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FE-regularization of non-smooth vibrations due to friction and impacts

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Abstract Friction and impacts during oscillations lead to discontinuities of the velocity and of the internal forces in the time-domain and to changes in the number of degrees of freedom, Ibrahim (1994). The analytical procedure for the integration of such non-smooth motions is to compute the history dependent separation times and to patch together a sequence of solutions for successive smooth problems, Popp (1998). However, this very accurate procedure has limits even for a relatively low number of generalized coordinates because of the required computational effort. Regularization techniques as usually used with FE allow to avoid the exact computation of all discontinuities by smoothing. But there is a big uncertainty in the choice of the regularization parameters needed for a sufficiently correct description of the oscillations under investigation. Stationary solutions of two forced mass–spring oscillators are used to calibrate the regularization parameters by comparing analytical results with regularized ones. This allows to compute the self-excitation of a continuous system and to prove the phenomena with known experimental data.

Keywords Friction, Impact, Finite elements, Non-smooth vibration

1 Introduction

A non-smooth discrete system changes its mechanical properties during the course of time. Examples are the possibility of sliding or sticking at contact points or the change from a motion in contact to a free motion or vice versa. Generally there exists a set of possible states which can differ in their degrees of freedom. At the beginning of a motion it is neither known how many possible states will become active in the future, nor the sequence of the active states is predetermined. However, we have to note, that each state represents itself a smooth problem. If $\mathbf{X}$ denotes the vector of generalized coordinates, the total solution in the time domain is given in general by a sequence of smooth solutions

$$\mathbf{X} = \left\{ \begin{array}{ll} \mathrm{X}_1, & t_1 \le t < t_2 \\ \mathrm{X}_2, & t_2 \le t < t_3 \\ \vdots & \\ \mathrm{X}_i, & t_i \le t < t_{i+1} \end{array} \right\} \quad i = 1, 2, 3, \ldots$$

Let us assume, the solution X_i corresponding to a distinct element of the set of possible states is known by integration up to a certain time $t > t_i$. Then a condition must be given, to determine the end of this state at t_{i+1}. This condition is found by checking switching conditions for each $t \ge t_i$. Knowing t_{i+1} a switching decision allows to choose the new state $\mathbf{X}_{i+1}$ out of the set of all possible states. The complete information about the mechanical quantities at the end of the state $\mathbf{X}_i$ is used as initial conditions for X_{i+1}. By sequential application the total solution is patched together at the points of discontinuities t_i.

The spatial discretization of a continuum by the FE-method leads to equations of motions with a finite number of degrees of freedom. Therefore all dynamic contact problems lead to non-smooth mechanical systems. In principle, analytical solutions can be computed as described above. In practice such an exact solution can only be obtained if the number of all possible states is relatively low, which leads essentially to the restriction to a small number of coordinates and contact points, Vielsack and Kammerer (1999). FE discretizations, however, lead to large numbers of coordinates. Therefore regularizations of all discontinuities are vital to achieve a solution at all. First the number of degrees of freedom of all possible smooth states should be the same for all t; secondly the switching conditions and switching decisions must be eliminated or at least transformed into a weaker form. Finally, the correct computation of the separation times should be avoided.

From the mechanical point of view all regularizations imply changes in the mechanical model of a non-smooth system. Restricting to the so-called penalty method as regularization, additional stiffnesses or dampers are introduced. Therefore, the solution of a regularized system can only be an approximation of the exact solution. The quality of the solution depends on a reasonable choice of the regularization parameters.

2 Calibration of the regularization parameters for friction

In the following a discrete mass–spring system (Fig. 1) is considered. This simple model allows a detailed discussion about the consequences of regularizing Coulomb's friction law.

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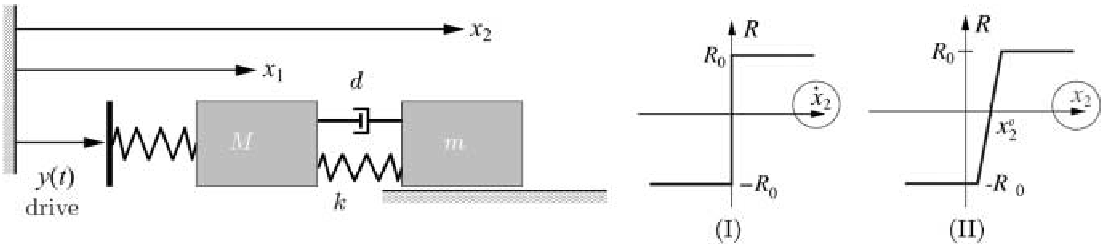

Fig. 1. Mechanical model for friction benchmark example (left) and considered friction laws – (I) Coulomb (center), (II) Regularization (right)

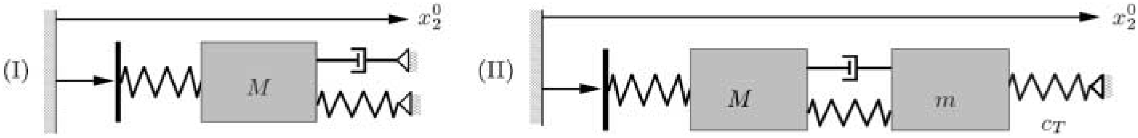

Fig. 2. Mechanical models for sticking of mass m: (I) non-smooth model, (II) regularized model with spring

The theoretical background of the analytical treatment can be found in Vielsack (2001). The non-smooth system has three possible smooth states: (a) the mass m sticks (1 DOF), (b) the mass moves to the left (2 DOF) or (c) the mass moves to the right (2 DOF). What really happens is depending on the property of the excitation $y(t)$, the drive.

To compare both approaches, first states of sticking are considered. Both situations are shown in Fig. 2. In the analytical approach the degrees of freedom are reduced to one. The displacement $x_2 = x_2^0$ of the mass m remains constant. Its value is known from the end of the preceding sliding state. In the case of regularization an additional spring with stiffness c_T (the penalty, Pfeiffer and Glocker 1999) is added to the mass m at the unloaded state $x_2 = x_2^0$. This penalty method leads to a modified system with the same number of degrees of freedom, two. It is obvious that the mass m can move despite the fact that sticking requires a complete stop. Theoretically the displacements $x_2(t)$ of the mass m could be made arbitrarily small by choosing c_T sufficiently large. However, this would lead to the well-known numerical problems in integrating stiff differential equations.

Secondly states of sliding are considered. In both approaches the system has two degrees of freedom. The analytical procedure is based on Coulomb's law $R = R_0 \operatorname{sgn}(\dot{x}_2)$ which shows a constant friction force R_0 opposite to the direction of the relative velocity $\dot{x}_2$ (see circle in Fig. 1, I) between the rigid support and the mass m. Within the regularized formulation, see Engleder and Vielsack (2001), $R = R_0 \operatorname{sgn}(H)$ is used with H incrementally updated in each time step by $H(t + \Delta t) = H(t) + c_T[x_2(t) - x_2(t + \Delta t)]$, with an upper bound $|H| \le |R_0|$. Now the friction force depends on the absolute displacement x_2 (see circle in Fig. 1, II) of the mass m.

The regularization leads to a friction law which is identical to the description of linear elastic-ideal plastic behaviour and can therefore be called the elasto-plastic regularization model. Its advantage lies in the fact that it is well established in F.E.M.

To follow stationary periodical oscillations long time integration is needed, mainly because the influence of the initial conditions at the beginning of the instationary motion must be eliminated by internal damping. Within the analytical approach a Runge–Kutta integration scheme with an accurate calculation of all separation points, see Meijard (1999) is taken. The regularized model, however, is integrated by the Newmark-method with uniform time steps. No iteration of separation times is performed. This leads to permanent numerical disturbances during the course of time with often detrimental effects. If the time step is not chosen sufficiently small, irregular response of the system will occur, see Vielsack and Hartung (1999). The major differences between both procedures are as follows:

Analytical	Regularization
– alternating number of degrees of freedom	– constant number of degrees of freedom
– exact description of all smooth states	– modified mechanical model
– exact determination of all switching conditions	– weak satisfaction of switching conditions
– exact computation of separation times	– no computation of separation times
– switching decisions	– no switching decisions needed
– high effort for time integration	– efficient integration (constant time steps)

An example is presented to illustrate the consequences of the regularization. The parameters – nondimensional – of the system are $R_0 = 1$, $M/m = 25$, $K/k = 1$ and $D = 0.01$. The drive has a constant velocity $v = 0.5$ superimposed by a harmonic part with an amplitude $A = 0.75$ and a frequency ratio of $\eta = 0.2$. The regularization parameter ϵ is defined by $\epsilon = k/c_T$. The interaction

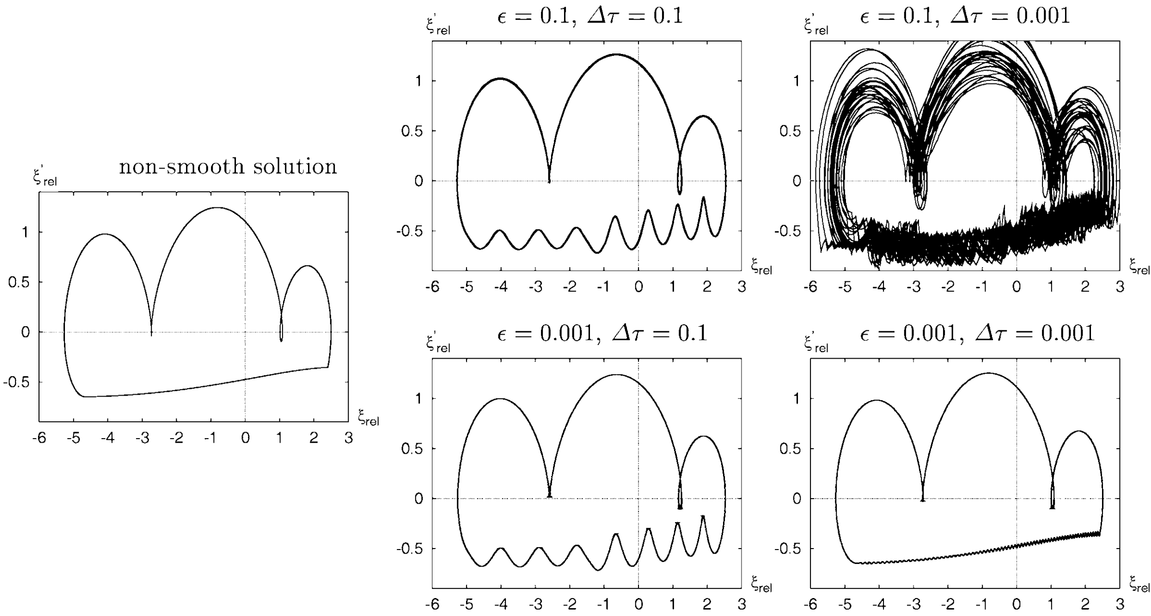

Fig. 3. Effects of regularization: phase curves in relative coordinates (ξ_{rel}, ξ'_{rel}) for m using $\epsilon = 0.1$ and 0.001 in combination with the time steps $\Delta\tau = 0.1$ and 0.001

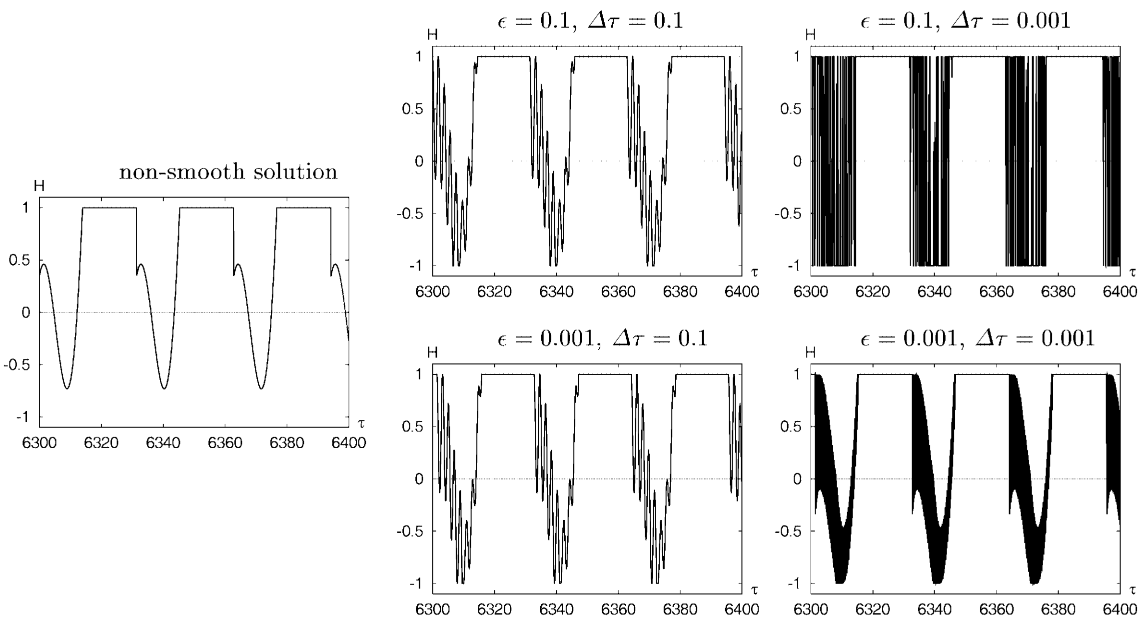

Fig. 4. Effects of regularization: contact forces for m using $\epsilon = 0.1$ and 0.001 in combination with the time steps $\Delta\tau = 0.1$ and 0.001

of the regularization parameter and the time step of the Newmark integration with average acceleration assumptions is discussed by combining $\epsilon = 0.1$ and 0.001 with the time steps $\Delta\tau = 0.1$ and 0.001. Figure 3 shows the phase curves for m in relative coordinates and Fig. 4 the corresponding contact forces.

Both, regularization parameter and time step size appear to have interacting effects on the results. The contact force shows oscillations during each state of sticking. The frequencies of the oscillations depend on the value of c_T. The combination of a small regularization parameter ϵ (large contact stiffness) with a large time step give rise to structural errors in the phase curves as well as in the contact forces. Only a small time step in combination with a small regularization parameter leads to sufficiently good results at least for the phase curve. Nevertheless, the high frequency oscillations of the contact force cannot be avoided then. It should be mentioned, that the contact force can be improved by adding a viscous damper parallel to the contact spring c_T, see Vielsack (1996). However, this would increase the number of regularization parameters to be calibrated and the

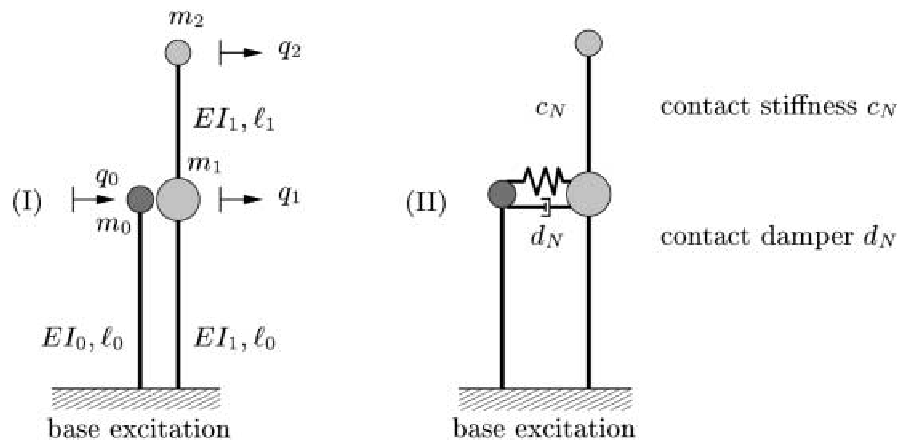

Fig. 5. Mechanical model for impact: (I) non-smooth model (Newtows impact law), (II) regularized model (linear spring c_N and linear damper d_N)

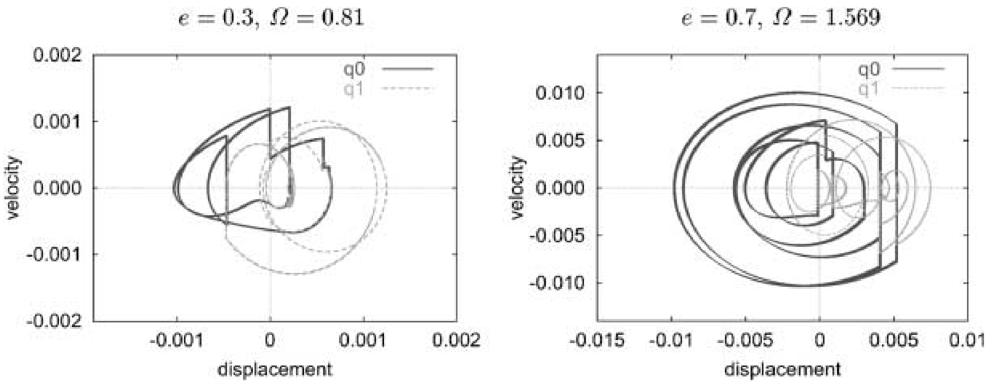

Fig. 6. Non-smooth model: phase curves for q_0 and q_1 using the coefficients of restitution $e = 0.3$ ($\Omega = 0.81$) and $e = 0.7$ ($\Omega = 1.569$)

applicability of the elasto-plastic regularization model would be lost.

3 Calibration of regularization parameters for impacts

Mechanical systems consisting of several subsystems can exhibit intermittent constraints. During oscillations the subsystems can move separately or come into contact with each other. Then impacts occur, Brogliato (1996). Generally the duration of an impact is much smaller than a representive periodical time of a stationary oscillation. Then the concept of Newton's impact law is often sufficient for a description. Impact is considered as one smooth state of the non-smooth problem with vanishing time of duration. Only the velocities are affected by this model. This effect can be controlled by a restitution factor $0 \leq e \leq 1$.

Again a mass-spring system is considered (Fig. 5) which is subjected to harmonic base excitation. It consists of two beam-subsystems, with length l_0 and $l_0 + l_1$, lumped masses m_0 and m_1, m_2 and bending stiffness EI_0 and EI_1, respectively. In addition a small internal damping is assumed to provide stationary responses when starting the oscillations with arbitrary chosen initial conditions. During oscillation three smooth states are possible: (a) the mass m_0 and m_1 are in permanent contact (2 DOF), (b) both subsystems move separately from each other (3 DOF), and (c) an impact between m_0 and m_1 occurs. A detailed discussion of the analytical treatment of the problem can be found in Engleder, Vielsack and Spiess (1998).

The regularization of the impact is performed similar to the previous model by the penalty method discussed before. For $q_0 > q_1$ a contact spring with stiffness c_N only acting in compression is added between the masses m_0 and m_1. The restitution coefficient is modelled by a contact damper with constant d_N only acting in compression. The sudden impact is enlarged to a finite time interval with unknown time-dependent damping force and displacement of the damper. Therefore, a correlation of the energy loss between a restitution coefficient and its corresponding constant d_N of damping cannot be given.

Thus, the introduction of both regularization parameters leads to the fact that the number of DOF is kept constant for all possible states (3 DOF). The intrinsic problem is the appropriate choice of the values of both regularization parameters. The finite duration of the impact requires sufficiently small time steps using the Newmark-method, as the integration with constant time steps leads to similar problems concerning the separation times as discussed before. Knowing the solution of the non-smooth model for a given problem, extensive numerical experiments are necessary to calibrate c_N and d_N and to find the acceptable time step size Δt for the regularized model.

As an example for the regularization results for varying coefficients of restitution respectively damping factors are given in Figs. 6 and 7. Both figures show essentially the same stationary oscillations. The phase curves for the contacting masses m_0 and m_1 are plotted as two separate lines in all diagrams. An overlapping of q_0 and q_1 indicates

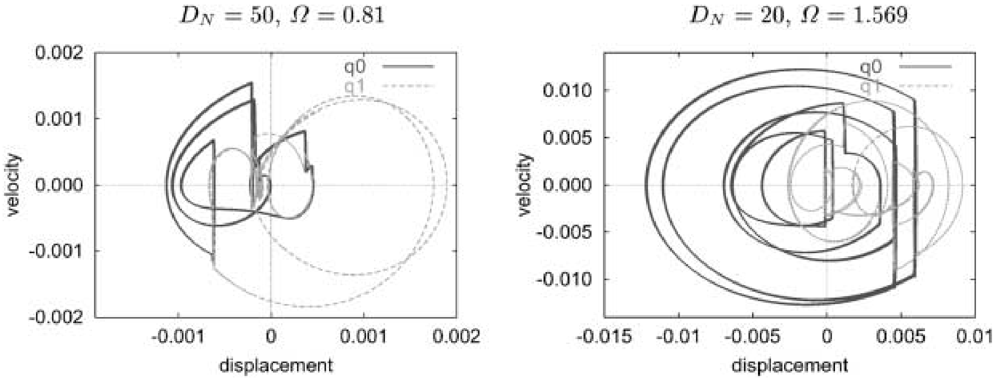

Fig. 7. Regularized model: phase curves for q_0 and q_1 using damping values $D_N = 50$ ($\Omega = 0.81$), $D_N = 20$ ($\Omega = 1.569$), contact stiffness $c_N = 10000$ and time step size $\Delta t = 10^{-4}$

a state of permanent contact. Following the non-dimensional representation of the system's parameters given in Engleder, Vielsack and Spiess (1998) the values $A = 0.001$ and $D = 0.001$ are used.

The exact result for $e = 0.3$ (close to ideal plastic impact) contains periodic oscillations with three states: (a) free motion, (b) impact and (c) motion in contact. In the case of a large restitution coefficient $e = 0.7$ (close to ideal elastic impact) only two states are present: free motion and impact. Moreover, the oscillation is modulated in the amplitudes but remains stationary. All regularized solutions (Fig. 7) are calculated with a constant time step $\Delta t = 10^{-4}$. Compared to the lowest non-periodical time 4.0 of the excitation this value – necessary to obtain a reasonable solution – is very small. The contact stiffness is taken as $c_N = 10000$. Taking a contact damper value $d_N = 50$ for the case $e = 0.3$ and a value $d_N = 20$ for $e = 0.7$ the regularized solutions agree fairly well with the exact responses. It must be stated, however, that the parameters c_N, d_N and Δt cannot be chosen separately, because they have interacting effects on the solutions.

4
Self-excitation of an elastic frame due to friction and impacts

The paradigm for self-excitation is a mass-spring system in contact with a rough surface, the latter moving with constant velocity. Its trivial solution describes a state of static equilibrium where the contact points remain in sliding for all times. The nontrivial, stationary solution is a stick-slip motion. Much effort has been made to compute such self-excited oscillations. A large number of different friction laws has been tested, usually keeping the normal force constant. Surprisingly, in the case of Coulomb's law only the trivial motion will exist under this assumption. Oden and Martin (1985) have solved this discrepancy by taking time varying normal forces into account. Then the trivial solution becomes also unstable and shows a stick-slip motion.

The idea with non-constant normal forces will be extended in the following by considering lift-off and impact in addition to time-varying normal forces. The mechanical system chosen is a steel frame with a horizontal and vertical member. The constant rectangular cross section is 20/2 mm. The length of each side is 300 mm. The horizontal edge is clamped, the lower end touches a rough surface which moves with constant velocity v_0. The upper end can be moved vertically by q_1^0 to induce a defined prestressing. Thus we achieve a problem with the combined behavior of Example 1 and 2.

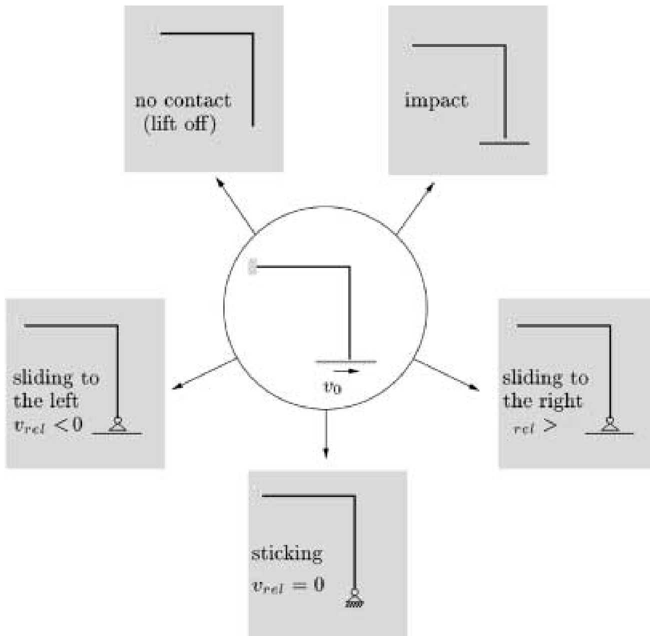

Fig. 8. Possible states during oscillation of the non-smooth model

The oscillations found in the structure can be described by five possible states. They are illustrated in Fig. 8. In addition experimental data are known, see Engleder (2000) about the main features of self-excited oscillations of the real frame.

The experiments show two kinds of phenomena: (a) self-excitation without lift off for low driving velocities and (b) oscillations where all five possible states are present for higher velocities.

Now, as in real practical situations, a finite element analysis with regularization should be performed without any information about the expected results. However, this would not lead to any reasonable result. Thus, all necessary parameters are calibrated in a way that the observed phenomena can be represented by the numerical analysis. Extensive numerical experiments lead to a time step size of $\Delta t = 10^{-3}$ for the standard Newmark-method (without numerical damping). The penalty parameters are chosen

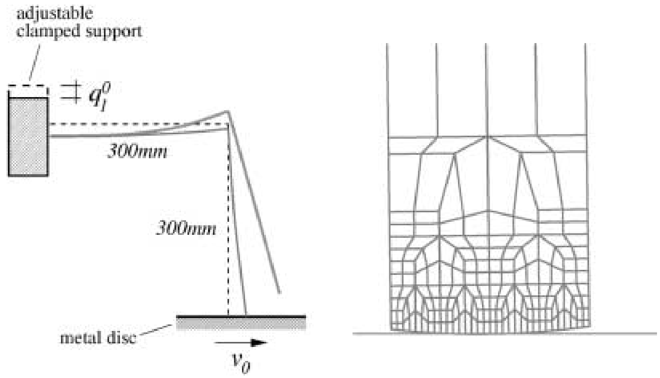

Fig. 9. Finite element model and discretization of the contact region

as $c_N = 10^5$, $c_T = 10^4$ and $D_N = 80$. Internal damping is described by Rayleigh-damping with $\alpha = 0.0301$ and $\beta = 1.841\mathrm{e}-5$. The FE discretization uses 9-node plane stress elements. The mesh at the contact region can be seen in Fig. 9. The rigid surface is modelled as a rigid body.

In addition the numerical calculations need information about the friction factor μ and the amount of dissipation during one impact. The friction factor μ has not been measured in the experiments and it would have been rather difficult to find such values. We will therefore restrict to a qualitative comparison between calculation and reality, which means whether the main features of the problem can be captured by the FE-regularization which are self-excitation without lift off for low driving velocities and motions with lift off for higher ones. Therefore in the following we are relatively free to choose the system parameters μ, v_0 and q_1^0 needed for the calculation in addition to the regularization parameters given above.

The time-displacement history in the first case without lift-off can be seen in Fig. 10. The horizontal line in both figures corresponds to a constant vertical relative displacement $q_1^0 = 2.5$ mm of the node in the middle of the contact region. To get the same result as observed in the experiments for its vertical motion, the driving velocity must be reduced from $v_0 = 250$ mm/s (experiment) to $v_0 = 100$ mm/s (simulation). The Coulomb friction factor is assumed to be $\mu = 0.1$.

In the second case the driving velocity $v_0 = 500$ mm/s is the same in both diagrams. Numerical experiments show that significant lift off occurs by increasing the prestress. Therefore $q_1^0 = 5$ mm is used for the analysis and a larger friction factor of $\mu = 0.2$ is chosen.

The depicted range of the time history of the computed displacement (Fig. 11) shows segments of irregularity. Obviously the existence of impacts can induce major

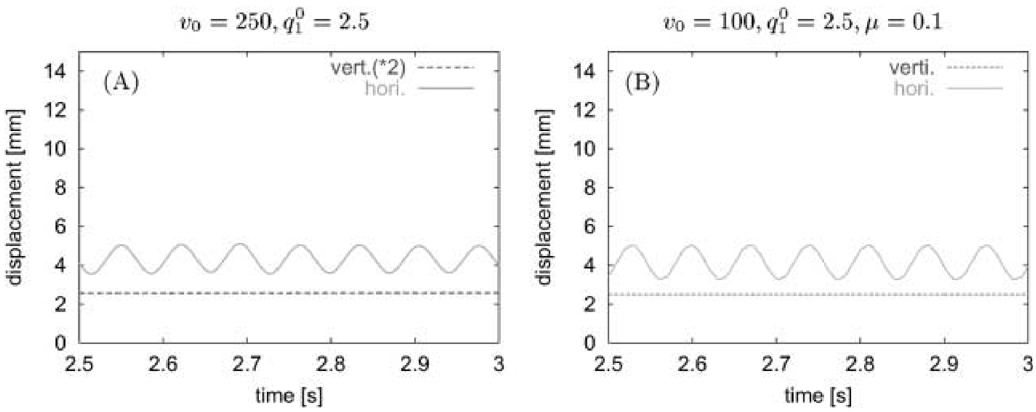

Fig. 10. Displacement-time relation for self-excitation without lift-off: (A) experimental and (B) numerical results

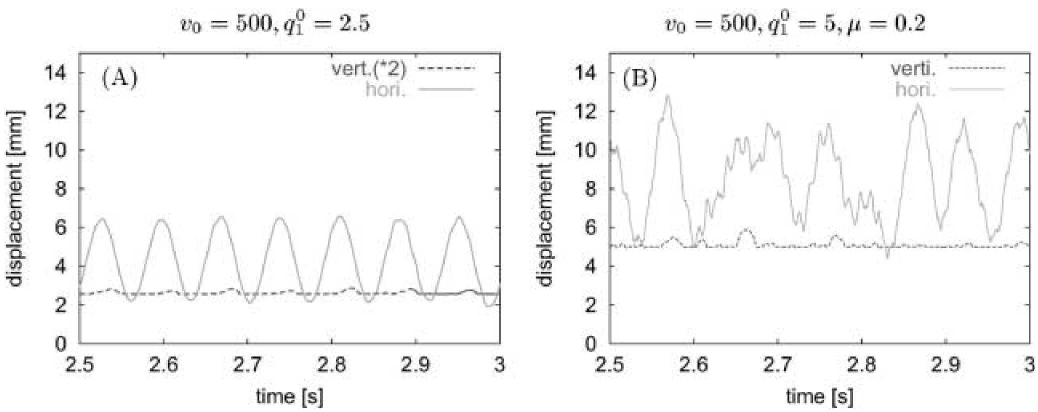

Fig. 11. Displacement-time relation for self-excitation with lift-off: (A) experimental and (B) numerical results

errors into the analysis. However, the amplitudes of the oscillations in these segments are limited and the irregularities vanish after some periods of the response. A coarse inspection shows a periodic motion in the total time domain.

5 Conclusions

Smooth continuous systems can be transformed to discrete systems by the FE method after choosing an appropriate discretization. Then vibrations can be investigated by integrating numerically the smooth discrete system. Oscillations of non-smooth continuous systems demand additional effort, because they consist of a priori unknown sequences of smooth states. Then within a solution with FE, regularizations of all irregularities at the impact and separation times are needed, thus always when the system's properties are changed. This implies the appropriate choice of

- the time step size for integration to capture the impact and separation times with sufficient accuracy and to allow the numerical computation of all regularized models
- the internal damping to avoid incorrect responses
- the penalty parameter for dry friction to describe the state of sticking
- the penalty parameter for an unilateral normal contact
- the damping parameter in case of a normal impact

The intrinsic problem is to choose all these numbers for a given problem. The fact of mutual dependence of all these essentially artificially introduced parameters makes a decision about the correctness of any result obtained with a model even more difficult than in standard FE analyses.

When treating smooth oscillations the experienced analyst is aware of the type of solution he can expect. A nice looking result gives mostly some confidence that all solution steps are correct. Non-smooth systems do not allow to follow this experience from smooth systems. In Vielsack (1999) it is shown for a harmonically excited single mass system with impacts, that a too large choice of the time step size leads to permanent numerical disturbances which causes a nice looking periodic response of the system, which is totally wrong. Summarizing it must be stated, that without the possibility of calibrating the regularization parameters and the time step size for the time integration procedure to some known real data, the computed results must be looked at with some suspicion.

Thus, calculating non-smooth oscillations of dynamical systems, the investigator must be aware of the strong nonlinearity of the problem. Extensive and brought variations of all regularization parameters are necessary to find out the sensivity of the systems response on these parameters.

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