

Balance of Liquid-phase Turbulence Kinetic Energy Equation for Bubble-train Flow

Milica ILIĆ*, Martin WÖRNER and Dan Gabriel CACUCI

Research Centre Karlsruhe, Institute for Reactor Safety, P .O. Box 3640, 76021 Karlsruhe, Germany

Abstract

In this paper the investigation of bubble-induced turbulence using direct numerical simulation (DNS) of bubbly two-phase flow is reported. DNS computations are performed for a bubble-driven liquid motion induced by a regular train of ellipsoidal bubbles rising through an initially stagnant liquid within a plane vertical channel. DNS data are used to evaluate balance terms in the balance equation for the liquid phase turbulence kinetic energy. The evaluation comprises single-phase-like terms (diffusion, dissipation and production) as well as the interfacial term. Special emphasis is placed on the procedure for evaluation of interfacial quantities. Quantitative analysis of the balance equation for the liquid phase turbulence kinetic energy shows the importance of the interfacial term which is the only source term. The DNS results are further used to validate closure assumptions employed in modelling of the liquid phase turbulence kinetic energy transport in gas-liquid bubbly flows. In this context, the performance of respective closure relations in the transport equation for liquid turbulence kinetic energy within the two-phase $k-\varepsilon$ and the two-phase $k-l$ model is evaluated.

KEYWORDS: *bubble-induced turbulence, turbulence kinetic energy, direct numerical simulation, bubble-train flow, turbulence modelling*

* Corresponding author, Tel. +49-7247-82-2949, Fax. +49-7247-82-3718, E-mail: ilic@irs.fzk.de

I. Introduction

While the modelling of turbulent single phase flows has already reached a certain level of maturity, models for the turbulence in bubbly flows are still under development. In approaches currently used turbulence of gas phase is commonly neglected and only the one in the liquid is modelled. For this purpose the respective transport equations of the well-established single-phase turbulence models are extended with more or less empirically established closure terms that account for interfacial effects (more details are given in section V). As it is not clear whether / how far the closure assumptions originally developed for single phase flows can retain their validity when the dispersed phase is present, such an approach might be argued as highly uncertain.

Most of the difficulties faced in the development of improved turbulence models for bubbly flows concern the pure understanding of mechanisms in which bubbles alter turbulence generation, redistribution and dissipation in the liquid phase. Analytically, these mechanisms were rigorously formulated by the derivation of basic balance equations for turbulence kinetic energy⁷⁾ and Reynolds stresses⁸⁾ in gas-liquid flows. Although known for more than a decade these equations could not be exposed to an appropriate quantitative analysis, because highly resolved data on the flow field and phase interface structure required for such an analysis have not been available.

Recent improvements in computer performances and positive experience from single phase flows suggest use of direct numerical simulations (DNS) for solving these problems. Based on computational grids fine enough to resolve all flow scales and auxiliary algorithms to track the gas-liquid interface, DNS of bubbly flow provides the full information on instantaneous three-dimensional flow field and phase interface topology. In spite of serious limitations concerning the magnitude of Reynolds number of the liquid flow and number of bubbles that can be tracked, DNS opens a new promising way to get a detailed insight into mechanisms governing the turbulence in bubbly flows. Namely, in various industrial processes involving slow dispersed two-phase flows no shear turbulence occurs and the main flow features such as phase distribution and mixing are controlled only by agitation of the liquid phase by moving bubbles. This agitation is called bubble-induced turbulence (BIT). Important information on the phenomenon of BIT can be obtained monitoring the effects of the dispersed phase on well investigated single phase flows with low Reynolds numbers. Among these, the simplest case concerns studying fluctuations of the liquid phase quantities induced by injection of gas bubble(s) into originally stagnant liquid.

This paper reports the use of DNS in statistical analysis of BIT. DNS are performed for a bubble-driven liquid motion induced by a regular train of ellipsoidal bubbles rising within a plane channel. Based on results of the DNS a quantitative analysis for the balance equation of liquid phase turbulence kinetic energy (k_L) is performed. The paper is organised as follows. In section II an outline of the methodology employed to perform DNS of the bubble-train flow is presented. Further, geometrical and physical parameters of the present numerical experiment are given.

Section III deals with theoretical considerations of the exact k_L equation. The methodology used for the evaluation of balance terms in this equation and corresponding results obtained using DNS data on the bubble-train flow are presented in section IV. In section V the validity of closure assumptions in modelled form of k_L equation employed in both the two-phase $k-\varepsilon$ and the two-phase $k-l$ model is tested against respective balance terms in the exact k_L equation. The paper is completed by conclusions.

II. Direct Numerical Simulation of Bubble-train Flow

1. Governing Equations

The direct numerical simulations are performed with our in-house computer code TURBIT-VoF¹⁵. The code is based on a single set of balance equations that express the conservation of mass (equation 1) and momentum (equation 2) for an incompressible isothermal flow of two immiscible Newtonian fluids¹⁹:

$$\nabla \cdot \mathbf{u} = 0 \quad (1)$$

$$\frac{\partial(\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \frac{1}{Re_{ref}} \nabla \cdot \boldsymbol{\tau} - \frac{(1-f)E\ddot{o}_{ref}}{We_{ref}} \frac{\mathbf{g}^*}{|\mathbf{g}^*|} + \frac{\kappa a_{in}}{We_{ref}} \mathbf{n} . \quad (2)$$

The above equations are given in non-dimensional form. The following scaling applies: distance, $\mathbf{x} = \mathbf{x}^* / L_{ref}^*$, velocity, $\mathbf{u} = \mathbf{u}^* / U_{ref}^*$, time, $t = t^* U_{ref}^* / L_{ref}^*$, density, $\rho = \rho^* / \rho_L^*$, viscosity, $\mu = \mu^* / \mu_L^*$ and pressure, $p = (p^* - \rho_L^* \mathbf{g}^* \cdot \mathbf{x}^*) / (\rho_L^* U_{ref}^{*2})$. The superscript $*$ indicates a dimensional variable, while L_{ref}^* and U_{ref}^* are reference length and velocity, respectively. As a result of scaling reference Reynolds number, $Re_{ref} = \rho_L^* U_{ref}^* L_{ref}^* / \mu_L^*$, reference Weber number, $We_{ref} = \rho_L^* L_{ref}^* U_{ref}^{*2} / \sigma^*$, and reference Eötvös number, $E\ddot{o}_{ref} = (\rho_L^* - \rho_G^*) |\mathbf{g}^*| L_{ref}^{*2} / \sigma^*$, appear in the momentum equation. In equation (2) $\boldsymbol{\tau}$ is the stress tensor given by $\boldsymbol{\tau} = \mu (\nabla \mathbf{u} + \nabla \mathbf{u}^T)$. Subscripts L and G indicate the liquid and gas phase, respectively. Surface tension is denoted with σ^* and gravity with \mathbf{g}^* . The last term in equation (2) expresses the contribution of the surface tension force. There, κ is twice the mean interface curvature, $\mathbf{n} = \mathbf{n}_G = -\mathbf{n}_L$ is the unit normal vector to the interface pointing from the gas into the liquid and a_{in} is the interfacial area concentration.

To distinguish between phases the liquid volumetric fraction, f , is introduced. Therefore, a cell is filled with liquid when $f = 1$ or with gas when $f = 0$. If $0 < f < 1$, an interface exists within a cell. In such cells the model of a homogeneous two-phase mixture is employed, i.e. the equality of phase velocities and pressures is assumed. The density and viscosity are expressed as: $\rho = 1 + (1-f) \rho_G^* / \rho_L^*$ and $\mu = 1 + (1-f) \mu_G^* / \mu_L^*$, respectively.

Phase interface evolution is tracked employing the transport equation for the liquid volumetric fraction:

$$\frac{\partial f}{\partial t} + \nabla \cdot (f\mathbf{u}) = 0. \quad (3)$$

The equation (3) is solved employing a Volume-of-Fluid procedure¹⁵. This procedure involves two steps. In the first one the interface orientation and location within each mesh cell is reconstructed using PLIC (Piecewise Linear Interface Calculation) method EPIRA that locally approximates the interface by a plane. In the second step the liquid fluxes across the faces of the mesh cell are computed. The methodology is verified comparing numerical results with experimental data for the rise of an ellipsoidal bubble ($E\ddot{o}_B=3.07$, $M=3.1 \cdot 10^{-6}$) and an oblate ellipsoidal cap bubble ($E\ddot{o}_B=243$, $M=266$).

2. Numerical Setup

Direct numerical simulations are performed for a two-phase mixture with a simple flow pattern that is called ‘regular bubble train’. The term regular bubble train indicates a quasi-steady flow where bubbles rise with the same velocity through the channel whose length is much larger than its hydraulic diameter. Bubbles have an identical shape and are uniformly distributed along the channel. In such a situation a unit cell containing one bubble can be extracted (Figure 1), that fully characterises the entire bubble-train flow. Taking the dimensions of the computational domain equal to the size of the unit cell, the simulation of the bubble-train flow can successfully be performed keeping the domain fixed and letting bubbles move through it.

(Appropriate place for Figure 1)

The computational domain in our DNS is prescribed to be a cube of non-dimensional size $1 \times 1 \times 1$. The domain is discretized by 64^3 uniform mesh cells. Boundary conditions are no-slip ones at the lateral rigid walls ($x_3=0$ and $x_3=1$) and periodic ones in vertical (x_1) and span-wise (x_2) direction. In this way an infinite number of bubble-trains shifted in lateral direction is simulated. The following parameters are specified: reference length $L_{ref}^*=4\text{m}$, reference velocity $U_{ref}^*=1\text{m/s}$, density ratio $\rho_G^*/\rho_L^*=0.5$, viscosity ratio $\mu_G^*/\mu_L^*=1$, bubble Eötvös number $E\ddot{o}_B=3.065$, and Morton number $M=3.06 \cdot 10^{-6}$. According to these values reference dimensionless numbers are: $E\ddot{o}_{ref}=49.05$, $We_{ref}=2.5$ and $Re_{ref}=100$.

Initially a spherical bubble with the diameter $D_b=0.25$ is positioned in the middle of the channel filled with stagnant liquid. Using the time step width $\Delta t=0.0001$ in total 65,000 time steps are computed. Within this time the gas-liquid system has reached a quasi-steady state, where the mean velocity of the liquid phase and the bubble rise velocity can be considered as approximately constant. The shape of the bubble is steady, an axisymmetric ellipsoid with the axis aspect ratio 1.635 (see Figure 2). Bubbles rise along an almost rectilinear path.

(Appropriate place for Figure 2)

III. Balance Equation of Liquid Turbulence Kinetic Energy

Among various turbulence quantities we chose to deal with the liquid phase turbulence kinetic energy because this quantity plays an important role in turbulence models for bubbly flows (see section V). Under the assumption of incompressibility the non-dimensional liquid phase turbulence kinetic energy is defined as:

$$k_L = \overline{\mathbf{u}'_L{}^2} / 2.$$

For gas-liquid flows, liquid phase turbulence kinetic energy generation, dissipation and transport as well as its interplay with flow parameters such as velocity field, phase distribution and interfacial structures is mathematically described by the following balance equation⁷⁾:

$$\begin{aligned} \frac{\partial}{\partial t} (\alpha_L k_L) + \nabla \cdot (\alpha_L k_L \overline{\mathbf{u}}_L) = & \underbrace{\frac{1}{Re_{ref}} \nabla \cdot (\alpha_L \overline{\overline{\tau}'_L \cdot \mathbf{u}'_L}) - \nabla \cdot \left[\alpha_L \left(\overline{\overline{p}'_L \mathbf{u}'_L} + \frac{1}{2} \overline{\overline{\mathbf{u}'_L{}^2 \mathbf{u}'_L}} \right) \right]}_{DIFFUSION} \\ & \underbrace{-\alpha_L \overline{\overline{\mathbf{u}'_L \mathbf{u}'_L}} : \nabla \overline{\mathbf{u}}_L}_{PRODUCTION} \underbrace{- \frac{1}{Re_{ref}} \alpha_L \overline{\overline{\tau}'_L} : \nabla \overline{\mathbf{u}}_L}_{DISSIPATION} + \underbrace{\left[\frac{1}{Re_{ref}} \overline{\overline{\tau}'_{Lin}} - \overline{\overline{p}'_{Lin}} \mathbf{I} \right] \cdot \overline{\mathbf{u}'_{Lin}} \cdot \mathbf{n}_{Lin} a_{in}}_{INTERFACIAL TERM}. \end{aligned} \quad (4)$$

The following notation is used. Subscript *in* denotes liquid phase quantities at the gas-liquid interface and α_L is mean liquid volumetric fraction defined via the characteristic function of the liquid phase, $\alpha_L = \overline{\Phi}_L$. The single overbar $\overline{\quad}$ indicates averaging. The double overbar $\overline{\overline{\quad}}$ denotes so-called phase-weighted averaging. For an arbitrary physical quantity A_L , this averaging is defined as $\overline{\overline{A_L}} = \overline{A_L \Phi_L} / \overline{\Phi_L}$. Fluctuating parts of physical quantities are evaluated as $A'_L = A_L - \overline{A_L}$ and $A'_{Lin} = A_{Lin} - \overline{A_L}$.

On the right-hand-side of equation (4) two distinctive groups of terms appear. The first one is the group of terms associated with the mean liquid volumetric fraction, α_L . Except for being multiplied with α_L , these terms are basically of the same form as the ones involved in the single-phase turbulence kinetic energy equation, i.e. the diffusion, production and dissipation term can be recognized. For this reason these terms are called single-phase-like terms. The last term, that is associated with the interfacial area concentration a_{in} , represents a source of liquid turbulence attributed to the presence of bubble interfaces and is called interfacial term.

Equation (4) is derived based on the local instant formulation of the mass and momentum conservation laws for two-phase flow, i.e. no model assumptions are made. In this context, equation (4) is called 'exact' k_L equation in order to distinguish it from the modelled k_L equation employed in turbulence models.

IV. Evaluation of Balance Terms in the Exact k_L Equation

1. Averaging Procedure

In evaluation of the mean and fluctuating quantities only the part of time signals representing the developed flow regime is of interest. As in this regime the bubble velocity changes slightly, the time interval $\Delta T=5.5-6.35$ during which the bubble passed the computational domain five times is considered when the time averaging is concerned. Since the procedure of time averaging requires that respective quantities are available for all mesh cells at every time instant, a problem of data storage appears. Namely, for the time averaging within the time interval $\Delta T=5.5-6.3$ in total 8,400 full data sets should be stored. In order to check if this problem can be avoided, the relationship that exists between temporal and spatial averaging is examined in detail. This examination has shown that the ergodic theorem is valid in vertical (x_1) direction and that the time averaging can successfully be replaced with the averaging along vertical (x_1) lines. In the context of equation (4) the line averaging corresponds to the variables denoted with single overbar. Note that the characteristic function of the liquid phase, Φ_L , is taken to be equal to the local liquid volumetric fraction, f . In principle, for the line averaging it is sufficient to consider a single instant in time within the fully developed flow regime. However, with the goal of getting smoother profiles evaluations are performed for every 200th time step. In total 42 sets of results based on line averaging are evaluated within the considered time interval ΔT and then the arithmetic mean of these profiles is computed.

2. Evaluation of Liquid-phase Interfacial Quantities

While the raw data obtained by DNS of the bubble-train flow are sufficient to evaluate the single-phase-like terms, the evaluation of the interfacial term requires some additional information. Namely, since the model of the homogeneous mixture is employed in cells containing interface, corresponding DNS data on the velocity and pressure cannot be used as representative liquid phase interfacial quantities.

The problem of determining the liquid interfacial pressure, p_{Lin} , is solved assuming that it is equal to the pressure in an adjacent cell which is fully occupied with liquid phase.

The procedure used for the evaluation of the velocity on the liquid side of the interface, \mathbf{u}_{Lin} , is more complicated and is shortly outlined here. Since no phase change is considered, the phase interfacial velocities are equal: $\mathbf{u}_{Lin} = \mathbf{u}_{Gin} = \mathbf{u}_{in}$. The velocity \mathbf{u}_{in} can be split into its tangential, $\mathbf{u}_{in,t}$, and normal, $\mathbf{u}_{in,n}$, component:

$$\mathbf{u}_{in} = \mathbf{u}_{in,t} + \mathbf{u}_{in,n}. \quad (5)$$

The tangential component, $\mathbf{u}_{in,t}$, is defined to be equal to the tangential velocity of a fluid particle lying at the interface⁴⁾. As in our case interfacial cells contain two-phase mixture, $\mathbf{u}_{in,t}$ is assumed to be equal to the tangential

component of the mixture velocity, $\mathbf{u}_{in,t} = \mathbf{u}_t = \mathbf{u} - (\mathbf{u} \cdot \mathbf{n}_L)\mathbf{n}_L$. The normal component of the interfacial velocity is defined as⁶⁾:

$$\mathbf{u}_{in,n} = \frac{\partial F / \partial t}{|\nabla F|} \mathbf{n}_L, \quad (6)$$

where $F(\mathbf{x}, t) = 0$ is the equation of the interfacial surface. In TURBIT-VoF gas-liquid interface is for any time instant, t , defined via the unit normal vector, \mathbf{n}_L , and a point lying on the interface $M(\mathbf{b})$:

$$F(\mathbf{x}, t) = (\mathbf{b} - \mathbf{x}) \cdot \mathbf{n}_L = 0. \quad (7)$$

Since function $F(\mathbf{x}, t)$ given by equation (7) is not explicit with respect to time, the problem of determining $\partial F / \partial t$ arises. The procedure applied for solving this problem is as follows. Consider two subsequent instants in time t_0 and $t_0 + \Delta t$ in which the interfacial surface passes through the point $M_0(\mathbf{b}_0)$ and $M_1(\mathbf{b}_0 + \delta \mathbf{x})$, respectively, i.e. the surface equation satisfies: $F(\mathbf{b}_0, t_0) = 0$ and $F(\mathbf{b}_0 + \delta \mathbf{x}, t_0 + \Delta t) = 0$. If the distance between M_0 and M_1 is small, the function $F(\mathbf{b}_0 + \delta \mathbf{x}, t_0 + \Delta t)$ can be expanded into a Taylor series. Neglecting terms of the second and higher order in this expansion and after some simple mathematical rearrangements, one obtains

$$\frac{\partial F}{\partial t} = -\frac{1}{\Delta t} \mathbf{n}_{L0} \cdot \delta \mathbf{x}, \quad (8)$$

where \mathbf{n}_{L0} is the unit normal vector at the time instant t_0 .

3. Balance of the Exact k_L Equation for Bubble-train Flow

In Figure 3 wall-normal profiles of the balance terms on the right-hand-side of equation (4) and of the volumetric gas fraction, $\alpha_G = 1 - \alpha_L$, are presented for two span-wise positions. The following can be observed.

Profiles of all the terms are symmetric with respect to the channel axis. Non-zero values are noticed only in the central part of the channel, i.e. in the domain where bubbles rise. Strong gradients of the liquid phase quantities in the region between the part of the channel through which bubbles move and the one permanently occupied with the liquid phase cause sharp peaks of all the terms at such locations. These peaks are especially remarkable for the diffusion term. As it is seen in Figure 3, profiles of the mean gas volumetric fraction, α_G , are in the case of the bubble-train flow continuous, but not continuously differentiable. Therefore, the peaks of the balance terms are expected to be reduced in a bubbly flow with smoother profiles of α_G , e.g. when the case of a bubble-swarm flow is considered.

The term that is called production, and in shear flows is always positive, is negative here. Although this result may seem surprising and bring into discussion the name of this term, the physics lying behind it is easy to understand taking into account that the motion of the liquid phase is driven by rising bubbles, i.e. that the energy in the liquid flow is transferred from the fluctuating velocity field caused by moving bubbles to the mean flow.

Since the diffusion has no net contribution, but represents the redistribution of turbulence kinetic energy over the flow domain, the following can be stated. In the case of the bubble driven liquid motion considered here the turbulence kinetic energy of the liquid phase is gained only by the interfacial term, while it is lost not only through the dissipation, but also through the production term. This conclusion gives rise to the importance of studying the interfacial term.

(Appropriate place for Figure 3)

V. Exact versus Modelled k_L Equation

In engineering applications liquid phase turbulence in bubbly flows is calculated using various statistical models. Among these two approaches involve a modelled form of the liquid phase turbulence kinetic energy equation: the two-phase $k-\varepsilon$ model^{1-3,11-14,17,18)} and the two-phase $k-l$ model¹⁰⁾. Modelled k_L equation is in these models derived by an extension of the respective single phase equation with a model term that accounts for the existence of gas-liquid interfaces. The objective of this section is to perform scrutiny and validation of closure assumptions adopted in the modelled form of the k_L equation employed in the two-phase $k-\varepsilon$ as well as in the two-phase $k-l$ model. In this context, both single-phase-like and interfacial terms are evaluated using DNS data on bubble-train flow and results are compared with respective balance terms from the exact k_L equation.

1. Validation of Closure Assumptions for Single-phase-like Terms

The commonly used model for the production term is based on the assumption that turbulent stresses are proportional to the mean strain rate in the liquid phase:

$$\Pi = \alpha_L \nu_L^{eff} \left[\nabla \bar{\mathbf{u}}_L + \left(\nabla \bar{\mathbf{u}}_L \right)^T \right] : \nabla \bar{\mathbf{u}}_L,$$

where ν_L^{eff} denotes so-called effective viscosity. When the two-phase $k-\varepsilon$ model is concerned, approaches used to evaluate ν_L^{eff} can be classified into the following three groups:

- Only eddy viscosity evaluated by two-phase $k-\varepsilon$ model, $\nu_L^{k\varepsilon}$, is considered^{1, 3, 12, 13, 18)}:

$$\nu_L^{eff} = \underbrace{C_\mu k_L^2 / \varepsilon_L}_{\nu_L^{k\varepsilon}}.$$

- Beside $\nu_L^{k\varepsilon}$ molecular viscosity of the liquid phase, ν_L , is taken into account^{2, 14, 17)}:

$$\nu_L^{eff} = \underbrace{C_\mu k_L^2 / \varepsilon_L}_{\nu_L^{k\varepsilon}} + \underbrace{1/Re_{ref}}_{\nu_L}.$$

- In addition to $\nu_L^{k\varepsilon}$ bubble-induced eddy viscosity, ν_L^B , evaluated by model of Sato et al.¹⁶⁾ is taken into consideration¹¹⁾:

$$\nu_L^{eff} = \underbrace{C_\mu k_L^2 / \varepsilon_L}_{\nu_L^{k\varepsilon}} + \underbrace{0.6\alpha_G D_b \left| \overline{\mathbf{u}_R} \right|}_{\nu_L^B}.$$

In the above expressions for ν_L^{eff} the value of the coefficient C_μ is 0.09 and $\overline{\mathbf{u}_R} = \overline{\mathbf{u}_G} - \overline{\mathbf{u}_L}$ represents the mean relative velocity between phases.

In two-phase $k-l$ model¹⁰⁾ effective viscosity, ν_L^{eff} , is using coefficient $\beta_l=0.56$ related to the two-phase mixing length, l_{TP} :

$$\nu_L^{eff} = \beta_l \underbrace{l_{TP} \sqrt{k_L}}_{\nu_L^{kl}}.$$

In this model l_{TP} , defined as a sum of single phase mixing length, l_{SP} , and bubble-induced mixing length, l_B , is used. However, for our case of slow bubble-driven liquid motion it was reasonable to neglect l_{SP} . Further on, as bubbles move in the central part of the channel, relation $l_B = \alpha_G D_b / 3$ proposed for the core region of bubbly flow¹⁰⁾ is considered.

The common closure relation for the diffusion term is, like in single phase flows, based on the assumption that the diffusion flux of k_L is proportional to the gradient of k_L .

$$DIFF = \nabla \cdot [\alpha_L \nu_L^D \nabla k_L].$$

While in all two-phase $k-\varepsilon$ approaches^{1-3,11-14,17,18)} the diffusion coefficient is taken to be equal to the effective viscosity, $\nu_L^D = \nu_L^{eff}$, in two-phase $k-l$ model¹⁰⁾ it is given as: $\nu_L^D = 0.5 R_{ref}^{-1} + \beta_2 \underbrace{l_{TP} \sqrt{k_L}}_{\nu_L^{kl}}$, where $\beta_2=0.38$.

Since the dissipation of the liquid turbulence kinetic energy, ε_L , is in the two-phase $k-\varepsilon$ model evaluated by a separate transport equation, we considered only the closure assumption adopted in the two-phase $k-l$ model¹⁰⁾:

$$\varepsilon_L = \gamma_1 \alpha_L k_L^{3/2} / l_{TP},$$

where the coefficient $\gamma_1=0.18$.

Using DNS data on the bubble-train flow single-phase-like terms are evaluated employing the closure assumptions presented in the text above and the results are presented in Figure 4. The following can be seen. According to all closure assumptions for the effective viscosity, ν_L^{eff} , positive values of production term are evaluated, that is opposite to the exact one which is always negative. The diffusion term is strongly underestimated when conventional single-phase approaches for estimation of the diffusion coefficient ($\nu_L^D = \nu_L^{k\varepsilon}$, $\nu_L^D = \nu_L^{k\varepsilon} + \nu_L$ and

$\nu_L^D \propto \nu_L^{kl}$) are applied. One more argument for this statement can be drawn from an analysis of the dissipation term. Namely, although the two-phase $k-l$ model underestimated the dissipation, this underestimation is not so pronounced as in the case of the diffusion. This means that not only the mixing length in bubbly flows, l_{TP} , should be corrected, but also the way in which it is related to the eddy viscosity, ν_L^{kl} . Implementation of Sato's eddy viscosity, ν_L^B , in the diffusion coefficient did not improve modelling of the diffusion term significantly.

(Appropriate place for Figure 4)

2. Validation of Closure Assumptions for Interfacial Term

In contrast to the modelling of single-phase-like terms, where practically no specific two-phase closure assumptions have been developed, various models for interfacial term are proposed in the literature. An overview of these models is given in Table 1. The following can be observed. In the development of closure assumptions for interfacial term bubbly flows are generally considered to be drag dominated. Namely, as it can be seen in Table 1, work of the drag force, W^D , is included in all models, while in models 2, 3 and 5 it is even considered to be the only contribution. The evaluation of W^D is in models 2-5 based on the mean relative velocity, $\overline{\mathbf{u}_R}$, but in model 1 the terminal velocity of a single bubble, $U_T = 1.41E\dot{\sigma}_{ref}^{0.25}We_{ref}^{-0.5}$, is used. The drag coefficient in models 1-4 is evaluated via: $C_D = \frac{2}{3}E\dot{\sigma}_B^{-1/2} \left\{ \frac{1+17.67\alpha_L^{1.3}}{18.67\alpha_L^{1.5}} \right\}^2$, while the constant value $C_D=0.44$ is used in the model 5. Van Driest's function, f_w , used in model 1 is formulated in the same way as in single phase flows. For the definition of the coefficient C_t see corresponding reference³. Non-drag contributions, W^{ND} , are in model 1 included through an additional term that accounts for absorption of liquid phase turbulence by bubbles, while in model 4 the work of the added-mass force is considered.

(Appropriate place for Table 1)

The performance of closure assumptions presented in Table 1 for the case of bubble-train flow is illustrated in Figure 5 for two span-wise positions. Encouraging results concerning modelling of interfacial term are obtained. Namely, when the profiles of these terms evaluated from the model 4 are compared with the ones obtained according to the exact expression, great discrepancies are not observed. All other closure assumptions underestimated the magnitude of the interfacial term. Comparison of forms of the model 4 and model 5 revealed the importance of the proper choice of the drag coefficient, C_D . Namely, although the 'standard' definition of the work

of the drag force used in model 4 is in model 5 multiplied with $1.44\alpha_L$ (what is here greater than 1), due to inappropriate correlation for C_D (valid for particulate Reynolds number higher than 1000) this model underestimated interfacial term. Since the terminal bubble velocity, U_T , is of the same order of magnitude as the mean relative velocity, $\overline{u_R}$, the underestimation of interfacial term by the model 1 can be attributed to multiplication factor $0.075f_w$. Introduction of this correction for the work of the drag force is not documented¹⁰⁾, but surprisingly when the closure assumptions in the modelled k_L equation are derived by the same group of authors⁹⁾, it is not taken into account. Our calculations confirm that the assumption of the drag dominance is correct. Namely, both non-drag contributions, W^{ND} , given in Table 1 turned out to be an order of magnitude lower than the corresponding drag ones, W^D . However, one should keep in mind that we considered fully developed steady bubble motion. In the case of non-steady bubble rise the contribution of the work of added-mass force might become important.

(Appropriate place for Figure 5)

VI. Conclusions

The present paper deals with the quantitative analysis of the balance equation for the liquid phase turbulence kinetic energy (k_L equation) in gas-liquid bubbly flows. According to this equation there are two governing mechanisms which determine the turbulence characteristics. The first group of terms is associated with liquid phase volumetric fraction and includes turbulence energy diffusion, dissipation and production. The other group is associated with the interfacial area concentration and is peculiar to two-phase flow systems.

To provide the data for the analysis of the k_L equation, direct numerical simulations of a regular train of ellipsoidal bubbles rising through an initially stagnant liquid are performed.

The analysis of the budget of the liquid phase turbulence kinetic energy reveals the importance of the interfacial term. Namely, since the production term is found to be negative, this term is the only source of turbulence kinetic energy. In the diffusive term the contribution of the pressure correlation is dominant.

The information on balance terms in the exact k_L equation is further used for scrutiny and validation of closure assumptions employed in the k_L equation of two-phase $k-\varepsilon$ and $k-l$ models. As concerns the case of the bubble-train flow studied here, the conventional modelling of production and diffusion terms totally fails: the production term predicted by models is positive, while it should be negative and the diffusive term is strongly underestimated. Taking into account bubble-induced eddy viscosity by the model of Sato et al.¹⁶⁾ did not result in significantly better results. The interfacial term evaluated using the model of Morel¹³⁾ showed rather good agreement with the exact one, while closure assumptions used by other authors did not perform well.

Finally, one can argue that the case of regular bubble train considered in this paper is somewhat academic and that this can be the reason why certain models do not perform well. However, this flow configuration is convenient for developing the computational tool to perform the quantitative analysis of the exact k_L equation and the validation of closure relationships employed in its modelled form. In future work we intend to apply the presented methodology to the case of liquid motion induced by a rising bubble swarm.

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Figure 1. Geometry of the computational domain.

Figure 2. Visualisation of instantaneous bubble shape and velocity field for plane $x_2=0.5$. In vertical direction velocity vectors are shown for every third mesh cell.

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Figure 4. Wall-normal profiles of single-phase-like terms at span-wise position $x_2=0.492$.

Figure 5. Wall-normal profiles of interfacial term at two span-wise positions, x_2 .

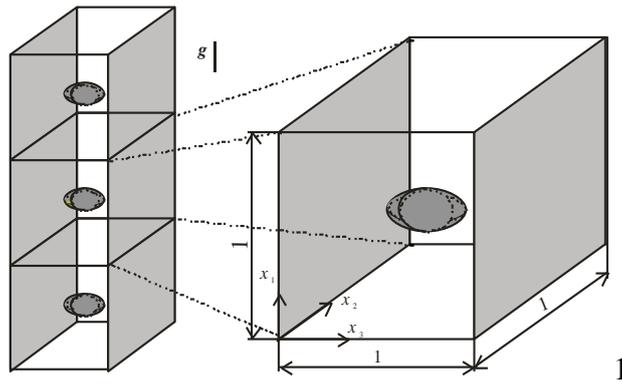


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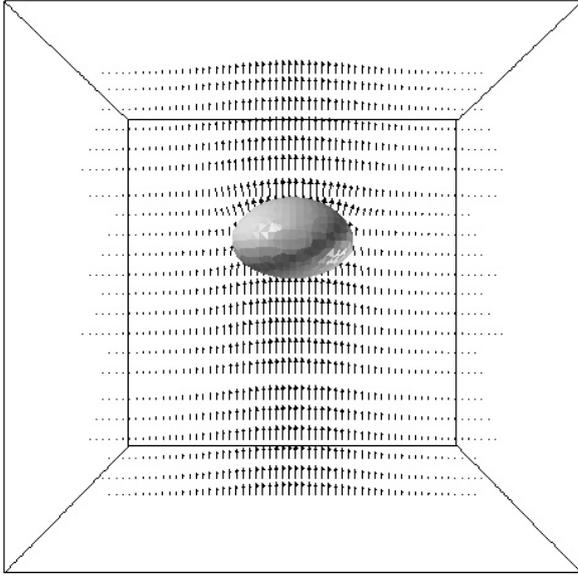


Figure 2. Visualisation of instantaneous bubble shape and velocity field for plane $x_2=0.5$. Velocity vectors are shown for every third mesh cell in vertical (x_1) direction.

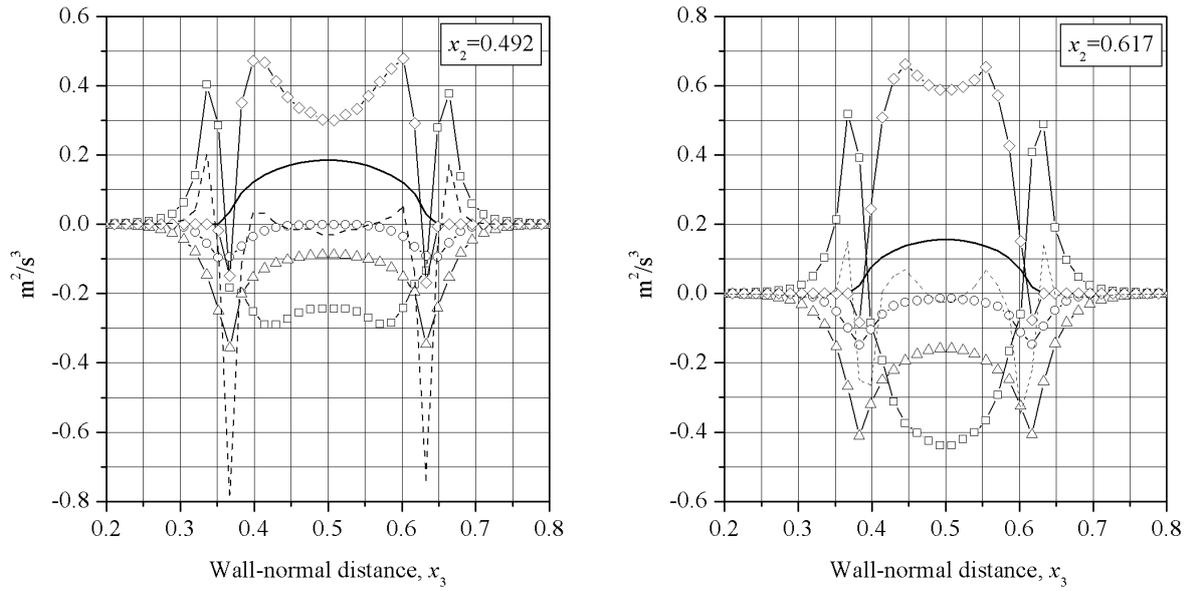


Figure 3. Wall-normal profiles of balance terms in equation (4) for two span-wise positions, x_2 .
 Legend: diffusion ∇ , production $-$, dissipation 8 , interfacial term M . Dashed line represents out-of-balance. Solid line represents mean gas volumetric fraction, α_G .

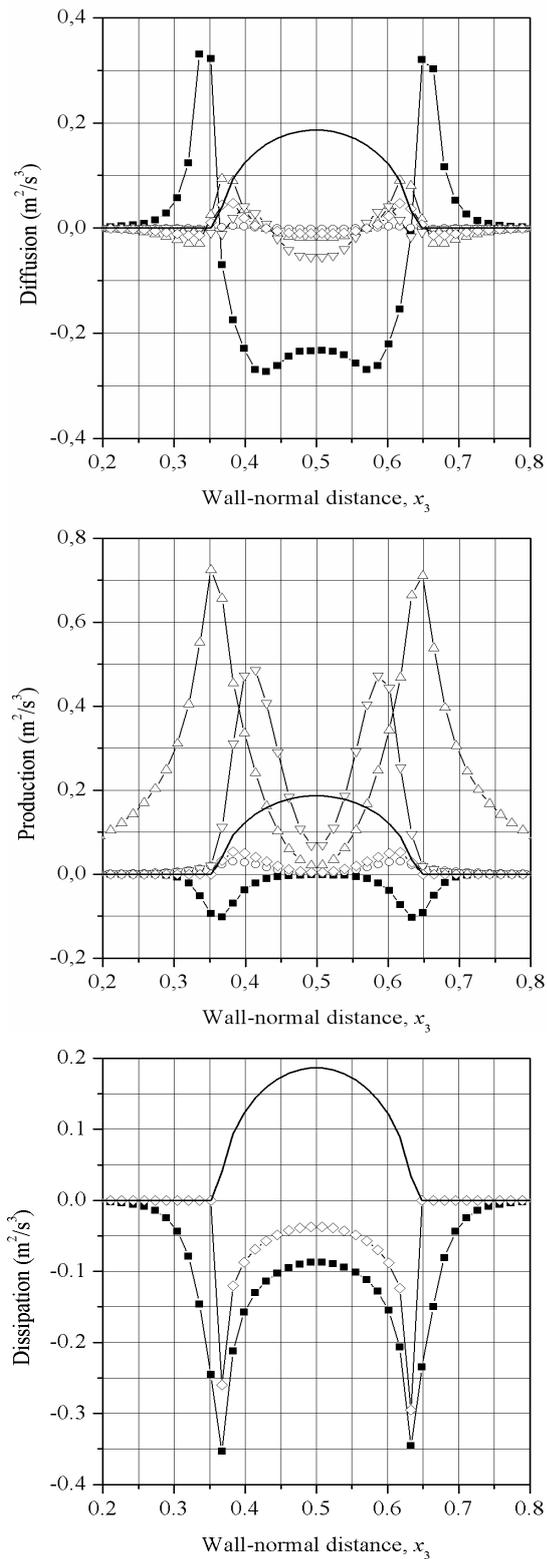


Figure 4. Wall-normal profiles of single-phase-like terms at span-wise position $x_2=0.492$. Legend: exact term \blacktriangle , $k-\varepsilon$ models with effective viscosity defined as: $\nu_L^{k\varepsilon}$ \blacksquare , $\nu_L^{k\varepsilon} + \nu_L$ \blacklozenge , $\nu_L^{k\varepsilon} + \nu_L^B$ \circ and $k-l$ model \circ . Solid line represents mean gas volumetric fraction, α_G .

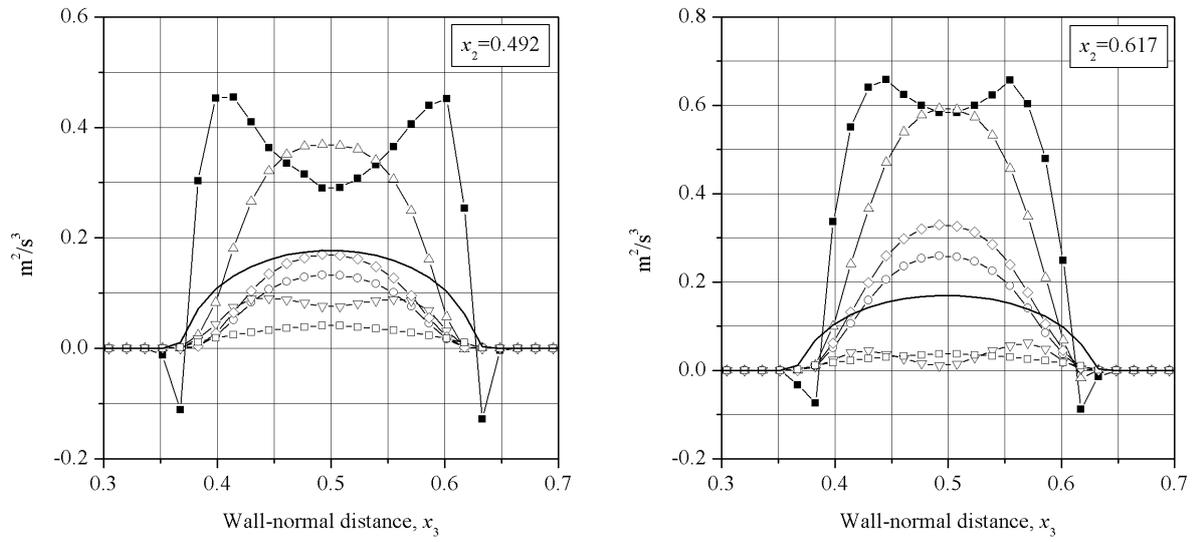


Figure 5. Wall-normal profiles of interfacial term at two span-wise positions, x_2 .
 Legend: exact terms !, model 1 ∇ , model 2 \times , model 3 \mathcal{M} , model 4 s and model 5 $-$ (see Table 1). Solid line represents mean gas volumetric fraction, α_G .

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Table 1. Closure assumptions for interfacial term in modelled k_L equation

Table 1. Closure assumptions for interfacial term in modelled k_L equation

| | Reference | Interfacial term (-) | |
|---------|-----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| | | Work of drag force, W^D | Other contributions, W^{ND} |
| Model 1 | 10) | $0.075 \left[\alpha_G \frac{3 C_D}{4 D_b} U_T^3 \right] f_w$ | $-\alpha_G \frac{k_L^{3/2}}{D_b}$ |
| Model 2 | 3) | $\frac{3}{4} \frac{\alpha_G C_D}{D_b} \left \overline{\mathbf{u}_R} \right \left\{ \frac{\overline{\mathbf{u}_R} \cdot \nabla \alpha_G}{0.3 Re_{ref} \alpha_L \alpha_G} + 2k_L (C_i - 1) \right\}$ | None |
| Model 3 | 11) | $0.25 \alpha_L (1 + C_D^{4/3}) \alpha_G \frac{\left \overline{\mathbf{u}_R} \right ^3}{D_b}$ | None |
| Model 4 | 13) | $\left[\frac{3}{4} \alpha_G \frac{C_D}{D_b} \left \overline{\mathbf{u}_R} \right ^3 \right]$ | $\frac{1 + 2\alpha_G}{2\alpha_L} \alpha_G \left\{ \frac{D_G \overline{\mathbf{u}_G}}{Dt} - \frac{D_L \overline{\mathbf{u}_L}}{Dt} \right\} \cdot \overline{\mathbf{u}_R}$ |
| Model 5 | 14) | $1.44 \alpha_L \left[\frac{3}{4} \alpha_G \frac{C_D}{D_b} \left \overline{\mathbf{u}_R} \right ^3 \right]$ | None |