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Calculation of normal forces in hangers of tied-arch-bridges based on frequency measurements considering model uncertainty and boundary condition sensitivity

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Abstract

Tied-arch bridges in railway infrastructure are increasingly exhibiting fatigue damage at the connection details of the vertical hangers. To assess their structural condition, short-term vibration measurements are taken on the hangers to determine their natural frequencies. This allows the normal forces acting on them to be calculated using prestressed modal analysis. Measurements conducted on seven tied-arch railway bridges revealed systematic deviations between the designed and actual normal forces, with discrepancies of up to 45% between hangers of identical length and construction. These deviations are attributed to complex fabrication and assembly processes during construction, including the effects of temperature, welding sequences, and support conditions during erection. As there is no independent validation method for the frequency-based approach, the accuracy of the back-calculated forces critically depends on the modelling assumptions. This paper investigates the sensitivity of normal force identification with respect to two key parameters: (i) the rotational connection stiffnesses at the arch and stiffening girder; and (ii) geometric imperfections in the form of sinusoidal pre-deflections. A parametric study based on Timoshenko beam theory shows that the uncertainty range between pinned and fully clamped boundary conditions remains significant, even for long hangers. An analytical investigation based on the Rayleigh–Ritz method shows that initial imperfections alter the natural frequencies through the coupling of longitudinal and transverse vibration modes. These findings lay the groundwork for the development of simplified, practical back-calculation recommendations.

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Keywords: Tied-arch bridge; Normal force identification; Vibration-based-monitoring; Boundary condition sensitivity; Imperfections

1. Introduction

In recent years, an increasing number of instances of damage to tied-arch bridges of rail infrastructure have been identified, even though many of these structures were only commissioned a few decades ago. The damage occurs in form of fatigue cracks in the connection details of the vertical hangers, particularly at the junctions between the arch and the stiffening girders, see Pfaffinger (2018). The occurrence of such fatigue damage at a relatively early stage of the structure's service life raises questions regarding the underlying structural behaviour and the accuracy of the original design assumptions. Along with the detected fatigue cracks, significant vibrations of the hangers are recorded during service inspections of many tied-arch bridge structures. In addition to the strong vibrations during vehicle passage, an unusually long decay process is also observed regarding these vibrations. Targeted field observations confirm these findings. Based on these observations, it is necessary to investigate the influence of traffic-induced vibrations. Accordingly, short-term measurements are currently being carried out on tied-arch railway bridges, both to determine the natural frequencies (see Fig. 1) and damping characteristics, and in turn to calculate the normal forces present in the hangers. As a result of these calculations, it was found that there are systematic deviations between the design normal forces and the actual normal forces in the hangers. Consequently, for hangers with an increasing length, the normal forces are often significantly lower compared to the design values, whereby higher normal forces are generally measured for short hangers. It should be noted at this point that, due to the nature of the static system, the relationship between the normal force and the natural frequencies is highly sensitive, and the assessment should therefore be treated with appropriate caution. Nevertheless, differing natural frequencies in hangers of the same length and construction indicate different utilisation of the normal force.

The reason for the deviations in the normal forces in the hangers from the design values is the complex assembly and fabrication processes involved in the construction of steel arch bridges. During construction, the structure passes a variety of construction stages in which the arches and deck girders are supported on temporary supports. The arrangement, stiffness and position of these supports generate internal stresses and deformations. The successive welding of the hanger components combined with a cutting to actual dimensions on-site takes place under the prevailing installation temperatures, which have a direct influence on the constraint forces in the structure, since the temperature-dependent expansion of the components is directly reflected by the normal forces in the hangers. Furthermore, in practice, the welding sequence is often determined by a parallel deployment of several welding teams, resulting in mutual dependencies: each welded hanger alters the current static system and thus the force distribution for all subsequent hangers to be installed. For structural engineers it is necessary to take these installation-related influences into account within the design process as well as to verify these influences after completion by measuring the hanger forces. For this reason, individual installation concepts are also required for all structures to take the described influences into account and to propose an appropriate installation concept.

A fundamental problem of frequency-based back-calculations is the lack of validation methods. As there are no alternative measurement methods available in practice for determining the normal force beside the frequency-based approach. Therefore, one must rely on numerical back-calculation and simplifications. In practical terms, there are many methods for estimating the normal force in a hanger based on experimentally determined natural frequencies. In addition to analytical solutions, such as those based on Ramberger (1978), which takes boundary conditions into account, numerical methods are used increasingly. These approaches usually use beam models that are independent of the main model. Alternatively, a numerical approach was pursued by Ries (2025). In this study, a tied-arch bridge structure was numerically modelled as a global shell model incorporating all structural details. Considering the design loads, both the natural frequencies and the normal forces were determined. However, this approach is not feasible in practice due to the considerable modelling and computational effort.

The relationship between the normal force in the hanger and the natural frequency forms the theoretical basis of all vibration-based identification methods. This relationship for a prismatic beam with simple supports and a constant cross-section is given in Geier and Petz (2004), Geier (2004) and Petersen (2017). The correlation between normal force and natural frequency is shown in Fig. 3. For slender and highly prestressed components, like cables, the expression reduces to the solution for a tensioned string. For short and stiff hangers, EI cannot be neglected without

introducing systematic errors. In cases where a shear deformation is not negligible, the Timoshenko beam theory must be applied (Petersen 2017). As the actual connection stiffness at the arch and stiffening members is generally unknown, this leads to a systematic uncertainty in the calculated normal forces, which increases as the length decreases.

This paper presents excerpts from ongoing research into the sensitivity of normal force calculations based on natural frequency measurements. Whilst the conservative calculation of normal forces is straightforward, obtaining accurate results requires a significant additional effort. Therefore, the sensitivity of the normal force with regard to the influencing parameters of connection stiffnesses and system imperfections is presented as an example. The aim of the investigations is to provide modelling and simplification recommendations so that accurate calculations can be carried out with significantly reduced effort. These recommendations are still pending.

2. Short-Term Measurements and Frequency-Based Normal Force Back-Calculation

2.1. Measurement and Back-Calculation procedure

To compare the design normal forces with the actual normal forces during service, short-term measurements are carried out directly on a railway bridge. These measurements are usually carried out at night to rule out interactions from artificial excitation induced by hammer impact and rail traffic, but also to ensure that the structure's temperature remains as constant as possible. During the short-term measurement, a triaxial ICP© miniature vibration sensor is positioned on the respective hanger under investigation. The sensor position must be chosen in such a way that a placement at the vibration nodes is avoided. Otherwise, certain natural frequencies cannot be recorded. Due to accessibility and ongoing operations, the sensor is usually positioned at a height of up to a maximum of two metres above the upper chord of the stiffening girder. This allows for accurate determination of the relevant natural frequencies. After installing the sensor, the hanger is manually excited by a hammer impact using a rubber-tipped hammer. The excitations are applied sequentially around both the strong and weak axes of the component. It should be noted that if the stiffness of the component is too high, it is not possible to determine the natural frequencies around the strong axis by manual excitation. An impulse is only applied once the hanger has returned to its rest position. The impulse from the hammer impact triggers a free vibration of the hanger.

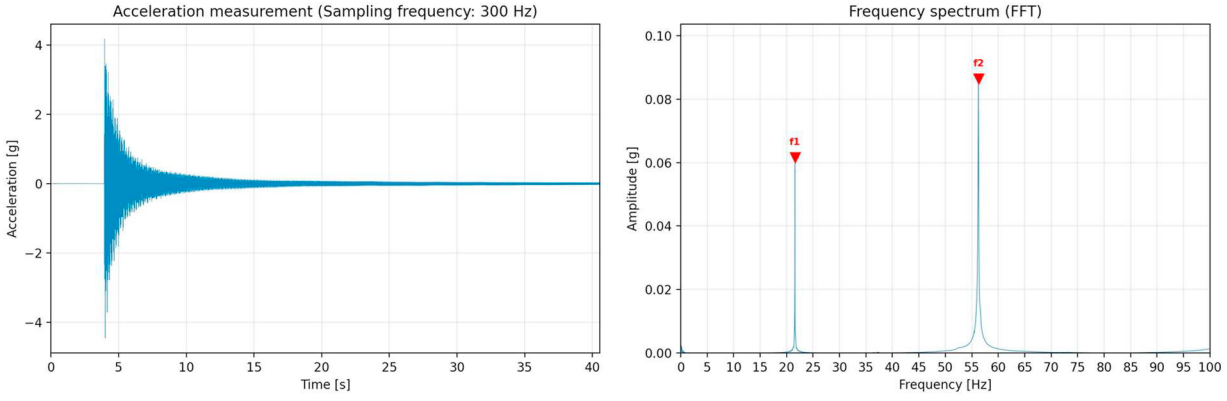


Fig. 1. (left) decaying vibration response; (right) frequency spectrum

Following this excitation, the decaying vibration response is recorded over a time interval of at least 35 seconds to ensure a sufficiently accurate frequency resolution (Fig. 1 (left)). As the frequency resolution is inversely proportional to the total measurement time, short recordings result in coarse frequency intervals and increased uncertainty in peak identification. The measured acceleration time series contains the dominant natural frequencies of the hanger in its current state of normal force. The natural frequencies are determined using a fast Fourier transform (FFT). By transforming the signal from the time domain to the frequency domain, the dominant frequency peaks and, consequently, the natural frequencies of the hanger can be identified (Fig. 1 (right)). In this process, the actual hanger

geometry, including the connections to the arch and stiffening girders, is represented in detail (see Fig. 2 (left)). The hanger lengths measured on the structure are, if available, adopted directly in form of model parameters. The required level of detail for the connection areas is evaluated in preliminary studies and simplified to achieve an optimal balance between model accuracy and computational effort. The normal force is determined within the framework of a prestressed modal analysis. In this process, the prestressing condition of the hanger is varied iteratively until a sufficient agreement between measured and calculated natural frequencies is achieved. To avoid potential sources of ambiguity and to ensure the required identification accuracy, several natural frequencies must always be considered simultaneously during this comparison.

The fundamental problem is also illustrated in Fig. 2 (right). It shows the correlation between the first to fourth natural frequencies and the associated normal force, based on a comprehensive finite-element parameter study of the volumetric model of the hanger six of structure 1 (see Fig. 2 and 3 (left)). It becomes obvious that, depending on the structure, this curve can become very shallow, implying that small changes in natural frequency result in large changes in normal force.

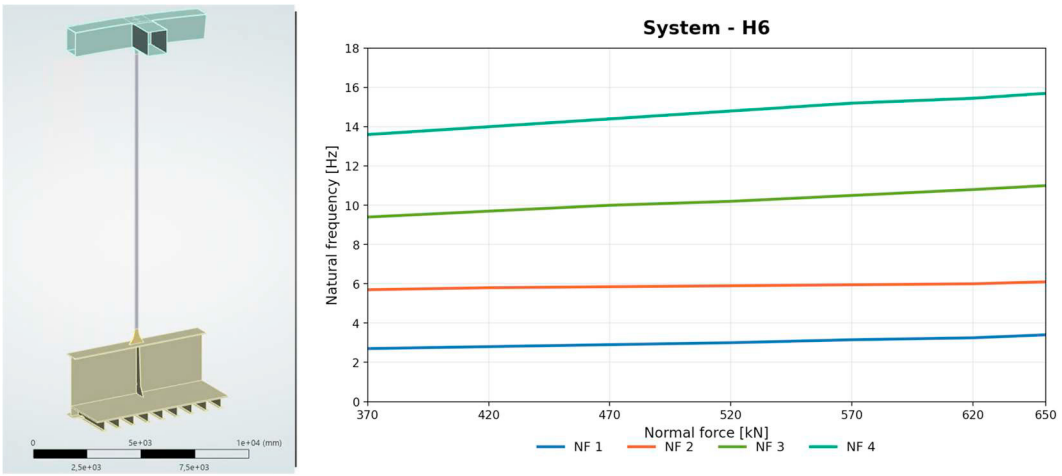


Fig. 2. Finite-element-model (left), sensitivity of the first four natural frequencies to variations in the normal force in the same hanger (right)

However, the detailed modelling requires significantly more time and resources, hence a common approach involves modelling the hanger as a simplified beam system with an equivalent spring stiffness to represent the boundary conditions. This highly simplified approach as well as its limitations is systematically analysed and evaluated within the scope of the present investigations (see section 3).

2.2. Results of a short-term measurement

As part of the investigations, seven tied-arch bridges have been analysed using short-term measurements until now. Fig. 3 compares the results of two railway bridge structures by way of example. Regardless of the size of the structure, which correlates with the number of hangers, a clear trend is observed in the distribution of natural frequencies within the hangers. Structure 1 (see Fig. 3 (left)) exhibits a very uneven distribution of natural frequencies in the range of the longer hangers (four to nine). Whereas in Structure 2 (see Fig. 3 (right)) there are hardly any differences in the natural frequencies for the symmetrical hangers. Even though the differences in natural frequencies appear minor at first glance, these differences should not be underestimated given the highly sensitive relationship between normal force and natural frequency. The sensitivity of the back-calculation is discussed in Chapter 3. For the structure in Fig. 3 (left), this means that hangers of the same length, which should theoretically have almost identical normal forces, exhibit normal forces that vary significantly in the range between 10% and 45% of the target value among the individual hangers of the same length. This implies an unplanned redistribution of the force flow regarding the load transfer, which may cause individual components to experience overloads. Consequently, rehabilitation concepts must be developed and implemented based on extensive recalculations to compensate for these overloads.

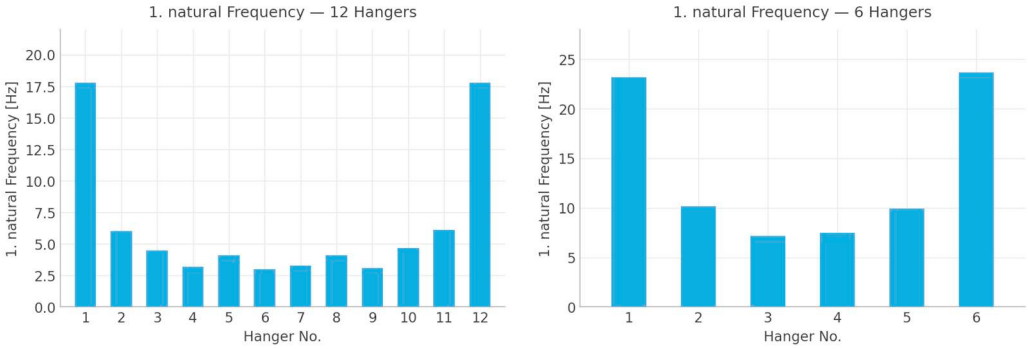


Fig. 3. (left) structure 1 with twelve hangers; (right) structure 2 with six hangers

3. Sensitivity of frequency-based Back-Calculation

3.1. Influence of connection stiffnesses

The following section provides a simplified illustration of the sensitivity of the calculation of hanger normal forces based on natural frequency measurements using the example of a flat steel hanger with dimensions of 200 mm × 50 mm in width and thickness. For the purposes of this parametric study, the connection geometries are not modelled in detail. The investigations serve solely to illustrate the sensitivity of the system. The calculation is carried out using second-order theory applying Timoshenko beam theory. It is assumed that interaction of the vibration about the weak and the strong axes is excluded. This influence will be quantified at a later stage. The investigations presented in this paper examine the influence of the following parameters on the identified normal force: the axial preload N (kN), the system length L (m), the rotational support stiffnesses at the arch $C_{\varphi,y,B}$ (kNm/rad) and at the stiffening girder $C_{\varphi,y,VT}$ (kNm/rad) of the weak axis as well as geometric imperfections about the weak axis Δx (mm).

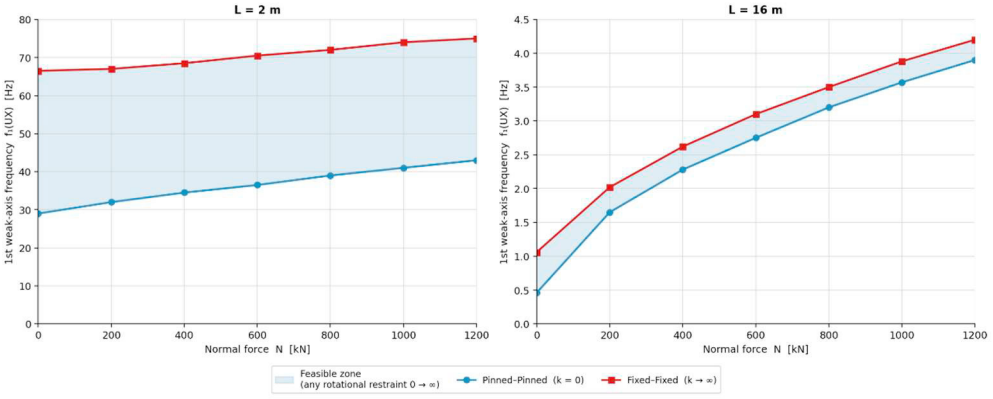


Fig. 4. First natural frequency for hanger systems of 2 m (left) and a 16 m (right) for pinned-pinned and fixed-fixed boundary conditions

The diagrams shown in Fig. 4 illustrate the theoretical lower and upper bounds of the bearing arrangement. Two different system lengths are used to assess the influence of the connection stiffness. The red curve corresponds to a full clamping at both ends with an infinite rotational stiffness. By halving the effective vibrational length, which is comparable to the reduction in buckling length for compression members, the natural frequency increases significantly. The blue curve represents the limit case of free end rotations, in which the vibrational length corresponds to the geometric system length and the results with a minimum natural frequency. The region between the two curves

represents all realistic and possible system states. The actual natural frequency therefore lies within this region. This simplified analysis assumes symmetrical bearing stiffnesses at both ends. In reality, the rotational stiffnesses at the arch and at the stiffening girder connections differ from one another. As expected, the absolute width of the highlighted region between the two limit cases decreases with increasing system length but remains within a range of approx. 0.3 to 0.4 Hz even for a length of L = 16 m. This illustrates that even for long members, an incorrect assumption regarding the boundary conditions leads to systematic deviations in the calculation of the normal force. Therefore, the connection stiffness in the computational model must be estimated as close to reality as possible.

3.2. Influence of structural imperfections

As structural imperfections cannot be ruled out during the manufacture of constructional products, the influence of imperfections in the form of a bar deflection (sinusoidal pre-curvature) is examined. For this purpose, a simplified Euler-Bernoulli beam, representing the hanger, based on second-order theory with initial imperfections is analysed.

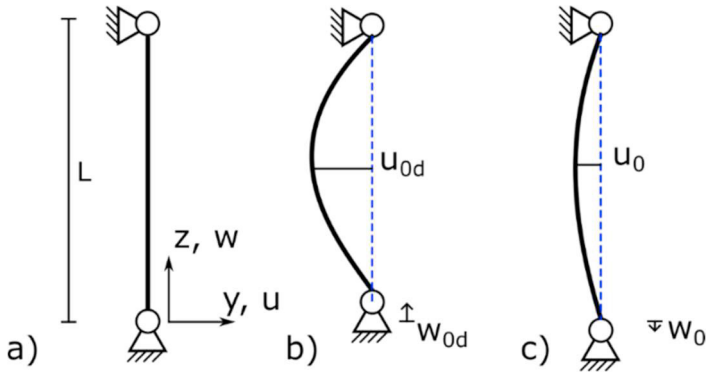


Fig. 5. Beam model of the hanger (a) in the deflected state, (b) and in the loaded state (c)

The load is applied via an initial axial displacement $w_0 = const. \neq 0$. It is assumed that the perfect Hanger Fig. 5 (a) is installed as shown in Fig. 5 (b), with some geometric imperfections $(w_{0d}; u_{0d})$. After the hanger is fixed in place, the load of the structure is applied (for example by applying ballast or removing temporary supports) in the form of another displacement. The hanger deforms accordingly as the support is lowered: the initial deflection $(w_{0d}; u_{0d})$ decreases and tensile stress develops (see Fig. 5 (c)). For further analysis it is assumed that the remaining initial deflection $(w_0; u_0)$, as seen in Fig. 5 (c), cannot be neglected. The example and the notation are based on Wauer (2014) and it is expanded by adding an initial deflection u_0. The Hanger is modelled as beam with pinned-pinned boundary conditions. It may be described with the differential equations (1) and (2)

$$EIu'''' - EAw_0'u'' + EA(w'u_0')' + \rho A\,\ddot{u} = 0, \tag{1}$$

$$EAw'' + EA(w'u_0')' - \rho A\,\ddot{w} = 0. \tag{2}$$

Where $(*)'$ and $(\dot{*})$ denote a derivative in space z or time t, respectively. The term EAw_0' represents the axial force (tension). It is assumed that the solution is isochrone and can be split into a product of a time and a space dependent term, as written in equation (3).

$$u = U(z)\,cos(\omega t - \alpha)\;and\;w = W(z)\,cos(\omega t - \alpha). \tag{3}$$

To describe the spatial part $U(z)$ and $W(z)$ the modes of vibration in equation (4) are chosen with a series of Sine-functions, as they fulfil the boundary conditions:

$$U(z) = \sum_n C_{1n} sin\left(\frac{n\pi z}{L}\right)\;and\;W(z) = \sum_n C_{2n}\,sin\left(\frac{n\,\pi z}{L}\right) \tag{4}$$

wherein C_{1n} and C_{2n} are the amplitudes of the n-th. mode, where n can be any natural number greater than zero. C_{1m} and C_{2m} are respectively the amplitudes of the m-th. mode, where m can be any natural number greater than zero. From here the Rayleigh-Ritz Method is employed to compute a solution. In the first step the Rayleigh quotient

$$R = \frac{\Sigma_n \left(C_{1n}^2 \left(EI \frac{\pi^4 n^4}{L^4} + Z \frac{\pi^2 n^2}{L^2} \right) + \Sigma_m (C_{1n} C_{2m} + C_{1m} C_{2n}) \frac{16EA}{L^3} \frac{mn(m^2+n^2)}{(m^2-n^2)^2} \left(1-(-1)^{n+m}\right) - C_{2n}^2 \left(EA \frac{\pi^2 n^2}{L^2} \right) \right)}{\Sigma_n C_{1n}^2 - C_{2n}^2} \tag{5}$$

is derived. It is an upper approximation for the angular frequency ω^2.The amplitude ratio (i.e. the relationship between the coefficients C_{1n} and C_{2n}) and the natural frequencies can be derived, by minimizing R . This leads to a set of equations, which can be solved for ω^2, this can be done by using a computer algebra-program, like MATLAB. The first three natural frequencies f_i are computed for the set of parameters given in Table 1 and compared to their corresponding values $f_{perf,i}$ of the perfect beam.

Table 1. Parameters of the numerical evaluation

E *(N/m²)*	A *(m²)*	I *(m⁴)*	L *(m)*	y_0 *(m)*	ρ *(kg/m³)*
$2.1 \cdot 10^{11}$	0.01	$2.083 \cdot 10^{-6}$	15.00	0.015-0.1	7850

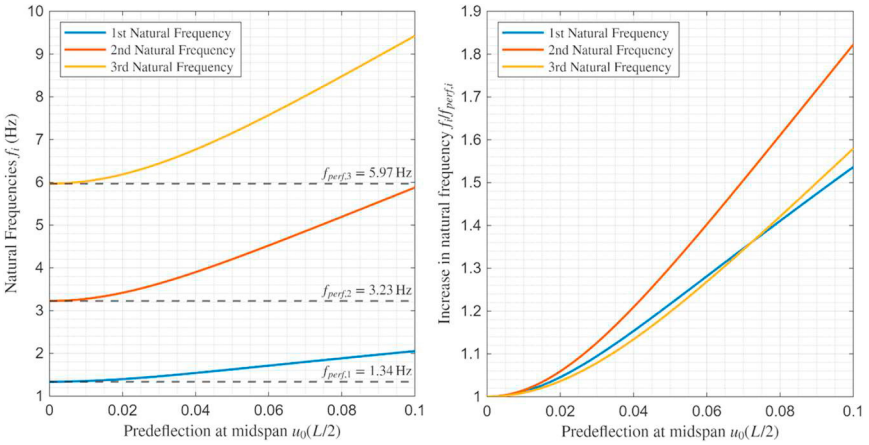


Fig. 6. Absolute (left) and relative (right) change in natural frequency

The result of this example is shown in Fig. 6 in terms of absolute and relative values. As frequencies increase, an increase in the stiffness of the system can be observed since the pre-deflection increases. The reason for this behaviour is the coupling of longitudinal and lateral vibration modes, which is in turn the consequence of the curvature respective the pre-deflection of the beam. This is further illustrated by computing the modes of vibration: In Fig. 7 the lateral displacement (blue) is plotted with the axial displacement over the length of the beam. The vibration modes are now coupled, since displacements in either z or y -direction are non-zero and can therefore no longer appear independent of each other.

In conclusion it is found that the natural frequencies of beams with imperfections deviate from their “perfect” counterparts. The difference can easily reach a value of 5%, which is significant when matching hanger-frequencies in the following step. In future research these phenomena should be investigated by evaluating the effect using real geometries and boundary conditions in a more complex numerical model and field tests as well. Additionally different pre-deflections should be considered, for example a sinusoidal representing a mutual twisting of the beam. This is important since the term $(w'u_0')'$ is not only dependent on the scale of the imperfection, but also on its shape along the beam.

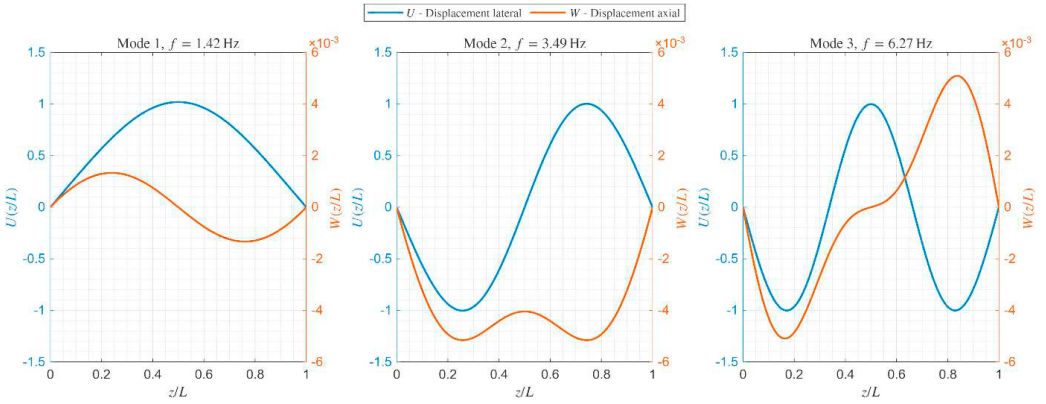


Fig. 7. Vibrational modes for the three lowest natural frequencies investigated

4. Outlook: Creation of a Back-Calculation recommendation

This paper demonstrates that vibration-based normal force back-calculation in hangers of tied-arch bridges is a highly sensitive procedure. The accuracy of this procedure depends critically on the realistic representation of the governing system parameters. Short-term measurements conducted on seven railway bridges reveal systematic deviations between design and actual normal forces of up to 45% among hangers of identical length and construction. The sensitivity analyses show that rotational connection stiffnesses at the arch and stiffening girder have a decisive influence on the back-calculated normal force. Even for long hangers (L = 16 m), the uncertainty region between the pinned–pinned and fixed–fixed limit cases span 0.3–0.4 Hz, corresponding to non-negligible deviations in the identified normal force. Furthermore, it is analytically demonstrated that geometric imperfections in the form of pre-deflections alter the natural frequencies relative to the ideal system through coupling of longitudinal and transverse vibration modes. This is an effect that must be accounted for during frequency matching.

Although the finite element model (see Fig. 3) presented in this work provide accurate results, they involve a level of modelling and computational effort that is rarely justifiable in engineering practice. The aim of further investigations is the development of a simplified, practical back-calculation procedure based on a beam model and measured natural frequencies. To this end, the relevant influencing parameters, in particular the rotational stiffnesses of the connections at the arch and stiffening girder, the influence of arch inclination in the connection zone, various cross-sectional geometries and a range of hanger lengths, will be systematically investigated and quantified through parametric studies. Based on the resulting sensitivity maps, modelling recommendations are to be derived that enable accurate normal force back-calculation with a justifiable level of effort. Since conservative simplifications offer no safety benefit in this context, both overestimation and underestimation of the normal force result in equally erroneous structural assessments; hence, maximizing identification accuracy is the primary objective.

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