Meteorol. Zeitschrift, N.F. **5**, 139–144 (August 1996)

Drop size distribution measurements in rain — a comparison of two sizing methods

M. Löffler-Mang, K. D. Beheng and H. Gysi, Karlsruhe

Summary. First results of raindrop size measurements obtained by two instruments whose operation principles are basically different are presented and the necessary procedure of comparison is described. The two devices are (i) a disdrometer which converts the mechanical momentum transferred by hydrometeors to a sensible surface into an electrical signal which allows a quantitative size classification and (ii) a Malvern Particle Sizer evaluating the particles' sizes in a specific volume by laser diffraction. The results differ in the range of small drop sizes, for large drops the agreement is quite good. Possible reasons for the differences are discussed.

Messung von Tropfengrößenverteilungen in Regen — ein Vergleich zweier Meßmethoden

Zusammenfassung. Mit zwei vom Prinzip her grundsätzlich verschiedenen Meßinstrumenten wurden erste Messungen von Tropfengrößenverteilungen in Niederschlag durchgeführt. Die Geräte sind (i) ein Disdrometer, welches den übertragenen Axialimpuls von Hydrometeoren auf einen Meßkörper in ein elektrisches Signal umwandelt und damit eine Größenklassifikation ermöglicht, und (ii) ein Malvern Particle Sizer, der die Teilchengrößenverteilung aus der Laserlichtbeugung am Partikelkollektiv im Meßstrahl optisch bestimmt. Die notwendige Prozedur zur Aufbereitung der Meßergebnisse für einen Vergleich wird beschrieben. Im Bereich kleiner Tropfen weichen die Partikelgrößenverteilungen voneinander ab, für größere Tropfen ist die Übereinstimmung recht gut. Mögliche Gründe für die Unterschiede werden diskutiert.

1. Introduction

The background of this investigation concerning raindrop spectra is to compare rain measurements obtained (i) by radar at low elevations above ground and (ii) by two ground-based instruments.

The reason for this comparison is obvious: Evidently the radar technique is an indirect method by which the power reflected by a number of atmospheric particles illuminated by a short electromagnetic pulse is detected. The received power strongly depends on the backscattering as well as on the attenuation of the radar signal due to particles present in the pulse volume. Both, backscattering and attenuation, are by definition functions of the size distribution of hydrometeors. So, in simply assuming the Rayleigh approximation of the backscattering and absorption coefficient, the reflectivity is proportional to the sixth moment of the number density of hydrometeors and the attenuation proportional to the third moment. Moreover, in deriving rain rates from radar reflectivities also the number density of raindrops — besides their fall velocities — comes into play. Thus, a detailed information on the number density of raindrops is crucial to the interpretation of radar measurements in terms of rain rates.

In contrast to the remote sensing technique by radar, the two instruments considered in this study are systems measuring the size distribution of hydrometeors at ground level in a direct way. They can therefore serve as in-situ truth experiments so that radar data can be adjusted. One device counts drops with an electro-mechanical system (Disdrometer) whereas the other is an optical system (Malvern Particle Sizer, MPS).

Since a long time raindrop size distributions have been measured. Wiesner (1895) was presumably the first collecting drops on filter paper. This method has then been used for a very long time (Diem 1942, Diem und Strantz 1971). In the last three decades new methods came up and the Joss-Waldvogel disdrometer became the meteorological standard device for measuring raindrop size distributions (Joss and Waldvogel 1967). Besides that new devices (optical systems and/or radar) were introduced e. g. by Donnadieu (1980), Steiner (1988) or Sheppard and Joe (1994) and comparative studies were performed. It should be noted that all these instruments were developed and used in the scope of meteorological applications with emphasis on the measurement of relatively large drops. In contrast, this paper introduces a standard device from engineering science which especially is capable to count also small drops.

The use of this instrument (unfamiliar to meteorology) makes it necessary to give a detailed explanation on interpretation and transformation of measured spectra. In the relevant literature some investigations can be found on size spectrum interpretation, manipulation, correction, calibration or truncation (Joss and Waldvogel 1969, Steiner and Waldvogel 1987, Sheppard 1990, McFarquhar and List 1993, Ulbrich 1994); but none of the studies points

DOI: 10.1127/metz/5/1996/139

Gebrüder Borntraeger Stuttgart, Germany

out the difference between flux-dependent and concentration-dependent measurements. Since for the two techniques considered here this difference is essential, a thorough discussion of measured quantities and their interrelation is presented in paragraph 3.

As a first step in this paper a case study has been performed to compare the results of the disdrometer and the Malvern Particle Sizer. Both devices are described, the comparison procedure is explained in detail and first results are shown.

2. Measuring techniques

2.1. Malvern Particle Sizer

The two sizing devices rely on in principle different methods. On the one hand, the Malvern Particle Sizer (MPS) is an optical method. The instrument is widely used for more than fifteen years, for example for spray investigations (SWITHENBANK et al. 1977, CHIGIER and STYLES 1978, LEFEBVRE 1980, WITTIG et al. 1983, LESCHONSKI 1984, KRÄMER 1988). The MPS is a laser diffraction system which is schematically sketched in Fig. 1: A helium-neon laser operates as light source with an expanded beam of 18 mm in diameter. The light, scattered by the drops in the beam, is focused by a lens to a concentric diode array in the distance f (focal length) behind the lens. The radial light intensity distribution detected by the diode array is a measure of the diameters of drops in the ensemble present within the laser beam since the scattering angle is a function of drop size only. In order to determine the actual drop size distribution the signals from the diode array are evaluated indirectly and iteratively: in assuming a certain initial drop size distribution the resulting light intensity distribution is calculated with the help of 15 definite sizing classes. By systematically varying the 15 parameters of the assumed drop size distribution the difference between the calculated and the actually measured light intensity is minimized resulting finally in the size distribution actually present in the laser beam.

For adaption of this measuring system to a wide range of applications specific lenses with different focal lengths can be mounted. The focal length of the configuration used is f = 1000 mm, leading to a sizing range of diameters from 18 µm to approx. 2 mm.

2.2. Disdrometer

The disdrometer is an electro-mechanical instrument for measuring raindrop size distributions continuously and automatically. It is a simple and unexpensive technique frequently used in meteorology since many years (JOSS and WALDVOGEL 1967, ATTMANNSPACHER and RIEDL 1974, WALDVOGEL 1974, KINNELL 1976, ANIOL et al. 1980).

The operation principle of the disdrometer is also described shortly (see also JOSS and WALDVOGEL 1967): The instrument (Fig. 2) consists of an electro-mechanical unit and a feedback amplifier. A conical styrofoam body with a cross sectional area of 50 cm^2 is used to pass the mechanical momentum of an impinging drop to a set of two moving coil systems. The styrofoam body and the two moving coils are fixed together rigidly.

By the impact of a drop the styrofoam body together with the two coils moves downwards and a voltage is induced in the sensing coil imbedded in magnet 2. This voltage is amplified and applied to the driving coil within magnet 1 such that a force counteracting the movement is produced. Since the amplitude is very small it needs very little time for the system to return to its original resting

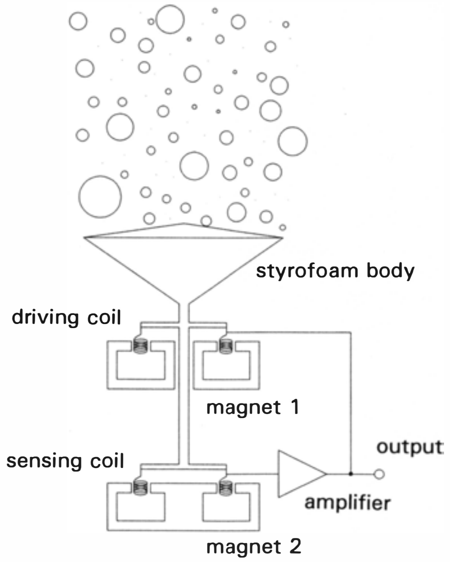

Fig. 2. Scheme of the disdrometer.

Abb. 2. Prinzip des Disdrometers.

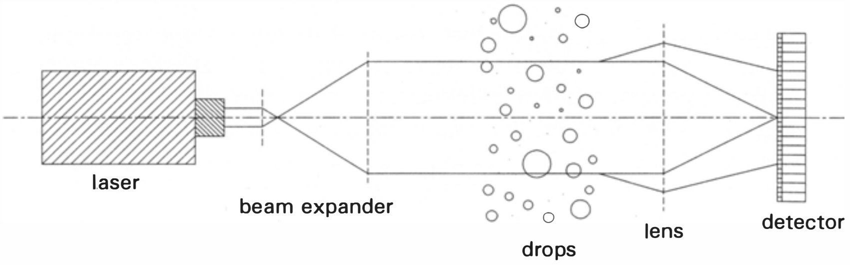

Fig. 1. Scheme of the Malvern Particle Sizer (MPS).

Abb. 1. Prinzip des Malvern Particle Sizer (MPS).

position and thus to get ready for the next impact of a drop. The amplitude of the pulse at the amplifier output, given as a certain voltage U, is a measure for the size of the drop. Each drop hitting the transducer produces a pulse of about 0.5 ms duration. The voltage U is roughly proportional to the mechanical momentum p of the hitting drop given as $p = mv = (4\,\pi\rho_w/3)x^3v$ where the drop's shape is assumed to be a sphere and ρ_w = density of water, x = drop diameter, v = terminal fall velocity so that

$$U \propto p \propto vx^3. \tag{1}$$

Since the terminal fall velocity is roughly proportional to the drop diameter (cf. GUNN and KINZER 1949, PRUPPACHER and KLETT 1978) it follows that

$$U \propto x^4. \tag{2}$$

For drops with diameter between 0.3 and 5.0 mm, which is the measuring range of the disdrometer, output pulses from 0.3 mV to 10 V are produced. After some electronical preprocessing the output signal is analysed directly by a conventional pulse height analyser, the results of which correspond to the number of drops in one of 20 size classes considered.

2.3. Fundamental difference

In comparing the results of the measurements obtained by the two methods under identical conditions, a fundamental difference has to be taken into account (UMHAUER et al. 1990):

- The Malvern Particle Sizer measures the size of all particles which are present within a certain measuring volume at a particular instant (concentration-dependent results); the same is valid for radar measurements.
- The disdrometer, however, measures the size of all particles which pass through (and hit) a defined sensor area within a predetermined time interval (flux-dependent results).

Note that each method gives different, but equivalent results. For demonstrating the basic difference between both methods we make use of a gedanken-experiment. If you look into a defined vertical cylinder filled with drops at one instant you will see the drops within this cylinder (concentration-dependent distribution). Now assume, that we close the top of the cylinder, let the drops fall with their terminal fall velocity and collect them at the bottom of the cylinder. After a fixed time interval the fastest (i.e. the largest) drops from the top of the cylinder have reached the bottom, but the slower (i.e. the smaller) drops are still missing. Therefore the so measured flux-dependent size distribution at the bottom is shifted towards larger drops.

Since between drop size and drop velocity a known relation exists as mentioned above, the flux-dependent results can in principle be converted to the concentration-dependent results and vice versa. The conversion method as well as some definitions and relations concerning the output data of both devices will be presented next.

3. Comparison procedure

As mentioned, the MPS has a wide measuring range and is most powerful for small droplet sizes. Thus it gives additional information on raindrop spectra containing drops below 0.3 mm diameter. This was motivation for introducing the MPS to meteorological investigations and should also be seen under the aspect that the knowledge on the size distribution of small drops will become increasingly important for experimental studies on scavenging and cloud microphysics.

The main drawback of the MPS is that total droplet number density cannot be derived due to the peculiarities of the measuring principle. So, as usual in engineering science, only the size class fractions (volume fractions) of the whole distribution are determined. The transformation of MPS data to flux-dependent results can therefore not be performed. However, the transformation of disdrometer data to concentration-dependent ones is possible. Note that in this chapter we refer to a notation common to evaluation of MPS data which we then apply in order to convert the disdrometer to MPS data for a comparison.

The disdrometer counts single drops of diameter x and provides as result a number density $q_0^j(x)$ where the superscript "j" indicates that this quantity (as others in the following) is a flux-dependent one. The subscript "0" identifies this function as a number density. Other subscripts used in the following refer to different types of density functions (see below). It is stressed that in meteorology drop size distributions are also commonly denoted by the term "number density", given e.g. by N(x). The difference between $q_0^j(x)$ and N(x) is that $q_0^j(x)$ is related to the total number, i.e. a dimensionless quantity, whereas N(x) is related to a unit volume, i.e. a concentration. In the following we present some definitions and relations concerning density functions as they are commonly used in engineering science. The superscript "j" is omitted, for convenience.

A **cumulative distribution $Q_r(x_i)$** refers to a quantity of particles that are smaller than or equal to a certain particle size x_i where the index r characterizes the kind of the described quantity: r = 0 → number, r = 1 → length, r = 2 → surface, r = 3 → volume or mass. Thus, it is

$$Q_r(x_i) = \frac{\text{quantity of all particles with } x \le x_i}{\text{total quantity}} \tag{3}$$

It obviously follows that if the total diameter interval is given by $x_{min} \le x \le x_{max}$ it yields

$$Q_r(x_{min}) = 0, \quad Q_r(x_{max}) = 1 \tag{4}$$

The **density distribution $q_r(x)$** is then defined by

$$q_r(x) = \frac{dQ_r(x)}{dx} \tag{5}$$

which, on the reverse, results in

$$Q_r(x_i) = \int_{x_{min}}^{x_i} q_r(x)dx \tag{6}$$

The number density is as usual convertable into a corresponding volume density $q_3(x)$ by

$$q_3(x) = \frac{x^3 q_0(x)}{\int_0^\infty x^3 q_0(x)dx} \qquad (7)$$

with the denominator being the third moment of the number distribution.

Since the Malvern Particle Sizer provides as primary result a concentration-dependent (denoted by the superscript "c") volume density $q_3^c(x)$, the flux-dependent disdrometer data have to be converted to concentration-dependent ones. The corresponding relation for the conversion results from dividing $q_0^j(x)$ by a normalized fall velocity $v^*(x) = v(x)/v(x_{max})$ for each drop size (UMHAUER et al. 1990):

$$q_0^c(x) = \frac{q_0^j(x)}{v^*(x)} \qquad (8)$$

and then calculating the volume density:

$$q_3^c(x) = \frac{x^3 q_0^c(x)}{\int_0^\infty x^3 q_0^c(x)dx} \qquad (9)$$

Thus, by the relations (8) and (9) the conversion of flux-dependent number densities into concentration-dependent volume densities can easily be carried out.

4. Results of measurements

For the measurements the MPS and the disdrometer were placed within a distance of less than one metre. For this first case study an artificial "test precipitation" with a size distribution suitable for both instruments was created by a very simple nozzle.

In Fig. 3a the disdrometer results are shown in terms of all cumulative distributions mentioned above. Note that the number density $q_0^j(x)$ is originally measured from which the volume distribution is derived.

It is seen that due to the conversion from flux-dependent to concentration-dependent values the number distribution $Q_0^c(x)$ tends to smaller drop sizes (dash-dotted line) compared to $Q_0^j(x)$ (dotted line), because $v^*(x)$ monotonically increases form 0 to 1 (cf. equation (8)). And, as also expected, the conversion of the number distribution (dash-dotted line) to the volume distribution (dashed line) results in a curve shift towards larger particle sizes.

Since $Q_3^c(x)$ resulting from the disdrometer measurements is the appropriate distribution which should be compared to the MPS measurements in Fig. 3b, both curves are drawn where the dashed curve is the same as in Fig. 3a. For drops larger than about 1400 μm in diameter both distributions show very good agreement. For smaller drop sizes the MPS data indicate an enhanced concentration of smaller drops.

This behaviour can also be recognized more pronounced in Fig. 4 where the volume densities $q_3^c(x)$ corresponding to the cumulative volume distributions of Fig. 3b are depicted. The two curves agree well for relatively large drops. But between drop diameters of about 500 and 1500 μm the differences ar quite large: The MPS curve attains its maximum at about 700 μm compared to the disdrometer data exhibiting a maximum at 900 μm. As already mentioned the MPS seems to detect more smaller drops than the disdrometer.

Some reasons for these differences may be:

- The MPS has its main measuring range at relative small drop sizes, the disdrometer, on the contrary, at relative large drop sizes due to an increasing accuracy of the method with increasing signal amplitude.
- The disdrometer may be less sensitive for small drops.

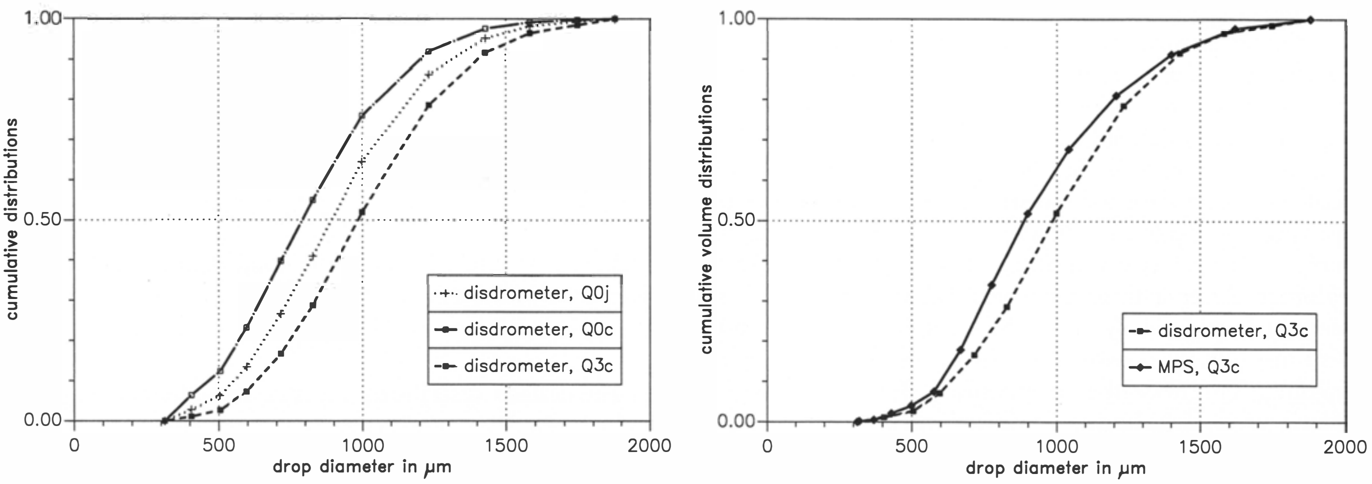

Fig. 3. (a) Cumulative distributions $Q_0^j(x)$ (dotted line), $Q_0^c(x)$ (dash-dotted line) and $Q_3^c(x)$ (dashed line) as a function of drop diameter in μm; (b) same as (a), but $Q_3^c(x)$ only for the MPS (solid line) and the disdrometer (dashed line).

Abb. 3. (a) Summenverteilungen $Q_0^j(x)$ (punktiert), $Q_0^c(x)$ (strichpunktiert) und $Q_3^c(x)$ (gestrichelt) als Funktion des Tropfendurchmessers in μm; (b) analog zu (a), aber nur $Q_3^c(x)$ für den MPS (durchgezogen) und das Disdrometer (gestrichelt).

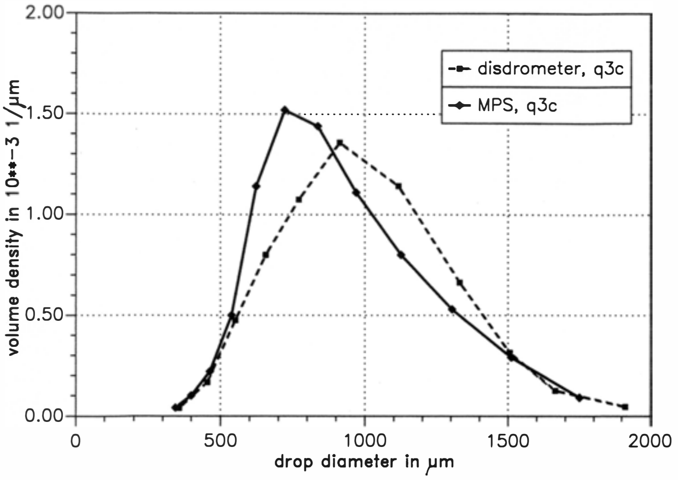

Fig. 4. Volume densities $q_3^c(x)$ in µm^{-1} as a function of drop diameter in µm for the MPS (solid line) and the disdrometer (dashed line).

Abb. 4. Volumendichten $q_3^c(x)$ in µm^{-1} als Funktion des Tropfendurchmessers in µm für den MPS (durchgezogen) und das Disdrometer (gestrichelt).

- Both instruments did not see exactly the same drop size distribution although they were placed rather close together.
- In case of a large concentration of small drops there exists the possibility that two drops impact at nearly the same time (i.e. within a millisecond) on the disdrometer's sensible surface so that they are interpreted as a single large drop.
- Photographic inspection of the disdrometer surface has shown that at the impact of a drop a portion of the water splashes back again, i.e. the impact is not totally inelastic. Consequently the transduced momentum is slightly higher than the original drop momentum and the size distribution therefore may be shifted towards larger drop sizes.
- Finally there could be an inaccuracy in the conversion from flux-dependent to concentration-dependent results. Eventually the normalized velocity factor $v^*(x)$ has slightly to be corrected.

5. Conclusions

First results of raindrop size measurements obtained by two instuments whose operation principles are basically different are presented and the procedure of comparison is described. The two devices are (i) a disdrometer which converts the linear momentum transferred by hydrometeors to a sensible surface into an electrical signal which allows a quantitative size classification and (ii) a Malvern Particle Sizer evaluating the particles' sizes in a specific volume by laser diffraction.

The results differ in that the MPS data show larger concentrations of smaller drops than the disdrometer data; for large drops the agreement is quite good. Some possible explanations for these differences are discussed. In principle the MPS seems to be a suitable instrument for measuring the small drop size range of rain.

This preliminary work will be continued, beginning with detailed investigations under well defined conditions in the laboratory. Thereafter measurements in natural rain will be performed. A further step will be investigations of drop size distributions with the MPS in ground-based clouds.

Acknowledgements

The authors wish to express their gratitude to the Engler-Bunte-Institut, Bereich Feuerungstechnik (Prof. LEUCKEL), and to the Institut für Lebensmittelverfahrenstechnik (Prof. SCHUBERT, Dipl.-Ing. HOGEKAMP) at the University of Karlsruhe. Both institutes have made available their Malvern Particle Sizers for this first comparison. Furthermore the helpful comments by an anonymous reviewer are gratefully acknowledged.

References

Aniol, R., J. Riedl, M. Dieringer, 1980: Über kleinräumige und zeitliche Variationen der Niederschlagsintensität. — Meteorol. Rdsch. **33**, 50–56.

Attmannspacher, W., J. Riedl, 1974: Ein Beitrag zur Niederschlagsstruktur in Praia, Santiago, Kap Verden. — Meteorol. Rdsch. **27**, 154–156.

Chigier, N. A., A. C. Styles, 1978: Temperature, concentration and velocity measurements in fuel spray free flames. — Amer. Chem. Soc., Advances in chemistry ser. **166**, No. 7, 111–120.

Diem, M., 1942: Messung der Größe von Wolkenelementen. — Ann. Hydr. **70**, 142–150.

Diem, M., R. Strantz, 1971: Typen der Regentropfenspektren II. Abhängigkeit von der Regenintensität. — Meteorol. Rdsch. **24**, 23–26.

Donnadieu, G., 1980: Comparison of results obtained with the VIDIAZ spectropluviometer and the Joss-Waldvogel rainfall disdrometer in a "rain of thundery type". — J. Appl. Meteorol. **19**, 593–597.

Gunn, R., G. D. Kinzer, 1949: The terminal velocity of fall for water drops in stagnant air. — J. Meteorol. **6**, 243–248.

Joss, J., A. Waldvogel, 1967: Ein Spektrograph für Niederschlagstropfen mit automatischer Auswertung. — Pure Appl. Geophys. **68**, 240–246.

—— 1969: Raindrop size distribution and sampling size errors. — J. Atmos. Sci. **26**, 566–569.

Kinnell, P. I. A., 1976: Some observations on the Joss-Waldvogel rainfall disdrometer. — J. Appl. Meteorol. **15**, 499–502.

Krämer, M., 1988: Untersuchung zum Bewegungsverhalten von Tropfen in turbulenter Strömung im Hinblick auf Verbrennungsvorgänge. — PhD Thesis, Universität Karlsruhe.

Lefebvre, A. H., 1980: Airblast atomization. — Prog. Energy Combust. Sci. **6**, 233 p.

Leschonski, K., 1984: Representation and evaluation of particle size analysis data. — Part. Charact. **1**, 89–98.

McFarquhar, G. M., R. List, 1993: The effect of curve fits for the disdrometer calibration on raindrop spectra, rainfall rate, and radar reflectivity. — J. Appl. Meteorol. **32**, 774–782.

Pruppacher, H. R., J. D. Klett, 1978: Microphysics of clouds and precipitation. — D. Reidel, Dordrecht.

Sheppard, B. E., 1990: Effect of irregularities in the diameter classification of raindrops by the Joss-Waldvogel disdrometer. — J. Atmos. Ocean. Technol. **7**, 180–183.

Sheppard, B. E., P. I. Joe, 1994: Comparison of raindrop size distribution measurements by a Joss-Waldvogel disdrometer, a PMS 2DG spectrometer, and a POSS Doppler radar. — J. Atmos. Ocean. Technol. **11**, 874–887.

Steiner, M., 1988: Bericht über Vergleichsmessungen mit verschiedenen Niederschlagsmeßinstrumenten. — Ber. LAPETH, Zürich, **28**, 90 p.

Steiner, M., A. Waldvogel, 1987: Peaks in raindrop size distributions. — J. Atmos. Sci. **44**, 3127–3133.

Swithenbank, J., J. M. Beer, D. S. Taylor, D. Abbot, C. G. McGreath, 1977: A laser diagnostic technique for the measurement of particle size distribution. — AIAA, Prog. in Astron. and Aeron. **53**, 421–447.

Ulbrich, C. W., 1994: Corrections to empirical relations derived from rainfall disdrometer data for effects due to drop size distribution truncation. — J. Atmos. Res. **34**, 207–215.

Umhauer, H., M. Löffler-Mang, P. Neumann, W. Leuckel, 1990: Pulse holography and phase-doppler technique. A comparison when applied to swirl pressure-jet atomizers — Part. Part. Syst. Charact. **7**, 226–232.

Waldvogel, A., 1974: The N_0 jump of raindrop spectra. — J. Atmos. Sci. **31**, 1067–1078.

Wiesner, J., 1895: Beiträge zur Kenntnis des tropischen Regens. — Sitz. Ber. Math. Nat. Kl., Wien, No. 104.

Wittig, S., M. Aigner, K. Sakbani, T. Sattelmayer, 1983: Optical measurements of droplet size distributions. Special considerations in the parameter definition for fuel atomizers. — AGARD, 62nd PEP-Symposium on Combustion Problems in Turbine Engines, Turkey, Conf.-Proc. No. 353.

MARTIN LÖFFLER-MANG
K. D. BEHENG
H. GYSI
Institut für Meteorologie und
Klimaforschung
Forschungszentrum Karlsruhe
Postfach 3640
D-76021 Karlsruhe

Received 11 September 1995, in revised form: 28 February 1996