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Current noise and Coulomb effects in superconducting contacts

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Abstract. We derive an effective action for contacts between superconducting terminals with arbitrary transmission distribution of conducting channels. In the case of normal-superconducting (NS) contacts we evaluate interaction correction to Andreev conductance and demonstrate a close relation between Coulomb effects and shot noise in these systems. In the case of superconducting (SS) contacts we derive the electron-electron interaction correction to the Josephson current. At $T = 0$ both corrections are found to vanish for fully transparent NS and SS contacts indicating the absence of Coulomb effects in this limit.

1. Introduction

Low energy electron transport across the interface between normal metals and superconductors (NS) is provided by the mechanism of Andreev reflection [1]. This mechanism involves conversion of a subgap quasiparticle entering the superconductor from the normal metal into a Cooper pair together with simultaneous creation of a hole that goes back into the normal metal. Each such act of electron-hole reflection corresponds to transferring twice the electron charge $e^* = 2e$ across the NS interface and results, e.g., in non-zero conductance of the system at subgap energies [2].

Andreev reflection is also responsible for dc Josephson effect in superconducting weak links without tunnel barriers. Suffering Andreev reflections at both NS interfaces, quasiparticles with energies below the superconducting gap are effectively "trapped" inside the junction forming a discrete set of levels which can be tuned by passing the supercurrent across the system [3]. At the same time, these subgap Andreev levels themselves contribute to the supercurrent [3, 4, 5, 6] thus making the behavior of superconducting point contacts and SNS junctions in many respects different from that of tunnel barriers.

Note that the above theories remain applicable if one can neglect Coulomb effects. In small-size superconducting contacts, however, such effects can be important and should in general be taken into account. A lot is known about interplay between fluctuations and charging effects in superconducting tunnel barriers [7]. Here we examine the properties of superconducting junctions going beyond the tunneling limit. Below we will demonstrate that Coulomb blockade in such junctions weakens with increasing barrier transmissions and eventually disappears in the limit of fully open superconducting contacts. We will also argue that in superconducting systems – similarly to normal contacts [8] – there also exists a direct relation between Coulomb effects and current fluctuations.
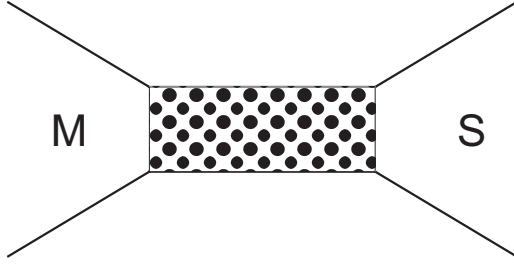
Figure 1. The system under consideration. Two big metallic reservoirs – one superconducting (S) and another one either normal or superconducting (M=N,S) – are connected by a short normal conductor.

2. The model and effective action

As it is shown in Fig. 1, we will consider big metallic reservoirs one of which is superconducting while another one could be either normal or superconducting. These two reservoirs are connected by a rather short normal bridge (conductor) with arbitrary transmission distribution T_n of its conducting modes and normal state conductance $G_N \equiv 1/R_N = (e^2/\pi) \sum_n T_n$. Both phase and energy relaxation of electrons may occur only in the reservoirs and not inside the conductor which length is assumed to be shorter than dephasing and inelastic relaxation lengths. As usually, Coulomb interaction between electrons in the conductor area is accounted for by some effective capacitance C.

In order to analyze electron transport in the presence of interactions we will make use of an approach based on the effective action formalism combined with the scattering matrix technique [8, 9, 10, 11, 12]. This approach can be conveniently generalized to superconducting systems. In fact, the structure of the effective action remains the same also in the supeconducting case, one should only replace normal propagators by 2×2 matrix Green functions which account for superconductivity, as it was done, e.g., in [7, 13, 14].

Following the standard procedure we express the kernel J of the evolution operator on the Keldysh contour in terms of a path integral over the fermionic fields which can be integrated out after the standard Hubbard-Stratonovich decoupling of the interacting term. Then the kernel J takes the form

$$J = \int \mathcal{D}\varphi_1 \mathcal{D}\varphi_2 \exp(iS[\varphi]), \tag{1}$$

where $\varphi_{1,2}$ are fluctuating phases defined on the forward and backward parts of the Keldysh contour and related to fluctuating voltages $V_{1,2}$ across the conductor as $\dot{\varphi}_{1,2}(t) = eV_{1,2}$. Here and below we set $\hbar = 1$.

The effective action consists of two terms, $S[\varphi] = S_c[\varphi] + S_t[\varphi]$, where

$$iS_c[V] = \frac{C}{2e^2} \int_0^t dt' (\dot{\varphi}_1^2 - \dot{\varphi}_2^2) \equiv \frac{C}{e^2} \int_0^t dt \, \dot{\varphi}^+ \dot{\varphi}^- \tag{2}$$

describes charging effects and the term $S_t[V]$ accounts for electron transfer between normal and superconducting reservoirs. It reads [14]

$$S_t[\varphi] = -\frac{i}{2} \sum_n \text{Tr} \ln \left[1 + \frac{T_n}{4} \left(\left\{ \check{G}_M, \check{G}_S \right\} - 2 \right) \right], \tag{3}$$

where $\check{G}_M$ and $\check{G}_S$ are 4×4 Green-Keldysh matrices of M- and S-electrodes which product implies time convolution and which anticommutator is denoted by curly brackets. In Eq. (2) we

also introduced "classical" and "quantum" parts of the phase, respectively $\varphi_+ = (\varphi_1 + \varphi_2)/2$ and $\varphi_- = \varphi_1 - \varphi_2$.

For later purposes we also express the average current and the current-current correlator via the effective action as

$$\langle \hat{I}(t) \rangle = ie \int \mathcal{D}\varphi_\pm \frac{\delta}{\delta\varphi_-(t)} e^{iS[\varphi]}, \tag{4}$$

$$\frac{1}{2}\langle \hat{I}\hat{I} \rangle_+ = -e^2 \int \mathcal{D}\varphi_\pm \frac{\delta^2}{\delta\varphi_-(t)\delta\varphi_-(t')} e^{iS[\varphi]}, \tag{5}$$

where $\langle \hat{I}\hat{I} \rangle_+ = \langle \hat{I}(t)\hat{I}(t') + \hat{I}(t')\hat{I}(t) \rangle$.

Let us introduce the matrix $\check{X}_0[\varphi_+] = 1 - T_n/2 + (T_n/4)\left\{\check{G}_M, \check{G}_S\right\}|_{\varphi_-=0}$. As the action S_t vanishes for $\varphi_-(t) = 0$ one has $\mathrm{Tr}\ln\check{X}_0 = 0$. Making use of this property we can identically transform the action (3) to

$$S_t = -\frac{i}{2}\sum_n \mathrm{Tr}\ln\left[1 + \check{X}_0^{-1} \circ \check{X}'\right], \tag{6}$$

where $\check{X}' = 1 + (T_n/4)\left(\left\{\check{G}_M, \check{G}_S\right\} - 2\right) - \check{X}_0$. Now let us separately consider NS and SS interfaces.

2.1. NS interfaces

At temperatures and voltages well below the superconducting gap Andreev contribution to the action of NS system dominates. Hence, it suffices to consider the limit of low energies $\epsilon \ll \Delta$. In this limit we can define the Andreev transmissions [2] $\mathcal{T}_n = T_n^2/(2 - T_n)^2$ and Andreev conductance $G_A = (2e^2/\pi)\sum_n \mathcal{T}_n$. Let us assume that either dimensionless Andreev conductance $g_A = 4\sum_n \mathcal{T}_n$ is large, $g_A \gg 1$, or temperature is sufficiently high (though still smaller than Δ). In either case one can describe quantum dynamics of the phase variable φ within the quasiclassical approximation [8, 9] which amounts to expanding S_t in powers of (small) "quantum" part of the phase $\varphi_-(t)$. Employing the above equations and expanding S_t up to terms $\sim \varphi_-^2$ we arrive at the Andreev effective action [15]

$$iS_t = iS_R - S_I, \tag{7}$$

where

$$iS_R = -\frac{ig_A}{2\pi} \int_0^t dt' \, \varphi^-(t')\dot{\varphi}^+(t'), \tag{8}$$

$$S_I = -\frac{g_A}{4} \int_0^t dt' \int_0^t dt'' \frac{T^2}{\sinh^2[\pi T(t' - t'')]} \varphi^-(t')\varphi^-(t'')$$
$$\times [\beta_A \cos(2\varphi^+(t') - 2\varphi^+(t'')) + 1 - \beta_A] \tag{9}$$

and

$$\beta_A = \frac{\sum_n \mathcal{T}_n(1 - \mathcal{T}_n)}{\sum_n \mathcal{T}_n} \tag{10}$$

is the Andreev Fano factor defined in a complete analogy with the normal Fano factor $\beta_N = \sum_n T_n(1 - T_n)/\sum_n T_n$. We observe that the action S_t is expressed in exactly the same form as that for normal conductors [8, 9] derived within the the same quasiclassical approximation

for the phase variable $\varphi(t)$. In order to observe the correspondence between the action [8, 9] and that defined in Eqs. (7)-(9) one only needs to interchange normal and Andreev conductances as well as the corresponding Fano factors

$$G_N \leftrightarrow G_A, \quad \beta_N \leftrightarrow \beta_A \tag{11}$$

and to account for an extra factor 2 in front of the phase φ_+ under cos in Eq. (9). This extra factor implies doubling of the charge during Andreev reflection.

2.2. Superconducting contacts

Turning to superconducting contacts we assume that fluctuating phases $\varphi_\pm(t)$ are sufficiently small and perform regular expansion of the exact effective action in powers of these phases. Then we obtain

$$iS_t = -\frac{i}{e} \int\limits_0^t dt' I_S(\chi)\varphi_-(t') + iS_R - S_I, \tag{12}$$

where χ is the time-independent phase difference,

$$I_S(\chi) = \frac{e\Delta \sin\chi}{2} \sum_n \frac{T_n}{\sqrt{1 - T_n \sin^2(\chi/2)}} \tanh \frac{\Delta\sqrt{1 - T_n \sin^2(\chi/2)}}{2T}. \tag{13}$$

defines the supercurrent across the system [16, 5] and

$$S_R = \int\limits_0^t dt' \int\limits_0^t dt'' \mathcal{R}(t' - t'')\varphi^-(t')\varphi^+(t''), \tag{14}$$

$$S_I = \int\limits_0^t dt' \int\limits_0^t dt'' \mathcal{I}(t' - t'')\varphi^-(t')\varphi^-(t'') \tag{15}$$

with both kernels $\mathcal{R}(t)$ and $\mathcal{I}(t)$ being real functions. The complete expressions for these functions turn out to be somewhat lengthy and for this reason are not presented here. Below we only emphasize some of the properties of $\mathcal{R}(t)$ and $\mathcal{I}(t)$.

To begin with, it is straightforward to verify that in the lowest order in barrier transmissions T_n the result (12)-(15) reduces to the standard AES action [7] for tunnel barriers in the limit of small phase fluctuations. Qualitatively new features emerge in higher orders in T_n being directly related to the presence of subgap Andreev levels $\pm\epsilon_n(\chi)$ inside the contact. Consider, for instance, the kernel $\mathcal{I}(t)$. It can be split into three contributions of different physical origin

$$\mathcal{I}(t) = \mathcal{I}_1(t) + \mathcal{I}_2(t) + \mathcal{I}_3(t). \tag{16}$$

The first of these terms, $\mathcal{I}_1(t)$, represents the subgap contribution due to discrete Andreev states. The Fourier transform of this term has the form

$$\begin{aligned}
\mathcal{I}_{1\omega} = \frac{\pi\Delta^4}{4} \sum_n \Bigg\{ &\frac{T_n^2 \sin^2\chi}{2\epsilon_n^2(\chi) \cosh^2(\epsilon_n(\chi)/2T)} \delta(\omega) \\
&+ \frac{T_n^2(1 - T_n)\sin^4(\chi/2)}{\epsilon_n^2(\chi)} \left[1 + \tanh^2(\epsilon_n(\chi)/2T)\right] \times \left[\delta\left(\omega - 2\epsilon_n(\chi)\right) + \delta\left(\omega + 2\epsilon_n(\chi)\right)\right] \Bigg\}.
\end{aligned} \tag{17}$$

It is obvious that this contribution is not contained in the AES action at all. The second term $\mathcal{I}_2(t)$ can be interpreted as the "interference term" between subgap Andreev levels and

quasiparticle states above the gap. In the low temperature limit $T \to 0$ the Fourier transform of this term $\mathcal{I}_{2\omega}$ differs from zero only at sufficiently high frequencies $|\omega| > \Delta + \epsilon_n(\chi)$. At higher temperatures $T > \epsilon_n(\chi)$, however, $\mathcal{I}_{2\omega}$ vanishes only for $|\omega| < \Delta - \epsilon_n(\chi)$ and remains non-zero otherwise. In the limit of small barrier transmissions this term scales as $\mathcal{I}_2 \propto T_n^{3/2}$ and, hence, is not contained in the AES action either. Finally, the third term $\mathcal{I}_3(t)$ accounts for the contribution of quasiparticles with energies above the gap. In the high frequency limit $\omega \gg \Delta$ or for $\Delta \to 0$ this term reduces to the standard result for a normal conductor $\mathcal{I}_{3\omega} \to (\omega/2e^2 R_N)\coth(\omega/2T)$.

Turning now to the function $\mathcal{R}(t)$ in Eq. (14) we note that its Fourier transform can be represented as $\mathcal{R}_\omega = \mathcal{R}'_\omega + i\mathcal{R}''_\omega$, where both $\mathcal{R}'_\omega$ and $\mathcal{R}''_\omega$ are real functions. The function $\mathcal{R}'_\omega$ is even in ω while $\mathcal{R}''_\omega$ is an odd function of ω, thus implying that the function $\mathcal{R}(t)$ is real.

The functions $\mathcal{R}(t)$ and $\mathcal{I}(t)$ are not independent. For instance, the Fourier transform $\mathcal{R}''_\omega$ is related to $\mathcal{I}_\omega$ by means of the fluctuation-dissipation relation $\mathcal{R}''_\omega = 2\mathcal{I}_\omega \tanh(\omega/2T)$. The two functions $\mathcal{R}'_\omega$ and $\mathcal{R}''_\omega$ are in turn linked to each other by the causality principle: the function $\mathcal{R}(t)$ should vanish for $t < 0$.

Finally we would like to point out that with the aid of the above Gaussian effective action one can easily evaluate the phase-phase correlation functions for our problem. Combining Eqs. (12)-(15) with (2) one finds (cf., e.g. [17])

$$\langle \varphi_+(t_1)\varphi_+(t_2)\rangle = -\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \mathrm{Im}\left(\frac{1}{C\omega^2/e^2 + \mathcal{R}_\omega}\right)\coth\frac{\omega}{2T}e^{-i\omega(t_1-t_2)}, \tag{18}$$

$$\langle \varphi_+(t_1)\varphi_-(t_2)\rangle = i\int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\left(\frac{1}{C\omega^2/e^2 + \mathcal{R}_\omega}\right)e^{-i\omega(t_1-t_2)},$$

$$\langle \varphi_-(t_1)\varphi_-(t_2)\rangle = 0.$$

Now we will employ the above results in order to describe the effect of electron-electron interactions on transport properties of superconducting contacts.

3. Coulomb blockade of Andreev reflection

We start from NS systems. In this case in the absence of interactions we set $\dot{\varphi}_+ = eV$ and trivially recover the standard result $I = G_A V$. For the current fluctuations $\delta I(t)$ from Eqs. (7)-(9) and (5) analogously to [8] we obtain

$$\frac{\langle |\delta I|^2_\omega\rangle}{G_A} = (1-\beta_A)\omega\coth\frac{\omega}{2T} + \frac{\beta_A}{2}\sum_{\pm}(\omega\pm 2eV)\coth\frac{\omega\pm 2eV}{2T}. \tag{19}$$

This equation fully describes current noise in NS structures at energies well below the superconducting gap. For $eV \gg T, \omega$ Eq. (19) reduces to the result [18] while in the diffusive regime the correlator (19) matches with the semiclassical result [19].

Let us now turn on interactions. In this case one should add the charging term (2) to the action and account for phase fluctuations. Proceeding along the same lines as in [8], for $g_A \gg 1$ or $\max(T, eV) \gg E_C = e^2/2C$ we get

$$I = G_A V - 2e\beta_A T\mathrm{Im}\left[w\Psi\left(1+\frac{w}{2}\right) - iv\Psi\left(1+\frac{iv}{2}\right)\right]. \tag{20}$$

where $\Psi(x)$ is the digamma function, $w = g_A E_C/\pi^2 T + iv$ and $v = 2eV/\pi T$.

The last term in Eq. (20) is the interaction correction to the I-V curve which scales with Andreev Fano factor β_A in exactly the same way as the shot noise. Thus, we arrive at

an important conclusion: *interaction correction to Andreev conductance of NS structures is proportional to the shot noise power in such structures.* This fundamental relation between interaction effects and shot noise goes along with that established earlier for normal conductors [8] extending it to superconducting systems. In both cases this relation is due to discrete nature of the charge carriers passing through the conductor.

Another important observation is that the interaction correction to Andreev conductance defined in Eq. (20) has exactly the same functional form as that for normal conductors, cf. Eq. (25) in [8]. Furthermore, in a special case of diffusive systems we have $G_N = G_A$, $\beta_N = \beta_A = 1/3$ and the only difference between the interaction corrections to the I-V curve in normal and NS systems is the charge doubling in the latter case. As a result, the Coulomb dip on the I-V curve of a diffusive NS system at any given T is exactly *2 times narrower* than that in the normal case. We believe that this narrowing effect was detected in normal wires attached to superconducting electrodes [20], cf. Fig. 3c in that paper.

4. Interaction correction to supercurrent

Let us now turn to the electron-electron interaction correction to the equilibrium Josephson current (13). Previously such correction was analyzed in the case of Josephson tunnel barriers in the presence of linear Ohmic dissipation [7]. The task at hand is to investigate the interaction correction to the supercurrent in contacts with arbitrary transmission distribution.

In order to evaluate the interaction correction it is necessary to go beyond the Gaussian effective action (12)-(15) and to evaluate the higher order contribution $\sim \varphi^3$. It is easy to observe that the interaction correction to the supercurrent is provided by the following non-Gaussian terms in the effective action:

$$
\delta(iS_t) = \int \int \int dt_1 dt_2 dt_3 Y(t_1, t_2, t_3) \varphi_-(t_1) \varphi_+(t_2) \varphi_+(t_3)
$$
$$
+ \int \int \int dt_1 dt_2 dt_3 Z(t_1, t_2, t_3) \varphi_+(t_1) \varphi_-(t_2) \varphi_-(t_3). \tag{21}
$$

The function $Y(t_1, t_2, t_2)$ can be written as

$$
Y(t_1, t_2, t_3) = \int \int \frac{d\omega_1}{2\pi} \frac{d\omega_2}{2\pi} Y(\omega_1, \omega_2) e^{-i\omega_1(t_1-t_2)} e^{-i\omega_2(t_1-t_3)}, \tag{22}
$$

where $Y(\omega_1, \omega_2) = Y(\omega_2, \omega_1)$. The function $Z(t_1, t_2, t_3)$ can be expressed in a similar way.

Adding the non-Gaussian terms (21) to the action and employing Eq. (4) we arrive at the following expression for the interaction correction

$$
\delta I_S(\chi) = ie \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} Y(\omega, -\omega) \langle \varphi_+ \varphi_+ \rangle_\omega + 2ie \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} Z(0, -\omega) \langle \varphi_+ \varphi_- \rangle_\omega, \tag{23}
$$

where the phase-phase correlators are defined in Eq. (18).

Let us consider the first term in the right-hand side of Eq. (23). It is easy to see that in the limit of low temperatures only frequencies $|\omega| > \Delta + \epsilon_n(\chi)$ contribute to the integral in Eq. (18) for $\langle \varphi_+ \varphi_+ \rangle$ while the contribution from the frequency interval $|\omega| < \Delta + \epsilon_n(\chi)$ vanishes. Furthermore, the leading contribution from the first term in Eq. (23) is picked up logarithmically from the interval $2\Delta \ll |\omega| \ll 1/R_N C$ where

$$
\langle \varphi_+ \varphi_+ \rangle_\omega \simeq \frac{e^2 R_N}{|\omega|} \tag{24}
$$

and the function $Y(\omega, -\omega)$ tends to a frequency independent value.

After a straightforward but tedious calculation in the interesting frequency range $\omega \gg \Delta$ from Eq. (3) one finds

$$Y(\omega, -\omega) = \frac{i\Delta \sin \chi}{4} \sum_n \frac{T_n(1 - T_n)(2 - T_n \sin^2(\chi/2))}{(1 - T_n \sin^2(\chi/2))^{3/2}} F(\epsilon_n(\chi)). \tag{25}$$

This high-frequency term involves the factor $1 - T_n$, i.e. it vanishes for fully open conducting channels. Combining Eqs. (24), (25) with (23), we arrive at the expression for the supercurrent

$$I(\chi) = I_S(\chi) + \delta I_S(\chi). \tag{26}$$

In the limit of low temperatures the interaction correction reads

$$\delta I_S(\chi) = -\frac{e\Delta}{2g_N} \ln\left(\frac{1}{2\Delta R_N C}\right) \sin \chi \sum_n \frac{T_n(1 - T_n)}{(1 - T_n \sin^2(\chi/2))^{3/2}} \left(2 - T_n \sin^2 \frac{\chi}{2}\right), \tag{27}$$

where $g_N = 2\pi/(e^2 R_N)$ is the dimensionless normal state conductance of the contact. This result is justified as long as the Coulomb correction $\delta I_S(\chi)$ remains much smaller than the non-interacting term $I_S(\chi)$ (13). Typically this condition requires the dimensionless conductance to be large $g_N \gg \ln(1/2\Delta R_N C)$.

Note that Eq. (27) was derived only from the first term in Eq. (23). The second term in this equation involving the function $Z(0, -\omega)$ and the correlator $\langle \varphi_+ \varphi_- \rangle$ can be treated analogously. It turns out to be smaller than that of the first term by the logarithmic factor $\sim \ln(1/2\Delta R_N C)$.

Let us emphasize again an important property of the result (27): The interaction correction contains the factor $1 - T_n$ and, hence, vanishes for fully open barriers. In other words, *no Coulomb blockade of the Josephson current is expected in fully transparent superconducting contacts.*

The expression for the interaction correction (27) can further be specified in the case of diffusive contacts. In the absence of interactions the Josephson current in such contacts follows from (13) and takes the form corresponding to the zero-temperature limit of a well known Kulik-Omelyanchuk formula for a short diffusive wire

$$I_S(\chi) = \frac{\pi\Delta}{2e R_N} \cos \frac{\chi}{2} \ln \frac{1 + \sin \frac{\chi}{2}}{1 - \sin \frac{\chi}{2}}. \tag{28}$$

Including interactions and averaging (27) with the bimodal transmission distribution

$$P(T_n) \propto \frac{1}{T_n \sqrt{1 - T_n}}. \tag{29}$$

one finds

$$\delta I_S(\chi) = -\frac{e}{8} \Delta \ln\left(\frac{1}{2\Delta R_N C}\right) \cot(\chi/2) \left[\left(\sin \frac{\chi}{2} + \sin^{-1} \frac{\chi}{2}\right) \ln \frac{1 + \sin(\chi/2)}{1 - \sin(\chi/2)} - 2\right]. \tag{30}$$

Note that the result (27) can formally be reproduced if one substitutes $T_n \to T_n + \delta T_n$ into Eq. (13), where

$$\delta T_n = -\frac{2}{g_N} \ln\left(\frac{1}{2\Delta R_N C}\right) T_n(1 - T_n), \tag{31}$$

and then expands the result to the first order in δT_n. Interestingly, the same transmission renormalization (31) follows from the renormalization group (RG) equations [10, 11, 12]

$$\frac{dT_n}{dL} = -\frac{T_n(1 - T_n)}{\sum_k T_k}, \quad L = \ln\left(\frac{1}{\epsilon R_N C}\right) \tag{32}$$

derived for *normal* conductors. In order to arrive at Eq. (31) one should just start the RG flow at $\epsilon = 1/R_N C$ and stop it at $\epsilon = 2\Delta$. Thus, the result (27) can be interpreted in a very simple manner: Coulomb interaction provides high frequency renormalization $T_n + \delta T_n$ (31) of the barrier transmissions which should be substituted into the classical expression for the supercurrent (13). It should be stressed, however, that the last step would by no means appear obvious without our rigorous derivation since the Coulomb correction to the Josephson current originates from the term $\sim \varphi_- \varphi_+^2$ in the effective action which is, of course, totally absent in the normal case.

5. Summary

In this paper we derived a general expression for the effective action of superconducting contacts with arbitrary transmissions of conducting channels. In the case of NS systems we described the interplay between Coulomb blockade and Andreev reflection and demonstrated a direct relation between shot noise and interaction effects in these structures. The fundamental physical reason behind this relation lies in discrete nature of the charge carriers – electrons and Cooper pairs – passing through NS interfaces. Our results allow to explain recent experimental findings [20].

Superconducting contacts with arbitrary channel transmissions show qualitatively new features as compared to the case of Josephson tunnel barriers [7]. The main physical reason for such differences is the presence of subgap Andreev bound states inside the system. Our results for the interaction correction might explain a rapid change between superconducting and insulating behavior recently observed [21] in comparatively short metallic wires with resistances close to the quantum resistance unit ~ 6.5 KΩ in-between two bulk superconductors. Previously it was already argued [22] that such a superconductor-to-insulator crossover can be due to Coulomb effects. Our present results provide further quantitative arguments in favor of this conclusion.

Acknowledgment

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