injection of a fluid volume V_f at the origin of the coordinate system, represented by $Q(x, y, z, t) = V_f \delta(x, y, z) \delta(t)$, is given by

$$p(r, t) = \frac{V_f}{S(4\pi ct)^{3/2}} \exp\left(-\frac{r^2}{4ct}\right) \quad (4)$$

where $c = \frac{k}{\gamma S}$ is the hydraulic diffusivity.

For continuous injection at a constant volumetric rate q_v beginning at $t = 0$, the source term is $Q(x, y, z, t) = q_v \delta(x, y, z) H(t)$. The corresponding solution is obtained by integrating the instantaneous point-source response Eq. (4) over the injection history⁵⁵:

$$p(r, t) = \frac{q_v \gamma}{k} \frac{1}{4\pi r} \operatorname{erfc}\left(\frac{r}{2\sqrt{ct}}\right) \quad (5)$$

The diffusion equation in a hydraulically anisotropic medium with a directional permeability (Eq. (2)) can be transformed into an equivalent isotropic form by introducing new spatial coordinates, each scaled by $\sqrt{k/k_{ii}}$ ⁵⁶:

$$X = x\sqrt{\frac{k}{k_{xx}}}, \quad Y = y\sqrt{\frac{k}{k_{yy}}}, \quad Z = z\sqrt{\frac{k}{k_{zz}}} \quad (6)$$

The corresponding spatial derivatives satisfy

$$k_{xx} \frac{\partial^2 p}{\partial x^2} = k \frac{\partial^2 p}{\partial X^2}, \quad k_{yy} \frac{\partial^2 p}{\partial y^2} = k \frac{\partial^2 p}{\partial Y^2}, \quad k_{zz} \frac{\partial^2 p}{\partial z^2} = k \frac{\partial^2 p}{\partial Z^2} \quad (7)$$

Using the scaling property of the Dirac delta function $\delta(ax) = \delta(x)/|a|$, the Dirac distributions transform as

$$\begin{aligned} \delta(x)\delta(y)\delta(z) &= \sqrt{\frac{k}{k_{xx}}} \sqrt{\frac{k}{k_{yy}}} \sqrt{\frac{k}{k_{zz}}} \delta(X)\delta(Y)\delta(Z) \\ &= J \delta(X)\delta(Y)\delta(Z) \end{aligned} \quad (8)$$

where

$$J = \left| \frac{\partial(X, Y, Z)}{\partial(x, y, z)} \right| = \sqrt{\frac{k^3}{k_{xx}k_{yy}k_{zz}}} \quad (9)$$

is the Jacobian determinant of the coordinate transformation. Therefore, the transformed source coefficients for instantaneous and continuous injection are related to their corresponding values in the original coordinates by:

$$\frac{\tilde{V}_f}{V_f} = \frac{\tilde{q}_v}{q_v} = J = \sqrt{\frac{k^3}{k_{xx}k_{yy}k_{zz}}} \quad (10)$$

In the new coordinates of Eq. (6) and transformed source terms of Eq. (10), the solution is analogous to the solution for the isotropic medium: Eqs. (4) and (5).

$$p(X, Y, Z, t) = \frac{\tilde{V}_f}{S(4\pi ct)^{3/2}} \exp\left(-\frac{X^2 + Y^2 + Z^2}{4ct}\right) \quad (11)$$

Substituting the transformed coordinates and $\tilde{V}_f = J V_f$ in Eq. (11) gives the pore pressure solution for instantaneous fluid injection in the original coordinate system as:

$$p(x, y, z, t) = \frac{V_f}{S(4\pi t/(S\gamma))^{3/2} \sqrt{k_{xx}k_{yy}k_{zz}}} \exp\left(-\frac{S\gamma}{4t} \left(\frac{x^2}{k_{xx}} + \frac{y^2}{k_{yy}} + \frac{z^2}{k_{zz}}\right)\right) \quad (12)$$

Eq. (12) is similar to the solution for heat conduction in media with anisotropic thermal conductivity presented by Carslaw and Jaeger.⁵⁵

The corresponding solution for continuous injection at a constant volumetric rate q_v into an anisotropic medium is then given by:

$$\begin{aligned} p(x, y, z, t) &= \frac{q_v \gamma}{4\pi \sqrt{k_{xx}k_{yy}k_{zz}}} \frac{1}{\sqrt{\frac{x^2}{k_{xx}} + \frac{y^2}{k_{yy}} + \frac{z^2}{k_{zz}}}} \times \\ &\quad \operatorname{erfc}\left(\frac{1}{2\sqrt{t/(S\gamma)}} \sqrt{\frac{x^2}{k_{xx}} + \frac{y^2}{k_{yy}} + \frac{z^2}{k_{zz}}}\right) \end{aligned} \quad (13)$$

In the isotropic limit, $k_{xx} = k_{yy} = k_{zz} = k$, the denominator and the argument of the complementary error function in Eq. (13) simplify as

$$\begin{aligned} \sqrt{k_{xx}k_{yy}k_{zz}} \sqrt{\frac{x^2}{k_{xx}} + \frac{y^2}{k_{yy}} + \frac{z^2}{k_{zz}}} &= \sqrt{k^3} \sqrt{\frac{r^2}{k}} = kr, \\ \frac{1}{2\sqrt{t/(S\gamma)}} \sqrt{\frac{x^2}{k_{xx}} + \frac{y^2}{k_{yy}} + \frac{z^2}{k_{zz}}} &= \frac{r}{2\sqrt{kt/(S\gamma)}} = \frac{r}{2\sqrt{ct}} \end{aligned} \quad (14)$$

Hence, Eq. (13) reduces exactly to the isotropic continuous point-source solution given by Eq. (5).

After a sufficiently long time of continuous fluid injection, the steady-state limit of the pore pressure in the anisotropic medium is:

$$p \left(x, y, z, t \gg \left[\frac{x^2}{k_{xx}} + \frac{y^2}{k_{yy}} + \frac{z^2}{k_{zz}} \right] (S\gamma) \right) \rightarrow \bar{p}(x, y, z) = \frac{q_v \gamma}{4\pi \sqrt{k_{xx}k_{yy}k_{zz}} \sqrt{\frac{x^2}{k_{xx}} + \frac{y^2}{k_{yy}} + \frac{z^2}{k_{zz}}}} \quad (15)$$

The steady-state pore pressure with magnitude p_0 forms an ellipsoidal surface with the principal semiaxes:

$$\begin{aligned} A &= \frac{q_v \gamma}{p_0 4\pi} \frac{1}{\sqrt{k_{yy}k_{zz}}}, \\ B &= \frac{q_v \gamma}{p_0 4\pi} \frac{1}{\sqrt{k_{xx}k_{zz}}}, \\ C &= \frac{q_v \gamma}{p_0 4\pi} \frac{1}{\sqrt{k_{xx}k_{yy}}} \end{aligned} \quad (16)$$

where A , B and C are semiaxes corresponding to the principal axes x , y and z , respectively. The semiaxes along each direction are inversely proportional to the permeabilities in the two other perpendicular directions.

2.3. Numerical approach

Numerical solutions are obtained using the finite element method (FEM) in the COMSOL Multiphysics® software. The poroelasticity coupling interface was used to achieve a fully coupled fluid flow and elastic response of the porous material. A complete set of governing equations is given in Appendix A.

A fixed volume of 120 liters of water per second is continuously injected for 10 years into a saturated porous medium. The injection interval varies between 4 m and 250 m across different models and is centred at the origin of the coordinate system aligned with the z -axis to represent a vertical borehole section. The poroelastic moduli of the materials are given in Table 1. A zero initial pressure condition is assumed throughout the domain of each model.^c

We evaluate the temporal and spatial evolution of the pore pressure within a volume of 4 km × 4 km × 4 km considering various boundary conditions (Fig. 1) and material parameters. The numerical models are benchmarked against the given analytical solutions, and the sources of deviations between them are identified across different model configurations. In addition, we compare the computational times required for each model setup, providing insight into the efficiency of the numerical approach. For all models, a relative tolerance of 10⁻⁷ was used as convergence criterion for the nonlinear solver in COMSOL Multiphysics.

We distinguish three main classes of models: isotropic models with finite domains (class A), isotropic models with infinite elements (class B), and anisotropic models with infinite elements (class C). Each model contains (i) an injection interval and (ii) a physical volume of porous material where we investigate the results. The models of classes B and C are additionally surrounded by infinite elements. Those are virtual domain elements that are effectively stretched to infinity^{46,57} to simulate unbounded domains. The outermost boundaries of each model have fixed zero pore pressure and roller boundary conditions. The latter prohibits displacement along the normal vector of the surface. Table 2 summarizes the models used in this study, specifying permeability,

^c The terms pore pressure and pressure increment (or other equivalent terms) are used interchangeably in this paper since the initial conditions are set to zero in all models.

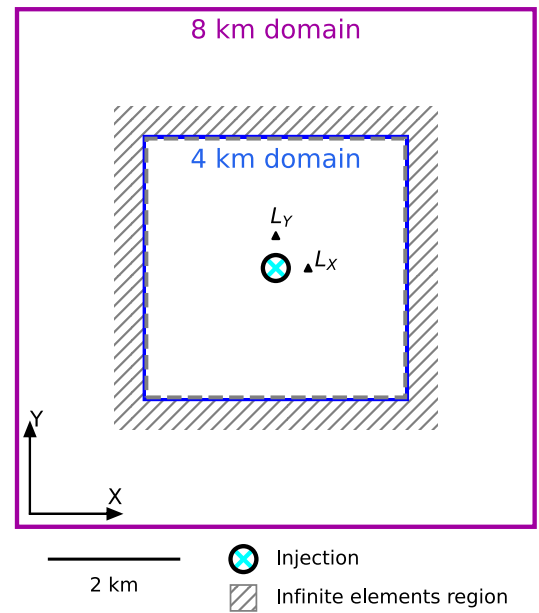


Fig. 1. Schematic x-y cross section of the model configurations used in this study. The solid lines indicate the boundaries of the conventional finite domains (4 km × 4 km × 4 km and 8 km × 8 km × 8 km models), whereas the dashed line marks the interface between the finite domain and the surrounding infinite element region (hatched). L_x and L_y denote observation points where the temporal evolution of pore pressure is shown in Section 3.

model dimensions, and length of the injection interval. Fig. 1 provides a schematic representation of the x-y cross sections of the models, illustrating configurations both with and without infinite elements. All COMSOL Multiphysics® model files (.mph) corresponding to each simulation setup are made available as the supplementary material in Appendix G.

Models of class A are designed to investigate boundary effects by varying the distance from the injection interval to the outer boundaries and the permeability of the porous medium. In particular, models *A-iso-4x4x4* and *A-iso-8x8x8* share identical parameter setups representing an isotropic porous medium of Berea sandstone with a continuous fluid injection from a 20 m long injection interval in the centre of the models. These models differ only in their spatial dimensions: 4 km × 4 km × 4 km and 8 km × 8 km × 8 km, respectively. Model *A-iso-8x8x8-k* maintains the same 8 km × 8 km × 8 km dimensions but is solved for two additional permeability values: four times lower (47.5 mD) and four times higher (760 mD) than the permeability of Berea sandstone, allowing evaluation of boundary effects under varying permeability.

Class B models simulate fluid flow in an isotropic medium while mitigating unwanted model boundary effects. Model *B-iso-inf* replicates the setup of model *A-iso-4x4x4*, but with the addition of infinite elements surrounding the finite domain, approximating point-source injection in an unbounded, isotropic, homogeneous medium. Models *B-iso-inf-ohio*, *B-iso-inf-ruhr* and *B-iso-inf-weber* share the same configurations as *B-iso-inf*, but use the material properties of Ohio, Ruhr, and Weber sandstones, respectively (Table 1).

Model *B-iso-inf-half-space* is a generic half-space model. It has a single boundary with zero pressure at finite distance of 2 km above the injection point. All other boundaries are stretched out with infinite elements. We used the model to show the perturbations in the pore pressure field due to the half-space boundary conditions. Model *B-iso-ext-inf* corresponds to model *B-iso-inf* but with an extended 250 m long injection interval, enabling an analysis of deviations arising from the influence of an elongated source.

Class C models are designed to simulate pressure diffusion in anisotropic media. They are modifications of the model *B-iso-inf* by

Table 2

Description of numerical models categorized into three classes based on their structural and material properties: Class A includes models *A-iso-4x4x4* and *A-iso-8x8x8* which represent isotropic media with finite volumes of different dimensions, while *A-iso-8x8x8-k* is a variation of the latter with modified permeability values. Class B consists of *B-iso-inf*, an isotropic medium surrounded by infinite elements, along with *B-iso-inf-ohio*, *B-iso-inf-ruhr*, and *B-iso-inf-weber*, which represent variations of model *B-iso-inf* with different material properties. Additionally, *B-iso-inf-half-space* represents an isotropic medium with infinite elements on all sides except the top, whereas *B-iso-ext-inf* includes an extended injection interval with infinite elements surrounding the physical domain. Class C introduces anisotropic effects, where *C-anis-inf* is an anisotropic medium enclosed by infinite elements, and *C-anis-inf-mesh1* and *C-anis-inf-mesh2* incorporate different mesh refinements. All models share a continuous injection rate of 120 l/s. L_{inj} denotes the length of the injection interval.

Model	Permeability (mD)	Model dimension	L_{inj} (m)
Class A - isotropic models with finite domain			
<i>A-iso-4x4x4</i>	190	4 km × 4 km × 4 km	20
<i>A-iso-8x8x8</i>	190	8 km × 8 km × 8 km	20
<i>A-iso-8x8x8-k</i>	47.5; 760	8 km × 8 km × 8 km	20
Class B - isotropic models with infinite elements			
<i>B-iso-inf</i>	190	Full space	20
<i>(B-iso-inf-) ohio/ruhr/weber</i>	190	Full space	20
<i>B-iso-inf-half-space</i>	190	Half space	20
<i>B-iso-ext-inf</i>	190	Full space	250
Class C - anisotropic models with infinite elements			
<i>C-anis-inf</i>	[190, 190 × n , 190] $n = 0.1; 1; 2; 10$	Full space	4
<i>C-anis-inf-mesh1</i>	[190, 190 × n , 190] $n = 0.1; 1; 2; 10$	Full space	4
<i>C-anis-inf-mesh2</i>	[190, 1900, 190]	Full space	4

varying the principal permeability component in the direction of the y -axis to investigate point-like fluid injection in an unbounded medium with permeability anisotropy. To avoid the superposition of hydraulic anisotropy effects with deviations from an extended source, we reduced the length of the injection interval to 4 m. Model *C-anis-inf* has a mesh structure similar to the isotropic model *B-iso-inf*, but incorporates permeability anisotropy characterized by scaling factors $n = 0.1; 2; 10$ of principal components of permeability, defined as $n = k_{yy}/k_{xx,zz}$. Models *C-anis-inf-mesh1* and *C-anis-inf-mesh2* were set to test different mesh structures.

Selecting an appropriate spatial discretization strategy is essential for achieving accurate results while maintaining efficient computation times. The high pore pressure gradients near the injection interval require fine discretization in the vicinity of the well. To accommodate this, the well is surrounded by a cylindrical domain reflecting source symmetry, with minimum element sizes ranging from 14 m to 25 m across the different models.

For models with higher degrees of anisotropy, we tested a refined version of the previously generated spherically symmetric mesh (model *C-anis-inf-mesh1*) and an alternative mesh with fewer elements but finer discretization along the fastest fluid flow direction (model *C-anis-inf-mesh2*) for the permeability of $k = [190, 1900, 190]$ mD. Figs. F.11(a), (b) and (c) depict the mesh element sizes in the x - y plane that crosses the centre of the injection interval for the models *C-anis-inf*, *C-anis-inf-mesh1* and *C-anis-inf-mesh2*, respectively. Detailed information on the mesh generation and the number of elements for each model is given in Appendix B.

The finite domains of the class B and C models were surrounded by a geometrically 100-m-thick infinite element domain, discretized using five swept mesh elements across its thickness. COMSOL Multiphysics®'s default rational coordinate stretching function was used⁴⁶:

$$f(\xi) = \frac{\Delta_p \xi}{\Gamma - \xi}, \quad \Gamma = \frac{\Delta_s + \Delta_p}{\Delta_s}, \quad (17)$$

where $\xi \in [0, 1]$ is the dimensionless coordinate across the infinite element domain, Δ_s is the scaled thickness of infinite element domain, and Δ_p is the pole distance. The default values $\Delta_s = 10^3 d_{\text{GeomChar}}$ and $\Delta_p = d_{\text{GeomChar}}$ were retained, where d_{GeomChar} is characteristic geometry dimension.

3. Results

We present numerical results for pore pressure in hydraulically isotropic and anisotropic media with varying material properties, meshes, boundary conditions, model sizes, and injection interval lengths, and compare them to the analytical solution of Rudnicki¹⁹ (Eq. (5)) and Eq. (13) derived in Section 2.2. Deviations from the analytical solution are defined as $(P_{\text{num}} - P_{\text{an}})/P_{\text{an}}$, where P_{num} and P_{an} are the pore pressure values of the numerical and analytical solutions, respectively. We evaluate the spatial distributions at distances greater than 50 m from the centre of the injection interval, and report temporal evolution ranging from 0.01 to 10 years. Computation times and number of degrees of freedom for the models of this study are provided in Appendix C.

3.1. Pressure evolution for varying model boundaries and isotropic material properties

Fig. 2 compares the pressure calculations from finite and infinite numerical models with the analytical solution for point-source injection in unbounded media (Eq. (5)). Fig. 2(a) shows the temporal evolution of the pore pressure at 500 m from the injection well (Location L_X) in the finite volume models (*A-iso-4x4x4* and *A-iso-8x8x8*) and in the model with infinite elements (*B-iso-inf*). Fig. 2(b) shows the corresponding relative deviations of numerical to analytical pressure calculations. The spatial distribution of the relative deviations is illustrated in Fig. 2(c). Overall, the numerical results of the models with finite volumes show good agreement with analytical solutions near the injection point and during the early stages of injection. However, discrepancies increase as the observation location approaches the model boundaries and as the system evolves towards steady-state conditions.

The pore pressure at location L_X in model *A-iso-4x4x4* diverges from the analytical solution after ~ 0.04 years (nearly half a month) and reaches a steady state after ~ 0.1 years (~ 1.2 months). The solution of the model *A-iso-8x8x8*, which has an eightfold larger volume, diverges and reaches the steady state after about half a year. After 10 years of continuous injection, the pore pressure level at location L_X is underestimated by about 20% in the model with volume of $4 \text{ km} \times 4 \text{ km} \times 4 \text{ km}$, and about 10% in the model with volume of $8 \text{ km} \times 8 \text{ km} \times 8 \text{ km}$ (Fig. 2(b)).

The relative deviations of the pore pressure along the x -axis after 0.1, 0.2 and 10 years of continuous injection are shown in Fig. 2(c). At 0.1 years, the model with 8 km of side length shows no signature of boundary effects along the principal x -axis within a 2 km distance from the injection point, with deviations remaining under 1.4%. By 0.2 years (~ 2.4 months), the same model yields a relative deviation of -1.69% at 0.5 km, -3.95% at 1 km, and -11.96% at 2 km. In 10 years, the deviation at 2 km reaches -40.8% . In the smaller model *A-iso-4x4x4*, at 1 km distance, the relative deviations at times 0.1, 0.2 and 10 years are -23.5% , -30.76% and -42.42% , respectively.

The influence of permeability is illustrated on Fig. 3. The relative deviations from the analytical solution are shown at location L_X for the models with identical geometry and model parameters as the model *A-iso-8x8x8* (green). The figure contains additional curves for four times smaller (blue) and larger (red) permeabilities (model *A-iso-8x8x8-k*). As permeability increases, the underestimation of pore pressure relative to the analytical solution occurs at earlier times: the model with a permeability of 760 mD passes the -1% limit after about 0.04 years, while the model with 47.5 mD after about 0.7 years.

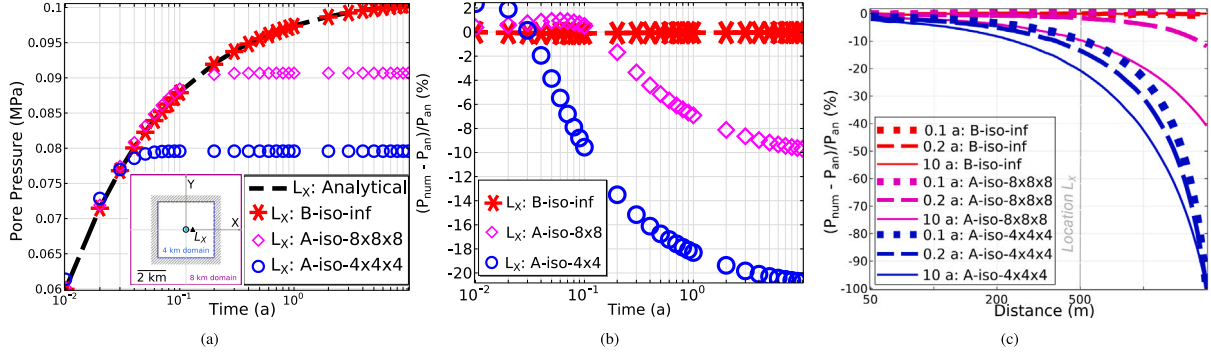


Fig. 2. The results illustrate (a) the temporal evolution of pore pressure at location L_X . The dashed line represents the analytical solution for a point source in an unbounded medium (Eq. (5)). The inset illustrates the injection point, observation point L_X , model boundaries for the finite volume domains (solid) and for the domain with infinite elements (dashed, hatched); (b) Corresponding relative deviations between numerical and analytical solutions; (c) Spatial distribution of relative deviations after 0.1, 0.2 and 10 years of continuous fluid injection. Each sub-figure contains solutions from three separate numerical models, which have identical model parameters except the distance to the boundaries of zero pore pressure constraint from the injection interval: (1) closest boundary at 2 km (blue); (2) closest boundary at 4 km (purple); and (3) stretched by infinite elements encompassing a 4 km $\times$ 4 km $\times$ 4 km physical volume (red).

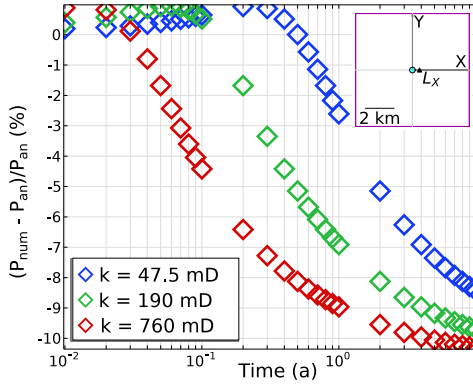


Fig. 3. Relative deviations of numerical results of pore pressure from the corresponding analytical solution at 500 m distance from the injection interval (location L_X) in the volume of 8 km $\times$ 8 km $\times$ 8 km with different permeabilities (model A-iso-8x8x8-k). The inset illustrates the x-y cross section of the model with positions of the injection point and the observation point L_X .

The isotropic model with infinite elements and Berea sandstone material parameters (*B-iso-inf*) shows no indication of boundary influence. Fig. 2(a) and (b) illustrate a close alignment between the numerical pore pressure values and the analytical solution of Eq. (5) at the location L_X , with the magnitudes of relative deviations remaining under 0.11% for simulation periods up to 10 years. Furthermore, the magnitudes of the deviations along the x-axis from 50 m to 2000 m remain below 0.56% at 0.1, 0.2 and 10 years (Fig. 2(c)). Additionally, benchmark simulations were performed to assess the robustness of the isotropic model setup *B-iso-inf* using three additional sets of material parameters listed in Table 1. In all cases, the numerical solution agrees well with the analytical solution, with relative deviations below 1.8% (Fig. D.7, see the details in Appendix D).

Realistic reservoir models require a half-space geometry and finite injection intervals. Model *B-iso-inf-half-space* represents point-source injection in a half-space, with the source located 2 km below a surface boundary maintained at zero pore pressure. Relative to the unbounded full-space solution Eq. (5), deviations increase with time and converge to steady-state values that depend on the proximity to the surface boundary (Fig. E.8, see the details in Appendix E). Real injection intervals commonly extend tens to hundreds of meters. The extended

source configuration of the model *B-iso-ext-inf* perturbs the spherical symmetry of the point-source response, with stronger effects along the extended source axis (z) than perpendicular to it (x) (Fig. E.9). Overall, the point-source approximation overestimates pore pressure in the x-direction up to approximately twice the source length, whereas it underestimates pore pressure in the z-direction up to approximately three times the source length (Fig. E.9, see Appendix E for the details).

3.2. Modelling hydraulically anisotropic media

A spatially uniform anisotropic permeability tensor was assigned throughout the model domain. The corresponding diffusion equation was analytically solved in Section 2.2, where we obtained the pore pressure solution for the continuous fluid injection given in Eq. (13).

Fig. 4 shows 0.1 MPa isolines of the pore pressure calculated by Eq. (13) with an injection rate of 120 l/s for the cross sections x – y (a) and x – z (b). The permeability matrix (Eq. (1)) has two equal principal components $k_{xx} = k_{zz} = k$, and varied three different values of k_{yy} : (i) k (isotropy), (ii) $2k$, (iii) $10k$. The corresponding solutions are shown at 0.01 years and 10 years after the beginning of the injection in the first and second columns of Fig. 4, respectively. Fig. 4(a) shows that pore pressure diffusion along the principal direction of the highest permeability (y-axis) is faster. However, the steady-state extent of the pressure contour along the y-axis is independent of k_{yy} and in each given case of anisotropic permeability matrix, converges to an isotropic value:

$$B = \frac{q_v \gamma}{p_0 4\pi} \frac{1}{\sqrt{k_{xx} k_{zz}}} = \frac{q_v \gamma}{p_0 4\pi} \frac{1}{k} \quad (18)$$

Conversely, along the principal directions where the corresponding permeability component is the same in all 3 cases, pore pressure isolines are constrained by the distance that is inversely proportional to the anisotropy scaling factor applied to the k_{yy} component ($n = [1, 2, 10]$, where $n = k_{yy}/k$)

$$A = C = \frac{q_v \gamma}{p_0 4\pi} \frac{1}{\sqrt{k_{yy} k_{xx} k_{zz}}} = \frac{q_v \gamma}{p_0 4\pi} \frac{1}{k \sqrt{n}} \quad (19)$$

The volume encompassed by the isosurface, where steady-state pore pressure values exceed p_0 , is proportional to $(k_{xx} k_{yy} k_{zz})^{-1}$ and, in the aforementioned 3 specific cases, decreases as n^{-1} .

These relationships describe the spatial extent of the steady-state pressure field. The rate of the pressure diffusion along each principal directions is instead determined by the permeability component

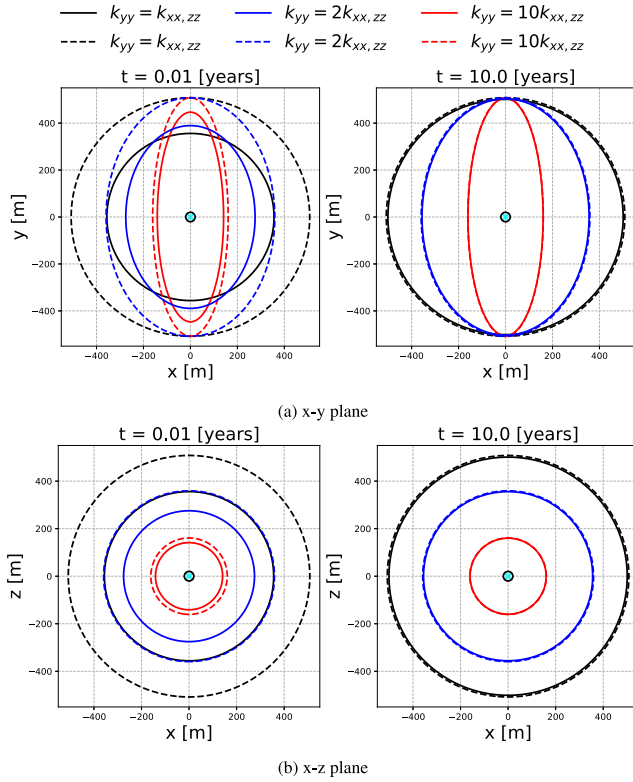


Fig. 4. (a) x-y and (b) x-z cross sections of 0.1 MPa isosurfaces of the pore pressure (Eq. (13)) at different times (0.01 and 10 years) from the beginning of the injection and with different factors of anisotropy: $k_{yy} = nk_{xx} = nk_{zz}$, $n = 1$ (isotropy, black), $n = 2$ (blue), $n = 10$ (red). The steady-state solutions (Eq. (15)) are displayed in dashed lines, with the colour of the corresponding permeability anisotropy factors. Material parameters (including permeability components k_{xx} and k_{zz}) correspond to Berea sandstone from Table 1.

corresponding to that same direction. For instance, the pore pressure evolution in the direction of the y -axis is given by:

$$p(0, y, 0, t) = \frac{q_v \gamma}{4\pi \sqrt{k_{xx} k_{zz}} |y|} \times \text{erfc} \left(\frac{1}{2\sqrt{tk_{yy}/(S\gamma)}} |y| \right) \quad (20)$$

where the temporal evolution is influenced by the k_{yy} component and the amplitude is proportional to $(k_{xx} k_{zz})^{-1/2}$.

Fig. 5 illustrates these directional dependencies by comparing the analytical solution with the results of the numerical model *C-anis-inf-mesh1* for different degrees of anisotropy $n = [0.1, 2, 10]$ ($k_{yy} = nk_{xx,zz}$) and $n = 1$ ($k_{xx} = k_{yy} = k_{zz}$) as a reference of the isotropic case. The uniformly refined mesh of this model provides a consistent discretization across all investigated anisotropy ratios. In the left panel of Fig. 5, the steady-state limit of the pore pressure values at location L_X differ for the different scaling factors. They are inversely proportional to the square root of the scaling factor of permeability along the y -axis, as can be deduced from Eq. (15). The right panel shows that the steady-state pore pressure limits at location L_Y are identical for all scaling factors. Additionally, the temporal evolution along the y -axis is the fastest for $n = 10$ and the slowest for $n = 0.1$.

A detailed assessment of the precision of the numerical results of pore pressure evolution in anisotropic media is described in Appendix F. For strong permeability anisotropy ($n = 0.1$ and $n = 10$), the largest deviations occurred along the direction of the highest permeability, where the numerical solution underestimated the analytical solution Fig. F.10. Furthermore, for the $n = 10$ case, two mesh refinement strategies were investigated. Uniform mesh refinement in all directions (*C-anis-inf-mesh1*) reduced the maximum relative deviation along the

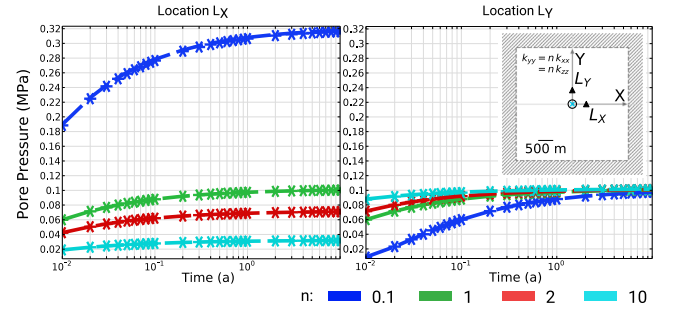


Fig. 5. Pore pressure over time at 500 m distance to the injection well along the x -axis (Location L_X) and along the y -axis (Location L_Y) in the model *C-anis-inf-mesh1*. Colours denote the degree of hydraulic anisotropy in the direction of the y -axis; asterisk data points represent numerical results, while dashed lines are values from the analytical expression (Eq. (13)). Green colour denotes the isotropic reference case; red corresponds to a permeability component k_{yy} that is twice as large as $k_{xx,zz}$ ($k_{yy} = 380$ mD); blue and cyan correspond to permeability values 10 times smaller ($k_{yy} = 19$ mD) and 10 times larger ($k_{yy} = 1900$ mD), respectively. The inset illustrates the injection interval, the location of observation points, the coordinate axes, and the boundaries of the model.

y -axis from 2.54% to 1.56%. Refining the mesh preferentially along the high-permeability direction (*C-anis-inf-mesh2*) further reduced the maximum deviation to 0.73%, while requiring approximately 40% fewer mesh elements and reducing the computation time by a factor of about three compared with *C-anis-inf-mesh1*.

4. Discussion

The scope of this study was to examine how key modelling assumptions influence the modelled evolution of pore pressure during fluid injection. Numerical models are particularly important in regions with sparse observations for estimating the spatial and temporal evolution of changes in pore pressure and stress. Although there are already significant uncertainties in estimated in-situ stress magnitudes, precise pressure calculations are essential, as pore pressure is a first-order control of the effective stress. Additionally, pore pressure changes induce total stress changes through poroelastic coupling.^{58,59} The importance of accurately predicting pressure evolution is underscored by the fact that even minor pressure variations may promote fault reactivation. Pore pressure perturbations of less than 1 MPa,¹² and in some cases as small as 1 kPa,²⁹ are sufficient to induce seismicity. This high sensitivity can be understood in terms of a crustal stress state close to frictional failure, consistent with the frictional limit concept⁶⁰ and the critically stressed fault hypothesis.⁶¹ We restrict our analysis to pressure variations. The pressure perturbations obtained in this study are in the order of 10 s of kPa (e.g. Fig. 2).

We extend the analysis of pore pressure diffusion in media with permeability anisotropy by deriving a closed-form solution for continuous point-source injection. In the numerical framework, particular attention is given to the role of outer model boundaries representing geological or modelling constraints, as these significantly influence the reliability of the results. Additionally, the study considers the impact of extended injection intervals within boreholes, and for anisotropic flow, provides suggestions for the improvement in the numerical discretization. These refinements are expected to enhance the robustness and applicability of poroelastic modelling for complex reservoir systems, thereby contributing to more accurate characterization of coupled hydro-mechanical processes in subsurface environments. Finally, the benchmarked model setup is applied to a geothermal doublet to isolate the effects of doublet alignment and permeability anisotropy on pore pressure evolution and to demonstrate their combined influence.

4.1. Study limitations

Despite the insights provided by this study, it is important to outline the framework and limitations of the analytical and numerical formulations. Although the numerical model solves the fully coupled poroelastic problem, the assumptions adopted here allow the fluid diffusion equation to be treated independently of stress changes (Appendix A, Section 2.2). This enables our analysis to be focused on pore pressure evolution within generic model configurations. Consequently, we do not discuss poroelastic or thermoelastic stresses, which may be relevant for induced seismicity.^{59,62,63} Thermal effects on pore pressure are also not included. For example, cooling during cold fluid injection may produce a pore pressure decrease near the wellbore during the early stage of injection, partly counteracting the injection induced pressure increase.⁶⁴ Furthermore, our models do not incorporate gravitational effects. These limitations could be addressed in future research.

The analytical closed-form solution is exact for a homogeneous, three-dimensional, unbounded medium with permeability anisotropy and a point-source representation of fluid injection. These assumptions enable an analytically tractable governing diffusion equation, but they do not account for heterogeneity, layering, internal hydraulic barriers such as low-permeability fault sections, stress-dependent permeability, or finite wellbore dimensions that may be relevant in field applications. In natural reservoirs, faults may act as internal hydraulic boundaries and substantially modify the pore pressure distribution. For example, a sealing fault may confine the pressure increase, whereas a hydraulically conductive fault may promote pressure migration along the fault.⁶² Such fault-specific hydraulic structures are not represented by the uniform effective permeability tensor considered here.

Similarly, the numerical models employing virtual infinite domains are evaluated for simple geometries under the same homogeneous conditions, such as idealized permeability structures. The use of unbounded domains should be understood as an idealized far-field representation, in which pressure perturbations are assumed to vanish at infinity, rather than as a representation of site-specific geological boundaries. The performance of infinite elements in more complex geological settings remains to be explored. Because the analysis is based on idealized scenarios, the reported pressure differences should be interpreted primarily as sensitivities to modelling assumptions rather than as universally applicable quantitative predictions. Although adding further complexity, such as stress-dependent permeability or heterogeneous material properties, could improve the physical realism of the models, it would also require additional assumptions and poorly constrained parameters. This would potentially increase model uncertainty and obscure the isolated effects examined here. The following discussion examines the agreement between analytical and numerical solutions and evaluates the influence of anisotropy and model boundary treatments on the spatio-temporal evolution of pore pressure.

Within these limitations, the analytical solution provides general directional trends and scaling relationships for pore pressure diffusion in anisotropic media. It also provides a computationally efficient means of obtaining first-order estimates of long-term pore pressure evolution and its direct contribution to effective stress over large spatial and temporal scales. Its low computational cost also enables rapid sensitivity analyses of the influence of hydraulic and poroelastic material properties, helping to identify influential parameters and relevant parameter ranges before more complex numerical models are developed. Such estimates may support preliminary screening of regions potentially affected by pressure diffusion, but they do not constitute a complete assessment of fault stability or induced seismicity hazard. The idealized formulation also enables to construct closely comparable analytical and numerical models. The analytical unbounded domain solution therefore provides an exact reference for quantifying sensitivity to outer boundary placement, evaluating infinite element performance, and assessing mesh discretization. In this sense, the model serves both as a tool for

developing process understanding and as a benchmark for numerical approaches to long-term diffusion problems.

The model can provide meaningful physical insight where fluid flow remains approximately diffusive, deformation is sufficiently small for linear poroelasticity to apply, hydraulic properties remain approximately constant, and reservoir-scale fracture effects can be represented by an effective permeability tensor. It is most applicable at distances sufficiently larger than the injection interval. For strongly heterogeneous reservoirs or where permeability evolves substantially during operation, the solution should instead be regarded as a baseline against which additional geological and nonlinear effects can be evaluated.

4.2. Impact of model boundaries and material parameters on pressure predictions

Long injection periods, common in subsurface operations, amplify model boundary artefacts in the evolution of pressure. Numerical models of subsurface fluid flow typically impose fixed-pressure or no-flow boundaries at finite distances, which can affect the computed pore pressure field. The magnitude of this influence depends on injection duration, distance to the model boundaries, and material properties such as permeability. To minimize these artefacts, domain-size independence checks are commonly performed by progressively enlarging the computational domain until the solution within the target region becomes insensitive to further increases.⁴⁴ However, this approach is both case-dependent and computationally costly, since the required model size scales with both permeability and injection time. For example, Chang and Segall⁴⁵ simulated a 30-day injection using a 2D domain scaled to the characteristic diffusive length $\sqrt{4c\Delta t}$, resulting in a 15 km lateral extent and a 2 km basement thickness. Extending such scaling to typical operational periods of several years in subsurface energy systems would require large computational domains.

To isolate model boundary-related effects, we used generic full-space models rather than geometry-specific reservoir configurations. The model with infinite elements reproduced the analytical solution with near-perfect accuracy across the studied temporal (0.01–10 years) (Fig. 2(a), (b)) and spatial (up to 2 km) ranges (Fig. 2(c)). In contrast, finite domain models exhibited clear boundary signatures that became more pronounced with time and proximity to the boundaries (Fig. 2(a), (b), (c)). Notably, finite domain models indicate that a steady-state pore pressure is reached already for earlier injection time intervals in comparison to the analytical full-space solution (Fig. 2(a)). Furthermore, the largest finite domain model tested in this study with the volume of 8 km × 8 km × 8 km was computationally more expensive as the infinite element model (factor 1.22, see Appendix C) and was unaffected by boundary artefacts only during the first 0.1 years of injection (Fig. 2(c)). These results highlight that infinite elements are advantageous for subsurface numerical simulation as they provide a robust and efficient means of approximating unbounded domains for time-scales of reservoir lifetime.

Our results show that permeability significantly affects the temporal evolution of boundary effects on pore pressure. Higher permeability leads to an underestimation of numerical results in comparison to the analytical solution already for shorter injection times (Fig. 3). However, the asymptotic behaviour of the relative deviations for the different permeability values indicates a tendency to converge towards the same long-term value. Our example for the position at a distance of 500 m to the injection shows a long-term underestimation of pore pressure of about 10%.

The material benchmark of a point injection into full space with infinite element boundaries was performed for sandstones (Ruhr, Berea) which are frequently used in geothermal circulation, hydrocarbon extraction and gas storage projects. The analytical and numerical solutions demonstrated satisfactory alignment for all sandstones Fig. D.7. This line of inquiry merits further investigation, particularly when applied to other rock types, such as those relevant to EGS.

Common boundary conditions in 3D numerical models of subsurface fluid injection, such as half-space geometries and extended injection intervals, are evaluated in this study by comparison with full-space point-source analytical solutions. In the numerical half-space models (see inset in Fig. E.8), boundary effects are minimal during the early stages of injection. The presence of the top boundary in the half-space configuration causes the pore pressure to reach a steady state for shorter injection times than in the full-space configuration (Fig. E.8). Furthermore, the use of half-spaces introduces an asymmetry in the pressure evolution: the pore pressure reaches larger values at an observation point located horizontally from the injection source in comparison to an observation point located at the same distance vertically, towards the surface of the half-space (Fig. E.8).

An extended vertical injection interval (inset in Fig. E.9) leads to a reduction in pressure in the horizontal direction compared to a point injection. Furthermore, our numerical solution demonstrates that the pressure evolution in the direction parallel to the injection interval is significantly higher than the pressure evolution in horizontal planes near the injection zone. The most significant deviations (>5%) from the point-source solution were observed at distances less than 1.5 times the length of the injection interval (Fig. E.9).

4.3. Impact of permeability anisotropy on the modelling of pressure diffusion

In reservoir fluid flow permeability characteristics are essential, e.g. for optimizing the design of injection and production well arrays.⁶⁵ As drilling is an expensive activity in field development, reducing the uncertainty in subsurface pore pressure distribution can lead to savings and optimization of the design of production and injection wells e.g. in fractured carbonates. Furthermore, pore pressure is required to select appropriate mud weights and depths for casings.³⁴ The preferential flow paths and thus the permeability anisotropy also influence thermal breakthrough in geothermal energy systems and must be carefully considered in nuclear waste disposal, where permeability anisotropy within the containment zone can impact long-term isolation and safety. Thus, permeability anisotropy of reservoir rocks has significant practical impact and has been considered in earlier studies. In their numerical 2D model, Vabishchevich and Grigoriev⁶⁶ focus on fracture porosity and permeability of carbonates. Cao et al.⁶⁷ studied the flow in a unit cube domain with permeability anisotropy and concluded that ignoring the anisotropy might result in significant errors in flux determination. Cao et al.⁶⁸ modelled near-well flow in anisotropic reservoirs. Our study investigated the effect of anisotropy in analytical as well as in numerical models.

4.3.1. Analytical approach for pressure diffusion in media with permeability anisotropy

We present a closed-form 3D solution for the pore pressure diffusion for a medium with varying principal components of permeability (Eq. (13)) and validate it against FEM results. The pore pressure evolution for media with permeability anisotropy exhibits elliptical isosurfaces in the steady state (Eq. (16)). The length of the semi-axis of the ellipsoid depends on the product of the permeability components in the perpendicular direction. Previous studies by Rotherth and Shapiro²⁹ and Shapiro⁵⁶ derived solutions for pore pressure resulting from continuous fluid injection into a medium with a diagonal permeability tensor. Their pore pressure expression follows the similar temporal evolution to that of Eq. (13), but the amplitude differs. In these studies, the pore pressure is inversely proportional to the arithmetic mean of the principal components of the permeability tensor instead of the square root of the product of the three permeability components $(k_{xx}k_{yy}k_{zz})^{-1/2}$ of the solution derived in this paper (Eq. (13)). The corresponding solution of

these earlier formulations for the semi-axes of long-term pore pressure p_0 isosurfaces can be derived as following:

$$\begin{aligned} A' &= \frac{q_v \gamma}{p_0 4\pi \left(\frac{k_{xx} + k_{yy} + k_{zz}}{3} \right)^{3/2}} \sqrt{k_{xx}}, \\ B' &= \frac{q_v \gamma}{p_0 4\pi \left(\frac{k_{xx} + k_{yy} + k_{zz}}{3} \right)^{3/2}} \sqrt{k_{yy}}, \\ C' &= \frac{q_v \gamma}{p_0 4\pi \left(\frac{k_{xx} + k_{yy} + k_{zz}}{3} \right)^{3/2}} \sqrt{k_{zz}} \end{aligned} \quad (21)$$

Consequently, the semi-axes of the isosurfaces representing the long-term limits of pore pressure depend on all three principal components of the permeability tensor according to Rotherth and Shapiro's solution. In contrast, in the solution presented here, the long-term limit of the pore pressure in a principal direction depends only on the permeability components of the perpendicular axes (Eq. (16)). The ratios of the semi-axis in both solutions correspond to $A : B : C = A' : B' : C' = \sqrt{k_{xx}} : \sqrt{k_{yy}} : \sqrt{k_{zz}}$.

Our solution satisfies the given diffusion equation exactly under the prescribed boundary conditions at the source. The results of absolute (non-normalized) analytical and numerical approaches fit in the spatio-temporal evolution (Figs. 5, F.10, F.11(d), (e), (f)). This highlights the strength of our analytical approach in comparison to earlier studies, where matches are provided for normalized values of pore pressure.

4.3.2. Numerical modelling using permeability anisotropy

For models with permeability anisotropy, we observed that the results using a mesh configuration validated under isotropic conditions were less accurate along the directions of highest permeability. For $n = 10$, the largest deviation occurred along the high-permeability y -axis, whereas for $n = 0.1$ the largest deviations occurred in the high-permeability x - and z -directions, as illustrated in Fig. F.10. To improve numerical precision, we tested different mesh-refinement strategies: uniform refinement in all directions and refinement aligned with the direction of higher permeability. The most efficient strategy was using the mesh structure adapted to the permeability tensor, which not only improved agreement with the analytical solution (Fig. F.11) but also significantly reduced computational cost Appendix C. However, because the directionally refined mesh was tested specifically for $n = 10$, its performance for other anisotropy ratios and orientations would require corresponding adaptation of the refinement direction.

We have used values of permeability anisotropy of one order of magnitude in this study and obtained good agreement of the numerical and analytical pore pressure changes with time. We compared our values with earlier investigations. Numerous studies^{23,69–73} show that the sandstone permeability depends on the orientation relative to bedding, with a higher permeability parallel to the bedding. Goupil et al.⁷⁴ summarize the permeability anisotropy ratios between 0.23 and 0.75 for various sandstones. Faulting and fracturing may introduce further directional contrasts. A worldwide compilation by Scibek⁷⁵ demonstrates the broad range of fault-zone permeabilities and the strong contrasts that may occur between fault zones and their surrounding host rocks.⁷⁶ Within faulted porous sandstones, Farrell et al.⁷⁷ measured permeability parallel to fault dip that was up to ten times higher than permeability along fault strike. Natural fractures can strongly influence reservoir-scale permeability and fluid flow by providing preferential flow pathways.⁷⁸ Permeability anisotropy of approximately one order of magnitude has also been inferred from hydraulic tests in fractured aquifers.⁷⁹ More broadly, Liu et al.⁸⁰ report horizontal-to-vertical reservoir permeability ratios ranging from approximately 1 to 100. Reported examples include the Krechba caprock,⁸¹ saline aquifers in the Bécancour region⁸² and the Matulla Sandstones⁸³. Therefore, the anisotropy factor of 10 selected for our study is considered representative of experimentally measured and field-inferred permeability contrasts, rather than as a site-specific estimate.

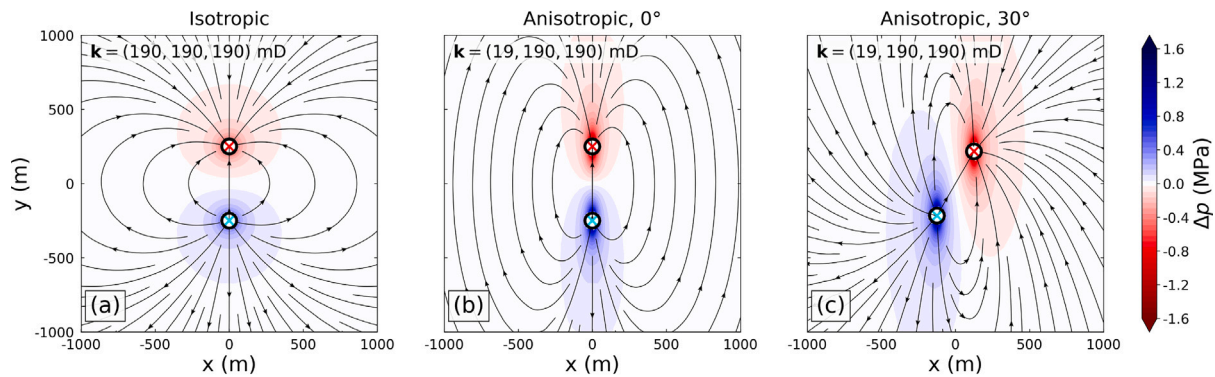


Fig. 6. Distribution of pressure changes after 10 years of continuous fluid circulation at a rate of 60 l/s in a geothermal doublet with a well spacing of 500 m. (a) Isotropic case. (b) Anisotropic case with the doublet aligned with the plane of high permeability. (c) Anisotropic case with the doublet aligned in the direction 30° clockwise from the high-permeability plane. The streamlines indicate Darcy's velocity field in each case. The crosses in cyan and red colours illustrate injection and production wells, respectively.

4.3.3. Influence of permeability anisotropy on geothermal doublet-induced pressure evolution

Geothermal wells are commonly drilled into or near major faults and fault zones to take advantage of naturally enhanced permeability. In this study, we do not resolve localized permeability variations associated with individual fault cores or damage zones. Instead, the homogeneous permeability tensor is interpreted as an effective representation of the directional hydraulic properties of a fractured reservoir. To simulate the influence of permeability anisotropy for geothermal applications, we extend the setup of model *C-anis-inf-mesh1* to a generic geothermal doublet configuration. The aim is to demonstrate how the permeability tensor influences the spatial distribution of induced pore pressure for different orientations of the pressure source–sink system. Fig. 6 presents the pressure distribution after 10 years of continuous doublet operation for isotropic and anisotropic media, where the anisotropic permeability is given by $\mathbf{k} = (19, 190, 190)$ mD. This tensor retains an anisotropy factor of 10 but represents a high-permeability yz -plane with lower permeability in the normal, x , direction. Because the doublet is located in the horizontal xy -plane, its orientation is described relative to the high-permeability y -direction.

Fig. 6.a shows the pressure distribution for the isotropic case. Positive and negative pore pressure changes superpose each other and exhibit nearly circular patterns around the injection and production wells, respectively. For the same circulation rate in an anisotropic medium, when the doublet is aligned with the plane of high permeability (Fig. 6.b), larger pressure changes develop along the high-permeability direction, extending over a larger distance in the same direction. In the case, when the doublet and the high-permeability plane are misaligned by 30°, the pressure distribution reflects the combined influence of the doublet geometry and the permeability tensor (Fig. 6.c). Resulting positive pressure changes extend to a larger area compared to the isotropic case and the case where the doublet was aligned in the direction of the preferential flow.

This has important implications for induced seismicity as it concerns effective stresses. Permeability anisotropy primarily controls the magnitude and spatial extent of these pressure perturbations, whereas the orientation of the geothermal doublet determines how the injection and production induced pressure fields overlap. As a result, pressure perturbations extend furthest along the high-permeability direction, reorients and occupies a larger area when the doublet is misaligned with the principal permeability axis than when it is aligned with the high-permeability direction (Fig. 6). This highlights the importance of accounting for permeability anisotropy when assessing the spatial pattern and likelihood of induced seismicity.

5. Conclusion and outlook

This study provides a comparison of key modelling assumptions influencing predictions of pore pressure evolution. It presents a closed-form three-dimensional analytical solution for pore pressure evolution in media with permeability anisotropy. The solution is validated against high-resolution finite element simulations using infinite elements. The systematic assessment of pore pressure evolution with both numerical and analytical methods led to practical insights and recommendations for future 3D simulations of geothermal, wastewater, and CO₂ injection scenarios:

(a) Directional flow behaviour in porous media strongly controls the temporal and spatial evolution of reservoir response. Application of the benchmarked framework to a generic geothermal doublet further demonstrates that the permeability anisotropy influences both the magnitude and the spatial distribution of effective stress changes, potentially enlarging and reorienting the zone in which critically stressed faults may be reactivated. Although a complete assessment of fault stability requires consideration of the coupled stress changes, fault hydraulic properties, and other site-specific conditions.

(b) The obtained analytical solution of the anisotropic diffusion equation provides a theoretical foundation and physical insight into the role of permeability anisotropy for pressure evolution. Its low computational cost also makes it suitable for rapid first-order estimates and initial sensitivity analyses in the early phases of geothermal project design before more complex numerical models are developed.

(c) Our findings demonstrate that with increasing factor of permeability anisotropy, numerical models tend to underestimate pore pressure, especially along the axis of highest permeability if the mesh is not adapted.

(d) High resolution poroelastic FEM simulations show excellent agreement with the analytical results, confirming the analytical model's suitability for preliminary assessments and for calibrating numerical models in anisotropic geological settings.

(e) Infinite elements at model boundaries significantly improve numerical stability, accuracy and computational efficiency, particularly for simulations covering injection periods from several months to multiple years, compared with conventional approaches where the model size and number of elements are increased with respect to the region of interest and are sensitive to model input parameters like permeability. We therefore recommend the use of infinite elements to simulate unbounded media for reservoir scale models. Alternatively, finite-domain models should be applied with careful consideration of

their range of validity, and with the modeller's awareness of the associated limitations, particularly in long-term fluid-injection simulations where model effects may substantially affect the evolution of pore pressure.

The present results are the base for future work considering coupling of pore pressure evolution to total stress to quantify resulting changes in effective normal and shear stresses in hydraulically anisotropic media, particularly under long-term operational conditions. Accurate modelling of long-term fluid flow is also essential for simulating advection-driven heat transport processes. Incorporating stress-dependent and time-varying permeability as well as geological complexities such as fault zones and heterogeneous rock properties will further enhance predictive capability. The approach developed here can be applied to assess reservoir performance and injection-induced stress changes that might cause induced seismicity in anisotropic geological settings. Thus, it has direct relevance to geothermal energy and underground gas storage applications in fractured formations.

CRediT authorship contribution statement

Tatia Sharia: Writing – review & editing, Writing – original draft, Visualization, Software, Methodology, Investigation, Formal analysis, Conceptualization. **Thomas Niederhuber:** Writing – review & editing, Software, Methodology, Investigation, Conceptualization. **Birgit Müller:** Writing – review & editing, Writing – original draft, Supervision, Methodology, Investigation, Conceptualization. **Andreas Rietbrock:** Writing – review & editing, Supervision, Resources, Methodology, Funding acquisition, Conceptualization.

Declaration of generative AI and AI-assisted technologies in the manuscript preparation process

During the preparation of this work the authors used ChatGPT (OpenAI) in order to improve clarity and conciseness of the text. After using this tool/service, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

The authors would like to thank Serge Shapiro for the valuable discussions on anisotropic modelling, and Karsten Reiter for reviewing the manuscript and providing insightful comments. This research was supported by the German Federal Ministry of Research, Technology and Space (BMFTR) under the name “CoolStress” with the grant number 03G0935A. The authors thank the Gauss Centre for Supercomputing e.V. (GCS) (www.gauss-centre.eu) for providing computing time on the Supercomputer JUWELS⁸⁴ at the Jülich Supercomputing Centre via the John von Neumann Institute for Computing. JUWELS is supported by GCS through funding by the German Federal Ministry of Research, Technology and Space and the Ministry of Culture and Science of the State of North Rhine-Westphalia.

Appendix A. Governing equations for the poroelastic numerical model

Here we present the governing equations of coupled poroelastic simulations solved using COMSOL Multiphysics[®].⁸⁵ The formulation is based on Biot¹⁶'s theory of poroelasticity.

The stress-strain relation contains an additional contribution from pore pressure:

$$\sigma = C : \epsilon - \alpha p I \quad (\text{A.1})$$

Here, σ denotes the total stress tensor, ϵ is the strain tensor, α is the Biot-Willis coefficient, p is the pore fluid pressure, and I is the identity tensor. C corresponds to the elasticity tensor evaluated under drained conditions.

The change in fluid content, ζ , is related to the pore pressure and volumetric strain, ϵ_{vol} , as

$$\zeta = \alpha \epsilon_{vol} + S_p p \quad (\text{A.2})$$

where the Biot-Willis coefficient relates the volumetric deformation of a porous material to changes in fluid content under constant pore pressure conditions. The poroelastic storage coefficient, S_p , characterizes the change of the fluid content with the change of the pressure under constant volumetric strain.

The poroelastic storage coefficient is given by

$$S_p = \frac{\epsilon}{K_f} + (\alpha - \epsilon) \frac{1 - \alpha}{K} \quad (\text{A.3})$$

where ϵ denotes porosity, K_f is the fluid bulk modulus, and K is the bulk modulus of drained porous material.

The momentum balance equation is

$$\nabla \cdot \sigma + \mathbf{f}_v = 0 \quad (\text{A.4})$$

where $\mathbf{f}_v$ is the body force.

Fluid flow and pore pressure field in porous medium are governed by mass conservation equation:

$$\frac{\partial}{\partial t}(\rho_f \epsilon) + \nabla \cdot (\rho_f \mathbf{v}_d) = Q_m \quad (\text{A.5})$$

where Q_m is the mass sink or source and ρ_f is density of the fluid. Darcy's velocity $\mathbf{v}_d$ is given by

$$\mathbf{v}_d = -\frac{\mathbf{k}}{\gamma}(\nabla p - \rho_f \mathbf{g}) \quad (\text{A.6})$$

where $\mathbf{k}$ is the permeability tensor.

Combining Eq. (A.2), Eq. (A.5) and Eq. (A.6) yields the governing pressure equation:

$$\rho_f S_p \frac{\partial p}{\partial t} + \nabla \cdot (\rho_f \mathbf{v}_d) = -\rho_f \alpha \frac{\partial \epsilon_{vol}}{\partial t} + Q_m \quad (\text{A.7})$$

In the present model, gravitational effects are neglected. Consequently, the corresponding terms in Eq. (A.4) and Eq. (A.6) vanish. In addition, permeability and dynamic viscosity are assumed constant. For point-source injection into a homogeneous unbounded poroelastic medium, the displacement field can be treated as irrotational.⁵⁰ Mechanical equilibrium then gives

$$\frac{\partial \epsilon_{vol}}{\partial t} = \frac{\alpha}{K + 4G/3} \frac{\partial p}{\partial t} \quad (\text{A.8})$$

Substitution of Eq. (A.8) into Eq. (A.7) eliminates the volumetric strain variable and gives

$$\rho_f S \frac{\partial p}{\partial t} + \nabla \cdot (\rho_f \mathbf{v}_d) = Q_m \quad (\text{A.9})$$

where

$$S = S_p + \frac{\alpha^2}{K + 4G/3} \quad (\text{A.10})$$

Thus, S_p is the poroelastic storage coefficient at constant volumetric strain, whereas S is the effective storage coefficient governing pressure diffusion after accounting for poroelastic deformation. The latter is equivalent to Eq. (3) and is used in the uncoupled pressure diffusion equation Eq. (2).

Appendix B. Additional information on mesh generation

For the isotropic models, the general procedure for mesh generation involved: firstly, creating a surface mesh of the cylinder using triangular elements; secondly, sweeping the surface mesh along the vertical direction inside the cylinder; thirdly, filling the remaining physical volume with tetrahedral mesh elements; and finally, extruding the outermost surface mesh of the physical volume throughout the virtual domain of infinite elements.

In isotropic point-source models (*B-iso-inf* and *B-iso-inf-half-space*), the cylinder surrounding the injection well has a radius of 100 m and a height of 60 m. Inside the cylinder, the mesh element sizes range from 14 m to 27 m. In model *B-iso-ext-inf*, which features a 250 m long injection well, the cylinder's radius and height are increased to 250 m and 500 m, respectively. Inside this larger cylinder, the mesh element sizes vary from 25 m to 52 m. In the remaining regions of the models, the mesh element sizes increase up to 550 m.

The resulting number of mesh elements is 78 796 in model *B-iso-inf*, 71 250 in model *B-iso-inf-half-space*, and 114 950 in model *B-iso-ext-inf*.

For the case of permeability anisotropy, we examined 3 variations of mesh structures: (1) a mesh generated similarly to model *B-iso-inf* with 78 809 mesh elements (Fig. F.11(a), model *C-anis-inf*); (2) a refined version of the original mesh by splitting the longest sides. This resulted in a total of 235 723 elements, with element sizes ranging from 7 m to 400 m, increasing uniformly in all directions (Fig. F.11(b), model *C-anis-inf-mesh1*); (3) a mesh in which element sizes increase with the square of the distance from the y -axis. This mesh contains 135,511 elements, with side lengths along the principal y -axis mostly under 50 m, ranging from 8 m (minimum) to 130 m (maximum). In the x - and z - axes, element sizes increase with distance up to 700 m (Fig. F.11(c), model *C-anis-inf-mesh2*).

Appendix C. Model sizes and computation times

We present an overview of the computation times and degrees of freedom for the different models used in this study. Detailed descriptions of model setups and mesh configurations can be found in Section 2.3 and Appendix B. Table C.3 summarizes the results. The first column lists the runtime (in seconds) for each model executed on 24 cores, the second shows the number of degrees of freedom (DoF), while the last two columns display the computation time and DoF ratios relative to the isotropic model with infinite elements (*B-iso-inf*), which serves as the reference. The largest model with finite volume *A-iso-8x8x8* we have tested requires approximately 1.22 times more computation time than the reference isotropic model with infinite elements. The computation times of the models with similar setup but different material parameters (*B-iso-inf-weber*, *B-iso-inf-ruhr*, *B-iso-inf-ohio* *B-iso-inf*) are nearly similar. The model with an extended injection interval (*B-iso-ext-inf*) has 35% more degrees of freedom than the reference model but requires nearly twice longer computation time. Among the three anisotropic models with permeability increased tenfold along the y -axis, the initial model *C-anis-inf* and the model with mesh refinement along the y -axis *C-anis-inf-mesh2* exhibit similar runtimes, whereas the uniformly refined anisotropic model *C-anis-inf-mesh1* requires about three times longer computational time.

In addition, the sensitivity of the numerical solution to the infinite element configuration was assessed using the setup of the model *B-iso-inf*. The geometrical thickness of the infinite element domain was varied between 50, 100, and 200 m using five swept mesh elements. The discretization was also varied between three, five, and seven swept elements while maintaining a geometrical thickness of 100 m. None of the tested configurations showed a systematic increase in relative pore pressure error with distance that would indicate the influence of the outer boundary within the investigated range of 50 m to 2 km after 10 years. However, the configurations differed in their numbers of degrees of freedom and computation times. The 100-m-thick infinite

Table C.3

Computation times (T in seconds) and degrees of freedom (DoF) of the numerical models. The point-source injection model with infinite elements (model *B-iso-inf*) is taken as a reference, and the last two columns show the ratios of computation times and degrees of freedom of the other models compared to the corresponding values of the reference model.

Model	T (s)	DoF	T/T_0	DoF/DoF_0
<i>B-iso-inf</i>	9245	698 922	1.00	1.00
<i>A-iso-4x4x4</i>	1835	208 884	0.20	0.30
<i>A-iso-8x8x8</i>	11 318	859 012	1.22	1.23
<i>B-iso-inf-weber</i>	9064	698 922	0.98	1.00
<i>B-iso-inf-ruhr</i>	9550	698 922	1.03	1.00
<i>B-iso-inf-ohio</i>	9314	698 922	1.01	1.01
<i>B-iso-inf-half-space</i>	7702	596 659	0.83	0.85
<i>B-iso-ext-inf</i>	17 598	941 872	1.90	1.35
<i>C-anis-inf</i>	8573	698 982	0.93	1.00
<i>C-anis-inf-mesh1</i>	25 778	1 640 980	2.79	2.35
<i>C-anis-inf-mesh2</i>	8137	806 046	0.88	1.15

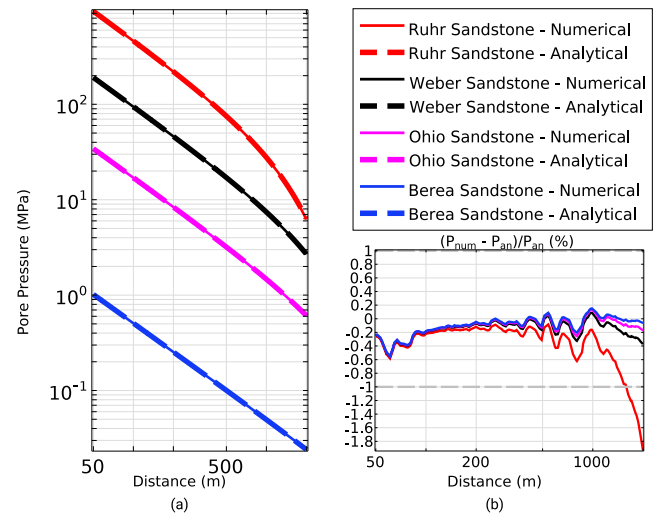


Fig. D.7. (a) Comparison of numerical and analytical results for the spatial distribution of pore pressure after 10 years of continuous injection for four different materials (see Table 1) and (b) the corresponding relative deviations of numerical results from the analytical solution.

element domain discretized using five swept elements was retained for the simulations presented in this study. A detailed description of these sensitivity tests, including the relative errors of pore pressure solution and results of the numerical performance, is provided as supplementary material in Appendix G.

Appendix D. Benchmarking infinite domain models on different material parameters

Fig. D.7 illustrates the spatial distribution of pore pressure over the distance from the injection well after 10 years and the corresponding relative deviations from the analytical solution (Eq. (5)). The offset on the logarithmic y -axis reflects the scaling factor in permeability among different materials. The magnitudes of the relative deviations are highest in the case of the lowest permeability. However, deviations remain below 1.8% for all materials after 10 years and distances between 50 m and 2000 m from the injection interval.

Appendix E. Infinite domain models with different boundary conditions

The model *B-iso-inf-half-space* simulates the point-source injection into a half space, with the source located 2 km beneath the surface

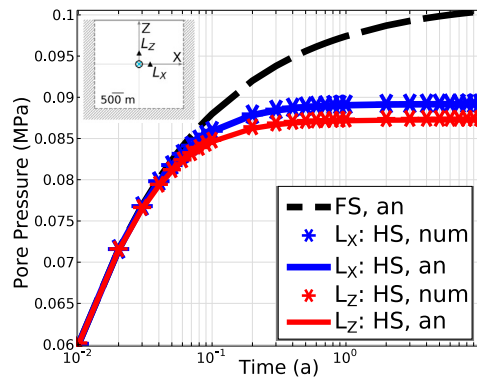


Fig. E.8. Temporal evolution of pore pressure at observation points L_x and L_z , each located 500 m from the centre of the injection interval in the half-space (HS) model. The model has a zero pore pressure boundary condition at the surface situated 2 km above the injection interval. For comparison, the dashed line represents the analytical solution (an) for the full space (FS). Blue and red asterisk data points are numerical results (num) of half-space model in x and z directions accordingly. The blue and red solid lines represent the full-space analytical solutions combined with a mirror-image source to reproduce the half-space geometry. The inset illustrates the injection interval, the locations of the observation points, and the boundaries of the models at finite distance, as well as those extended by infinite elements.

boundary prescribed with a zero pore pressure condition. This boundary condition can be satisfied analytically by introducing an additional mirror-image point source located 2 km above the ground. Fig. E.8 shows the temporal evolution of the pore pressure at observation points L_x and L_z , positioned 500 m from the injection point in the horizontal and vertical directions, respectively. The observation point along the vertical axis, which is closest to the surface, deviates from the analytical solution earliest and in 10 years produces approximately 13% less pore pressure level, while the one along the horizontal axis results in approximately 11% less pore pressure than the full-space solution.

Fig. E.9 shows the pore pressure values of the model with 250 m long injection interval (model *B-iso-ext-inf*) compared to the point-source solution. The insets of the figure show the relative deviations from the point-source solution after 10 years of injection and a schematic illustration of the injection interval, coordinate axes and model boundaries. These deviations are less pronounced in the perpendicular (x) direction than in the parallel (z) direction. For example at 150 m from the centre of the injection interval, the relative deviation is approximately -9% in the x -direction and $+44\%$ in the z -direction. The deviation reaches $\pm 5\%$ at distances of 213 m (x) and 337 m (z), and falls below $\pm 1\%$ at 520 m (x) and 710 m (z), respectively.

Appendix F. Numerical results with different mesh structures

The results of the model *C-anis-inf* with a mesh structure similar to the isotropic model *B-iso-inf*, show the highest deviations along the y - and z -axis with different factors of anisotropy (Fig. F.10). For a scaling factor of 2, the numerical model performs similarly to the isotropic one with the magnitudes of deviations below 0.37% in both y - and z -directions. However, with larger degrees of anisotropy, the magnitudes of deviations increase and vary depending on the axes. For $n = 0.1$, pore pressure diffusion is fastest in x - and z -directions, resulting in iso-surfaces elongated along the same axes, with numerical solution underestimating analytical solution by up to 3.5% along the z -axis. Similarly, for $n = 10$, larger deviation magnitudes (up to 2.54%) are observed along the y -axis, where pore pressure diffusion is fastest and iso-surfaces are also elongated along the y -axis compared to x - and z -axes. In both cases (scaling factors of 0.1 and 10), a higher degree of

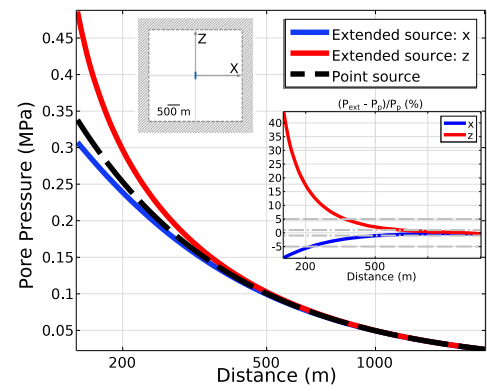


Fig. E.9. Comparison of pore pressure evolution for point injection (analytical results) and extended source injection (numerical results of model *B-iso-ext-inf*). For the extended source the pore pressure evolution is shown along the z -axis, parallel to the injection interval and along the x -axis, perpendicular to it. The insets show the relative deviations of the numerical results of the extended source model from the analytical solution of the point source (Eq. (5)) in both x - and z -directions, as well as a schematic illustration of the model.

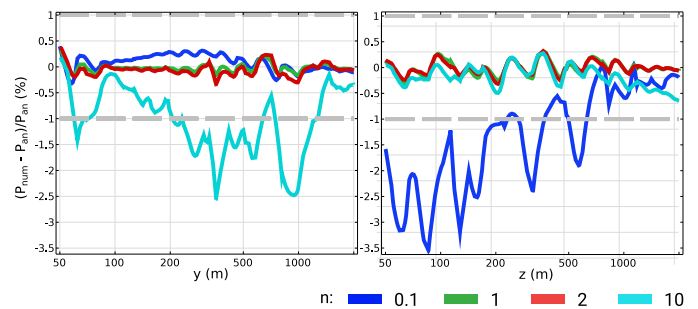


Fig. F.10. Relative deviation of numerically solved pore pressure (model *C-anis-inf*) with respect to the analytical solution Eq. (13) along the y - (left figure) and the z -axis (right figure) for different cases of permeability anisotropy. The colours denote the scaling factors of permeability components $k_{yy}/k_{xx,zz} = n$.

anisotropy led to a greater underestimation of the analytical solution along the direction of faster fluid flow.

Since the results of the model *C-anis-inf* showed elevated deviations against the analytical solution along the axis of the higher permeability component, we investigated the effect of mesh refinement on model accuracy. Specifically, we applied different meshing strategies for the case where $n = k_{yy}/k_{xx,zz} = 10$. In the first approach (model *C-anis-inf-mesh1*), the mesh was uniformly refined in all directions, resulting in total of 235723 mesh elements. In the second approach (model *C-anis-inf-mesh2*), the mesh was refined along the principal axis of higher permeability, yielding 135511 mesh elements, approximately 40% less elements than in the first approach. Nevertheless, results of the second approach (Fig. F.11) demonstrated better agreement with the analytical solution along the y axis, with the maximum relative deviation magnitudes of 0.73% compared to 1.56% for the first approach. Along the x and z directions, the performance of the both models was similar, with relative deviations ranging between -0.65% and 0.15% . In addition, the model *C-anis-inf-mesh2* was about 3 times more computationally efficient than the model *C-anis-inf-mesh1* (see Appendix C).

Appendix G. Supplementary data

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.gete.2026.100913>.

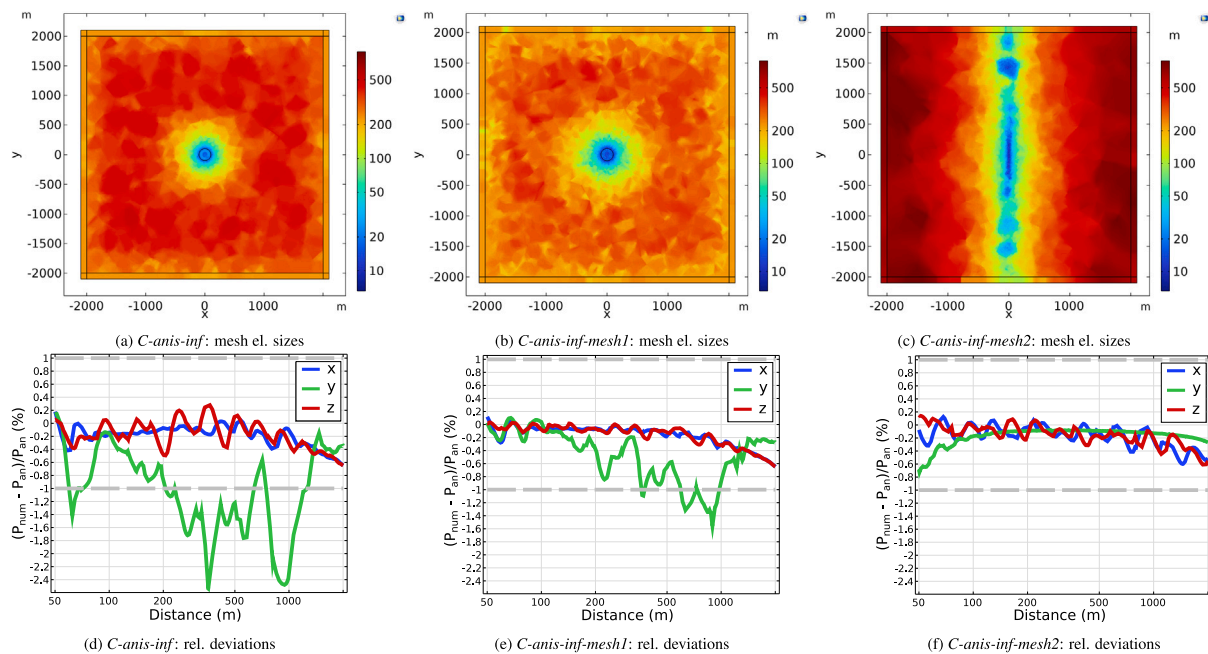


Fig. F.11. Comparison of results for different model discretizations: the top row shows mesh sizes on the x-y cross section at $z = 0$; the colour scale denotes the length of the longest edge of a mesh element. The bottom row presents the corresponding relative deviations between numerical and analytical results for a permeability anisotropy of $k_{yy}/k_{xx,zz} = 10$, plotted as a function of distance from the injection point along the x-, y-, z- axes. (a) and (d) correspond to the initial model, (b) and (e) - to the model with uniformly refined mesh structure, (c) and (f) - to the model with a mesh refinement along the y-axis.

Data availability

No data was used for the research described in the article.

References

- Raleigh CB, Healy JH, Bredehoeft JD. An experiment in earthquake control at Rangely, Colorado. *Science*. 1976;191(4233):1230–1237. <http://dx.doi.org/10.1126/science.191.4233.1230>.
- Suckale J. Induced seismicity in hydrocarbon fields. *Adv Geophys*. 2009;51:55–106. [http://dx.doi.org/10.1016/S0065-2687\(09\)05107-3](http://dx.doi.org/10.1016/S0065-2687(09)05107-3).
- Ellsworth WL. Injection-induced earthquakes. *Science*. 2013;341(6142):1225942. <http://dx.doi.org/10.1126/science.1225942>.
- Keranan K, Savage H, Abers G, Cochran E. Potentially induced earthquakes in Oklahoma, USA: Links between wastewater injection and the 2011 Mw 5.7 earthquake sequence. *Geology*. 2013;41:699–702. <http://dx.doi.org/10.1130/G34045.1>.
- Ruiz-Barajas S, Sharma N, Convertito V, Zollo A, Benito B. Temporal evolution of a seismic sequence induced by a gas injection in the eastern coast of Spain. *Sci Rep*. 2017;7. <http://dx.doi.org/10.1038/s41598-017-02773-2>.
- Van Wees J, Buijze L, Thienen-Visser K, Nepveu M, Wassing B, Orlie B, Fokker P. Geomechanics response and induced seismicity during gas field depletion in the Netherlands. *Geothermics*. 2014;52:206–219. <http://dx.doi.org/10.1016/j.geothermics.2014.05.004>.
- McGarr A. Maximum magnitude earthquakes induced by fluid injection. *J Geophys Res: Solid Earth*. 2014;119(2):1008–1019. <http://dx.doi.org/10.1002/2013JB010597>.
- Häring M, Schanz U, Ladner F, Dyer B. Characterisation of the Basel 1 enhanced geothermal system. *Geothermics*. 2008;37:469–495. <http://dx.doi.org/10.1016/j.geothermics.2008.06.002>.
- Buijze L, van Bijsterveldt L, Cremer H, Paap B, Veldkamp H, Wassing BB, van Wees J-D, van Yperen GC, ter Heege JH, Jaarsma B. Review of induced seismicity in geothermal systems worldwide and implications for geothermal systems in the Netherlands. *Neth J Geosci*. 2019;98:e13. <http://dx.doi.org/10.1017/njg.2019.6>.
- Weingarten M, Ge S, Godt JW, Rubinstein JL. High-rate injection is associated with the increase in U.S. mid-continent seismicity. *Science*. 2015;348(6241):1336–1340. <http://dx.doi.org/10.1126/science.1251345>.
- Ge S, Saar MO. Review: Induced seismicity during geoelectric development—A hydromechanical perspective. *J Geophys Res: Solid Earth*. 2022;127(3). <http://dx.doi.org/10.1029/2021JB023141>.
- Zoback MD, Harjes H-P. Injection-induced earthquakes and crustal stress at 9 km depth at the KTB deep drilling site, Germany. *J Geophys Res: Solid Earth*. 1997;102(B8). <http://dx.doi.org/10.1029/96JB02814>.
- Ziv A, Rubin AM. Static stress transfer and earthquake triggering: No lower threshold in sight? *J Geophys Res: Solid Earth*. 2000;105(B6):13631–13642. <http://dx.doi.org/10.1029/2000JB900081>.
- Grasso J-R, Sornette D. Testing self-organized criticality by induced seismicity. *J Geophys Res: Solid Earth*. 1998;103(B12). <http://dx.doi.org/10.1029/97JB01344>.
- Biot MA. General theory of three-dimensional consolidation. *J Appl Phys*. 1941;12(2):155–164. <http://dx.doi.org/10.1063/1.1712886>.
- Biot MA. Mechanics of deformation and acoustic propagation in porous media. *J Appl Phys*. 1962;33(4):1482–1498. <http://dx.doi.org/10.1063/1.1728759>.
- Rice J, Cleary M. Some basic stress diffusion solutions for fluid-saturated elastic porous media with compressible constituents. *Rev Geophys Space Phys*. 1976;14. <http://dx.doi.org/10.1029/RG014i002p00227>.
- Cleary MP. Fundamental solutions for a fluid-saturated porous solid. *Int J Solids Struct*. 1977;13(9):785–806. [http://dx.doi.org/10.1016/0020-7683\(77\)90065-8](http://dx.doi.org/10.1016/0020-7683(77)90065-8).
- Rudnicki JW. Fluid mass sources and point forces in linear elastic diffusive solids. *Mech Mater*. 1986;5(4):383–393. [http://dx.doi.org/10.1016/0167-6636\(86\)90042-6](http://dx.doi.org/10.1016/0167-6636(86)90042-6).
- Segall P, Lu S. Injection-induced seismicity: Poroelectric and earthquake nucleation effects. *J Geophys Res: Solid Earth*. 2015;120(7):5082–5103. <http://dx.doi.org/10.1002/2015JB012060>.
- Goebel T, Weingarten M, Chen X, Haffner J, Brodsky E. The 2016 Mw5.1 Fairview, Oklahoma earthquakes: Evidence for long-range poroelastic triggering at >40 km from fluid disposal wells. *Earth Planet Sci Lett*. 2017;472:50–61. <http://dx.doi.org/10.1016/j.epsl.2017.05.011>.
- Kim T, Avouac J-P. Stress-based and convolutional forecasting of injection-induced seismicity: Application to the Otaniemi geothermal reservoir stimulation. *J Geophys Res: Solid Earth*. 2023;128(4):e2022JB024960. <http://dx.doi.org/10.1029/2022JB024960>.
- Clavaut J-B, Mainault A, Zamora M, Rasolofosaon P, Schlitter C. Permeability anisotropy and its relations with porous medium structure. *J Geophys Res: Solid Earth*. 2008;113(B1). <http://dx.doi.org/10.1029/2007JB005004>.
- Heffer K, Lean K. Earth stress orientation - a control on, and guide to, flooding directionality in a majority of reservoirs. In: Linville B, ed. *Reservoir Characterization III*. PennWell Books; 1993.
- Kirkby A, Massiot C. Direction-dependent permeability and resistivity of fractured rocks tuned to New Zealand geothermal reservoirs. *Geothermics*. 2026;136:103620. <http://dx.doi.org/10.1016/j.geothermics.2026.103620>.
- Shapiro SA, Huenges E, Borm G. Estimating the crust permeability from fluid-injection-induced seismic emission at the KTB site. *Geophys J Int*. 1997;131(2):F15–F18. <http://dx.doi.org/10.1111/j.1365-246X.1997.tb01215.x>.
- Shapiro SA, Audigane P, Royer J-J. Large-scale in situ permeability tensor of rocks from induced microseismicity. *Geophys J Int*. 1999;137(1):207–213. <http://dx.doi.org/10.1046/j.1365-246x.1999.00781.x>.

28. Mukuhira Y, Yang M, Ishibashi T, Okamoto K, Moriya H, Kumano Y, Asanuma H, Shapiro SA, Rubinstein JL, Ito T, Yan K, Zuo Y. Scaling microseismic cloud shape during hydraulic stimulation using in situ stress and permeability. *J Geophys Res: Solid Earth*. 2023;128(8). <http://dx.doi.org/10.1029/2023JB026839>.
29. Rothert E, Shapiro SA. Statistics of fracture strength and fluid-induced microseismicity. *J Geophys Res: Solid Earth*. 2007;112(B4). <http://dx.doi.org/10.1029/2005JB003959>.
30. Carcione JM, Gei D. Theory and numerical simulation of fluid-pressure diffusion in anisotropic porous media. *Geophysics*. 2009;74(5). <http://dx.doi.org/10.1190/1.3192911>.
31. Fokker PA, Salina Borello E, Verga F, Viberti D. Harmonic pulse testing for well performance monitoring. *J Pet Sci Eng*. 2018;162:446–459. <http://dx.doi.org/10.1016/j.petrol.2017.12.053>.
32. Li BQ, Khoshmanesh M, Avouac J-P. Surface deformation and seismicity induced by poroelastic stress at the Raft River geothermal field, Idaho, USA. *Geophys Res Lett*. 2021;48(18). <http://dx.doi.org/10.1029/2021GL095108>.
33. Rutqvist J, Majer EL, Oldenburg CM, Peterson J, Vasco DW, Lawrence. Integrated modeling and field study of potential mechanisms for induced seismicity at the geysers geothermal field, California. Lawrence Berkeley National Laboratory; 2006 <https://api.semanticscholar.org/CorpusID:250480302>.
34. Atashbari V, Tingay M. Pore pressure prediction in a carbonate reservoir. In: *SPE oil and gas India conference and exhibition*. Mumbai, India: 2012. <http://dx.doi.org/10.2118/150836-MS>.
35. Xia Y, Gao P, Jiang Z, Fan Q, Wei R, Li Q, Zhang L, Yu T, Zhao J, Yang L, Song Y. Numerical simulation of CO₂ storage in subseabed sediments in the Shenhu Area, South China Sea: The effect of reservoir permeability anisotropy. *Energy Fuels*. 2024;38(11):9933–9946. <http://dx.doi.org/10.1021/acs.energyfuels.4c00824>.
36. Zhang G, Lu P, Ji X, Zhu C. CO₂ plume migration and fate at Sleipner, Norway: Calibration of numerical models, uncertainty analysis, and reactive transport modelling of CO₂ trapping to 10,000 years. *Energy Procedia*. 2017;114:2880–2895. <http://dx.doi.org/10.1016/j.egypro.2017.03.1410>, 13th International Conference on Greenhouse Gas Control Technologies, GHGT-13, 14–18 November 2016, Lausanne, Switzerland.
37. Feng Y, Chen L, Suzuki A, Kogawa T, Okajima J, Komiya A, Maruyama S. Numerical analysis of gas production from reservoir-scale methane hydrate by depressurization with a horizontal well: The effect of permeability anisotropy. *Mar Pet Geol*. 2019;102:817–828. <http://dx.doi.org/10.1016/j.marpetgeo.2019.01.041>.
38. Bakker M, Bot B. The effective vertical anisotropy of layered aquifers. *Groundwater*. 2025;63(1):68–75. <http://dx.doi.org/10.1111/gwat.13432>.
39. Renard P, Ababou R. Equivalent permeability tensor of heterogeneous media: Upscaling methods and criteria (review and analyses). *Geosciences*. 2022;12(7). <http://dx.doi.org/10.3390/geosciences12070269>.
40. Cappa F, Rutqvist J. Impact of CO₂ geological sequestration on the nucleation of earthquakes. *Geophys Res Lett*. 2011;38(17). <http://dx.doi.org/10.1029/2011GL048487>.
41. Boyet A, De Simone S, Vilarrasa V. Physics-based modeling to understand and to propose forecasting methods of induced seismicity. *Seismol Res Lett*. 2023;94(6):2666–2678. <http://dx.doi.org/10.1785/0220230109>.
42. Panja P, McLennan J, Green S. Influence of permeability anisotropy and layering on geothermal battery energy storage. *Geothermics*. 2021;90:101998. <http://dx.doi.org/10.1016/j.geothermics.2020.101998>.
43. Baisch S, Vörös R, Rothert E, Stang H, Jung R, Schellschmidt R. A numerical model for fluid injection induced seismicity at Soultz-sous-Forêts. *Int J Rock Mech Min Sci*. 2010;47(3):405–413. <http://dx.doi.org/10.1016/j.ijrmms.2009.10.001>.
44. Blank L, Rioseco E, Caiazza A, Wilbrandt U. Modeling, simulation, and optimization of geothermal energy production from hot sedimentary aquifers. *Comput Geosci*. 2021;25:1–38. <http://dx.doi.org/10.1007/s10596-020-09989-8>.
45. Chang KW, Segall P. Injection-induced seismicity on basement faults including poroelastic stressing. *J Geophys Res: Solid Earth*. 2016;121(4):2708–2726. <http://dx.doi.org/10.1002/2015JB012561>.
46. COMSOL AB. Infinite elements, perfectly matched layers, and absorbing layers. In: *COMSOL Multiphysics Reference Manual*. Stockholm, Sweden: 2022:424–427. COMSOL Multiphysics v. 6.1. <https://doc.comsol.com/6.1/docserver>.
47. Haji TK, Faramarzi A, Metje N, Chapman D, Rahimzadeh F. Development of an infinite element boundary to model gravity for subsurface civil engineering applications. *Int J Numer Anal Methods Geomech*. 2020;44(3):418–431. <http://dx.doi.org/10.1002/nag.3027>.
48. Gharti HN, Tromp J, Zampini S. Spectral-infinite-element simulations of gravity anomalies. *Geophys J Int*. 2018;215(2):1098–1117. <http://dx.doi.org/10.1093/gji/ggy324>.
49. Dong W, Selvadurai A. A combined finite and infinite element approach for modeling spherically symmetric transient subsurface flow. *Comput Geosci*. 2009;35(3):438–445. <http://dx.doi.org/10.1016/j.cageo.2008.02.037>.
50. Wang HF. *Theory of Linear Poroelasticity with Applications to Geomechanics and Hydrogeology*. 2000 <http://www.jstor.org/stable/j.ctt1jkttr4>.
51. Gassmann F. Über die elastizität poroser medien. *Vierteljahr der Naturforschenden Ges Zur*. 1951;96:1–23.
52. Nelson PH, Kibler JE. A catalog of porosity and permeability from core plugs in siliciclastic rocks. U.S. Geological Survey; 2003. <http://dx.doi.org/10.3133/ofr03420>.
53. Schmoker JW, Krystinik KB, Halley RB. Selected characteristics of limestone and dolomite reservoirs in the United States. *Am Assoc Pet Geol Bull*. 1985;69(5):733–741. <http://dx.doi.org/10.1306/AD4627F9-16F7-11D7-8645000102C1865D>.
54. Lucia F. *Carbonate Reservoir Characterization: An Integrated Approach*. 2007. <http://dx.doi.org/10.1007/978-3-540-72742-2>.
55. Carslaw HS, Jaeger JC. *Conduction of Heat in Solids*. Oxford Science Publications; 1959.
56. Shapiro SA. *Fluid-Induced Seismicity*. Cambridge University Press; 2015. <http://dx.doi.org/10.1017/CBO9781139051132>.
57. Zienkiewicz OC, Emson C, Bettess P. A novel boundary infinite element. *Internat J Numer Methods Engng*. 1983;19(3):393–404. <http://dx.doi.org/10.1002/nme.1620190307>.
58. Addis M. The stress-depletion response of reservoirs. In: *SPE annual technical conference and exhibition*. San Antonio, Texas: 1997. <http://dx.doi.org/10.2118/38720-MS>.
59. Altmann JB. *Poroelastic Effects in Reservoir Modelling* (Ph.D. thesis). Karlsruhe Institut für Technologie (KIT); 2010. <http://dx.doi.org/10.5445/IR/1000021248>.
60. Sibson R. Frictional constraints on thrust, wrench and normal faults. *Nature*. 1974;249:542–544. <http://dx.doi.org/10.1038/249542a0>.
61. Barton CA, Zoback MD, Moos D. Fluid flow along potentially active faults in crystalline rock. *Geology*. 1995;23(8):683–686. [http://dx.doi.org/10.1130/0091-7613\(1995\)023<0683:FFAPAF>2.3.CO;2](http://dx.doi.org/10.1130/0091-7613(1995)023<0683:FFAPAF>2.3.CO;2).
62. Fan Z, Eichhubl P, Newell P. Basement fault reactivation by fluid injection into sedimentary reservoirs: Poroelastic effects. *J Geophys Res: Solid Earth*. 2019;124(7):7354–7369. <http://dx.doi.org/10.1029/2018JB017062>.
63. Cao W, Durucan S, Shi J-Q, Cai W, Korre A, Ratouis T. Induced seismicity associated with geothermal fluids re-injection: Poroelastic stressing, thermoelastic stressing, or transient cooling-induced permeability enhancement? *Geothermics*. 2022;102:102404. <http://dx.doi.org/10.1016/j.geothermics.2022.102404>.
64. Fan Z, Parashar R. Analytical solutions for a wellbore subjected to a non-isothermal fluid flux: Implications for optimizing injection rates, fracture reactivation, and EGS hydraulic stimulation. *Rock Mech Rock Eng*. 2019;52(11):4715–4729. <http://dx.doi.org/10.1007/s00603-019-01867-9>.
65. Bell J, Babcock E. The stress regime of the Western Canadian Basin and implications for hydrocarbon production. *Bull Can Pet Geol*. 1986;34(3):364–378. <http://dx.doi.org/10.35767/gscpgbull.34.3.364>.
66. Vabishchevich PN, Grigoriev AV. Numerical modeling of fluid flow in anisotropic fractured porous media. *Numer Anal Appl*. 2016;9:45–56. <http://dx.doi.org/10.1134/S1995423916010055>.
67. Cao J, Gao H, Dou L, Zhang M, Li T. Modeling flow in anisotropic porous medium with full permeability tensor. *J Phys: Conf Ser*. 2019;1324. <http://dx.doi.org/10.1088/1742-6596/1324/1/012054>.
68. Cao J, Zhang N, James LA, Johansen TE. Modeling semi-steady state near-well flow performance for horizontal wells in anisotropic reservoirs. *Comput Geosci*. 2018;22:725–744. <http://dx.doi.org/10.1007/s10596-018-9722-z>.
69. Benson P, Meredith P, Platzman E. Relating pore fabric geometry to acoustic and permeability anisotropy in Crab Orchard Sandstone: A laboratory study using magnetic ferrofluid. *Geophys Res Lett*. 2003;30. <http://dx.doi.org/10.1029/2003GL017929>.
70. Benson P, Meredith P, Platzman E, White R. Pore fabric shape anisotropy in porous sandstones and its relation to elastic wave velocity and permeability anisotropy under hydrostatic pressure. *Int J Rock Mech Min Sci*. 2005;42:890–899. <http://dx.doi.org/10.1016/j.ijrmms.2005.05.003>.
71. Baud P, Meredith P, Townend E. Permeability evolution during triaxial compaction of an anisotropic porous sandstone. *J Geophys Res: Solid Earth*. 2012;117(B5). <http://dx.doi.org/10.1029/2012JB009176>.
72. Gehne S, Benson P. Permeability and permeability anisotropy in Crab Orchard sandstone: experimental insights into spatio-temporal effects. *Tectonophysics*. 2017;712–713:589–599. <http://dx.doi.org/10.1016/j.tecto.2017.06.014>.
73. Meng T, Yongbing X, Ma J, Yue Y, Liu W, Zhang J, Erbing L. Evolution of permeability and microscopic pore structure of sandstone and its weakening mechanism under coupled thermo-hydro-mechanical environment subjected to real-time high temperature. *Eng Geol*. 2021;280. <http://dx.doi.org/10.1016/j.enggeo.2020.105955>.
74. Goupil M, Heap M, Baud P. Permeability anisotropy in sandstones from the Soultz-sous-Forêts geothermal reservoir (France): implications for large-scale fluid flow modelling. *Geotherm Energy*. 2022;10. <http://dx.doi.org/10.1186/s40517-022-00243-1>.
75. Scibek J. Multidisciplinary database of permeability of fault zones and surrounding protolith rocks at world-wide sites. *Sci Data*. 2020;7. <http://dx.doi.org/10.1038/s41597-020-0435-5>.
76. Scibek J. Multidisciplinary database of permeability of fault zones and surrounding protolith rocks at world-wide sites. *Sci Data*. 2020. <http://dx.doi.org/10.6084/m9.figshare.10052033.v2>.
77. Farrell N, Healy D, Taylor C. Anisotropy of permeability in faulted porous sandstones. *J Struct Geol*. 2014;63:50–67. <http://dx.doi.org/10.1016/j.jsg.2014.02.008>.
78. Nelson R. *Geologic Analysis of Naturally Fractured Reservoirs*. Gulf Professional Pub.; 2001 Chemical, petrochemical & process; <https://books.google.de/books?id=VhfXIAEACAAJ>.

79. Maréchal JC, Dewandel B, Subrahmanyam K. Use of hydraulic tests at different scales to characterize fracture network properties in the weathered-fractured layer of a hard rock aquifer. *Water Resour Res.* 2004;40(11). <http://dx.doi.org/10.1029/2004WR003137>.
80. Liu J, Dempsey D, Nicol A, Parker M, Titus K. Impacts of reservoir permeability anisotropy on hydrogen migration and recovery efficiency in geological storage. *Fuel.* 2026;408:137596. <http://dx.doi.org/10.1016/j.fuel.2025.137596>.
81. Armitage PJ, Faulkner DR, Worden RH, Aplin AC, Butcher AR, Iliffe J. Experimental measurement of, and controls on, permeability and permeability anisotropy of caprocks from the CO₂ storage project at the Krechba Field, Algeria. *J Geophys Res: Solid Earth.* 2011;116(B12). <http://dx.doi.org/10.1029/2011JB008385>.
82. Tran Ngoc TD, Lefebvre R, Konstantinovskaya E, Malo M. Characterization of deep saline aquifers in the Bécancour area, St. Lawrence Lowlands, Québec, Canada: Implications for CO₂ geological storage. *Environ Earth Sci.* 2014;72. <http://dx.doi.org/10.1007/s12665-013-2941-7>.
83. Farouk S, Sen S, Saada S, Mohammed Eldosouky A, Abdelhamed R, Kassem A, Al-Kahtany K, Abdeldaim A. Characterization of Upper Cretaceous Matulla and Wata clastic reservoirs from October field, Central Gulf of Suez, Egypt. *Geomech Geophys Geo-Energy Geo-Resources.* 2023;9:1–23. <http://dx.doi.org/10.1007/s40948-023-00648-7>.
84. Jülich Supercomputing Centre. JUWELS Cluster and Booster: Exascale Pathfinder with Modular Supercomputing Architecture at Juelich Supercomputing Centre. *Journal of large-scale research facilities.* 2021;7(A183). <http://dx.doi.org/10.17815/jlsrf-7-183>.
85. COMSOL AB. Theory for the poroelasticity interfaces. In: *The Subsurface Flow Module User's Guide*. Stockholm, Sweden: 2022:282–286. COMSOL Multiphysics v. 6.1, <https://doc.comsol.com/6.1/docserver>.