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TURBULENT HEAT FLUX BALANCE FOR NATURAL CONVECTION IN AIR AND SODIUM ANALYSED BY DIRECT NUMERICAL SIMULATIONS

ABSTRACT

Direct simulation data of turbulent Rayleigh-Bénard convection in air and liquid sodium are used to investigate fundamental statistical properties of turbulent heat transfer in natural convection. The complete transport equation of turbulent heat flux is analysed in its analytical form. The results indicate that in natural convection diffusive transport of turbulent heat flux is very important and is mainly caused by the combined action of pressure and temperature fluctuations. Furthermore it is found that in buoyant flows the molecular destruction is an important sink of turbulent heat flux. This is in contrast to forced flows in which pressure diffusion and molecular destruction are small. Therefore in turbulence models based on the transport equation of turbulent heat flux both contributions are usually neglected. Nevertheless, the results show that for natural convection in air and sodium adequate modelling of pressure diffusion and molecular destruction will be essential.

1. Introduction

Many flows in engineering practice are influenced by buoyancy forces or are caused entirely by such forces. In order to predict and possibly control the flow phenomena influenced or dominated by buoyancy, engineers need to be able to calculate these phenomena. Though the exact equations describing turbulent buoyant flows, i.e. the Navier-Stokes and the thermal energy equation, are well known, direct solution of these equations with todays computer power is impossible for practical problems. At present only the statistical approach where the instantaneous velocity and temperature are separated into statistical mean and fluctuating quantities is feasible. However, the averaging procedure introduces unknown correlations between various fluctuating quantities. These so called Reynolds stresses and turbulent heat fluxes represent the transport of momentum and heat by turbulent fluctuations and have to be modelled to obtain a closed system of equations.

The most commonly used model for calculation of turbulent flow in engineering problems is the k-ε model [1]. It was originally developed for forced flows and uses the eddy-viscosity and turbulent Prandtl number concept to model the Reynolds stresses and turbulent heat fluxes. Thus turbulence is described as a gradient diffusion process. However, gradient transport is unrealistic for natural convection where often turbulent heat

transfer in isothermal regions [2] or countergradient heat flux does occur. Furthermore, in the k-ε model an isotropic turbulent viscosity and a strict analogy between Reynolds stresses and turbulent heat fluxes is assumed. Therefore the k-ε model cannot give a realistic description of the strongly anisotropic turbulence in natural convection [3].

From these shortcomings of the eddy viscosity concept it appears, that models based on the transport equations of Reynolds stresses and turbulent heat fluxes are needed to obtain correct predictions of turbulence in natural convection. In buoyant flows diffusive transport of turbulent quantities is of great importance [4] and the assumption of local equilibrium is not valid in natural convection [3, 5]. In algebraic models diffusion is neglected and thus full differential Reynolds stress models should be used. This implies that unknown higher order correlations have to be modelled to close the exact transport equations of Reynolds stresses and turbulent heat fluxes. Measurements of some of these correlations are - if at all possible - only of limited accuracy [6]. Nevertheless, detailed information about all terms in the transport equations of Reynolds stresses and turbulent heat fluxes is needed in order to calibrate and improve existing models.

The objective of this paper is to provide this information in a first step for the turbulent heat flux. We use direct simulation results of turbulent Rayleigh-Bénard convection in air and liquid sodium to analyse the complete transport equation of turbulent heat flux in its analytical form. We discuss the physical meaning of the different terms and their relevance for turbulence modelling of natural convection and give special attention to identify the influence of molecular Prandtl number. Although our analysis is restricted to low turbulence levels and a simple geometry we believe that our results are of general interest since they illuminate fundamental statistical features of turbulence in natural convection.

2. Simulation Model

The simulation model used is the TURBIT-code [7]. It allows for direct numerical simulation of turbulence in simple channel geometries and is based on the complete time-dependent three-dimensional conservation equations of mass, momentum and thermal energy. These equations are solved in dimensionless form. For normalization the channel height H, velocity $u_o = (g\beta\Delta T_W H)^{1/2}$, time H/u_o, pressure ρu_o^2 and temperature difference between walls ΔT_W are used (where g = gravity, β = volume expansion coefficient and ρ = density). The Boussinesq approximation is assumed to be valid. For spatial discretization second order central finite differences are used. The momentum equation is integrated in time by the explicit Euler-Leapfrog scheme whereas the semi-implicit Leapfrog Crank-Nicholson scheme is used for the thermal energy equation [8]. The simulations are performed on grids which resolve both the largest and smallest scales of turbulence. Thus no turbulence model is needed and the simulations do not depend on any model coefficients.

Physically the Rayleigh-Bénard convection is considered which is the convection in an infinite horizontal fluid layer heated at the lower and cooled at the upper wall. It is characterized by two dimensionless numbers, the Prandtl number $Pr = \nu/\kappa$ and the Rayleigh number $Ra = g\beta\Delta T_W H^3/(\nu\kappa)$ (where ν = viscous diffusivity, κ = thermal diffusivity). Here we consider two simulations, one for air (Ra = 630 000, Pr = 0.71) and one for liquid sodium (Ra = 24 000, Pr = 0.006). The turbulence level of the velocity field is similar for both simulations as the Grashof number Gr = Ra/Pr is of simi-

lar magnitude [5]. For air the velocity and temperature field are also similar according to Pr $\approx$ 1, whereas in liquid sodium the highly turbulent velocity field is contrasted by a temperature field that is largely laminar because it is governed by the large thermal diffusivity.

In both simulations periodic boundary conditions are used in the horizontal directions whereas at the lower and upper wall no slip conditions and constant wall temperatures are specified. The simulation of sodium was started from the final data of a previous simulation (Ra = 12 000, Pr = 0.006) with the same spatial discretization. For the simulation of air final results of a former simulation (Pr = 0.71, Ra = 380 000) were interpolated to a finer grid and used as initial data. Parameters and grid data of the simulations are given in Table 1. For verification of the numerical results by experimental data see [5].

Pr	Ra	Gr	$X_{1,2}$	$\Delta x_{1,2}$	Δx_{3W}	$N_{1,2}$	N_3
0.006	24 000	$4 \cdot 10^6$	8.0	0.032	0.005	250	39
0.71	630 000	$8.9 \cdot 10^5$	7.92	0.0396	0.005	200	39

Table 1: Parameter and grid data of the simulations: $X_{1,2}$ horizontal extension of channel in terms of H, $\Delta x_{1,2}$ mesh width in horizontal direction, Δx_{3W} mesh width near the walls in vertical direction, $N_{1,2,3}$ number of mesh cells in horizontal and vertical directions.

3. Results

During the simulation the full three-dimensional results for the three velocity components, pressure and temperature are stored at certain time intervals. For statistical analysis only results within the fully developed flow regime are used. To obtain reasonable statistical data averages $\overline{y}$ for a calculated variable y are formed over horizontal planes and these are averaged over several time steps. From this averaging procedure vertical profiles are resulting, where $x_3 = 0$ corresponds to the lower and $x_3 = 1$ to the upper wall.

3.1 Mean Temperature and vertical turbulent heat flux

Before we consider the balance of turbulent heat flux we first give results for the mean temperature and the turbulent heat flux itself. Figure 1 shows the vertical profiles for the mean temperature $\overline{T}$ for both simulations. For air the well-known behaviour is found with thin thermal boundary layers corresponding to a Nusselt number of about 7.2 and an isothermal core. In liquid sodium the thermal boundary layers extend almost over the whole channel and a roughly linear temperature profile is found. The Nusselt number is only about 1.4 and convection has therefore only small influence on heat transfer. The strongly different temperature fields in both simulations become also apparent in the vertical profile of the vertical turbulent heat flux $\overline{u_3'T'}$, see figure 2. In sodium the maximum of $\overline{u_3'T'}$ is only reached at channel midwidth whereas in air outside the thin boundary layers a constant value of $\overline{u_3'T'}$ is found. However, due the isothermal core observed in the simulation with air no gradient model can correctly predict both the mean temperature and turbulent heat flux profile.

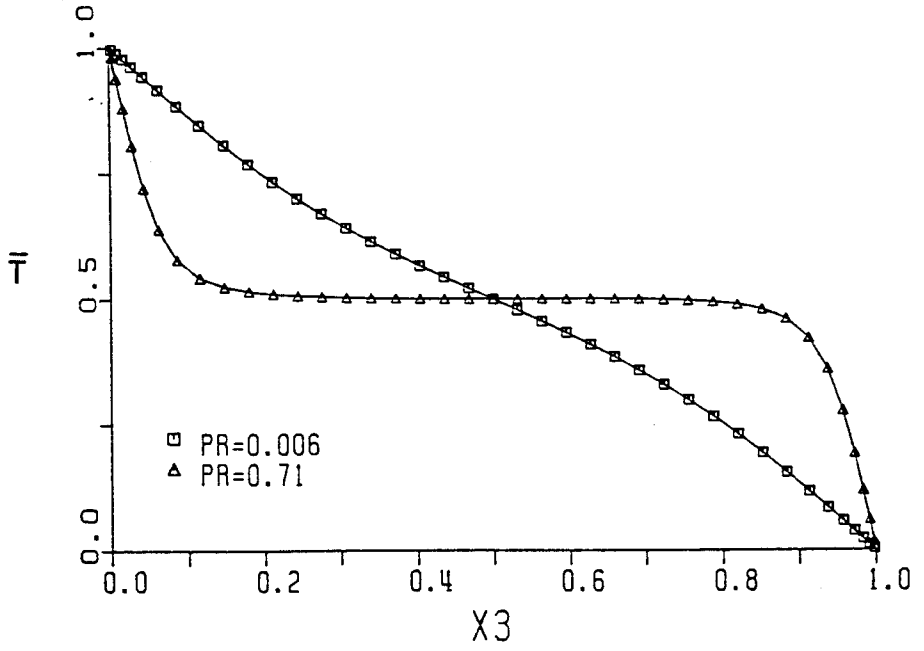

Fig. 1: Vertical mean temperature profile $\overline{T}$: Δ air, □ sodium.

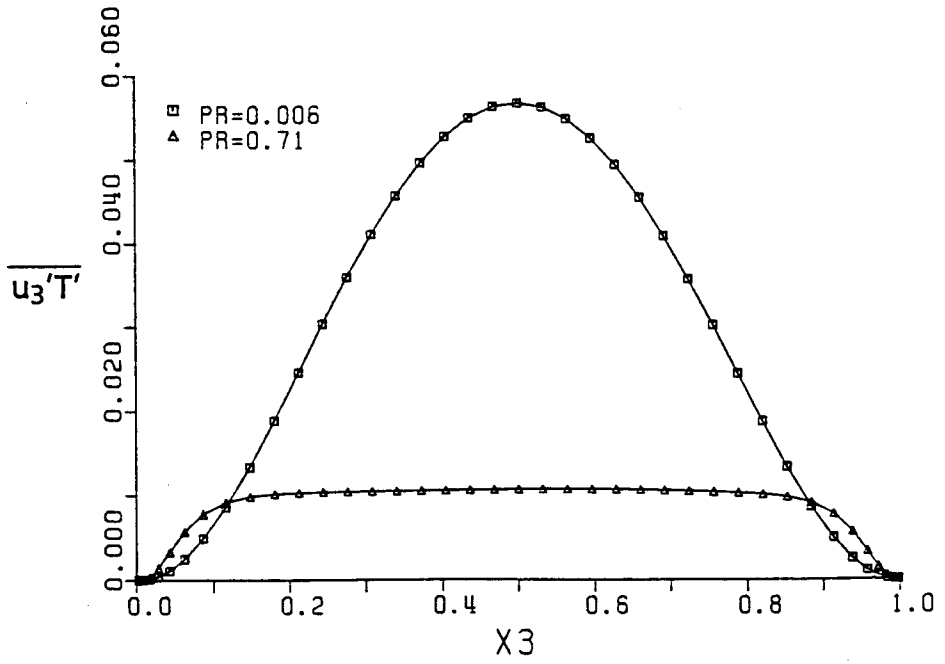

Fig. 2: Vertical profile of turbulent heat flux $\overline{u_3'T'}$: Δ air, □ sodium.

3.2 Balance of turbulent heat flux

In turbulent Rayleigh-Bénard convection long-time averaged mean velocities are zero. Therefore in the transport equation of turbulent heat flux [9] no convective transport and no production by the mean velocity field is present. In fully developed flow the balance equation for the vertical turbulent heat flux $\overline{u_3'T'}$ reduces to

$$0 = \underbrace{-\frac{\partial}{\partial x_3}\left(\overline{u_3'^2 T'} + \overline{p'T'} - \frac{1}{Pr\sqrt{Gr}}\,\overline{u_3'\frac{\partial T'}{\partial x_3}} - \frac{1}{\sqrt{Gr}}\,\overline{T'\frac{\partial u_3'}{\partial x_3}} \right)}_{D}$$

$$\underbrace{-\overline{u_3'^2}\frac{\partial \overline{T}}{\partial x_3} + \overline{T'^2}}_{P} + \underbrace{\overline{p'\frac{\partial T'}{\partial x_3}}}_{PS}$$

$$\underbrace{-\frac{1}{\sqrt{Gr}}\left(1+\frac{1}{Pr}\right)\overline{\frac{\partial u_3'}{\partial x_i}\cdot\frac{\partial T'}{\partial x_i}}}_{MD} . \qquad (1)$$

Here D denotes the diffusion of $\overline{u_3'T'}$, consisting of a turbulent part (first two terms) and a molecular part (last two terms). The production term P consists of contributions due to mean temperature field (first part of P) and due to buoyancy force (second part of P). The molecular destruction MD vanishes only for isotropic turbulence [9]. In this case the pressure scrambling term PS is the only sink of turbulent heat flux.

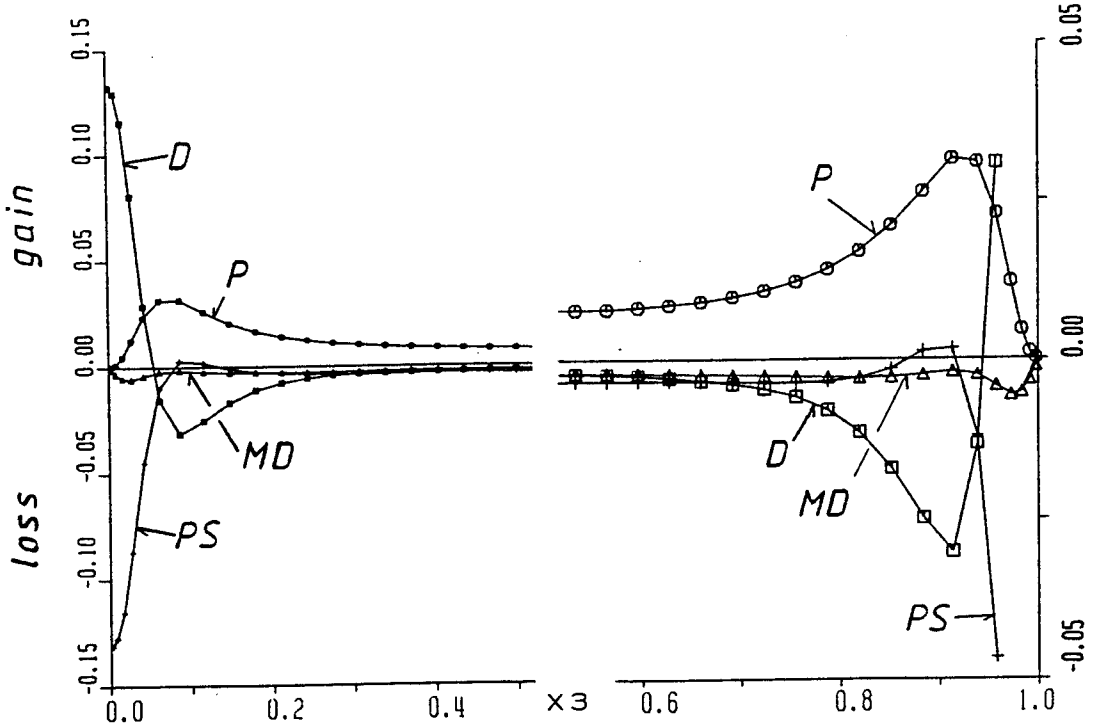

Fig. 3: Terms of the transport equation of turbulent heat flux in air: □ diffusion, ○ production, Δ molecular destruction, + pressure scrambling term

Figure 3 shows the vertical profiles of the different terms for the simulation with air. Different axes of ordinates are used for the lower and upper half of the channel in order to achieve a better resolution of the various curves. The production term P is zero at the walls, increases with increasing wall distance and reaches its maximum at the edge of the boundary layers. In the centre of the channel a constant value is approached. In this region the production is balanced half by the pressure scrambling term and about one quarter by molecular destruction. The remaining surplus of P in the centre of the channel is redistributed by diffusive transport towards the boundary layers. Near the walls diffusion is balanced by the pressure scrambling term.

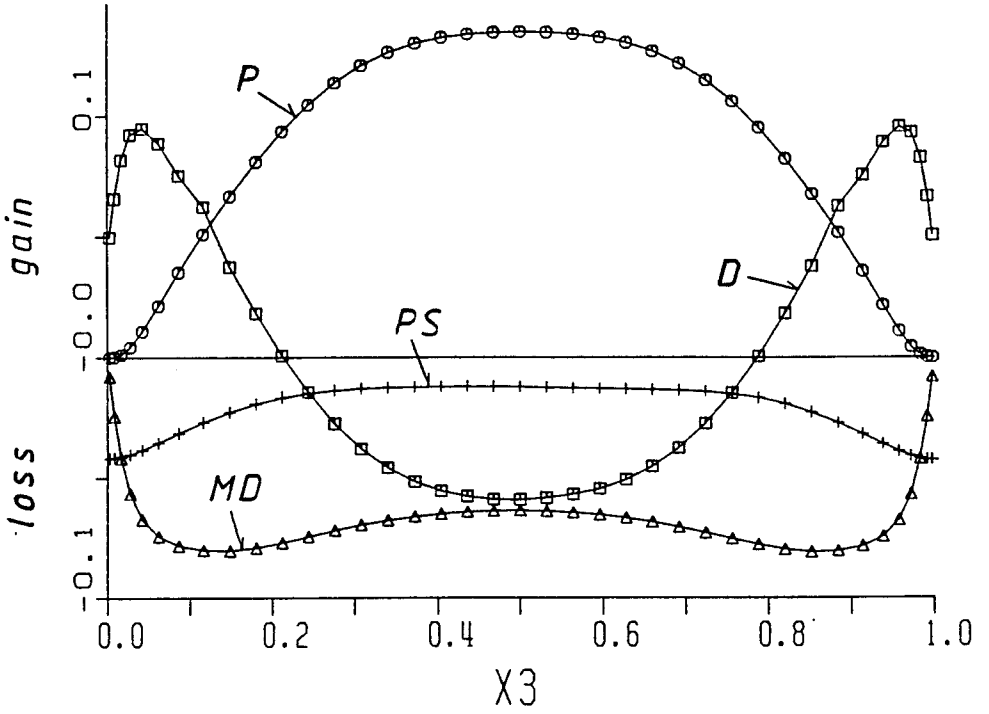

Fig. 4: Terms of the transport equation of turbulent heat flux in liquid sodium: □ diffusion, ○ production, Δ molecular destruction, + pressure scrambling term

The balance of vertical turbulent heat flux for the simulation with sodium is given in figure 4. Due to the thick thermal boundary layers the vertical profile of the production does not show any peaks. The balance reveals that at the Rayleigh number considered here in contrast to general assumption not the pressure scrambling term but the molecular destruction is the main sink of turbulent heat flux. Furthermore figure 4 shows that the vertical redistribution of turbulent heat flux by diffusion is not only of relevant size for air (see fig. 3) but also for liquid sodium.

The closure terms in the transport equation of turbulent heat flux are the molecular destruction, the pressure scrambling term and the complete diffusion term. Usually molecular destruction is neglected as isotropic turbulence is assumed [10]. However, as our results reveal, turbulence in buoyant flows is not really isotropic and in the simulations considered here, the molecular destruction is an important sink. Although a decreasing relevance of the molecular destruction can be expected with increasing turbulence level, this term should not be neglected in natural convection but be modelled adequately.

Standard diffusion models neglect molecular contributions and pressure fluctuations and account only for the triple correlation of velocity and temperature fluctuations [10]. Further inspection of our results indicates that in air outside the boundary layers molecular diffusion is in dead negligible, whereas it is of great importance near the walls. However, the results for sodium indicate that for this type of fluid at the Rayleigh number considered here diffusive transport of turbulent heat flux is in the complete channel mainly due to molecular processes.

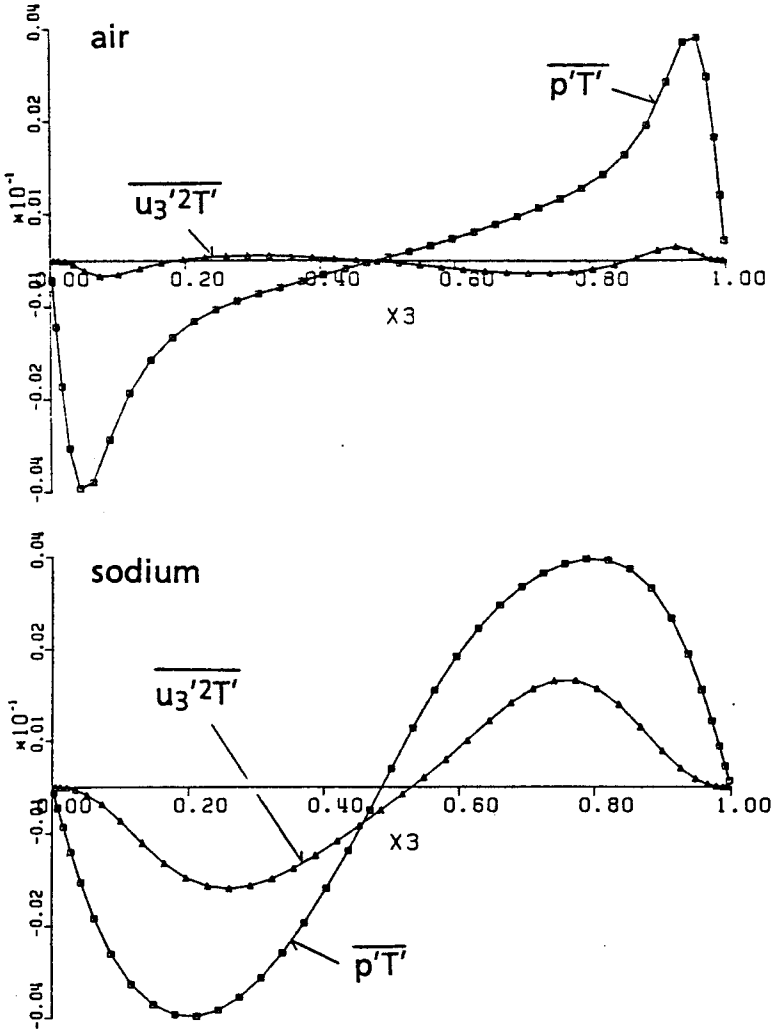

Fig. 5: Vertical profiles of turbulent diffusion terms in air (top) and sodium (bottom): □ $\overline{p'T'}$, Δ $\overline{u_3'^2T'}$

In figure 5 we give vertical profiles for the correlations $\overline{p'T'}$ and $\overline{u_3'^2T'}$ from which turbulent diffusion is calculated. Although the results for air and sodium are quite different they show that in natural convection gradients of $\overline{p'T'}$ are much larger than that of $\overline{u_3'^2T'}$ and thus turbulent diffusion is mainly caused by pressure fluctuations and not by velocity fluctuations. In the simulation with air $\overline{p'T'}$ and $\overline{u_3'^2T'}$ act - with exception of the near wall regions - in opposite direction. This is also expected to occur in liquid sodium, however at much larger Rayleigh numbers. Thus, closure schemes that neglect pressure diffusion are not able to reproduce the diffusion profile given in figure 3 even qualitatively.

4. Conclusions

Data bases of turbulent Rayleigh-Bénard convection in air and sodium were used to analyse terms of the transport equation of vertical turbulent heat flux. The results indicate that in buoyant flows diffusive transport is very important as it is responsible for redistribution of turbulent heat flux from the centre of the channel towards the boundary layers. We found that in natural convection the essential physical mechanism for turbulent diffusion is due to the action of pressure fluctuations, not due to velocity fluctuations. Because of the strongly anisotropic turbulence in natural convection the molecular destruction does not vanish for this type of flow but it is an important sink of turbulent heat flux. For this reasons turbulence models based on the transport equation of turbulent heat fluxes have to account for adequate modelling of molecular destruction and diffusion by pressure fluctuations to give realistic predictions of turbulent heat transfer in natural convection. For application to liquid metal convection at moderate and most technical Rayleigh numbers in addition modelling of molecular diffusion is essential.

5. References

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