Superconducting current in narrow proximity wires

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The critical supercurrent in proximty-wires shows a surprising temperature dependence in the experiment [1]. This observation has lead to speculation whether there are new effects involved and/or the usual criterion for the dirty limit is not valid for this type of systems. We propose a simple SNS-model for this system and show analytically, that this unexpected scaling is already contained in a conventional description of this model by quasiclassical Green's functions in the dirty limit. Based n further numerical calculations, excellent agreement with the experimental results is obtained.

1. Introduction

Recent experiments [1] have shown, that a thin normal diffusive wire upon which an array of superconducting stripes is laid can carry a supercurrent. This is due to the well-known proximity effect [2]. As the S-N contacts are very good, the penetration depth of the pair amplitude into the normal wire $\xi_N = \sqrt{\frac{D}{2\pi T}}$ is much bigger than the wire thickness e_N at all experimental temperatures. This implies, that the pair amplitude in the regions covered by the stripes is identical to the bulk value and can be taken as constant over the cross section of the wire. Therefore, this structure is equivalent to a simple chain of SNS-junctions, so the supercurrent is a Josephson current [3]. However, comparison to standard results for such structures [2] failed [1]. The observed critical current $I_c(T)$ resembled to a clean ($I_c(T) \propto e^{-\frac{T}{T_0}}$) rather than a dirty system. This has raised the question, whether the usual criterion between clean and dirty limit is correct for weak superconductivity or whether this effect has to be attributed to new quantum effects not contained in quasiclassical theory of superconductivity. However, we will demonstrate that neither of this is true, that the deviations are due to the use of Ginzburg/Landau theory in [2], which is not valid for the experimental range of temperatures and that a full quasiclassical calculation can explain the experiment quantitavely.

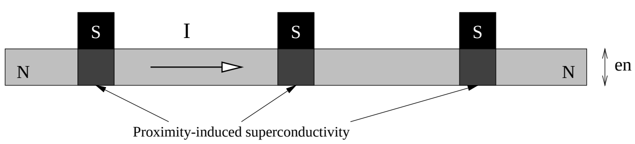

Figure 1: Experimental setup

2. Results

Obviously, the critical current is determined by the longest SNS-cell serving as a "bottleneck", so it is sufficient to study a single cell.

2.1. Analytical approximations

However, in the case $d \gg \xi_N$, where d is the distance of two superconducting banks, the mutual influence between the superconductors can be neglected and the solution can be decomposed into contributions from both sides, carrying a fixed phase. From this approximation, we get for the current

$$I = \frac{64\pi T}{eR_N} \sum_{\omega_m} \frac{d}{\xi_{N,\omega}} w(\omega,\Delta) e^{-\frac{d}{\xi_{N,\omega}}} \sin\phi \quad (1)$$

$$w(\omega,\Delta) = \frac{\Delta^2}{(\omega+\Omega+\sqrt{2(\Omega^2+\omega\Omega)})^2} \quad (2)$$

where ϕ is the phase difference between the superconductors, R_N the Drude resistance and $\omega_n = (2n+1)\pi T \quad n = 0,1,2\ldots$ are fermion Matsubara frequencies. Furthermore, $\xi_{N,\omega} = \sqrt{\frac{D}{2\omega}}$, Δ is the superconducting order parameter and $\Omega = \sqrt{\omega^2+\Delta^2}$. For $T \gg \epsilon_d = D/d^2$, all frequencies higher than the lowest one can be neglected. In the case $T \ll \Delta$, this gives the critical current by setting $\phi = \pi/2$:

$$I_c = \frac{64\pi}{3+2\sqrt{2}} \frac{T}{eR_N} \frac{d}{\xi_N} \exp\left(-\frac{d}{\xi_N}\right) \quad (3)$$

This result agrees with [4] and has the usual dirty limit form $I_c \propto T^q \exp\left(-\sqrt{\frac{T}{T_0}}\right)$ with $T_0 = \frac{D}{2\pi d^2}$

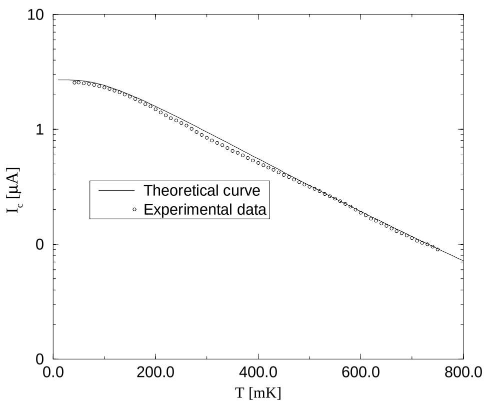

Figure 2:

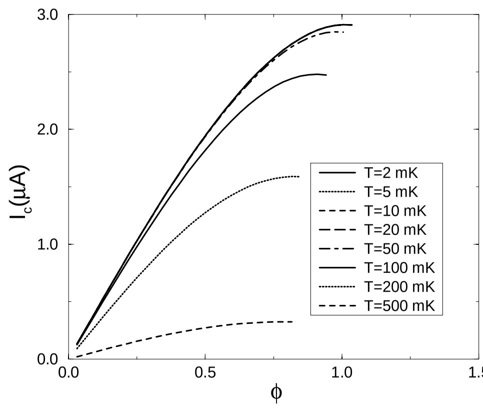

Figure 3:

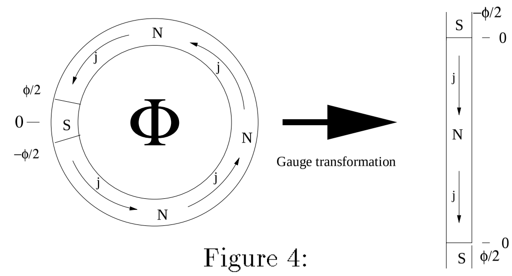

Figure 4:

(Thouless energy), however, we find $q = 3/2$ whereas in the Ginzburg-Landau limit $q = 1/2$.

Nevertheless, this has interesting properties. The logarithmical derivative $\frac{dI_c(T)}{I_c(T)dT} = \left(\frac{3}{2T} - \frac{1}{2\sqrt{TT_0}}\right)$ passes a minimum at $36T_0$ and varies very slowly at higher Temperatures, so $\log I_c$ is almost linear in T. This is just a remarkable mathematical artefact caused by the exponent $q = 3/2$. However, we have demonstrated that "quasi-clean" scaling can also occur in dirty systems. As a good approximation for the slope of the logplot, we can take the logarithmical derivative in the minimum and get as a reasonable approximation $I_c \propto e^{-T/T^*}$ where $T^* = 48\pi d^2/D$.

At $T = 0$, we can estimate the critical current using a rather crude approximation to be $I_c = \frac{D}{4R_N e d^2} = \frac{\pi T_0}{2eR_N}$ which again demonstrates the importance of T_0 as relevant energy scale for the Josephson current in SNS-junctions instead of Δ like for tunneling junctions. This estimate matches with the experiment.

2.2. Numerics

The solution of the full problem including Matsubara summation was obtained numerically. The resulting critical current matches quantitatively to the experiment, see fig. 2.

It is well known, that in Josephson junctions at very low temperatures, $I(\phi)$ deviates from sinusodiality. However, these deviations are small here (see Fig. 3) at all relevant temperatures.

2.3. An equivalent system

A "proximity loop" structure (see Fig. 4), can be mapped onto the same problem, by introducing a gauge invariant phase $\chi = \varphi - 2e\int_0 d\vec{x}\ \vec{A}(\vec{x})$ to the anomalous superconducting Green's function F. The Green's function have to be unique in every point of the ring, so after going round once, the "real" phase φ can only change by $2\pi n \quad n = \ldots -2, -1, 0, 1, 2\ldots$. Here, d is the circumference of the ring and the phase difference of the leads is $\phi = \frac{2e\Phi}{\hbar} \bmod 2\pi$, where Φ is the magnetic flux caught in the ring. As this setup allows to control ϕ by controlling Φ, the stability of the current-phase relation could be probed in a simple experiment.

3. Conclusions

We have introduced a simple model for a proximity-wire. We have demonstrated, that the unexpected experimental observations are all contained in standard quasiclassical theory and so there is no evidence for new effects. Finally, the analogy to a ring structure with a magnetic field was established.

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