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We develop a detailed analysis of electron transport in normal diffusive conductors in the presence of proximity induced superconductivity. A rich structure of temperature and energy dependencies for the system conductance, density of states and related quantities was found and explained. If the normal conductor forms a loop its conductance changes  $h/2e$ -periodically with the magnetic flux inside the loop. The amplitude of these conductance oscillations shows a reentrant behavior and decays as  $1/T$  at high  $T$ .

Presently transport properties of normal-superconducting proximity systems attract much experimental and theoretical interest. Various nontrivial features of such systems have been recently discovered [1,2]. The aim of this paper is to investigate the coherent electron transport in a normal diffusive conductor attached to a superconductor.

One can show [3] that complicated geometrical realizations of the system [1,2] can be essentially reduced to the following simple model: a normal diffusive wire is attached to a normal reservoir at  $x = 0$  and a superconducting one at  $x = d$ . In order to calculate the conductance of this wire we use the standard formalism of quasiclassical Green functions in the Keldysh technique (see e.g. [4]). The first step is to find the retarded normal and anomalous Green functions of the system  $g^R = \cosh \theta$  and  $f^R = \sinh \theta$ ,  $\theta = \theta_1 + i\theta_2$ . In the diffusive approximation this has been done with the aid of the Usadel equation (see [3] for details). The second step is to solve the kinetic equation. As a result for a differential conductance of the system (normalized to its Drude value  $G_N$ ) at low voltages and in the absence of tunnel barriers one finds [4]  $\tilde{G} = \frac{1}{2T} \int_0^\infty d\epsilon \operatorname{sech}^2(\epsilon/2T) D(\epsilon)$  where  $D(\epsilon) = \left( \int_0^1 d\bar{x} \operatorname{sech}^2 \theta_1(\bar{x}) \right)^{-1}$  is the transparency of the system at the energy  $\epsilon$ .

The conductance  $G(T)$  shows the reentrant behavior (Fig. 1) (see also [5]). At low temperatures  $T \ll \epsilon_d$  the correction is  $\delta G := \tilde{G} - 1 \propto (T/\epsilon_d)^2$ , at  $T \gg \epsilon_d$  we have  $\delta G \propto \sqrt{\epsilon_d/T}$ . The square-root-scaling of  $\delta G$  at high  $T$  has an obvious physical interpretation: as the part of the N-wire of the size  $\sim \xi_N = \sqrt{D/2\pi T}$  close to the NS boundary becomes effectively superconducting due to the proximity effect, only the rest of the wire contributes to the resistance. As a result it becomes smaller than  $1/G_N$ . As the temperature is lowered the conductance increases, reaches its maximum at  $\xi_N \sim d$  and then decreases again.

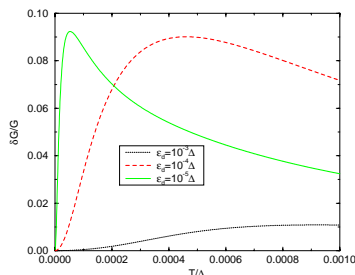


FIG. 1.

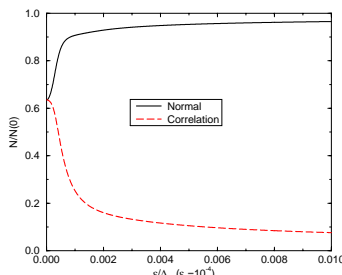


FIG. 2.

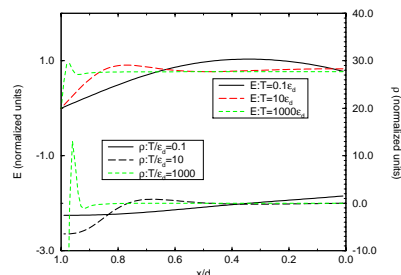


FIG. 3.

In order to understand the reentrant behavior of  $G$  at low  $T < \epsilon_d$  we calculated the density of states (DOS) averaged over the length of the wire. The normal density of states  $N_N = N(0) \int_0^1 d\bar{x} \Re(g)$  shows a soft pseudogap below  $\epsilon_d$  (see Fig. 2). At the first sight at low  $T$  this would lead to a decrease of  $\tilde{G}$  above 1. This is, however, not the case because of an additional contribution of correlated electrons present in the N-wire due to the proximity effect. The DOS for such electrons in the wire  $N_S = N(0) \int_0^1 d\bar{x} \Im(f)$  becomes larger for small  $\epsilon$  (Fig.2). These two effects exactly compensate each other at  $T = 0$ , in which case  $\tilde{G} = 1$ . For  $T > 0$  we always have  $(\Re g)^2 + (\Im f)^2 = \cosh^2 \theta_1 > 1$ , i.e. the pseudogap effect never dominates the correlation-induced enhancement and  $G(T > 0) > 1$ . On the other hand, due to the presence of this pseudogap at  $\epsilon \sim \epsilon_d$  the total transparency  $D(\epsilon)$  decreases with  $\epsilon$  for  $\epsilon < \epsilon_d$  resulting in the corresponding reentrant behavior of  $G$  at low  $T$ .

Our theory also allows to calculate the profile of the electric field  $E(x)$  and the charge  $\rho(x)$  inside the N-metal (Fig.3). At  $T = 0$  we have (in normalized units)  $E(\bar{x}) = \cos(\bar{x}\pi/2) - \frac{\pi}{2}(\bar{x} - 1)\sin(\bar{x}\pi/2)$  and at  $T \gg \epsilon_d$ :  $E(\bar{x}) \propto (1 - \bar{x})\sqrt{T/\epsilon_d}$  near the superconductor ( $1 - \bar{x} \ll \xi_T$ ) and  $E(\bar{x}) = 1$  near the normal reservoir ( $\bar{x} \ll \xi_T$ ). As we see, the electric field distribution is essentially nonuniform: it monotonously decreases with  $T$  close to a superconductor and overshoots its normal state value further from it.

Let us also point out that the reentrant behaviour of  $G(T)$  takes place only in the absence of low transparent tunnel barriers at the boundaries of the N-wire. In the presence of such barriers the field distribution in the system becomes entirely different and  $G(T)$  monotonously increases with  $T$  for most of the situations [3]. Depending on the sample both increasing or decreasing  $G(T)$  has been found in the experiments [1].

In the experiments [2] the conductance of a ring-shaped proximity wire was investigated in the presence of the magnetic flux  $\Phi$  inside the ring. This system shows  $h/2e$ -periodic in  $\Phi$  magnetoresistance-oscillations. At high temperatures the amplitude of these decays  $\propto 1/T$  [2].

For simplicity, we have chosen a system where the ring has circumference  $2d$  and the connections to the reservoirs are of the length  $d$ , so the Thouless energies  $\epsilon_d = \frac{D}{d^2}$  and  $\epsilon_{3d} = \frac{D}{(3d)^2}$  become important. We introduced a phase by a gauge transformation and used a ‘‘Kirchhoff law’’ for the Green’s functions at branching points [6,3].

The transparency of the system  $D(\epsilon)$  (Fig. 4) shows an interesting structure. E.g. the oscillations change their sign at intermediate energies. This effect could possibly be probed by measurements at very low temperatures ( $T \ll \epsilon_{3d}$ ) at finite voltages. At high energies ( $\epsilon \gg \epsilon_d$ ), however, the amplitude of the oscillations of  $D(\epsilon)$  is exponentially suppressed because for such energies no superconducting correlations (which can only originate  $h/2e$ -oscillations) are present in the ring.

Thus – even at high temperatures – only the states with low energies  $\epsilon < \epsilon_d$  are responsible for the effect of conductance oscillations. At  $T \gg \epsilon_d$  the amplitude of these oscillations can be estimated as  $\Delta \bar{G} = \bar{G}_{\Phi=0} - \bar{G}_{\Phi=h/4e} \approx \frac{1}{2T} \int_0^{\epsilon_c} d\epsilon \operatorname{sech}^2(\epsilon/2T)(D_0 - D_{h/4e}) \approx \frac{\epsilon_c}{2T} \Delta D_{av}$  where  $\epsilon_c \approx \epsilon_d$  and  $\Delta D_{av}$  is the average amplitude of the oscillations of the transparency below  $\epsilon_c$ . This estimate demonstrates that – in a complete agreement with the experimental results [2] – the  $1/T$ -scaling persists at all temperatures  $T > \epsilon_d$ . Also the numerical results show that the  $1/T$ -scaling of the oscillation amplitude is excellently fulfilled at sufficiently high  $T$ . (see Figs. 5 and 6). At lower temperatures the amplitude of the conductance oscillations shows the reentrant behavior reaching the maximum at  $T \sim \epsilon_d$  and vanishing completely at  $T = 0$  (Fig. 5). This reentrant behavior has a similar physical origin to that of  $G(T)$  discussed above in the absence of the ring.

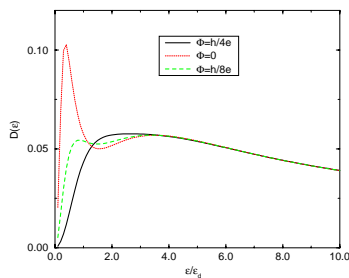


FIG. 4.

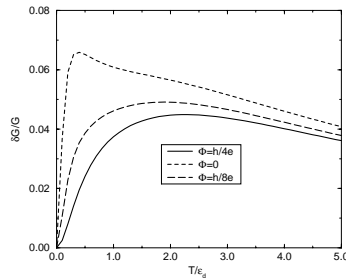


FIG. 5.

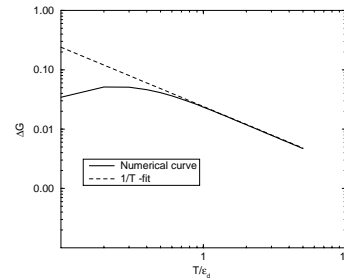


FIG. 6.

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## References

- [1] V.T. Petrashov et al., Phys. Rev. Lett. **74**, 5268 (1995).
- [2] H. Courtois et al., Phys. Rev. Lett **76**, 130 (1996).
- [3] A.A. Golubov, F. Wilhelm and A.D. Zaikin, preprint cond-mat/9601085; submitted to Phys. Rev. B.
- [4] A.F. Volkov, A.V. Zaitsev and T.M. Klapwijk, Physica C **210**, 21 (1993) and references therein.
- [5] Yu. V. Nazarov and T.H. Stoof, Phys. Rev. Lett. **76**, 823 (1996); preprint cond-mat/9601071.
- [6] A.V. Zaitsev, Physica B **203**, 274 (1994).

## Transport cohérent dans un fil diffusif entre deux réservoirs

Nous développons une analyse détaillée du transport des électrons dans des conducteurs normaux en présence de supraconductivité induite par l’effet de proximité. On a pu trouver et expliquer la dépendance riche de la conductance, de la densité d’état et d’autres grandeurs en fonction de la température et de l’énergie. Si le conducteur normal contient une boucle, sa conductance oscille avec la période  $h/2e$  en fonction du flux magnétique à travers la boucle. L’amplitude de ces oscillations en fonction de la température fait apparaître un phénomène ‘‘ré-entrant’’ et diminue comme  $1/T$  pour  $T$  grand.