Coherent transport in a normal wire between reservoirs

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We develop a detailed analysis of electron transport in normal diffusive conductors in the presence of proximity induced superconductivity. A rich structure of temperature and energy dependencies for the system conductance, density of states and related quantities was found and explained. If the normal conductor forms a loop its conductance changes $h/2e$-periodically with the magnetic flux inside the loop. The amplitude of these conductance oscillations shows a reentrant behavior and decays as $1/T$ at high T.

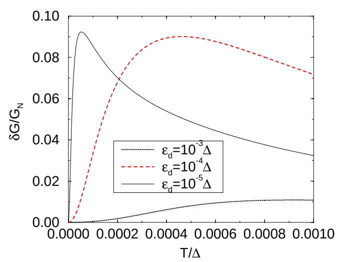

Fig.1

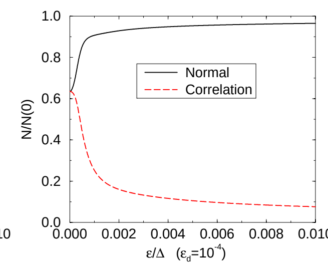

Fig.2

Presently the transport properties of normal/superconducting proximity systems attract much experimental and theoretical interest. Various nontrivial features of such systems have been recently discovered [1, 2] The aim of this paper is to investigate the coherent electron transport in a normal diffusive conductor attached to a superconductor.

One can show [3] that complicated geometrical realizations of the system [1, 2] can be essentially reduced to the following simple model: a normal diffusive wire is attached to a normal reservoir at $x = 0$ and a superconducing one at $x = d$. In order to calculate the conductance of this wire we use the standard formalism of quasiclassical Green functions in the Keldysh technique (see e.g. [4]). The first step is to find the retarded normal and anomalous Green functions of the system $g^R = \cosh\theta$ and $f^R = \sinh\theta$, $\theta = \theta_1 + i\theta_2$. In the diffusive approximation this has been done with the aid of the Usadel equation (see [3] for details). The second step is to solve the kinetic equation. As a result for a differential conductance of the system (normalized to its Drude value G_N) at low voltages and in the absence of tunnel barriers one finds [4] $\bar{G} = \frac{1}{2T}\int_0^\infty d\epsilon\, \mathrm{sech}^2(\epsilon/2T)D(\epsilon)$ where $D(\epsilon) = \left(\int_0^1 d\bar{x}\, \mathrm{sech}^2\theta_1(\bar{x})\right)^{-1}$ is the transparency of the system at the energy ϵ.

The conductance $G(T)$ shows the reentrant behavior (Fig. 1) (see also [5]). At low temperatures $T \ll \epsilon_d$ the correction is $\delta G := \bar{G} - 1 \propto (T/\epsilon_d)^2$, at $T \gg \epsilon_d$ we have $\delta G \propto \sqrt{\epsilon_d/T}$. The square-root-scaling of δG at high T has an obvious physical interpretation: as the part of the N-wire of the size $\sim \xi_N = \sqrt{D/2\pi T}$ close to the NS boundary becomes effectively superconducting due to the proximity effect, only the rest of the wire contributes to the resistance. As a result it becomes smaller than $1/G_N$. As the temperature is lowered the conductance increases, reaches its maximum at $\xi_N \sim d$ an then decreases again.

In order to understand the reentrant behavior of G at low $T < \epsilon_d$ we calculated the density of states (DOS) averaged over the length of the wire. The normal density of states $N_N = N(0)\int_0^1 d\bar{x}\, \Re(g)$ shows a soft pseudogap below ϵ_d (see Fig. 2). At the first sight at low T this would lead to a decrease of $\bar{G}$ below 1. This is, however, not the case because of an additional contribution of correlated electrons present in the N-wire due to the proximity effect. The DOS for such electrons in the wire $N_S = N(0)\int_0^1 d\bar{x}\, \Im(f)$ becomes larger for small ϵ (Fig.2). These two effects exactly compensate each other at $T = 0$, in which case $\bar{G} = 1$. For $T > 0$ we always have $(\Re g)^2 + (\Im f)^2 = \cosh^2\theta_1 > 1$, i.e. the pseudogap effect never dominates the correlation-induced enhancement and $G(T > 0) > 1$. On the other hand, due to the presence of this pseudogap at $\epsilon \sim \epsilon_d$ the total transparency $D(\epsilon)$ decreases with ϵ

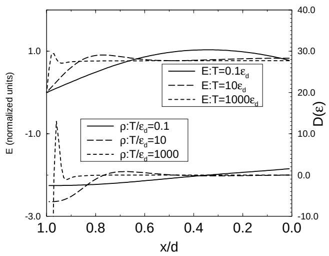

Fig.3

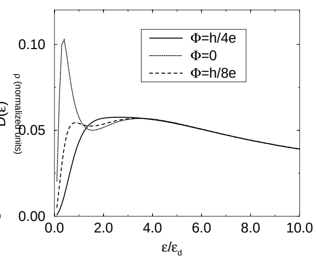

Fig.4

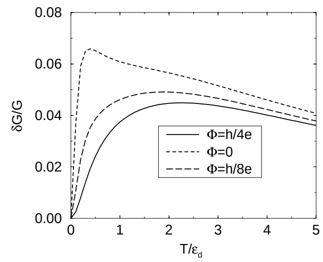

Fig.5

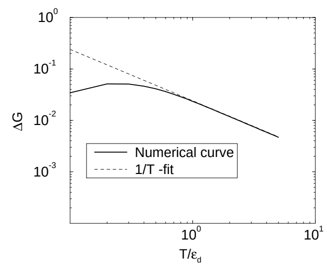

Fig.6

for $\epsilon < \epsilon_d$ resulting in the corresponding reentrant behavior of G at low T.

Our theory also allows to calculate the profile of the electric field $E(x)$ and the charge $\rho(x)$ inside the N-metal (Fig.3). At $T = 0$ we have (in normalized units) $E(\bar{x}) = \cos(\bar{x}\pi/2) - \frac{\pi}{2}(\bar{x} - 1)\sin(\bar{x}\pi/2)$ and at $T \gg \epsilon_d$: $E(\bar{x}) \propto (1 - \bar{x})\sqrt{T/\epsilon_d}$ near the superconductor $(1 - \bar{x} \ll \xi_T)$ and $E(\bar{x}) = 1$ near the normal reservoir $(\bar{x} \ll \xi_T)$. As we see, the electric field distribution is essentially nonuniform: it monotoneously decreases with T close to a superconductor and overshoots its normal state value further from it. Let us also point out that the reentrant behaviour of $G(T)$ takes place only in the absence of low transparent tunnel barriers at the boundaries of the N-wire. In the presence of such barriers the field distribution in the system becomes entirely different and $G(T)$ monotoneously increases with T for most of the situations [3]. Depending on the sample both increasing or decreasing $G(T)$ has been found in the experiments [1].

In the experiments [2] the conductance of a ring-shaped proximity wire was investigated in the presense of the magnetic flux Φ inside the ring. This system shows $h/2e$-periodic in Φ magnetoresistance-oscillations. At high temperatures the amplitude of these decays $\propto 1/T$ [2].

For simplicity, we have chosen a system where the ring has circumference $2d$ and the connections to the reservoirs are of the length d, so the Thouless energies $\epsilon_d = \frac{\mathcal{D}}{d^2}$ and $\epsilon_{3d} = \frac{\mathcal{D}}{(3d)^2}$ become important. We introduced a phase by a gauge transformation and used a "Kirchhoff law" for the Green's functions at branching points [6, 3]

The transparency of the system $D(\epsilon)$ (Fig. 4) shows an interesting structure. E.g. the oscillations change their sign at intermediate energies. This effect could possibly be probed by measurements at very low temperatures ($T \ll \epsilon_{3d}$) at finite voltages. At high energies ($\epsilon \gg \epsilon_d$), however, the amplitude of the oscillations of $D(\epsilon)$ is exponentially supressed because for such energies no superconducting correlations (which can only originate $h/2e$-oscillations) are present in the ring.

Thus – even at high temperatures – only the states with low energies $\epsilon < \epsilon_d$ are responsible for the effect of conductance oscillations. At $T \gg \epsilon_d$ the amplitude of these oscillations can be estimated as $\Delta\bar{G} = \bar{G}_{\Phi=0} - \bar{G}_{\Phi=h/4e} \approx \frac{1}{2T}\int_0^{\epsilon_c} d\epsilon\, \mathrm{sech}^2(\epsilon/2T)(D_0 - D_{h/4e}) \approx \frac{\epsilon_c}{2T}\Delta D_{av}$ where $\epsilon_c \approx \epsilon_d$ and ΔD_{av} is the average amplitude of the oscillations of the transparency below ϵ_c. This estimate demonstrates that – in a complete agreement with the experimental results [2] –the $1/T$-scaling persists at all temperatures $T > \epsilon_d$. Also the numerical results show that the $1/T$-scaling of the oscillation amplitude is excellently fulfilled at sufficiently high T. (see Figs. 5 and 6). At lower temperatures the amplitude of the conductance oscillations shows the reentrant behavior reaching the maximum at $T \sim \epsilon_d$ and vanishing completely at $T = 0$ (Fig. 5). This reentrant behavior has a similar physical origin to that of $G(T)$ discussed above in the absence of the ring.

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