A key parameter to characterize Taylor flow in narrow circular and rectangular channels

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Abstract

In this paper we show that the ratio ψ between the bubble velocity (U_B) and the total superficial velocity (J_{tot}) is a key parameter in Taylor flow. Depending on the value of ψ the streamlines in the liquid slug show a recirculation pattern or complete bypass flow. Among the quantities that are related to ψ are the mean liquid velocity, the relative velocity, the gas hold-up, the cross-sectional area of the bypass and recirculation flow region, the non-dimensional recirculation time in the liquid slug, the thickness of the liquid film and the bubble diameter. In experiments and technical applications J_{tot} is often known or prescribed, whereas U_B is unknown. Thus, when ψ is known, U_B and all the other above quantities can be directly computed. By a similitude analysis we show that ψ may depend on up to ten non-dimensional groups. However, the evaluation of literature data for Taylor flow indicates that it mainly depends on the capillary number. When inertial and gravitational effects are important additionally the Laplace number and the Eötvös number may be of some influence. We thus suggest focusing in future experimental and theoretical studies on further clarification of these functional relationships and propose to correlate ψ with the capillary number Ca_J, which is based on J_{tot} as velocity scale.

Introduction

Taylor flow is a special kind of slug flow in small channels, where the liquid slugs which separate the elongated bullet-shaped bubbles (Taylor bubbles) are free from gas entrainment. Taylor flow occurs in micro-fluidic devices for applications in life sciences (lab-on-a-chip), material synthesis and chemical process engineering, e.g. in catalytic multiphase capillary and monolithic reactors (Kreutzer et al., 2005). Taylor flow is attractive because of its well defined interfaces and flow conditions which are easier to control than in macroscopic devices and because of its advantageous mass transfer properties. The later stems from (i) the high interfacial area per unit volume, (ii) the thin liquid film which separates the body of the bubble from the channel wall, and (iii) the recirculation in the liquid slug which accounts for good mixing and a wall-normal convective transport in laminar low.

In Taylor flow, the liquid film thickness and the recirculation in the liquid slug depend mainly on the capillary number $Ca_B = \mu_L U_B / \sigma$, where U_B is the bubble velocity, μ_L is the liquid viscosity and σ is the coefficient of surface tension. For given gas and liquid superficial velocities J_L and J_G the total superficial velocity $J_{tot} \equiv J_L + J_G$ and the volumetric flow rate ratio $\beta \equiv J_G/J$ are known, whereas the bubble velocity (and thus Ca_B) and the gas-holdup $\varepsilon = J_G / U_B$ are unknown.

In this contribution we perform a similitude analysis for incompressible Taylor flow in straight circular and rectangular channels. We show that many characteristic quantities in Taylor flow are related to a single key parameter, namely to the ratio between the (unknown) bubble velocity and the (given) total superficial velocity. Thulasidas et al. (1997) denoted this ratio by $\psi \equiv U_B / J_{tot}$. We compare theoretical and experimental results from literature to elucidate the functional dependence of ψ on the capillary Ca_J, which is based on J_{tot} as velocity scale.

Problem Description

We consider the pressure-driven flow of two immiscible fluids with constant physical properties in a straight channel. The channel cross-section with area A_{ch} is either circular (diameter D, radius R, area $A_{ch} = \pi D^2/4$) or rectangular with width B and height H (area $A_{ch} = BH$). For the rectangular channel we assume $H \leq B$ so that the aspect ratio $\chi \equiv H/B$ is in the range $0 < \chi \leq 1$. We further define the hydraulic diameter $D_h \equiv 2BH/(B+H)$.

We assume that the Taylor flow consists of a sequence of alternating gas bubbles and liquid slugs, where the length of all gas bubbles and that of all liquid slugs is the same. Then, the flow hydrodynamics is fully described by a single unit cell of length $L_{uc} = L_B + L_{slug}$. In Figure 1 we show a sketch of such a “perfect” Taylor flow.

We denote the constant volumetric flow rates of both phases by Q_G and Q_L. Then, the gas and liquid superficial velocities are $J_G \equiv Q_G / A_{ch}$ and $J_L \equiv Q_L / A_{ch}$, respectively. The bubble velocity and the mean liquid velocity are given by $U_B \equiv J_G / \varepsilon$ and $U_L \equiv J_L / (1 - \varepsilon)$. Here, $\varepsilon = V_B / V_{uc}$ is the fractional volumetric gas content in the unit cell, and

V_B and $V_{uc} = L_{uc} A_{ch}$ denote the volumes of the bubble and the unit cell, respectively. With these definitions, the total superficial velocity is given by

$$J_{tot} \equiv \frac{Q_G + Q_L}{A_{ch}} = J_G + J_L = \varepsilon U_B + (1-\varepsilon) U_L \qquad (1)$$

For any cross-section at a certain axial position y it is

$$A_{ch} = A_L(y) + A_B(y) \qquad (2)$$

where A_L and A_B are the cross-sectional areas occupied by the liquid and the bubble, respectively.

Similitude Analysis

In this section we perform a similitude analysis of the problem described above and begin with a list of the relevant quantities for a rectangular channel.

First, there are six (constant) physical properties, namely

- Gas and liquid density (ρ_G, ρ_L)
- Gas and liquid viscosity (μ_G, μ_L)
- Coefficient of surface tension (σ)
- Gravitational acceleration (g)

Next, there are three flow specific quantities, namely

- Gas and liquid volumetric flow rate (Q_G, Q_L)
- Pressure drop along the unit cell (Δp_{uc})

Finally, there are five geometrical quantities, namely

- Channel height (H)
- Channel width (B)
- Length of the unit cell (L_{uc})
- Bubble volume (V_B)
- Angle of channel axis with respect to the gravitational field (φ)

These are in total 14 variables with three basic dimensions, namely kg, m, s. According to the π-theorem there are eleven independent non-dimensional groups that characterize the problem. (For a circular channel it is one less because the two length scales H and B are replaced by the single length scale D).

From the above 14 quantities, in an experiment usually the following eleven quantities are given or prescribed: ρ_G, ρ_L, μ_G ,μ_L ,σ ,g ,Q_G ,Q_L ,H ,B ,φ. The three main unknown parameters are then the length of the unit cell, the gas holdup in the unit cell and the pressure drop along the unit cell. The respective three non-dimensional groups are:

$$\Pi_1 \equiv \frac{L_{uc}}{D_h} \equiv \Lambda \qquad (3)$$

$$\Pi_2 \equiv \frac{V_B}{A_{ch} L_{uc}} \equiv \varepsilon \qquad (4)$$

$$\Pi_3 \equiv \frac{\Delta p_{uc}}{\rho_L} \left(\frac{A_{ch}}{Q_G + Q_L} \right)^2 = \frac{\Delta p_{uc}}{\rho_L J_{tot}^2} \equiv Eu_{uc} \qquad (5)$$

As the eight other independent non-dimensional groups we choose

$$\Pi_4 \equiv \frac{\rho_G}{\rho_L} \equiv \rho' \qquad (6)$$

$$\Pi_5 \equiv \frac{\mu_G}{\mu_L} \equiv \mu' \qquad (7)$$

$$\Pi_6 \equiv \frac{Q_G}{Q_G + Q_L} \equiv \beta \qquad (8)$$

$$\Pi_7 \equiv \frac{\mu_L}{\sigma} \frac{Q_G + Q_L}{A_{ch}} = \frac{\mu_L J_{tot}}{\sigma} \equiv Ca_J \qquad (9)$$

$$\Pi_8 \equiv \frac{\sigma \rho_L D_h}{\mu_L^2} \equiv La \qquad (10)$$

$$\Pi_9 \equiv \frac{g(\rho_L - \rho_G) D_h^2}{\sigma} \equiv E\ddot{o} \qquad (11)$$

$$\Pi_{10} \equiv \frac{H}{B} = \chi \qquad (12)$$

$$\Pi_{11} \equiv \varphi \qquad (13)$$

Thus in an experiment $\Pi_4 - \Pi_{11}$ are (usually) given, while the non-dimensional groups $\Pi_1 - \Pi_3$ are unknown.

We note that the non-dimensional unit cell length Λ depends in a significant way on the device and mechanism used to generate the Taylor bubbles. Thus, it is often possible to adjust an experiment so that Λ is within a certain range.

The main global quantities of interest are then the gas holdup ε and the Euler number Eu_{uc}, which represents the non-dimensional pressure drop. For both quantities the following functional relations hold:

$$\varepsilon = F_\varepsilon(Eu_{uc}, \Lambda, \rho', \mu', \beta, Ca_J, La, E\ddot{o}, \chi, \varphi) \qquad (14)$$

$$Eu_{uc} = F_{Eu}(\varepsilon, \Lambda, \rho', \mu', \beta, Ca_J, La, E\ddot{o}, \chi, \varphi) \qquad (15)$$

The key parameter ψ

From the above eleven independent non-dimensional groups further non-dimensional groups can be defined. Examples are the Reynolds number

$$Re_{\mathrm{J}} \equiv \frac{\rho_{\mathrm{L}} D_{\mathrm{h}} J_{\mathrm{tot}}}{\mu_{\mathrm{L}}} = LaCa_{\mathrm{J}} \tag{16}$$

and the Weber number

$$We_{\mathrm{J}} \equiv \frac{\rho_{\mathrm{L}} D_{\mathrm{h}} J_{\mathrm{tot}}^2}{\sigma} = Ca_{\mathrm{J}} Re_{\mathrm{J}} = LaCa_{\mathrm{J}}^2 \tag{17}$$

Here, we have chosen the Laplace number as group Π_8 instead of the Reynolds and Weber number because for given fluid properties and a certain channel hydraulic diameter *La* is a constant. The same holds for the Eötvös number *Eö*.

In this paper we suggest as an especially useful non-dimensional group the ratio

$$\psi \equiv \frac{U_{\mathrm{B}}}{J_{\mathrm{tot}}} = \frac{\beta}{\varepsilon} \tag{18}$$

so that

$$\begin{aligned} \psi &= \frac{\beta}{F_{\varepsilon}} \\ &\equiv F_{\psi}(Eu_{\mathrm{uc}}, \Lambda, \rho', \mu', \beta, Ca_{\mathrm{J}}, La, E\ddot{o}, \chi, \varphi) \end{aligned} \tag{19}$$

and

$$Eu_{\mathrm{uc}} = G_{Eu}(\psi, \Lambda, \rho', \mu', \beta, Ca_{\mathrm{J}}, La, E\ddot{o}, \chi, \varphi) \tag{20}$$

In this paper we will not consider relations for the Euler number Eu_{uc} but focus only on the functional relationship for ψ, which is of fundamental importance for Taylor flow. Namely, when ψ is known, the following quantities can be directly computed from other given or prescribed quantities:

- The bubble velocity

$$U_{\mathrm{B}} = \psi J_{\mathrm{tot}} \tag{21}$$

- The mean liquid velocity (this follows from Eq. (1))

$$\frac{U_{\mathrm{L}}}{J_{\mathrm{tot}}} = \frac{1-\beta}{1-\beta/\psi} \tag{22}$$

- The gas holdup

$$\varepsilon = \frac{\beta}{\psi} \tag{23}$$

- The capillary number

$$Ca_{\mathrm{B}} = \psi Ca_{\mathrm{J}} \tag{24}$$

- The bubble Reynolds number

$$Re_{\mathrm{B}} = \psi Re_{\mathrm{J}} = \psi LaCa_{\mathrm{J}} \tag{25}$$

- The non-dimensional relative velocity, both in the form

$$W \equiv \frac{U_{\mathrm{B}} - J_{\mathrm{tot}}}{U_{\mathrm{B}}} = 1 - \frac{1}{\psi} \tag{26}$$

and in the form

$$Z \equiv \frac{U_{\mathrm{B}} - J_{\mathrm{tot}}}{J_{\mathrm{tot}}} = \psi - 1 \tag{27}$$

Next, we show that ψ is also related to important local quantities in Taylor flow such as the liquid film thickness.

Dependence of bubble diameter and liquid film thickness on ψ

A mass balance for the liquid phase in a frame of reference moving with the bubble yields

$$(J_{\mathrm{tot}} - U_{\mathrm{B}}) A_{\mathrm{ch}} = \left[U_{\mathrm{L,cs}}(y) - U_{\mathrm{B}} \right] A_{\mathrm{L}}(y) \tag{28}$$

Here, $U_{\mathrm{L,cs}}(y)$ is the mean cross-sectional liquid velocity at a certain axial position y. In the liquid slug it is $A_{\mathrm{L}} = A_{\mathrm{ch}}$ so that for this case it follows immediately from Eq. (28) the well known result $U_{\mathrm{slug}} = J_{\mathrm{tot}}$, which states that the mean axial velocity at any cross-section within the liquid slug is equal to the total superficial velocity. (For the flow of two incompressible phases through a straight channel with constant cross-section this result also follows from global mass continuity, see Suo & Griffith, 1964).

From Eq. (28) it follows with Eq. (2)

$$\frac{A_{\mathrm{L}}(y)}{A_{\mathrm{ch}}} = \frac{\psi - 1}{\psi - U_{\mathrm{L,cs}}(y)/J_{\mathrm{tot}}} \tag{29}$$

and

$$\frac{A_{\mathrm{B}}(y)}{A_{\mathrm{ch}}} = \frac{1 - U_{\mathrm{L,cs}}(y)/J_{\mathrm{tot}}}{\psi - U_{\mathrm{L,cs}}(y)/J_{\mathrm{tot}}} \tag{30}$$

If in the liquid film the velocity $U_{\mathrm{L,cs}}(y)$ is zero, then it follows from Eqs. (29) and (30) the result

$$\frac{A_{\mathrm{L}}}{A_{\mathrm{ch}}} = 1 - \frac{A_{\mathrm{B}}}{A_{\mathrm{ch}}} = 1 - \frac{1}{\psi} \tag{31}$$

Thus, in the case of a stagnant liquid film the cross-sectional area of the bubble and that of the liquid film is constant and axially uniform. An example for this case is the penetration of an inviscid gas phase into a capillary where it displaces a viscous liquid in the absence of gravity. Indeed, in such a situation the thickness of the liquid film becomes constant in a distance sufficiently far from the bubble tip (Bretherton, 1961; Giavedoni & Saita, 1997). In this case the fraction of liquid, m, left behind the semi-infinite bubble is of interest (Taylor, 1961). Since this fraction is equal to the ratio A_L / A_{ch} it follows from Eq. (31) and definition (26) that it is $m = W$ in this special case.

For a <u>circular channel</u> where the thickness of the liquid film, δ_F, is uniform at the bubble circumferential it is

$$\frac{A_L}{A_{ch}} = \frac{\frac{\pi}{4}D^2 - \frac{\pi}{4}(D-2\delta_F)^2}{\frac{\pi}{4}D^2} = \frac{4\delta_F(D-\delta_F)}{D^2} \tag{32}$$

With this result and Eq. (31) we obtain for the thickness of the stagnant liquid film the quadratic equation

$$\frac{\delta_F}{D}\left(1-\frac{\delta_F}{D}\right) = \frac{1}{4}\left(1-\frac{1}{\psi}\right) \tag{33}$$

which has the solution

$$\frac{\delta_F}{D} = \frac{1}{2}\left(1-\frac{1}{\sqrt{\psi}}\right) \tag{34}$$

For $\delta_F << D$ it follows from Eq. (33) the approximation

$$\frac{\delta_F}{D} \approx \frac{1}{4}\left(1-\frac{1}{\psi}\right) \tag{35}$$

so that

$$\psi \approx \frac{1}{1-\frac{4\delta_F}{D}} \approx 1+\frac{4\delta_F}{D} \tag{36}$$

For the bubble diameter in a circular channel we obtain - under the assumption of a stagnant liquid film - from Eq. (31) the result

$$\frac{D_B}{D} = \frac{1}{\sqrt{\psi}} \tag{37}$$

For a <u>rectangular channel</u> and an axisymmetric bubble surrounded by a stagnant liquid film it follows from Eq. (31) for the bubble diameter

$$\frac{D_B}{B} = 2\sqrt{\frac{\chi}{\pi\psi}}, \quad \frac{D_B}{H} = \frac{2}{\sqrt{\pi\chi\psi}} \tag{38}$$

or, in terms of the hydraulic diameter

$$\frac{D_B}{D_h} = \frac{1+\chi}{\sqrt{\pi\chi\psi}} \tag{39}$$

It is important to note that in inclined or vertical channels and in the presence of gravity, the velocity in the liquid film $U_{L,cs}(y)$ will be in general different from zero and will vary with the axial position. Then, also the liquid film thickness and the bubble diameter become, according to Eqs. (29) and (30), a function of y and are no longer uniform along the body of the bubble. In this case ψ will depend on the Eötvös number and on the angle φ.

Recirculation flow and bypass flow

In incompressible Taylor flow in a frame of reference moving with the bubble a recirculation pattern in the liquid slug occurs when the bubble velocity is lower than the liquid velocity on the channel axis, i.e. for $U_B < U_{L,max}$ (Taylor, 1961; Cox, 1964). For a liquid slug with a fully developed laminar velocity profile the maximum liquid velocity is given by $U_{L,max} = CU_{slug}$, where C is a constant. The value of C depends on the shape of the channel cross-section. For a circular channel it is $C_\circ = 2$ while for a square channel it is $C_\square = 2.096$. Since in incompressible Taylor flow it is $U_{slug} = J_{tot}$, the condition for recirculation flow becomes $U_B < CJ_{tot}$, or $\psi < C$, respectively.

The cross-sectional regions with bypass flow (close to the walls) and recirculation flow (in the channel center) are separated by the “dividing streamline” (Thulasidas et al., 1997), see Figure 1. The position of the dividing streamline is obtained from the condition that the total flow rate within the recirculation area is zero in the moving frame of reference. Thulasidas et al. (1997) showed that in a circular channel the radial position r_1 of the dividing streamline is given by

$$\frac{r_1}{R} = \sqrt{2-\psi} \tag{40}$$

while the radial position r_0, where the velocity in the moving frame of reference is zero, is given by

$$\frac{r_0}{R} = \sqrt{1-\frac{\psi}{2}} \tag{41}$$

Thus, in a circular channel the fractional recirculation area is $A_1/A_{ch} = 2 - \psi$ and it is $A_1/A_0 = 2$ for any value of ψ.

The intensity of the recirculation can be quantified by the time needed for the liquid to move from one end of the liquid slug to the other end. A second characteristic time scale is the time needed by the liquid slug to travel a distance of its own length. Thulasidas et al. (1997) defined the ratio of both time scales as the non-dimensional recirculation time, τ. For a circular channel the recirculation time is (Kececi et al. 2009)

$$\tau_{\bigcirc} = \left(\frac{1}{\psi} - \frac{1}{2} \right)^{-1} \qquad (42)$$

For $\psi = C_{\bigcirc} = 2$ it is $A_1/A_{ch} = 0$ and the recirculation area vanishes. For $\psi > 2$ complete bypass flow occurs.

For rectangular channels, the fractional areas A_1/A_{ch} and A_0/A_{ch} as well as the non-dimensional recirculation time τ have been studied theoretically by Kececi et al. (2009). Based on the assumption that the liquid slug is sufficiently long to form a fully developed laminar velocity profile, the authors used an approximation to the exact laminar velocity profile in a rectangular channel proposed in literature and showed that A_1/A_{ch}, A_0/A_{ch} and τ all depend in a unique way on ψ and on the channel aspect ratio χ. Kececi et al. (2009) evaluated theses relations numerically and displayed the results in graphical form as function of ψ for different values of χ.

The transition to complete bypass flow occurs in a square channel for $\psi > 2.096$ and in a planar channel, formed by two parallel plates, for $\psi > 1.5$. For rectangular channels the transition to complete bypass flow occurs for $\psi > C_{\square}$. The value of $C_{\square}$ can be determined from the relation

$$C_{\square} = \frac{3}{2}\Big(1 + 0.54669\chi + 1.55201\chi^2 - 4.05943\chi^3 + 3.21493\chi^4 - 0.85731\chi^5\Big) \qquad (43)$$

Eq. (43) was proposed by Spiga & Morini (1994) who determined for laminar single phase flow in a rectangular channel exact values of U_{max} / U_{mean} for ten different values of the aspect ratio in the range $0 \le \chi \le 1$ and fitted these data by Eq. (43) with an accuracy of 0.06%.

Correlations for ψ

In the previous section we showed that ψ is related to other quantities by means of the following identities

$$\psi \equiv \frac{U_B}{J_{tot}} = \frac{\beta}{\varepsilon} = \frac{1}{1-W} = 1 + Z \qquad (44)$$

In this section we collect and compare literature data for different expressions in Eq. (44).

Liu et al. (2005) performed experiments in capillaries with circular and square cross-section with hydraulic diameters in the range of 0.9 – 3 mm using air and three different liquids in co-current upward flow. They fitted their experimental data for the ratio of bubble velocity to total superficial velocity by the correlation

$$\frac{U_B}{J_{tot}} = \psi = \frac{1}{1 - 0.61 Ca_J^{0.33}} \qquad (45)$$

This correlation is valid in the range $0.0002 \le Ca_J \le 0.39$.

Correlations from relative bubble velocity

From experiments in circular tubes, Fairbrother & Stubbs (1935) suggested the following correlation for m

$$m = 1.0 Ca_B^{0.5} \qquad (46)$$

which is valid in the range $7.5 \times 10^{-5} < Ca_B < 0.014$. Later Taylor (1961) extended the validity of Eq. (46) for Ca_B up to 0.1. With Eq. (26) and with $Ca_B = \psi Ca_J$ Eq. (46) becomes an implicit relation for ψ as function of Ca_J

$$1 - \frac{1}{\psi} = \psi^{0.5} Ca_J^{0.5} \qquad (47)$$

Bretherton's (1961) analytical approach at low Ca_B resulted in the following expression

$$m = 2.68 Ca_B^{2/3} \qquad (48)$$

Giavedoni & Saita (1997) found that their numerical results for the film thickness match the theoretical correlation of Bretherton for $Ca_B < 10^{-3}$. Thus, we adopt this range also for the validity of Eq. (48). This equation is equivalent to

$$1 - \frac{1}{\psi} = 2.68 \psi^{2/3} Ca_J^{2/3} \qquad (49)$$

For large values of Ca_B, Taylor (1961) found that m acquires a value of 0.58, while theoretically it was found to be equal to 0.6 (Cox, 1964). This was confirmed by the numerical simulations of Giavedoni & Saita (1997) who found $m = 0.559$ for $Ca_B = 2$ and $m = 0.592$ for $Ca = 10$. The value of $m = 0.559$ for $Ca_B = 2$ corresponds to $\psi = 1/(1-0.559) = 2.268$ for $Ca_J = 2 / 2.268 = 0.882$ while that of $m = 0.592$ for $Ca_B = 10$ corresponds to $\psi = 1/(1-0.592) = 2.451$ for $Ca_J = 10 / 2.451 = 4.08$.

Correlations from liquid film thickness and bubble diameter

For a stagnant liquid film Eqs. (34), (36) or (37) can be used to determine correlations for ψ from correlations for δ_F or D_B, respectively. As an example, we mention the experimental correlations of Marchessault & Mason (1960) for the thickness of the liquid film in a circular channel with various inclinations

$$\frac{2\delta_F}{D} = \sqrt{\frac{\mu_L}{\sigma}} \left[-A + B\sqrt{U_B} \right] \qquad (50)$$

where A and B are coefficients (the dimension of A is $cm^{0.5}s^{0.5}$ while B is dimensionless). With Eq. (35) the latter correlation translates into

$$m = 1 - \frac{1}{\psi} = 2\psi^{0.5} Ca_{\mathrm{J}}^{0.5} \left(B - \frac{A}{\sqrt{U_{\mathrm{B}}}} \right) \tag{51}$$

where U_{B} is in cm/s. For $A = 0$ and $B = 0.5$ Eq. (51) becomes identical with Eq. (47) from Bretherton (1961).

By scaling arguments for a semi-infinite bubble Aussilous & Quere (2000) derived the following equation for the liquid film thickness in a circular tube

$$\frac{\delta_{\mathrm{F}}}{D} = \frac{0.66 Ca_{\mathrm{B}}^{2/3}}{1 + 3.33 Ca_{\mathrm{B}}^{2/3}} \tag{52}$$

which is valid in the range $10^{-3} \leq Ca_{\mathrm{B}} \leq 1.4$. Inserting Eq. (52) in Eq. (34) gives

$$\frac{1}{\sqrt{\psi}} = 1 - \frac{1.32 Ca_{\mathrm{B}}^{2/3}}{1 + 3.33 Ca_{\mathrm{B}}^{2/3}} \approx \frac{1 + 2 Ca_{\mathrm{B}}^{2/3}}{1 + 3.33 Ca_{\mathrm{B}}^{2/3}} \tag{53}$$

In terms of Ca_{J} the latter equation becomes

$$\psi = \left(\frac{1 + 3.33 \psi^{2/3} Ca_{\mathrm{J}}^{2/3}}{1 + 2\psi^{2/3} Ca_{\mathrm{J}}^{2/3}} \right)^2 \tag{54}$$

For square capillaries Kreutzer et al. (2005) proposed the following correlation for the bubble diameter in the diagonal direction

$$\frac{D_{\mathrm{B,sq}}}{D_{\mathrm{h}}} = 0.7 + 0.5 \exp\left(-2.25 Ca_{\mathrm{B}}^{0.445} \right) \tag{55}$$

For $Ca_{\mathrm{B}} > 0.04$ the bubble is axisymmetric so that Eq. (39) can be used to obtain the following implicit equation for ψ

$$\begin{aligned} \psi &= \frac{4}{\pi} \left(\frac{D_{\mathrm{h}}}{D_{\mathrm{B}}} \right)^2 \\ &= \frac{4}{\pi} \left[0.7 + 0.5 \exp\left(-2.25 \psi^{0.445} Ca_{\mathrm{J}}^{0.445} \right) \right]^{-2} \end{aligned} \tag{56}$$

In Eqs. (45), (47), (49), (54) and (56) taken from literature, the velocity ratio ψ is correlated only with $\Pi_7 = Ca_{\mathrm{J}}$ but not with any other non-dimensional group. In Figure 2 we display the functional relations $\psi = \psi(Ca_{\mathrm{J}})$ as obtained by Eqs. (45), (47), (49), (54) and (56). Also shown are experimental data of Thulasidas et al. (1995) for co-current upward Taylor flow in a square channel (D_{h} = 2 mm). These data show a large scatter band for $Ca_{\mathrm{J}} < 0.01$ while for higher values of Ca_{J} the scatter of the data is reduced and with increase of Ca_{J} an increase of ψ can be observed. For $Ca_{\mathrm{J}} < 0.03$ all correlations which are valid in this range give about similar values for ψ and the deviations are in general small. Also the value of ψ is less than 1.2 in this range, indicating that the bubble moves only slightly faster than the total superficial velocity. For $Ca_{\mathrm{J}} > 0.03$ a rather good agreement is found between the correlations of Aussilous & Quere (2000), Eq. (54), and Kreutzer et al. (2005), Eq. (56), and the experimental data of Thulasidas et al. (1995), whereas the correlation proposed by Liu et al. (2005) for their experimental data, Eq. (45), yields much smaller values. The correlation of Kreutzer et al. (2005) extends to values up to Ca_{J} = 3.86 and approaches a limiting value of $\psi = 2.59$. So it seems a reasonable good fit for square channels for $Ca_{\mathrm{J}} \geq 0.03$ when the bubble is axisymmetric. A comparison of the correlations of Aussilous & Quere (2000) for a circular channel, Eq. (54), and the one of Kreutzer et al. (2005) for a square channel, Eq. (56), suggests that for a given value of Ca_{J} the value of ψ is slightly larger in the square than in the circular channel.

Conclusions

In this paper we showed that the ratio between the bubble velocity (U_{B}) and the total superficial velocity (J_{tot}), which is here denoted as ψ, is a key parameter in Taylor flow. Among the quantities that are uniquely related to ψ are the mean liquid velocity, the relative velocity, and the gas hold-up. Depending on the value of ψ, the flow in the liquid slug shows - in a moving frame of reference - a recirculation pattern in the channel center and bypass flow close to the walls or complete bypass flow. The cross-sectional area of the bypass and recirculation region, and the non-dimensional recirculation time in the liquid slug depend on ψ and, for rectangular channels, on the channel aspect ratio. The local thickness of the liquid film in a certain cross-section depends on the mean liquid velocity in this cross-section and on ψ. If the liquid film is stagnant, unique relations exists between the liquid film thickness and bubble diameter and ψ. A similitude analysis showed that ψ may depend on up to ten other non-dimensional groups. However, the evaluation of literature data for Taylor flow indicates that ψ mainly depends on the capillary number. While these data show some scatter, they nevertheless give a consistent picture of this dependence.

In experiments and technical applications the total superficial velocity is often known or prescribed, whereas the bubble velocity is unknown. Thus, when ψ is known, all the above quantities can be directly computed. When inertial and gravitational effects are important, ψ will in addition to the capillary number also depend on the Laplace number (La) and the Eötvös number ($Eö$). Also the volumetric flow rate ratio (β) might be of some influence. We thus propose to focus in future experimental and theoretical studies on further clarification of the functional relationship $\psi = \psi(Ca_{\mathrm{J}}, \beta, La, Eö)$. These four parameters are easy to vary and control in experiments. Furthermore, ψ can be determined from the bubble velocity, which is easy to measure in transparent channels.

Nomenclature

A_0	cross-sectional area where the velocity in the liquid slug is positive in a frame of reference moving with the bubble (m^2)
A_1	cross-sectional area of recirculation region (m^2)
A_B	cross-sectional area of bubble (m^2)
A_{ch}	channel cross-sectional area (m^2)
A_L	cross-sectional area occupied by liquid (m^2)
B	width of rectangular channel (m)
Ca	Capillary number (–)
D	diameter of circular channel (m)
D_h	hydraulic diameter of rectangular channel (m)
$E\ddot{o}$	Eötvös number (–)
Eu_{uc}	Euler number (–)
g	gravitational constant (m/s^2)
H	height of rectangular channel (m)
J_G	gas superficial velocity (m/s)
J_L	liquid superficial velocity (m/s)
J_{tot}	total superficial velocity (m/s)
La	Laplace number (–)
L_B	bubble length (m)
L_{slug}	length of the liquid slug (m)
L_{uc}	Length of the unit cell (m)
Δp_{uc}	pressure difference along the unit cell (Pa)
Q_G	gas volumetric flow rate (m^3/s)
Q_L	liquid volumetric flow rate (m^3/s)
R	radius of circular channel (m)
Re	Reynolds number (–)
U_B	bubble velocity (m/s)
U_L	mean liquid velocity within the unit cell (m/s)
U_{slug}	mean velocity in liquid slug (m/s)
V	volume (m^3)
W	non-dimensional relative velocity (–)
We	Weber number (–)
Z	non-dimensional relative velocity (–)

Greek letters

χ	aspect ratio of rectangular channel (–)
δ_F	liquid film thickness (m)
ε	gas holdup in unit cell (–)
φ	angle between channel axis and gravity vector (–)
Λ	non-dimensional length of the unit cell (–)
μ	dynamic viscosity (Pa s)
ρ	density (kg/m^3)
σ	coefficient of surface tension (N/m)
τ	non-dimensional recirculation time (–)
ψ	velocity ratio $\psi \equiv U_B / J_{tot}$ (–)

Subscripts

B	bubble
F	liquid film
G	gas phase
J	quantity is based on J_{tot} as velocity scale
L	liquid phase
slug	liquid slug
uc	unit cell

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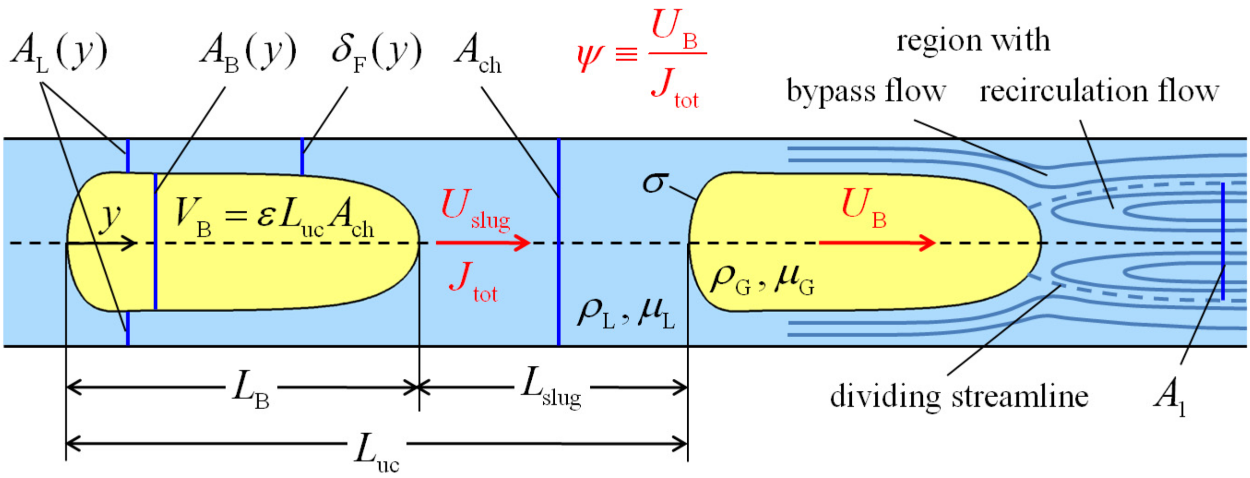

Figure 1: Sketch of Taylor flow with characteristic dimensions, areas, and streamlines in the recirculation flow regime (recirculation flow pattern after Taylor (1961)).

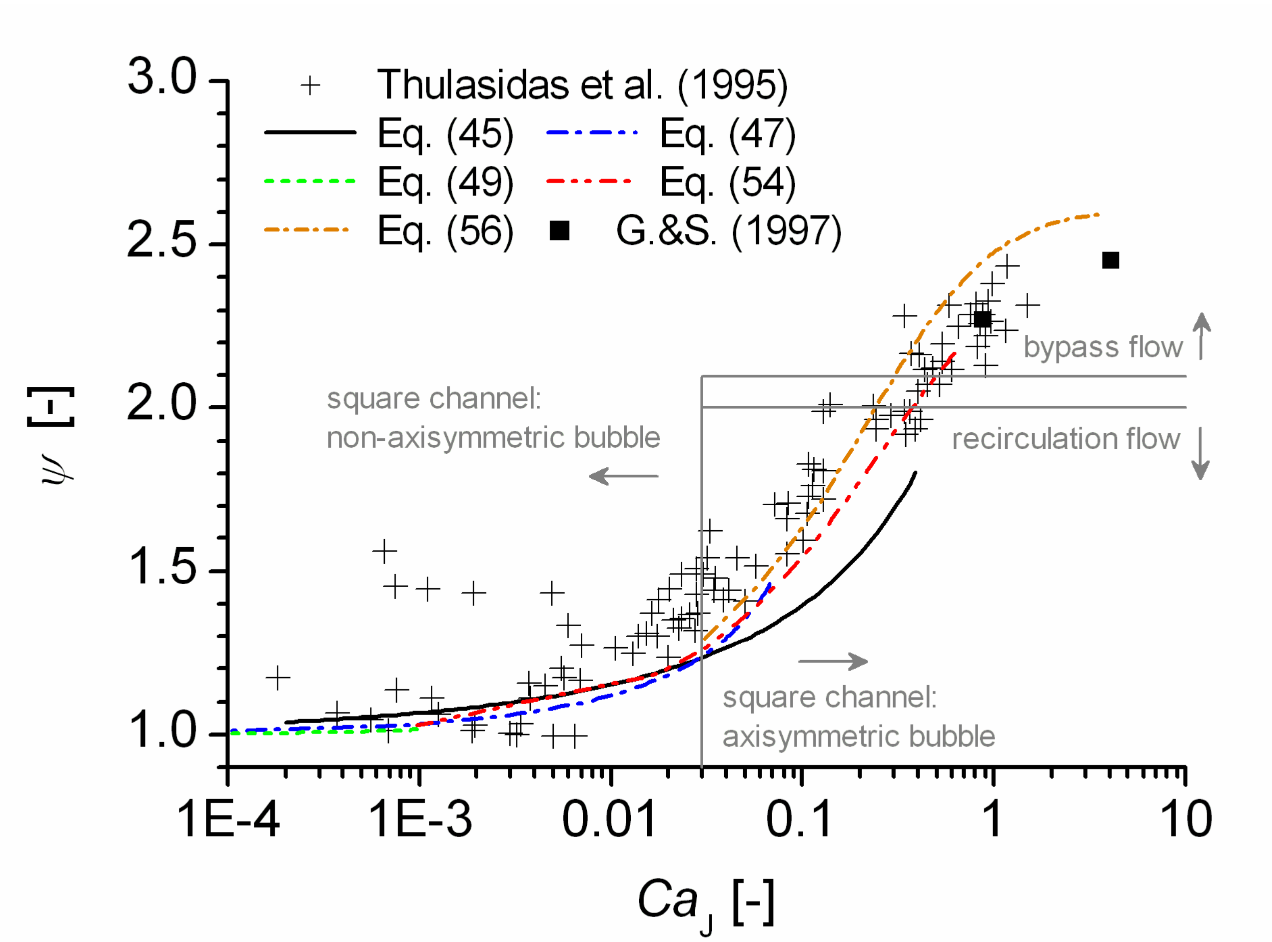

Figure 2: Literature correlations for ψ as function of the capillary number Ca_J. The symbol ■ corresponds to the numerical data of Giavedoni & Saita (1997) for Ca_B = 2 and 10, respectively. The thin grey horizontal lines indicate the transition from recirculation flow to complete bypass flow in a circular (lower line) and a square (upper line) channel. The thin grey vertical line indicates the transition from a non-axisymmetric to an axisymmetric bubble shape in a square channel.