Charge Transport in Voltage-Biased Superconducting Single-Electron Transistors

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Charge is transported through superconducting SSS single-electron transistors at finite bias voltages by a combination of coherent Cooper-pair tunneling and quasiparticle tunneling. At low transport voltages the effect of an "odd" quasiparticle in the island leads to a $2e$-periodic dependence of the current on the gate charge. We evaluate the $I-V$ characteristic in the framework of a model which accounts for these effects as well as for the influence of the electromagnetic environment. The good agreement between our model calculation and experimental results demonstrates the importance of coherent Cooper-pair tunneling and parity effects.

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The single-electron tunneling (SET) transistor has proven an ideal system to display the concepts of "single electronics". In this device an island is coupled via tunnel junctions to the leads. The island potential can be modulated by a capacitively coupled gate voltage V_g. In transistors with normal-conducting islands and leads the current depends e-periodically on the gate charge $Q_0 = C_g V_g$. Recently much attention has been devoted to single-electron transistors with superconducting islands and normal leads (NSN) and entirely superconducting systems (SSS). In a superconducting island, where Cooper pairs form the condensate, the addition of one extra electron - the "odd" one - costs the gap energy Δ [1]. Hence the physical properties of the system depend on the parity of the charge number in the island, and the $I-V$ characteristics is expected to be $2e$-periodic in the gate charge. The first clear signature of $2e$-periodicity was observed by Tuominen *et al.* [2]. However, no satisfactory explanation of these experimental results has been provided until now. In the meantime a number of experiments both in NSN transistors [3,4] and in SSS transistors [5–9] have demonstrated a rich variety of phenomena and show good agreement with theoretical results. It is now also well understood [4] that the difficulties in observing the $2e$-periodicity arise from the extreme sensitivity of the even-odd difference to effects of the electromagnetic environment which create non-equilibrium quasiparticles.

Parallel to the experiments the theoretical description of systems exhibiting parity effects made rapid progress. In Ref. [2] the authors present an equilibrium model which accounts for the temperature dependence of the even-odd asymmetry. A kinetic model was developed in Ref. [10] to describe transport. In this framework the $I-V$ characteristics of NSN transistors can be derived [4,11]. For SSS transistors, there exists a well-developed theory for the current-biased system [12,5,13]. The voltage-biased system has been considered in the absence of parity effects [14], while in Ref. [8] only resonant Cooper-pair tunneling has been studied. In this paper we investigate a model which includes Cooper-pair and single-particle tunneling as well as parity effects. First we study the $I-V$ characteristic of an SSS transistor in the absence of external impedances. In the transport voltage range $eV \lesssim E_C$ even-odd effects are observed (E_C denotes the scale of the charging energy, see below). We then discuss examples for the relevant transport processes. In order to compare with the experiment, we account for the influence of the electromagnetic environment. This rather complex model explains the experimental results of Ref. [2].

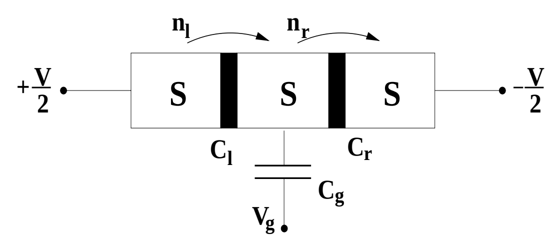

FIG. 1. The SSS transistor. $n_{l/r}$ is the number of electrons which have tunneled through the left and the right junction.

We consider a SET transistor (see Fig. 1) with superconducting electrodes and island (with energy gaps Δ) below the crossover temperature $T_{cr} = \Delta/k_B \ln N_{eff}$ where parity effects can be observed. Here $N_{eff} = 2N_I(0)\sqrt{2\pi\Delta k_B T}$ is the effective number of states available for the odd quasiparticle [2] and $N_I(0)$ is the density of states (per spin) in the island. For the moment we ignore the effect of the external impedance.

The system can be described by a sum of Hamiltonians for the left and right electrode and the island, the charging energy and the tunneling Hamiltonian $H = H_L + H_R + H_I + H_{ch} + H_T$. We will treat the Josephson tunneling non-perturbatively. Hence we start from the model Hamiltonian [14]

$$H_0 = \sum_{Q,\bar{Q}} \left(\left[\frac{(Q+Q_0)^2}{2C} - \frac{1}{2}\bar{Q}V \right] |Q,\bar{Q}\rangle\langle Q,\bar{Q}| - \frac{E_J}{2} \sum_{\pm}\sum_{\pm} |Q \pm 2e, \bar{Q} \pm 2e\rangle\langle Q,\bar{Q}| \right) \quad (1)$$

and we will account for the quasiparticle tunneling in perturbation theory. Here $Q \equiv (n_l - n_r)e$ denotes the island charge, and we defined a total charge which has passed through the system $\bar{Q} \equiv (n_l + n_r)e$ (with respect to a reference state). Further $C = C_l + C_r + C_g$ is the total island capacity and $Q_0 = C_g V_g$ the offset charge (here we assume $C_l = C_r$). The first two terms describe the charging energy and the energy gain in tunneling due to the transport voltage. They are diagonal in the basis of charge states $|Q,\bar{Q}\rangle$. The typical scale of the charging energy is $E_C = e^2/2C$. The last term describing Cooper-pair tunneling with coupling energy E_J is off-diagonal and, thus, mixes the charge states $|Q,\bar{Q}\rangle$. Therefore, the eigenstates of H_0 are linear combinations of these states

$$|\Psi_k\rangle = \sum_{n,m} a^k_{n,m} |ne, me\rangle \quad . \quad (2)$$

A dc-current requires a dissipation mechanism. In the case of zero external impedance the quasiparticle tunneling can cause transitions between different eigenstates $|\Psi_k\rangle$. It is accounted for by

$$H_T^{qp} = \sum_{k\in I, p\in L} T^{(l)}_{pk} |Q+e,\bar{Q}+e\rangle\langle Q,\bar{Q}| c_k^\dagger c_p + \text{h.c.} + \sum_{k\in I, p'\in R} T^{(r)}_{kp'} |Q-e,\bar{Q}+e\rangle\langle Q,\bar{Q}| c_{p'}^\dagger c_k + \text{h.c.} \quad (3)$$

The tunneling matrix elements in the left/right junction $T^{(l/r)}_{pk}$ are related to the conductance of the junctions by $1/R_{l/r} = 4\pi e^2/\hbar \; N_{L/R}(0) N_I(0) |T^{(l/r)}|^2$, where we approximated $T^{(l/r)}_{pk} \approx T^{(l/r)}$. If the junction resistances are large compared to the quantum resistance $R_{l/r} > R_K = h/e^2$ the transition rates can be calculated by the golden rule

$$\Gamma_{i\to f} = \frac{1}{e} \sum_{j,Q,\bar{Q}} \left(\frac{I^{(j)}_{qp}(\varepsilon_{if})}{1 - \exp(-\varepsilon_{if}/k_B T)} + e\gamma_{esc} \right) \cdot \sum_{j=l:\ \pm\pm} \sum_{j=r:\ \pm\mp} |\langle \Psi_f | Q \pm e, \bar{Q} \pm e\rangle\langle Q,\bar{Q}|\Psi_i\rangle|^2 \ . \quad (4)$$

Here $I^{(j)}_{qp}$ is the well-known $I-V$ characteristic for quasiparticle tunneling [15] between superconductors and ε_{if} is the energy difference between initial and final state. In order to allow for the parity effect, we have to include the escape rate γ_{esc} of an odd quasiparticle in the island [10]

$$\gamma_{esc} \simeq \begin{cases} \frac{1}{2e^2 R_{l/r} N_I(0)} \frac{\varepsilon_{if}+\Delta}{\sqrt{(\varepsilon_{if}+\Delta)^2 - \Delta^2}} \theta(\varepsilon_{if}) & \text{if } Q \text{ odd} \\ 0 & \text{if } Q \text{ even} \end{cases} \ . \quad (5)$$

In order to determine the dc-current, we follow the procedure described in Ref. [14]: First, we determine the eigenstates of H_0 either in perturbation theory (as we shall discuss below) or numerically taking into account a sufficient number of charge states. This procedure converges for not too large Josephson coupling energies $E_J < E_C$. Given the eigenstates $|\Psi_k\rangle$ we calculate the rates in Eq. (4), which then enter a master equation $\partial_t P_k = \sum_{n\neq k} (P_n \Gamma_{n\to k} - P_k \Gamma_{k\to n})$ for the probabilities P_k to find the system in the k-th eigenstate. The stationary solution $\partial_t P_k = 0$ is sufficient to evaluate the dc-current

$$I = \frac{e}{2} \sum_{k,n\neq k} P_k \Gamma_{k\to n} (\langle \Psi_n | \bar{Q} | \Psi_n\rangle - \langle \Psi_k | \bar{Q} | \Psi_k\rangle) \quad . \quad (6)$$

The results are shown in Fig. 2(a). We used the parameters $\Delta = 1.3 E_C$, $E_J^0 = 0.17 E_C$, $R_{l/r} = R \approx R_K$, $\gamma_{esc} = 2.5 \cdot 10^{-5} (RC)^{-1}$, which correspond to those in Ref. [2]. Here E_J^0 denotes the Ambegaokar-Baratoff expression for the Josephson energy. In Eq. (1) the generalized Josephson coupling energy in the presence of charging effects [16] enters, which in the present case is larger than E_J^0 by roughly 20 %. We note that the $I-V$ characteristic is $2e$-periodic and observe a rich structure deep in the subgap region. For transport voltages $eV \gtrsim 2.5 E_C$ the $2e$-periodic features disappear and the current becomes e-periodic in Q_0 again. This is not surprising since on a current scale $I \gg e\gamma_{esc}$ the unpaired quasiparticle in the island looses its importance.

The basis of eigenstates of H_0 is "appropriate" for the problem. On inspecting our numerical procedure we find that for low transport voltages only a few (two or three) states $|\Psi_k\rangle$ are noticeably populated. Similar behavior is found in systems without coherent tunneling like NNN or NSN transistors. Therefore, we can calculate transition rates and the current analytically if we know the eigenvalues of H_0 and the corresponding eigenstates. To this end, we determine the coefficients $a^k_{n,m}$ in Eq. (2) by using perturbation theory in E_J. Away from certain resonant situations, the k-th eigenstate has only one coefficient $a^k_{q,p}$ of order unity, whereas all other coefficients are considerably smaller. To fix ideas, let us consider the state $|\Psi_0\rangle$ in the range of gate charges $Q_0 \in [0, e/2]$. In this eigenstate the most likely charge state is $|Q = 0e, \bar{Q} = 0e\rangle$, i.e. $a^0_{0,0} \approx 1$. Due to coherent tunneling of one Cooper pair, there is a small amplitude $a^0_{\pm 2,\pm 2} \propto E_J/E_C$ to find the system in the charge states $|Q = \pm 2e, \bar{Q} = \pm 2e\rangle$. Since also several Cooper pairs can tunnel coherently, the system can be, e.g., in the charge state $|2e, 6e\rangle$ with a very small amplitude $a^0_{2,6} \propto (E_J/E_C)^3$.

At resonance lines, however, it is possible that this amplitude is much larger. Let us consider the solid straight line in Fig. 2 (b), which is given by

$$3eV = 4E_C(1 - Q_0) \quad . \quad (7)$$

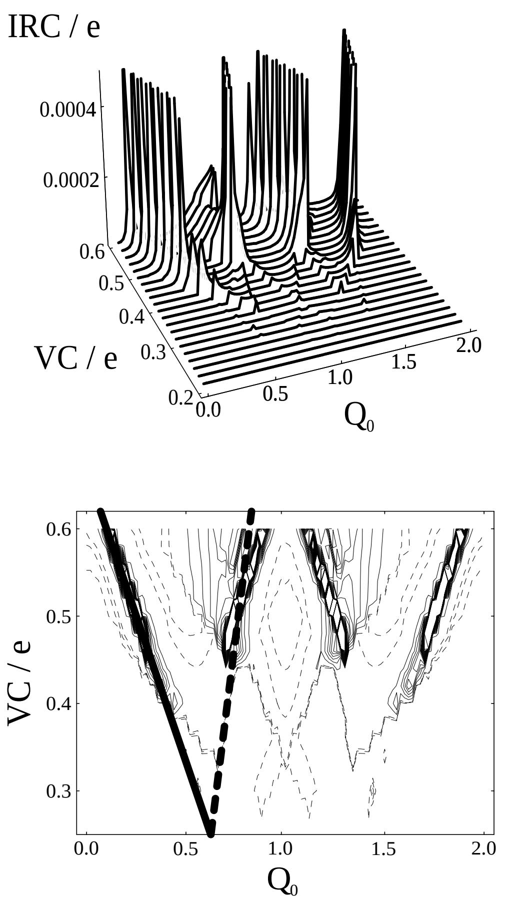

FIG. 2. a) $I-V$ characteristic of a SSS transistor for vanishing external impedance (parameters see text). b) Contour plot of the same data. There are three dashed contour lines in the current range $I = 0\ldots10^{-5}\ e/(RC)$ and 20 lines for $I \leq 10^{-3}\ e/(RC)$. A pronounced resonance is found along the straight solid line $3eV = 4E_C(1-Q_0)$. The straight dashed line $eV = 4E_C(Q_0 - 1/2)$ marks the edge of the shoulder-like structure.

Along this line the charge states $|0e, 0e\rangle$ and $|2e, 6e\rangle$ have the same energy, i.e., three Cooper pairs tunnel resonantly there. The resonance results in a drastically increased amplitude $a^0_{2,6} \propto (E_J/E_C)$.

A transition from $|\Psi_0\rangle$ to another eigenstate can occur if it is energetically favorable and the matrix element of the final state with $|\Psi_0\rangle$ according to Eq. (4) is nonzero. On analyzing which transitions due to quasiparticle tunneling are energetically favorable, we find that a process

$$|\Psi_0\rangle \approx |0e, 0e\rangle \longrightarrow |\Psi_1\rangle \approx |1e, 7e\rangle \qquad \text{(process a)}$$

is possible. Out of resonance the rate of process a) is of the order $(E_J/E_C)^6$. In a narrow strip (whose width is characterized by E_J) around the resonance line Eq. (7), however, we find

$$\Gamma_a \propto \left(\frac{E_J/2}{4E_C(1-Q_0)+eV}\right)^2 \propto \left(\frac{E_J}{E_C}\right)^2 \quad . \qquad (8)$$

The line in Fig. 2(b) corresponding to Eq. (7) marks the most significant resonance in the $I-V$ characteristic. We are, thus, led to the conclusion that the dominant transport process in the subgap region is tunneling of quasiparticles accompanied by simultaneous tunneling of several Cooper pairs. Due to this combination enough energy is gained to overcome the quasiparticle tunneling gap 2Δ. The importance of this type of transport mechanism was first noted by Fulton *et al.* [17]. Although the rates for these processes in general are small, they are considerably enhanced in situations, where resonant transfer of Cooper pairs is possible.

So far we have studied the conditions for the system to leave the initial state. However, a dc charge transport through the system requires cycles, after which the island returns to a state equivalent to the initial one. The simplest version is a two-step cycle of subsequent transitions of the same type in the left and right junction. Such cycles dominate in NNN or NSN transistors at low bias voltages. The cycles which lead to the pronounced features in Fig. 2 arise due to two-step cycles as well, but the second step is different from the first one. The transition completing the cycle which starts with process a) is

$$|\Psi_1\rangle \approx |1e, 7e\rangle \longrightarrow |\Psi_2\rangle \approx |0e, 12e\rangle \qquad \text{(process b)} \quad ,$$

i.e., quasiparticle transfer accompanied by only two Cooper pairs tunneling. The latter process is not in resonance and, therefore, the rate is $\Gamma_b \propto (E_J/E_C)^4$. Whereas off-resonance the process a) is the bottleneck for the current, at resonance the process b) has the smaller rate. This explains that at the resonance the current increases by roughly two orders of magnitude.

Another interesting feature in the $I-V$ characteristic is the shoulder-like structure between the high resonances for gate charges $Q_0 \in [e/2, e]$. It is directly related to the escape rate γ_{esc} of the odd particle. The first step in the relevant cycle is a process similar to process b)

$$|\Psi_0\rangle \approx |0e, 0e\rangle \longrightarrow |\Psi_3\rangle \approx |1e, 5e\rangle$$

with a rate $\propto (E_J/E_C)^4$, which is relatively large (as discussed before). The current, however, is limited by the second step

$$|\Psi_3\rangle \approx |1e, 5e\rangle \longrightarrow |\Psi_4\rangle \approx |0e, 6e\rangle \quad .$$

This is a pure quasiparticle transition without Cooper pairs, which can occur because the escape rate γ_{esc} has no gap. In Fig. 2(b) it is seen that the cycle sets in for transport voltages $eV \geq 4E_C(Q_0 - 1/2)$ (the dashed straight line). This is exactly the condition for the odd quasiparticle to gain energy on leaving the island.

So far we have considered the ideal case of a vanishing external impedance Z. In order to compare our results with experimental data it is necessary to account for the effect of the electromagnetic environment. An external impedance gives rise to incoherent Cooper-pair transitions. For not too low transport voltages the rate of these transitions is given by (see, e.g., Refs. [14,19])

$$\Gamma^{env}_{i\to f} = \frac{1}{\hbar^2}|\langle\Psi_f|\bar{Q}/2|\Psi_i\rangle|^2 \frac{2\mathrm{Re}Z(\omega)\ \varepsilon_{if}}{1-\exp\left(-\varepsilon_{if}/k_BT\right)} \quad . \tag{9}$$

For appropriate parameters they lead to a pronounced resonance structure in the $I-V$ characteristic [20].

Furthermore, in an experiment the temperature of the environment is not necessarily the same as the electron temperature [21,4]. High temperature fluctuations in the environment induce quasiparticle tunneling and, thus, can cause qualitative changes in the $I-V$ characteristic. In order to take into account this effect, we have to add to the single-electron tunneling rate $\Gamma_{i\to f}$ the term

$$\Gamma^{e,qp}_{i\to f} = \frac{1}{e^2R_t}\frac{2R_e}{R_K}k_BT_e\frac{2\Delta}{2\Delta-\varepsilon_{if}}e^{-(2\Delta-\varepsilon_{if})/k_BT_e} \tag{10}$$

for $\varepsilon_{if} < 2\Delta$. The Ohmic resistance R_e describes the environment, which fluctuates at the high temperature $T_e \gtrsim E_C$. Since there is no detailed information about the electromagnetic environment in Ref. [2], we fix the parameters by comparing with similar experiments. In Fig. 3 the $I(Q_0)$ dependence for several transport voltages is plotted. The calculation reproduces remarkably well both the general shape of the experimental curves and the order of magnitude of the current. We have cut the current resonances for the bias voltages $eV = 167\mu V$, $200\mu V$, since the maximum differential conductance, which can be observed in an experiment, is limited [8]. In a model calculation like ours we cannot expect perfect correspondence between theory and experiment. The reason is that the system is very sensitive to even small amounts of quasiparticle tunneling which can overcome a threshold for cycles in lower order of E_J/E_C. (We have discussed a similar effect in connection with the shoulder-like structure). Different relevant temperatures in the environment (which is very likely) can cause subtle changes in the $I-V$ characteristics. In this case Eq. (10) is only a rough approximation. Therefore, we have to adjust the parameter R_e for different bias voltages. Finally we mention that one can speculate about processes of higher order in the quasiparticle tunneling, e.g., co-tunneling or coherent two-electron tunneling across one junction, accompanied by Cooper pairs. However, we come to the conclusion that in the present case processes of higher order in the quasiparticle tunneling can be neglected. There are two possibilities: i) The process creates excitations. Then the rate has a gap $\geq 2\Delta$ and becomes important only for higher transport voltages, where also first order quasiparticle tunneling is possible. ii) The process does not create excitations. But then the phase space is reduced by at least one small factor $1/(N_I(0)\Delta)$ [1].

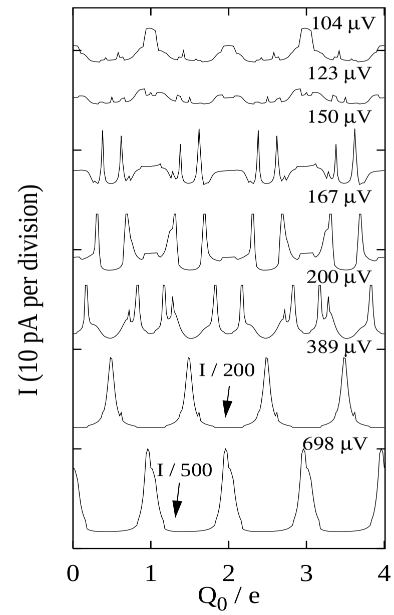

FIG. 3. The current $I(Q_0)$ through an SSS SET transistor, including both resonant Cooper-pair transitions due to an external impedance Z and high temperature fluctuations in the electromagnetic environment. Parameters are: $E_C = 180\mu$eV, $\Delta = 240\mu$eV, $E_J^0 = 30\mu$eV, $R_l + R_r = 50$kΩ, $\gamma_{esc} \simeq 2\cdot 10^6 s^{-1}$, Re$Z(0)$=80$\Omega$, $R_e = 0.8\Omega$ (for $V \leq 123\mu$V), $R_e = 3\Omega$ (for $V \geq 150\mu$V), T_e=3.6K. We have assumed a slight asymmetry of the junction resistances $R_l : R_r = 4 : 3$.

Summarizing, we can say that our non-perturbative model adequately describes charge transport in voltage biased SET transistors under various conditions in a large range of transport voltages. Due to the parity effect the $I-V$ characteristics are $2e$-periodic in the gate charge for not too large transport voltages. The dominant transport mechanism is quasiparticle tunneling, accompanied by coherent tunneling processes of several Cooper pairs. An external impedance at low temperatures causes incoherent Cooper-pair transitions, whereas high temperature fluctuations in the environment “poison” the ideal structure, thus restoring e-periodicity of the current.

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[20] We note that in Ref. [8] resonant Cooper-pair tunneling has been observed in a wide range of transport voltages. The results have been explained there within a less complex model for resonant Cooper-pair tunneling. We can reproduce the experimental data of Ref. [8] in a range $eV = 0.2E_C \ldots 4E_C$ (not shown here), although the Josephson coupling energy in the experiments is of the order of the charging energy.
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