PRIORITY-RULE METHODS FOR THE RESOURCE-CONSTRAINED PROJECT SCHEDULING PROBLEM WITH MINIMAL AND MAXIMAL TIME LAGS — AN EMPIRICAL ANALYSIS —

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We consider heuristics for the resource-constrained project scheduling problem with minimal and maximal time lags minimizing the project duration (RCPSP/max). The classical RCPSP represents a special case of RCPSP/max where the minimal time lags correspond to the respective activity durations and no maximal time lags have to be observed. If activities and (generalized) precedence constraints are modelled by an activity-on-node (AoN) network, the maximal time lags are represented by backward arcs weighted with the negative value of the maximal time lag (Brinkmann & Neumann 1995). Thus, we obtain a cyclic AoN project network with cycles of non-positive length. In opposite to RCPSP, the feasibility problem of RCPSP/max is NP-hard (Bartusch et al. 1988). In the following, we sketch several priority-rule-based methods for RCPSP/max. Thereafter, we briefly present an empirical analysis.

Priority-rule-based heuristics for RCPSP/max

For RCPSP/max an exact branch-and-bound procedure proposed by Bartusch et al. (1988), a truncated B&B depicted by Brinkmann & Neumann (1995), and a priority-rule-based method offered by Neumann & Zhan (1995) are known so far. We improve and extend the approach of Neumann & Zhan (1995). Priority-rule-based heuristics can be divided into methods being based on a serial and a parallel dispatching scheme. Using a serial scheme we obtain active schedules whereas a parallel scheme generates non-delay schedules (Kolisch 1995). Neumann & Zhan (1995) make use of a parallel scheme. If no maximal time lags have to be observed any activity can be scheduled arbitrarily late. In the case of RCPSP/max, this is not possible for activities whose latest start times are affected by maximal time lags.
Heuristics for RCPSP/max can be classified into direct methods and contraction methods. An empirical study by Brinkmann & Neumann (1995) shows that the contraction method provides better results than the direct method. The contraction method schedules each cycle structure (strong component) independently from other activities of the project. If there is a feasible subschedule for every cycle structure, the whole project is feasible (Bartusch et al. 1988). By shrinking each cycle structure to a single node (contracted node), the cyclic network can be contracted to an acyclic one (cf. Franck 1996, Brinkmann & Neumann 1995). Minimal time lags referring to contracted nodes have to be calculated with respect to the start time of the corresponding cycle structure.

Contraction method:
Step 1: Determine all cycle structures
Step 2: Schedule each cycle structure separately:
  2.1: Expand the cycle structure by a dummy source and a dummy sink
  2.2: Determine the earliest and the latest start time by a temporal analysis
  2.3: Schedule the cycle structure by a priority-rule-based heuristic
Step 3: Contract the cyclic network
Step 4: Schedule the acyclic contracted network by a priority-rule-based heuristic.

The determination of the resource requirements of a contracted cycle structure can be done in two ways. Either compute the maximum resource requirement of each resource or compute the actual time-varying resource profile. Brinkmann & Neumann (1995) and Neumann & Zhan (1995) use the maximum resource requirement. Therefore, they exclude many good solutions. We are using the actual resource profile for each cycle structure to guarantee better solutions. To handle time-varying resource requirements, we have to adapt the serial and the parallel generation scheme. If there is a sufficient amount of resources at period t to execute an activity j, using a parallel scheme we cannot conclude that there is a sufficient amount of resources for the entire duration of activity j. Therefore, we have to check resource availability for each period.
We have to put the main emphasis on step 2.3. During scheduling of a cycle structure we have to consider the maximal time lags. Those can cause several rescheduling steps. If an activity is scheduled, the maximal latest start time $maxaz_j$ of each unscheduled activity j of the cycle structure has to be updated. If there is a $maxaz_j$ which is greater than an earliest precedence and resource feasible start time, we have to perform a rescheduling step. In this rescheduling step all activities which have a start time equal to or greater than the activity i whose start time ST_i implies the $maxaz_j$, have to be unscheduled. Activity i is assigned an earliest start time greater than ST_i.
In opposite to the contraction method the direct method schedules the entire cyclic network without a divide and conquer approach.

Direct method:
Step 1: Determine all cycle structures
Step 2: Perform a temporal analysis for the cyclic network and the cycle structures
Step 3: Schedule the cyclic network by a priority-rule-based heuristic.

Using the direct method we have to cope with new problems. Additional to problems with maximal time lags inside a cycle structure, several cycle structures can blockade each other or single activities can prevent that a cycle structure can be scheduled. Therefore, it is possible that, for each cycle structure there is a feasible solution but we cannot find a feasible solution with the direct method. The number of rescheduling steps is substantially greater than in the contraction method. In the direct method two kinds of rescheduling steps may occur. The first kind is invoked if a single activity in a cycle structure has to be right-shifted. The second kind occurs if the whole cycle structure has to be right-shifted. If two cycle structures have a (direct or indirect) precedence relation they can blockade each other. In order to resolve the conflict the successive cycle structure has to wait until the other cycle structure is scheduled. Another reason for shifting the whole cycle structure is, that a single activity is a predecessor of a node belonging to a cycle structure. This single activity can lift the earliest precedence feasible start time beyond $maxaz_j$ of the cycle structure activity j.
The advantage of the direct method is that the activities of a cycle structure are not in a fixed relation to each other. Therefore, we have more possibilities to fit in a cycle structure into a partial schedule. One of the drawbacks of the direct method consists of the large computation time due to the large number of rescheduling steps. In order to combine the advantage of the direct and contraction method the following hybrid approach could be chosen: First, a sequence of activities (instead of schedule) is determined for each cycle structure. After that, the (time) schedules of the cycle structures are fixed during the scheduling of the contracted network, observing the given activity sequence.

Computational results

We have tested our heuristics using a problem set with 1440 problem instances generated by the problem generator ProGen/max (cf. Schwindt 1995). The problem set is characterized as follows:

no. of activities: 100	no. of renewable resources R= {5,...,8}
resource factor {0.25, 0.5, 0.75, 1}	no. of required resources {1,...,R}
resource strength {0.2, 0.5, 0.75}	rel. no. of max.time lags{[0.05,0.15], [0.15,0.25]}
restrictiveness {0.35, 0.5, 0.65}	no. of cycle structures {{2,...,7}, {8,...,13}}
units of required resources {1, 2, 3}	no. of nodes in cycle structure {2,...,15}

Both the contraction method and the direct method have been tested with the following priority rules: Latest Start Time (LST), Latest Finish Time (LFT), Minimum Slack Time (MSLK), Least Float per Successor (LFS), Worst Case Slack (WCS), Resource Scheduling Method (RSM), Improved Resource Scheduling Method (IRSM), Most Immediate Successors (MIS), Most Total Successors (MTS), Longest Path Following (LPF), and Random (Rand) (cf. Kolisch 1995, Neumann & Zhan 1995). The rules WCS, RSM and IRSM had to be adapted to the serial scheme because they have been defined only for the parallel scheme so far. The mean deviation from the lower bound is summarized in the following table. The lower bound is the maximum of the length of a longest path from the beginning to the end of the project and a resource constrained lower bound.

SS: contracted network: serial scheme; cycle structures: serial scheme
PS: contracted network: parallel scheme; cycle structures: serial scheme
PS: contracted network: parallel scheme; cycle structures: serial scheme
SP: contracted network: serial scheme; cycle structures: parallel scheme
PP: contracted network: parallel scheme; cycle structures: parallel scheme

%	LST	WCS	MTS	LPF	LFT	RSM	MSLK	LFS	MIS	IRSM	Rand	Min.
SS	15.5	15.8	16.7	16.9	16.9	17.8	18.4	18.6	21.2	21.4	22.2	14.4
PS	16.8	16.9	17.1	17.2	16.9	17.2	16.9	17.4	19.5	19.5	20.1	15.9
SP	16.6	16.9	17.9	18.0	18.1	18.9	19.5	19.7	22.4	22.6	23.2	14.3
PP	17.9	17.9	18.1	18.2	18.0	18.2	17.9	18.5	20.6	21.3	21.4	15.7

Using a Friedmann and a Wilcoxon signed-rank test we obtain the following results for the different combinations of the generation schemes. $x \succ\succ y$ indicates that x is significantly better than y by an α-niveau of 1%. $x \succ y$ denotes that x has a higher signed rank-sum than y.

$$SP \succ\succ SS \succ\succ PP \succ\succ PS$$

For the priority rules we obtain the following ranking depending on the scheduling schemes ($\alpha = 10\%$):

SS: $LST \succ\succ WCS \succ\succ MTS \succ LPF \succ LFT \succ\succ RSM \succ\succ MSLK \succ\succ LFS \succ\succ MIS \succ\succ IRSM \succ\succ Rand$
PS: $WCS \succ MSLK \succ LST \succ LFT \succ\succ LPF \succ MTS \succ\succ LFS \succ RSM \succ\succ IRSM \succ MIS \succ\succ Rand$
SP: $LST \succ\succ WCS \succ\succ MTS \succ LPF \succ LFT \succ\succ RSM \succ\succ MSLK \succ\succ LFS \succ\succ MIS \succ\succ IRSM \succ\succ Rand$
PP: $WCS \succ LST = MSLK \succ LFT \succ\succ MTS \succ LPF \succ\succ LFS \succ RSM \succ\succ IRSM \succ MIS \succ\succ Rand$

In opposite to results obtained by Kolisch (1995) for RCPSP the serial scheme seems to be more efficient for solving the RCPSP/max. One reason may be the long durations

of the shrunk cycle structures compared to regular activities. As the best rules we can identify the LST and WCS rule. Using the serial scheduling scheme for the cycle structures we have been able to solve 1333 problem instances. With the parallel scheme we have obtained feasible solutions for 1395 problem instances. The serial method solved 6 problem instances which could not be solved by the parallel scheme. All in all, 1401 of the 1440 problem instances could be solved. From our obtained results, we suggest to use the SP-scheme with LST or WCS. A further result of our investigations is that the IRSM-rule recommended by Kolisch (1995) has turned out to be one of the worst rules for RCPSP/max, whereas the rules WCS and LST are the best rules as stated by Kolisch. The LFT-rule as one of the best rules for the RCPSP yields not the same good results by solving the RCPSP/max.
The above empirical results refer to single-pass methods. Now, we will focus on multi-pass methods (cf. Kolisch 1995). We distinguish two possibilities of multi-pass heuristics. Both approaches solve the same problem instance several times and select the best result. The regret-based biased random sampling sets the selection probabilities depending on priority values favouring those activities which seem to be a more adequate choice (Kolisch 1995). This approach uses the same priority rule for each sampling. The second possibility is to combine several priority rules (Neumann & Zhan 1995). With every rule we solve the problem instance one time. To guarantee the comparability we have investigated the multi-pass methods with 5 samplings. As a result we can state that the combination of priority rules has provided significantly better results than the regret-based biased random sampling. We obtained the best results by the combination of MSLK, WCS, RSM, IRSM, and Random.
A first investigation of the direct method shows that the direct method seems to yield significantly better results than the contraction method. On the other hand, the direct method necessities more rescheduling steps. The number of rescheduling steps can easily be several hundreds. A reliable statement whether the serial or the parallel scheme is better suited for the direct method is not possible so far because research is up to show this. Definitely, it can be stated that the direct method is consuming much more time than the contraction method because there are significantly more rescheduling steps.

References

Bartusch, M., Möhring, R.H., Radermacher, F.J. (1988): Scheduling project networks with resource constraints and time windows; *Annals of Oper. Res. 16*, 210 – 240

Brinkmann, K., Neumann, K. (1995): Sequential and contraction B&B-based heuristics for RLP/max and RCPSP/max; *Report WIOR 471*, Institut für Wirtschaftstheorie und Operations Research, Universität Karlsruhe; submitted to J. Decision Systems

Franck, B. (1996): Empirische Untersuchung von Prioritätsregeln für die ressourcenbeschränkte Projektplanung mit zeitlichen Minimal- und Maximalabständen; In: Kleinschmidt, P. (Ed.): *Operations Research Proceedings 1995*; Springer, Berlin

Kolisch, R. (1995): *Project Scheduling under Resource Constraints;*Physica, Heidelberg

Neumann, K., Zhan, J. (1995): Heuristics for the minimum project-duration problem with minimal and maximal time lags under fixed resource constraints; *J. Intelligent Manufacturing 6*, 145 – 154

Schwindt, C. (1995): ProGen/max: A new problem generator for different resource-constrained project scheduling problems with minimal and maximal time lags; *Report WIOR 449* , Institut für Wirtschaftstheorie und Operations Research, Universität Karlsruhe