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A New Identification Method for Wiener and Hammerstein Systems

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Abstract

System identification is very important to technical and nontechnical areas. All physical systems are nonlinear to some extent and it is natural better to use nonlinear model to describe a real system. The Wiener and Hammerstein systems are proved to be good descriptions of nonlinear dynamic systems in which the nonlinear static subsystems and linear dynamic subsystems are separated in different order. Descriptions of different nonlinear systems need different Wiener and Hammerstein model structures.

The aim of this doctoral dissertation is to develop an unified new recursive identification method in the prediction error method and model scheme for Wiener and Hammerstein systems; to derive the identification algorithms for a class of Wiener and Hammerstein model structures with continuous and discontinuous nonlinearities and to implement and test the algorithms with simulation examples in a MATLAB/Simulink environment.

With the definition and extraction of intermediate variables by using the key term separation principle, a Wiener and Hammerstein system can be described by a nonlinear pseudo-regression model. If some suitable submodel structures are selected, such a nonlinear pseudo-regression model could be pseudo-linear and can be approximately transformed into a pseudo-linear MISO system.

The intermediate variables can be estimated recursively. The errors in estimated parameters and in intermediate variables affect strongly the identification procedure and results. Therefore, the estimated parameters or rather the intermediate variables should be smoothed by using smoothing techniques. Under some common assumptions and by using the adaptive recursive pseudo-linear regressions (RPLR), satisfied parameter estimates of the Wiener and Hammerstein system can be obtained in the presence of a white or a coloured measurement noise without parameter redundancy.

The new method gives good results for all considered Wiener and Hammerstein systems and for some comparable examples, the results are also better. The major advantage of the new method is its unity and efficiency. It can be easily extended to identify other block-oriented nonlinear dynamic systems.

Eine neue Identifikationsmethode für Wiener und Hammerstein Systeme

Zusammenfassung

Systemidentifikation ist sehr wichtig für technische und nichttechnische Bereiche. Alle physikalischen Systeme sind mehr oder weniger nichtlinear, und es ist natürlich besser, mit einem nichtlinearen Modell ein reales System zu beschreiben. Die Wiener und Hammerstein Systeme beschreiben solche nichtlinearen dynamischen Systeme gut, bei denen die nichtlinearen statischen Teilsysteme und die linearen dynamischen Teilsysteme immer getrennt und in verschiedenen Ordnungen angeordnet sind. Beschreibungen der unterschiedlichen nichtlinearen Systeme brauchen unterschiedliche Wiener und Hammerstein Modell Strukturen.

Das Ziel und die Aufgaben der vorliegenden Dissertation sind: eine neue Identifikationsmethode und einige Konzepte im Rahmen der Fehler-Vorhersage-Methode und –Modellstrukturen zu entwickeln; rekursive Identifikationsalgorithmen für eine Klasse Strukturen von Wiener und Hammerstein Systemen mit kontinuierlichen und diskontinuierlichen Nichtlinearitäten abzuleiten; alle abgeleiteten Algorithmen mit MATLAB/Simulink zu implementieren und Simulationsbeispiele durchzuführen. Die Ergebnisse werden auch ausgewertet.

Durch die Definitionen und das Herausziehen der Zwischenvariablen mit dem sogenannten "key term separation" Prinzip kann ein Wiener und Hammerstein System durch ein nichtlineares Pseudoregressionsmodell beschrieben werden. Wenn geeignete Teilmodellstrukturen gewählt werden, kann das nichtlineare Pseudoregressionsmodell pseudo-linear sein und es kann näherungsweise zu einem pseudo-linearen MISO System umgeformt werden.

Die Zwischenvariablen können rekursiv geschätzt werden. Aber die Schätzungsfehler der Parameter und Zwischenvariablen wirken stark auf die Identifikationsverfahren und die Ergebnisse. Deshalb sollen die geschätzten Parameter bzw. die geschätzten Zwischenvariablen durch Glatt- und Filtertechnik geglättet werden. Unter etwas allgemeinen Voraussetzungen und durch adaptive rekursive pseudo-lineare Regression (RPLR) kann man zufriedenstellende Parameterschätzungen vom Wiener und Hammerstein System mit der Anwesenheit von einem weißen oder verfärbten Ausgangsrauschen ohne Parameterredundanz erhalten.

Die neue Identifikationsmethode gibt gute Ergebnisse für alle berücksichtigten Wiener und Hammerstein Systeme und für manche vergleichbaren Beispiele sind die Ergebnisse besser.

Der Hauptvorteil der neuen Methode ist ihre Einheitlichkeit und Anwendbarkeit für viele unterschiedliche Modelltypen und ihre Wirksamkeit. Sie kann leicht erweitert werden, um andere blockorientierte nichtlineare dynamische Systeme zu identifizieren.

Inhaltsverzeichnis

Notation

Symbol	Explanation
$u(t)$	System input
$u_j(t)$	$j-th$ system input signal to a MISO Wiener and Hammerstein system
$y(t)$	System output
$y^*(t)$	System output without measurement noise
$\hat{y}(t)$	Predicted system output
$\tilde{y}(t)$	Separated part from a Feedback-Hammerstein system
$w(t)$	Intermediate variable
$w_k(t)$	$k-th$ intermediate variable
$\boldsymbol{\theta}$	Parameter vector of linear dynamic subsystem
$\boldsymbol{\theta}^*$	Parameter vector of linear dynamic subsystem without key term
$\boldsymbol{\theta}_k$	Parameter vector of the $k-th$ linear dynamic subsystem
$\boldsymbol{\eta}$	Parameter vector of nonlinear static subsystem
$\boldsymbol{\eta}^*$	Parameter vector of nonlinear static subsystem without key term
$\boldsymbol{\eta}_k$	Parameter vector of the $k-th$ nonlinear static subsystem
ϑ	Parameter vector
ϑ_t	Parameter vector at time t
$\overline{\vartheta_t}$	Smoothed parameter vector at time t
$\boldsymbol{\varphi}(t)$	Pseuso-regression vector
$g(t)$	Impulse response
$g(u(t),\vartheta)$	A general model of nonlinear system
$g^m(\cdot)$	Nonlinear polynomial function of degree m
$\Omega_k(\cdot)$	$k-th$ basis function of nonlinear static subsystem
$\chi(t)$	Input of nonlinear static subsystem
$\gamma(t)$	Output of nonlinear static subsystem
$G(q^{-1})$	Transfer function
$G^*(q^{-1},\boldsymbol{\theta}^*)$	Transfer function without key term
$G_k(q^{-1},\boldsymbol{\theta}_k)$	$k-th$ linear dynamic subsystem
$N_k(\cdot,\boldsymbol{\eta}_k)$	$k-th$ nonlinear static subsystem
$N^*(\cdot,\boldsymbol{\eta}^*)$	Nonlinear static subsystem without key term
$A(q^{-1})$	System output polynomial
$A^*(q^{-1})$	Another form of system output polynomial
$B^*(q^{-1})$	Nominator polynomial linear dynamic subsystem without the key term
$B_k(q^{-1})$	Nominator polynomial of the $k-th$ linear dynamic subsystem
$B_k^*(q^{-1})$	Nominator polynomial of the $k-th$ linear dynamic subsystem without the key term
$F(q^{-1})$	Denominator polynomial of linear dynamic subsystem
$F_k(q^{-1})$	Denominator polynomial of the $k-th$ linear dynamic subsystem

a_k	$k-th$ coefficient of $A(q^{-1})$
b_k	$k-th$ coefficient of $B(q^{-1})$
b_{jk}	$k-th$ coefficient of $B_j(q^{-1})$
β_k	$k-th$ coefficient of a nonlinear polynomial $N(\cdot,\boldsymbol{\eta})$
β_{jk}	$k-th$ coefficient of a nonlinear polynomial $N_j(\cdot,\boldsymbol{\eta}_j)$
$e(t)$	White measurement noise
$\varepsilon(t)$	Colored measurement noise
$H(q^{-1},\boldsymbol{\xi})$	Noise filter
$\boldsymbol{\xi}$	Parameter vector of $H(q^{-1},\boldsymbol{\xi})$
$C(q^{-1})$	Nominator of $H(q^{-1},\boldsymbol{\xi})$
$D(q^{-1})$	Denominator of $H(q^{-1},\boldsymbol{\xi})$
n_c	Order of the nominator $C(q^{-1})$
n_d	Order of the denominator $D(q^{-1})$
q^{-1}	Shift operator
i	Multi input number to a MISO system
m	Order of a nonlinear system
n	Order of a linear system
m_k	Order of the $k-th$ nonlinear static subsystem
n_k	Order of the $k-th$ linear dynamic subsystem
γ	An integer in the interval $[0, n-1]$
p_k	Discrete frequency response of the linear system
ϖ_k	$k-th$ frequency
$\psi_k(t)$	Output of the $k-th$ frequency sampling filter
ρ	Constant parameter
λ	Forgetting factor
mov	Moving window length
Δp	Parameter identification error
$\lvert\Delta\rvert$	Avarage parameter identification error

Abbreviation	**Explanation**
ARX	AutoRegressive with eXogenous (or eXternal) input
ARMAX	AutoRegressive Moving Average with eXogenous inputs
NARMAX	Nonlinear AutoRegressive Moving Average with eXogenous inputs
BJ	Box-Jenkins
FIR	Finite Impulse Response
FSF	Frequency Sampling Filter
SISO	Single Input-Single Output system
MISO	Multi Input-Multi Output system
OE	Output Error
PEM	Prediction Error Minimization
RTF	Rational Transfer Function
RPEM	Recursive Prediction Error Minimization
PLR	Pseudo-Linear Regressions
RPLR	Recursive Pseudo-Linear Regressions

1. Introduction

1.1. General problems in system identification

System identification is of fundamental importance in automatic control. The key task of system identification is to find out a best suitable mathematical model between the inputs, outputs and disturbances of a real system. Models can be useful for gaining a better understanding of the system and to predict or simulate a system's behavior. Advanced control techniques for the design and analysis of controllers, optimization, supervision, fault detection and diagnosis components are also based on models of real systems. The quality of the model typically determines an upper bound on the quality of the final problem solution. Therefore, there is a strong demand for system modeling and identification schemes.

If the physical laws governing the behavior of the system are known, it is so called ***a white-box model*** in which all parameters and variables can be interpreted in terms of physical entities and all parameters are known. On the other hand, ***a black-box model*** is constructed only from system input and output data without any knowledge of physical insight. But in many practical cases, it often occurs that one knows only a little bit about the system, that is, the system modeling is based on the recorded input and output data with some prior knowledge about the system, e.g., the structure and order of the system. By analyzing and extracting information from the system and using the identification methods for black-box model, ***a gray-box model*** will be constructed.

System identification is concerned with a black-box model or a gray-box model which has the following basic items (Zadeh, 1962; Aström and Eykhoff, 1971): observed data and prior information from the real system, a model set and an identification criterion. That is, according to some identification criterion and guided by prior information, a "best" model is chosen from the model set to fit the observed data best. It can not be said this model is the best and an unique model for a system. There must be some approximations in system identification. A best suitable model is only under the meaning of a definite optimization criterion.

System identification must be a procedure with analysis, synthesis, selection and optimization. To identify a system, i.e., to establish a model for the system, one should get a physical insight into the system as much as possible. It depends greatly on how much details one knows about the system a prior, e.g., if it is a linear system or a nonlinear system, a time-invariant system or a time-variant system, a continuous system or a discontinuous system, a single input-single output (SISO) system or a multi input-multi output (MIMO) system, an open-loop system or a closed-loop system and so on. One should also know how the system can be influenced, i.e., which input and output signals can be selected to measure, which disturbance signals can disturb the system and where they can appear in the system and how measurement experiments can be designed and realized. Therefore, different systems need different model structure selections. In most cases it is convenient and sufficient to use some standard model structures. Actually, identification methods and algorithms are always related directly with model structures and optimization criterions to get a better data fit.

After selecting the model structure and possible identification strategies, based on the inputs and outputs, the parameters of the model will be determined by an optimization process with minimization or maximization of a linear or nonlinear criterion to solve a least-squares (data-fitting) problem. It is clear that most optimization problems benefit from good starting points, i.e., initial values which improve the execution efficiency and can help to locate the global minimum instead of a local minimum.

System identification can carry out off-line or on-line recursively. Most system identifications are related with process control purpose to get on-line better control performance. Then the identification and control criterion will be considered at the same time, i.e., so called control-relevant system

identification. Therefore, on-line recursive identification methods are more significant. At last, the identification results should be verified and improved iteratively till they are satisfactory.

Therefore, a system can be identified with the following iterative steps:

1. ***Optimal experiment design and data collection***. Choice of the excitation signals, the sampling time and the interesting inputs and outputs to be measured.

2. ***Model structure selection***. Select a suitable model structure with suitable order.

3. ***Model estimation***. Given a suitable model structure and measured data, some suitable identification methods and optimal algorithms are available and the parameters in the model structure can be estimated by optimizing some criterion or loss function.

4. ***Model validation***. The model is simulated using "fresh" data and the estimated outputs are compared with the measured outputs. Verify if the identified model is valid and if it is exact or suitable enough for special purposes.

System identification is a well-established field with a number of approaches and algorithms. The methods for black-box identification of linear, time-invariant dynamical systems with given discrete-time data are broadly studied. They can be classified into: the prediction error methods (e.g., Ljung. 1987); the subspace methods (e.g., Van Overschee and De Moor, 1993); the nonparametric correlation and spectral analysis methods (e.g., Billings and Fakhouri, 1978; G. Bretthauer,1983). Other references can be found in Schwarz, 1967; Eykhoff, 1974; Strobel, 1975; Isermann, 1988 and Wernstedt, 1989.

1.2. Identification of nonlinear systems

Identification of linear systems has become a routine task. A number of successful methods are available to solve the problem in the time or in the frequency domain, using iterative and non iterative identification schemes. The basic reason for this success is the appealing simplicity of linear models. They give a lot of insight and are often used as the basis for many design techniques. The price for this "simplicity" is the use of a strong assumption: the underlying physical process exhibits qualitatively similar dynamic behavior to the linear model in the operating area of interest.

Actually, all physical systems are nonlinear to an extent. A system is called nonlinear if the input-output steady state relation is nonlinear. Because nonlinear models are able to describe the system behavior in a much larger operating region than corresponding linear models, it is reasonable and necessary to characterize or predict the behavior of real nonlinear processes directly using nonlinear models to improve identification performance over their whole operating range. Therefore, it leads to the development of approaches for nonlinear modeling and analyzing of nonlinear systems. This Ph.D. work is concerned with nonlinear system identification.

The most difficult task in nonlinear system identification is to deal with the curse of dimensionality. This is a common characteristic of nonlinear model structures since nonlinear systems usually exhibit a variety of complex dynamic behavior. From the nonlinear regression perspective, there is no much difference in identifying a linear system or a nonlinear system. The linear identification methods are like a root of the tree of system identification. Any other nonlinear identification methods can be seen, to some extent, coming from or ending at that root. As in linear cases, the crucial step in the identification of nonlinear systems is to select the model structure and to establish suitable identification schemes and some parameterization (e.g., function expansion) of the predictor.

The basic nonlinear system identification concept is depicted from a general modeling point of view in Fig. 1.1.

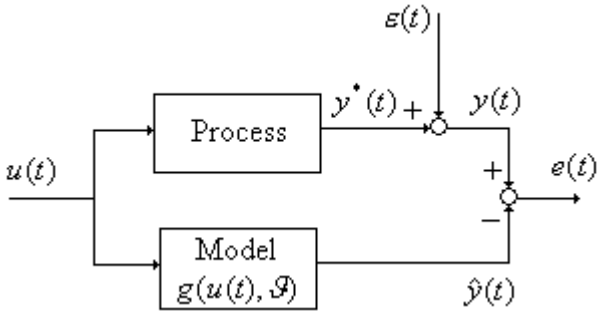


Fig. 1.1 Process and model

A nonlinear model $g(u(t),\vartheta)$ maps the input $u(t)$ to the measured output $y(t)$ which is corrupted with noise $\varepsilon(t)$. The model is parameterized by minimizing the error $e(t) = y(t) - \hat{y}(t)$ to get the parameter vector ϑ such that $\hat{y}(t) = g(u(t),\vartheta)$.

It is useful to subdivide general nonlinear system identification into two categories:

- ***Structure-identification***. This deals with so-called structure optimization techniques and the problem of searching an optimal model structure, i.e., the optimal kind of function $g(u(t),\vartheta)$ and the optimal number of parameters. This typically leads to a combinatorial optimization problem which grows rapidly in complexity with the problem size.

- ***Parameter-identification***. Having decided the type and size of the nonlinear model structure, it remains to find reasonable parameter values. The goal of a parameter optimization technique is to find the "best" approximation $\hat{y}(t)$ to the measured output $y(t)$, which may be disturbed by noise $\varepsilon(t)$, by adapting the parameter vector ϑ. It leads to the nonlinear local and global optimization problems and methods, especially prediction error minimization (PEM) methods.

For general nonlinear systems there are no universal identification techniques. All of them depend on prior knowledge of the system, i.e., of its mathematical representation. Differences in the dynamic behavior of these models can be extremely significant, and they are entirely due to the different ways these model components are combined. A survey of it is given by Unbehauen (1996). Leontaritis and Billings (1985) proposed a general approach for the identification and structural determination of nonlinear systems approximated by dynamic polynomial representations. The available nonlinear identification techniques have been subdivided by several authors (Billings and Fakhouri, 1978; Korenberg, 1985) into three basic classes:

1. Cascade or block-oriented structured approaches.

2. Kernel or nonparametric approaches. (e.g., Wiener and Volterra representations).

3. Parametric approaches. (e.g., NARMAX models).

Parametric additive approaches split the high-dimensional problem into a sum of lower dimensional problems. The justification of the additive structure can be drawn from a Taylor series expansion of the process. Thus, any (smooth) process can be approximated by an additive model structure. The important issue in practice is, of course, how fast the additive approximation converges to the true process behavior if the model complexity increases. This depends on the usually unknown structure of the process and the particular construction algorithm applied for building the additive model. The world of expansions in terms of artificial neural networks, wavelet transforms, fuzzy models, etc., are

also important for nonlinear dynamic systems which will not be considered in this thesis. We only concern with the known parametric additive structures to identify the system parameters and consider the nonlinear parameter identification problems in prediction error method schemes.

The type of PEM algorithms to be applied depends on whether the parameters enter the model structure in a linear or in a nonlinear way. The latter situation leads to a nonlinear least-squares problem. When all parameters enter the structure in a linear form, one usually talks about a pseudo linear least-squares problem, i.e., linear-in-the-parameters-identification. Among the identification and optimization techniques, linear identification and optimization techniques are the most mature and most straightforward to be applied. It offers a number of highly desirable features such as an analytic one-shot solution, an unique global optimum, and a recursive formulation that allows an online application. Many robust and fast linear identification and optimization implementations are available in toolboxes, e.g., MATLAB Identification Toolboxes (Ljung, 1997). Powerful and very efficient structure identification and optimization techniques and software packages are also available.

The polynomial autoregressive moving average model with exogenous inputs (NARMAX) model provides an important general representation of nonlinear time-invariant systems. And roughly speaking, it is always valid for systems with analytic nonlinearities. If a finite number of past inputs $u(t)$ and outputs $y(t)$ with measurement noise $\varepsilon(t)$ are collected, then the NARMAX model is given by

$$y(t) = g^m[u(t), u(t-1), \cdots, u(t-n), y(t-1), y(t-2), \cdots, y(t-n)] + \varepsilon(t) \tag{1.1}$$

where g^m is a nonlinear function of degree m. For $m = 1$ the resulting model is a linear autoregressive moving average model with exogenous inputs (ARMAX) model.

There is no restriction in the nature of the excitation in the NARMAX procedure. But it should be note that the estimation of the parameters can be computationally expensive due to the number of parameters that increase exponentially with the degree of the kernel. And the discontinuous nonlinearities such as saturation, backlash, hysteresis and dead zone cannot be modeled using it.

The NARMAX representation includes a family of other nonlinear representations and provides an alternative to block oriented structured model (Pearson, 1999), such as:

- ***Kolmogorov-Gabor polynomial***

It is also the general NARMAX representation as shown in Eq. (1.1).

- ***Non-parametric Volterra-series***

$$y(t) = g^m[u(t), u(t-1), \cdots, u(t-n)] + \varepsilon(t)\,. \tag{1.2}$$

Since no feedback is involved the Volterra-series model is guaranteed to be stable.

- ***Parametric Volterra-series***

$$y(t) = g^m[u(t), u(t-1), \cdots, u(t-n)] - a_1 y(t-1) - a_2 y(t-2) - \cdots - a_n y(t-n) + \varepsilon(t)\,. \tag{1.3}$$

It is a simplified version of the Kolmogorov-Gabor polynomial and realizes a linear feedback (the first order output) and models a nonlinearity only for the inputs. Its stability can be easily proven by checking the dynamics of the linear feedback. It can also be seen as an extension of the Volterra-series model if the order n is chosen large. It can be argued that in this case the additional linear feedback would help to reduce the dynamic order compared with the non-parametric Volterra-series model.

- ***Nonlinear differential equation (NDE)***

$$y(t) = b_0 u(t) + b_1 u(t-1) + \cdots + b_n u(t-n) + g^m[y(t-1), \cdots, y(t-n)] + \varepsilon(t)\,. \tag{1.4}$$

It can be considered as the counterpart of the parametric Volterra-series model since it is linear in the inputs (the first order input) but nonlinear in the outputs. It arises frequently from modeling based on theoretical analysis and should be applied only if its structure matches the process structure really well.

1.3. Wiener and Hammerstein system

In order to describe adequately the nonlinear behavior of the system over the entire range of operating conditions, a nonlinear block-oriented model is often used and the identified system is generally subdivided into linear dynamic subsystems (or linear dynamic blocks) and nonlinear static subsystems (or nonlinear static blocks). The well-known Wiener systems and Hammerstein systems are nonlinear models that are used in many domains for their simplicity and physical meaning, where the system steady-state behavior is determined completely by the static-nonlinearities, while the system dynamic behavior is determined by both the nonlinearities and the linear dynamic model components. For example, a Wiener system (Figure 1.2) consists of a linear dynamic block followed by a nonlinear static block. A Hammerstein system (Figure 1.3) is just a Wiener system structurally reversed, that is, a nonlinear static block is followed by a linear dynamic block. The nonlinearities in Wiener and Hammerstein systems could be continuous and discontinuous. An advantage of the distinction into nonlinear and linear blocks is that the system stability is determined solely by the linear parts of the model, which can be easily checked. Sometimes, it is assumed that the steady-state behavior is known and uses this knowledge to determine the linear dynamic subsystem (Pearson, 2000).

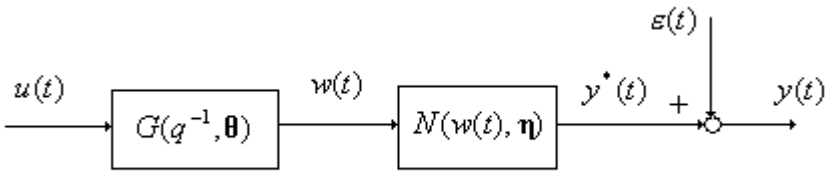


Fig. 1.2 Wiener system

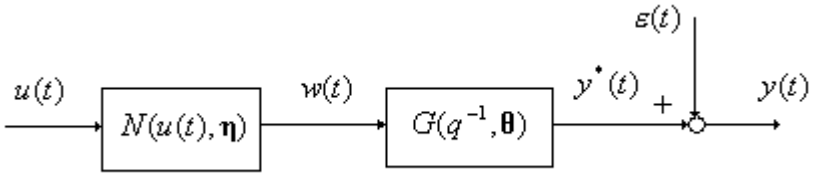


Fig. 1.3 Hammerstein system

Some other cascade or block-oriented structured systems include a MISO Wiener system, a MISO Hammerstein system, a mixed MISO Wiener and Hammerstein system, a Wiener-Hammerstein system (LNL) and a Hammerstein-Wiener system (NLN).

1.4. Aims and Outline

It is clear that there exists no general valid descriptions for the full class of nonlinear systems. For that reason, a very wide variety of models and identification methods is proposed in the literature. As a natural extension of the linear system identification approach, it allows to carry over many of the methods of the linear modelling approach to the nonlinear world, maintaining their simplicity, user friendliness, and "short" experiment time to identify them.

The main purpose of this thesis is to develop an unified and efficient identification methodology for a special class of block-oriented structured SISO, MISO and cascade nonlinear time-invariant systems, that is, a class of Wiener and Hammerstein systems with continuous or discontinuous nonlinearities. It is also from some common and suitable model structures and only based on the observed input and output data and a prior knowledge about the behavior of nonlinearities without the assumption that the steady-state behavior is known. Under this identification scheme, we derive the special algorithms for each possible case, and analyze and verify the correctness of the derived algorithms by simulation examples.

The outline of this thesis is as follows:

- In chapter 2, nonlinear system identification will be discussed from a new identification perspective. System descriptions, the key term separation principle and the new identification method for the identification of Wiener and Hammerstein systems will be introduced.

- In chapter 3, the identification algorithms for SISO and MISO Wiener and Hammerstein systems will be derived and their efficiency will be shown by simulation results.

- In chapter 4, identification algorithms for a Wiener system with general discontinuous nonlinearities will be developed. Again, some simulation results will be discussed.

- In chapter 5, the identification algorithms will be extended and derived for a class of cascade Wiener and Hammerstein systems and their efficiency will be illustrated by simulation results.

- In Chapter 6, the identification algorithms will be extended and derived for generalized Wiener and Hammerstein systems, which are simplified from a parametric Volterra-series. There, some simulation results will also be discussed.

- Finally, in Chapter 7, conclusions will be given.

2. Concept of a new identification method for Wiener and Hammerstein systems

2.1. General description

Many nonlinear dynamic systems can be approximated by Wiener and Hammerstein systems. An earliest identification algorithm of this kind of systems was developed by Narendra and Gallman (1966). They estimated separately and sequentially the linear dynamic transfer function and the nonlinear static polynomial by the iterative least squares scheme. A noniterative version of the method was proposed by Chang and Luus (1971). However, there is a certain amount of redundancy in the parameters to be estimated and each parameter of the static nonlinear model has several estimates. To obtain an unbiased estimator in the case of correlated noise and output, Stoica and Söderström (1982) developed instrumental variable techniques. Rosenthal (1985) studied identification algorithms and two-step strategy for open and closed loop Wiener system and Hammerstein system. Pearson (2000) introduced an identification method for a Wiener system, a Hammerstein system and a feedback-Hammerstein system but with known nonlinear blocks.

An approach based on the combination of the correlation analysis and the least squares (LS) method was developed by Haber (1979). For nonparametric methods, Billings and Fakhouri (1982, 1997) and Greblicki (1994, 1998, 1999) presented algorithms for identification based on correlation analysis. Schetzen (1981) and Hunter and Korenberg (1986) use Gaussian input and estimate the linear and nonlinear subsystems iteratively. The linear system is estimated from the cross-correlation function, and the nonlinear system is described with a polynomial.

In Wiener and Hammerstein systems, the nonlinear blocks are sometimes supposed to be invertible. It should be a strong assumption. This approach will be referred to as the internal error approach, since it aims to minimize the intermediate error between the output of the linear subsystem and the input of the nonlinear subsystem. Pajunen (1992) identified the static nonlinearity of the Wiener system in terms of its inverse. Greblicki (1992) followed a similar approach but did not impose any parametric restrictions on the functional form of the nonlinearity. Wigren (1993) proposed a recursive identification algorithm for the Wiener model, the linear block with a transfer function operator and the nonlinear block as piecewise linear and the static nonlinearity as opposed to its inverse and thus one can handle nonlinearities which are not single valued. Kalafatis (1995) used a least squares algorithm to simultaneously estimate the parameters of the linear subsystem and the inverse static nonlinearity and assumed that the static nonlinearity is continuous, differentiable to the polynomial order m and single valued in the region which the input-output data span. Hagenblad (1999) used finite impulse response (FIR) model for the linear subsystem and B-splines for the inverse of the system nonlinearity.

In a Wiener and Hammerstein system, the linear dynamical blocks and the nonlinear static blocks are always separated. One problem is that if the linear and nonlinear subsystems are parameterized separately, the Wiener and Hammerstein systems are over-parameterized in some presented algorithms (Boutayeb, 1994; Kalafatis 1997; Hagenblad, 1998; Zhu, 1998). Numerical problems will occur if the over-parameterization is not addressed. That is, a constant gain can be distributed arbitrarily between the linear and nonlinear subsystems. In order to get a unique solution, the gain of one or some subsystems must be fixed. A simpler solution is to just fix some of the parameters of the linear or nonlinear subsystems, without loss of generality, let them be constant during the minimization. Some other possible constraints in the minimization will not be considered here.

It can be concluded from the literature that the most important attempt is always trying to reduce parameter redundancy by using special linear and nonlinear model structures. Some other attempts are to select a parameterization and approximations or relax algorithms to simplify the computation procedures to fit the individual nonlinear model situations to process data.

2.2. Linear and nonlinear submodels

In a Wiener and Hammerstein system, many different linear and nonlinear submodel structures have been considered. Pajunen (1992) treated the problem of model reference adaptive control of a Wiener system. The linear system was represented as a transfer function and the inverse of the nonlinearity was represented with B-splines. Vörös (1995) used a transfer function for the linear subsystem and a polynomial for the nonlinearity. In Bruls et al. (1997), a state space model was used for the linear system and Chebyshev polynomials for the nonlinearity. In Kalafatis et al. (1997), FIR or the frequency sampling filter (FSF) model were suggested for the linear subsystem and a power series or B-splines for the nonlinearity. In Zhu (1999), high order autoregressive with exogenous input (ARX) model was used for linear subsystem and cubic splines for the nonlinearity.

2.2.1. Model structures for linear dynamic subsystems

2.2.1.1. Rational transfer function model (RTF)

Assume that the linear dynamic system is time invariant, causal, and stable. Such a system is completely described by its impulse response $g(t)$, $t = 1,2,\cdots,\infty$. For a given input $u(t)$, the linear dynamic system output $y(t)$ is as follows

$$y(t) = \sum_{k=0}^{\infty} g(k)u(t-k), \quad t = 1,2,\cdots,\infty. \tag{2.1}$$

The transfer function $G(q^{-1})$ of the system is described by

$$G(q^{-1}) = \sum_{k=0}^{\infty} g(k)q^{-k} \tag{2.2}$$

and the linear system output can then be written as $y(t) = G(q^{-1})u(t)$.

If $G(q^{-1})$ is stable, then

$$\sum_{k=0}^{\infty} \left| g(k) \right| < \infty. \tag{2.3}$$

Although a system is uniquely determined by its impulse response, it is no practical way to work with this in general infinite sequence. Furthermore we need an expression where the system $G(q^{-1})$ is characterized by a finite number of parameters. The parameters can be collected into a parameter vector $\boldsymbol{\theta}$, and the transfer function can be written as $G(q^{-1}, \boldsymbol{\theta})$.

A common choice is to select the transfer function as a rational function where the numerator and denominator coefficients are the parameters. Without loss of generality, we suppose the orders of numerator and denominator are the same, n.

$$G(q^{-1}, \boldsymbol{\theta}) = \frac{B(q^{-1})}{F(q^{-1})} \tag{2.4a}$$

where

$$B(q^{-1}) = b_0 + b_1 q^{-1} + b_2 q^{-2} + \cdots + b_n q^{-n} \tag{2.4b}$$

$$F(q^{-1}) = 1 + f_1 q^{-1} + f_2 q^{-2} + \cdots + f_n q^{-n}. \tag{2.4c}$$

$B(q^{-1})$ and $F(q^{-1})$ are coprime. It is also possible to redefine the $B(q^{-1})$ polynomial to include extra delays. The parameter vector is $\boldsymbol{\theta}^{\mathbf{T}} = [f_1, f_2, \cdots, f_n, b_0, b_1, \cdots, b_n]$.

2.2.1.2. Finite impulse response model (FIR)

As a special case of the low order rational transfer function model, a stable linear dynamic system may be represented by the FIR model of order n

$$\begin{aligned} y(t) &= G(q^{-1}, \boldsymbol{\theta}) u(t) \\ &= \sum_{k=0}^{n} b_k u(t-k) \end{aligned} \tag{2.5}$$

where the parameter vector $\boldsymbol{\theta}^{\mathbf{T}} = [b_0, b_1, \cdots, b_n]$.

A FIR model of order n can only describe a system whose impulse response has maximum length of n time steps, but if we let n tends to be infinity, any given stable system will be possible to describe accurately. That is, by selecting n large enough, a FIR model can always describe a stable $G(q^{-1}, \boldsymbol{\theta})$ accurately enough.

But FIR methods are inefficient parameter estimators because more parameters are necessary to represent the process, that is, more data will be required to estimate those parameters than for a parsimonious parametric model. We might need many parameters to describe a FIR model, and we do not beforehand know how many. Luckily, the cost for using more parameters is not too large in terms of time when we use the linear regression estimate. The limited number of data available in practice does, however, put a limit of the number of parameters we can estimate. Trying to estimate too many parameters may cause numerical problems, and the estimates are more influenced by noise if we have only a few data. Therefore, if an output variable is measured infrequently, a very time-consuming response test may be required to gather enough data to estimate FIR coefficients. But because of the simple linear regression algorithm a satisfied result can also be got. And, in addition, the FIR model can also be converted to another model structure if desired.

2.2.1.3. Frequency sampling filter model (FSF)

Alternatively, as another form of FIR model, the FSF model can be used to represent the linear system (Bitmead and Anderson, 1981). Since the FSF model is obtained from a linear transformation of the FIR model and consists of a set of narrow bandpass filters, they have a common model order. A FSF model of order n is given by

$$y(t) = \sum_{k=0}^{n-1} p_k \psi_k(t) \tag{2.6}$$

where p_k is the discrete frequency response of the linear system at $\varpi_k = \frac{2\pi k}{n}$ and $\psi_k(t)$ for $k = 1,2,\cdots,n$ is the output of the $k-th$ frequency sampling filter defined as

$$\psi_k(t) = \frac{1}{n} \cdot \frac{1-q^{-n}}{1-e^{j\varpi_k} q^{-1}} u(t). \tag{2.7}$$

The parameter vector is $\boldsymbol{\theta}^{\mathbf{T}} = [\, p_0, p_1 \cdots, p_{n-1} \,]$.

The same reason as in the FIR model case, the number of parameters of a FSF model to be estimated can also be quite large. However, only a limited amount of information about the linear system in the frequency domain may be required. In this case, we can make use of the orthogonal properties of the FSF model under periodic excitation to drastically reduce the number of parameters of the linear system to be estimated.

Goberdhansingh et al. (1992) showed that when a periodic input signal of the form $u(t) = \lambda e^{\frac{j2\pi\gamma t}{n}}$, where γ is an integer in the interval $[0, n-1]$, is passed through the bank of FSF filters, only the FSF filter with center frequency $2\pi\gamma / n$ will have a nonzero output. For example, if we choose the input signal to be a sinusoid consisting of a single frequency $\varpi_k = \frac{2\pi k}{n}$, which is the center frequency of the $k-th$ FSF filter, Eq. (2.6) can then be described by

$$y(t) = p_k \psi_k(t) + p_{n-k} \psi_{n-k}(t) \tag{2.8}$$

where the parameter vector is $\boldsymbol{\theta}^{\mathbf{T}} = [\, p_k, p_{n-k} \,]$.

That is, the model parameters can be dramatically reduced. This can be easily extended to the case where the input signal is composed of multiple sinusoids, i.e., each additional frequency will add two more terms, that is, two more parameters in Eq. (2.8). Then the number of parameters to be estimated is generally fewer than the number required by the FIR model to describe a system accurately enough.

2.2.2. Common linear MISO model structure

At first, we define the measurement noise. In a real system, there is always noise and the filter of this noise should also be identified. The model structure depends also on the structure of noise filter which is written as

$$\begin{aligned} \varepsilon(t) &= H(q^{-1}, \xi) e(t) \\ &= \frac{C(q^{-1})}{D(q^{-1})} e(t) \end{aligned} \tag{2.9a}$$

where

$$C(q^{-1}) = 1 + c_1 q^{-1} + \cdots + c_{n_c} q^{-n_c} \tag{2.9b}$$

$$D(q^{-1}) = 1 + d_1 q^{-1} + \cdots + d_{n_d} q^{-n_d}. \tag{2.9c}$$

The orders of $C(q^{-1})$ and $D(q^{-1})$ are n_c and n_d. Here, $e(t)$ is white noise. The noise filter parameter vector is $\boldsymbol{\xi}^{\mathbf{T}} = [d_1, d_2, \cdots, d_{n_d}, c_1, c_2, \cdots, c_{n_c}]$.

The most popular linear representation is the ARMAX model which is discussed by Ljung (1987)

$$y(t) = \sum_{j=1}^{n_a} a_j y(t-j) + \sum_{j=0}^{n} b_j u(t-j) + \sum_{j=0}^{n_c} c_j e(t-j) \tag{2.10}$$

where n_a is order of the system output polynomial, a_k for $j = 1, \cdots, n$ are the autoregressive parameters of the system and $c_0 = 1$. The parameter vector is $\boldsymbol{\theta}^{\mathbf{T}} = [a_1, a_2, \cdots, a_{n_a}, b_0, b_1, \cdots, b_n]$.

Note that this model can be extended directly to multiple-input ARMAX models by adding terms to the second sum, corresponding to the delayed values of additional exogenous inputs.

A general linear MISO model structure of i inputs is described by

$$\begin{aligned} &A(q^{-1}, \boldsymbol{\theta}_a) y(t) = G_1(q^{-1}, \boldsymbol{\theta}_1) u_1(t) + G_2(q^{-1}, \boldsymbol{\theta}_2) u_2(t) + \cdots + G_i(q^{-1}, \boldsymbol{\theta}_i) u_i(t) + \varepsilon(t) \\ &= \frac{B_1(q^{-1})}{F_1(q^{-1})} u_1(t) + \frac{B_2(q^{-1})}{F_2(q^{-1})} u_2(t) + \cdots + \frac{B_i(q^{-1})}{F_i(q^{-1})} u_i(t) + \frac{C(q^{-1})}{D(q^{-1})} e(t) \end{aligned} \tag{2.11a}$$

where

$$A(q^{-1}, \boldsymbol{\theta}_a) = 1 + a_1 q^{-1} + \cdots + a_{n_a} q^{-n_a} \tag{2.11b}$$

$$B_j(q^{-1}) = b_{j0} + b_{j1} q^{-1} + \cdots + b_{jn_j} q^{-n_j} \tag{2.11c}$$

$$F_j(q^{-1}) = 1 + f_{j1} q^{-1} + \cdots + f_{jn_j} q^{-n_j}. \tag{2.11d}$$

$\boldsymbol{\theta}_a$ and $\boldsymbol{\theta}_j$ for $j = 1,2,\cdots,i$ are the corresponding parameter vectors. $B_j(q^{-1})$ and $F_j(q^{-1})$ are coprime and have the same order n_j for $j = 1,2,\cdots,i$. It is also possible to redefine the $B_j(q^{-1})$ polynomial to include extra delays. Eq. (2.11a) contains several special model structures, some of them are listed in Table 2.1.

Table 2.1 Some linear MISO model structures as special cases of Eq. (2.11)

Polynomials used in Eq. (2.11a)	Name of the MISO model structure
$B_j(q^{-1})$	FIR
$A(q^{-1}, \boldsymbol{\theta}_a)$, $B_j(q^{-1})$	ARX
$A(q^{-1}, \boldsymbol{\theta}_a)$, $B_j(q^{-1}), C(q^{-1})$,	ARMAX
$B_j(q^{-1})$, $F_j(q^{-1})$	OE
$B_j(q^{-1})$, $F_j(q^{-1})$, $C(q^{-1})$, $D(q^{-1})$	BJ

In Table 2.1 the acronyms FIR, ARX, ARMAX, OE and BJ denote: Finite Impulse Response, AutoRegressive with eXogenous inputs, AutoRegressive Moving Average with eXogenous inputs, Output Error and Box-Jenkins, respectively.

Actually, Eq. (2.11a) can be rewritten as

$$y(t) = A^*(q^{-1}, \boldsymbol{\theta}_a) y(t-1) + G_1(q^{-1}, \boldsymbol{\theta}_1) u_1(t) + G_2(q^{-1}, \boldsymbol{\theta}_2) u_2(t) + \cdots + G_i(q^{-1}, \boldsymbol{\theta}_i) u_i(t) + \varepsilon(t) \tag{2.12}$$

where $A^*(q^{-1}, \boldsymbol{\theta}_a) = -a_1 - a_2 q^{-1} - \cdots - a_{n_a} q^{-n_a+1}$ and $y(t-1)$ at the right side of Eq. (2.12) can also be regarded as a pseudo-input of the system.

2.2.3. Model structures for nonlinear static subsystems

In the following we consider two cases:

- ***Continuous nonlinearities***

To express the continuous nonlinear static subsystem $N(\cdot, \boldsymbol{\eta})$, one can use a function expansion of order m with basis functions $\Omega_k(\cdot)$ and parameters β_k for $k = 1,2,\cdots,m$. The main structure is given by

$$\begin{aligned} \gamma(t) &= N(\chi(t), \boldsymbol{\eta}) \\ &= \sum_{k=1}^{m} \beta_k \cdot \Omega_k(\chi(t)) \end{aligned} \tag{2.13}$$

where $\chi(t)$ is the input of nonlinear static subsystem. $\gamma(t)$ is the output nonlinear static subsystem. The parameter vector is $\boldsymbol{\eta}^{\mathrm{T}} = [\beta_1, \beta_2, \cdots \beta_m]$.

If the internal parameters of the basis functions are fixed, the output is a linear function of the parameters $[\beta_1, \beta_2, \cdots \beta_m]$. A simple case is a polynomial $\Omega_k(\chi(t)) = \chi^k(t)$ for $k = 1,2,\cdots,m$

$$\gamma(t) = \sum_{k=1}^{m} \beta_k \cdot \chi^k(t). \tag{2.14}$$

The polynomial representation has the advantage of more flexibility and of a simpler use. It is widely used in literature. Naturally, the nonlinearity can be approximated by a single polynomial. An alternative structure is to use a polynomial of the inverse of the nonlinear static subsystem for Wiener system (Kalafatis 1995).

Another useful case are splines which are also nice functions, since they are computationally very simple and can be made as smooth as desired. But it requires the choice of break points (knots). Hagenblad (1999) and Zhu (1999) used B-splines and cubic splines, respectively, to describe inverse of the nonlinear subsystem in Wiener system.

- ***Discontinuous nonlinearities***

By introducing some switching functions, one can get models for discontinuous nonlinearities, like direction-dependent nonlinearity, preload nonlinearity, dead-zone nonlinearity, saturation nonlinearity, and so on.

2.3. New concept of the recursive identification method for Wiener and Hammerstein systems

2.3.1. Recursive pseudo-linear regressions (RPLR)

Supposing a general SISO block-oriented time-invariant nonlinear system with input $u(t)$ and output $y(t)$, the data are assumed to be collected in discrete time. At time t, we have:

- the observed system input vector $\mathbf{u}^{\mathrm{T}}(t) = [u(1), u(2), \cdots, u(t)]$,
- the observed system output vector $\mathbf{y}^{\mathrm{T}}(t) = [y(1), y(2), \cdots, y(t)]$ and
- r unmeasurable intermediate variable vectors, $\hat{\mathbf{w}}^{\mathrm{T}}(t) = [\hat{\mathbf{w}}_1^{\mathrm{T}}(t), \hat{\mathbf{w}}_2^{\mathrm{T}}(t), \cdots, \hat{\mathbf{w}}_r^{\mathrm{T}}(t)]$, where $\hat{\mathbf{w}}_k^{\mathrm{T}}(t) = [\hat{w}_k(1), \hat{w}_k(2), \cdots, \hat{w}_k(t)]$ for $k = 1,2,\cdots,r$. They can be seen as states of the system and can be recursively estimated by using the estimated parameters.

The main problem is how to find a good parameterization and how to deal with it. The general form of such a regression model is given by

$$\begin{bmatrix} \hat{\mathbf{w}}(t+1) \\ \hat{y}(t) \end{bmatrix} = g(\mathbf{u}(t), \hat{\mathbf{w}}(t), \mathbf{y}(t-1), \vartheta) + e(t) \tag{2.15}$$

where ϑ is a constant whole parameter vector to be identified $\vartheta^{\mathrm{T}} = [\boldsymbol{\theta}, \boldsymbol{\eta}, \boldsymbol{\xi}]$. $e(t)$ is a random error in the measured output $y(t)$ values.

The function $g(\mathbf{u}(t), \hat{\mathbf{w}}(t), \mathbf{y}(t-1), \vartheta)$ can been considered as a concatenation of two mappings:

1. Taking the observation $\mathbf{u}(t)$, $\mathbf{y}(t-1)$ and estimation $\hat{\mathbf{w}}(t)$ and mapping them into a finite dimensional regression vector, $\boldsymbol{\varphi}(t) = \boldsymbol{\varphi}(\mathbf{u}(t), \hat{\mathbf{w}}(t), \mathbf{y}(t-1))$.
2. Taking this regression vector $\boldsymbol{\varphi}(t)$ by choosing $g(\boldsymbol{\varphi}(t), \vartheta)$ to the output space.

Then the estimated system output $\hat{y}(t)$ is given by

$$\hat{y}(t) = g(\boldsymbol{\varphi}(t), \vartheta). \tag{2.16}$$

We consider the identification problem in a prediction error method and model scheme. To identify the parameter vector, we compare the predicted output $\hat{y}(t)$ with the measured output $y(t)$ in the following prediction error criterion:

$$\vartheta = \arg\min_{\vartheta} \sum \frac{1}{2}[(y(t) - \hat{y}(t)]^2. \tag{2.17}$$

It is the same for all linear and nonlinear dynamical systems and according to the different model structures it turns out the choices of parameter-depended regression vector $\boldsymbol{\varphi}(t)$ and lead to different identification algorithms.

As Ljung (1987) pointed out: no matter how $\boldsymbol{\varphi}(t)$ is formed, it is the known data at time t and it can contain arbitrary transformations of measured and estimated data. Because $\boldsymbol{\varphi}(t)$ depends on $\hat{\mathbf{w}}(t)$ which contains information given by the model at early time instants, the model (2.16) is regressive. $g(\boldsymbol{\varphi}(t), \vartheta)$ can be pseudo-linear. A pseudo-linear regression is defined as a model structure where the

prediction is linear in the parameters ϑ but nonlinear in data $\boldsymbol{\varphi}(t)$. Then the regression parameter vector ϑ can be calculated by LS or PEM estimator. That is the so called pseudo-linear regressions (PLR)

$$\begin{aligned}\hat{y}(t) &= \boldsymbol{\varphi}^{\mathrm{T}}(t)\cdot\vartheta \\ &= \boldsymbol{\varphi}_1^{\mathrm{T}}(t)\cdot\vartheta_1 + \boldsymbol{\varphi}_2^{\mathrm{T}}(t)\cdot\vartheta_2 + \cdots + \boldsymbol{\varphi}_s^{\mathrm{T}}(t)\cdot\vartheta_s .\end{aligned} \tag{2.18}$$

Eq. (2.18) can be regarded as a finite-dimensional parameterization of a general, unknown nonlinear predictor. The problems are how to choose the regressors $\boldsymbol{\varphi}(t)$ according to the physical insight into the system and how to determine the predictor in some recursive fashion to arrive at the recursive pseudo-linear regressions (RPLR) estimates.

Eq. (2.18) can also be seen as a transformation result from a s pseudo inputs pseudo-linear system

$$\hat{y}(t) = G_1(q^{-1},\boldsymbol{\theta}_1)u_1(t) + G_2(q^{-1},\boldsymbol{\theta}_2)u_2(t) + \cdots + G_s(q^{-1},\boldsymbol{\theta}_s)u_s(t) . \tag{2.19}$$

A special example is to identify the parameters of a single nonlinear static model of order m

$$\hat{y}(t) = \beta_0 + \beta_1 u(t) + \beta_2 u^2(t) + \cdots + \beta_m u^m(t) . \tag{2.20}$$

Because the parameters enter the model linearly, Eq. (2.20) can be written directly into a pseudo-linear regression form

$$\hat{y}(t) = \boldsymbol{\varphi}^{\mathrm{T}}(t)\cdot\vartheta \tag{2.21}$$

where $\boldsymbol{\varphi}^{\mathrm{T}}(t) = [1, u(t), \cdots, u^m(t)]$ and $\vartheta^{\mathrm{T}} = [\beta_0, \beta_1, \cdots, \beta_m]$, namely transforming this problem into a linear MISO FIR model (Ljung, 1995) which has $m+1$ inputs: $1, u(t), u^2(t), \cdots, u^m(t)$. With the known LS or PEM algorithms for linear regression models, the parameter vector ϑ in Eq. (2.21) can be identified correctly.

Based on the analysis above, we transform a SISO cascade time-invariant nonlinear system into a pseudo-linear MISO system with multiple pseudo-inputs as shown in Eq. (2.19). Then a pseudo-linear MISO prediction error model can be formulated. With the same principle, it can be extended to MISO systems to obtain the corresponding identification algorithms. The aim of this thesis is to make such a unified and efficient identification methodology for a class of Wiener and Hammerstein systems available by transforming the nonlinear identification problem into a general pseudo-linear MISO identification problem.

2.3.2. Key term separation principle and estimation

To write out the description of the whole model with explicit parameters, it presents usually nonlinearities or redundancy in parameters, because of the substitutions from or into nonlinear or high order terms and products between parameters. This identification problem can be solved by LS or PEM method but suffering from the too much redundant parameters and its complicated computation.

Instead of direct substitution, one can compress or merge the redundant terms by introducing some possible intermediate variables to form a RPLR estimates. Because of the separation of linear subsystems and nonlinear subsystems in a Wiener and Hammerstein system, the intermediate variables become definitively. It is naturally sometimes to take the variables between the subsystems as the intermediate variables.

Vörös (1995) used the key term separation principle to identify Wiener system and Hammerstein system. The basic idea of the key term separation principle is a form of half-substitution, that is, only the separated key terms will be substituted with their front expressions. Then, with an analytic additive form, the system output $y(t)$ with a minimum number of parameters can be determined. We illustrate the key term separation principle by a Wiener system and a Hammerstein system in detail.

2.3.2.1. Wiener system

Considering a Wiener system shown in Fig. 1.2, the variable relationships can be written as

$$w(t) = G(q^{-1}, \boldsymbol{\theta})u(t) \tag{2.23}$$

and

$$y(t) = N(w(t), \boldsymbol{\eta}) + \varepsilon(t)\,. \tag{2.24}$$

Substituting Eq. (2.23) into Eq. (2.24) directly, the system output is given by

$$y(t) = N(G(q^{-1}, \boldsymbol{\theta})u(t), \boldsymbol{\eta}) + \varepsilon(t)\,. \tag{2.25}$$

But we can also estimate the intermediate variable $w(t)$ and use the key term separation principle to write an alternative form for Eq. (2.25). According the key term separation principle, the nonlinear subsystem function in Eq. (2.24) can be separated as

$$y(t) = \rho \cdot w(t) + N^*(w(t), \boldsymbol{\eta}^*) + \varepsilon(t) \tag{2.26}$$

where $w(t)$ in $\rho \cdot w(t)$ is the so called key term. ρ is a constant coefficient.

If the substitution of Eq. (2.23) into Eq. (2.24) is only done for the key term $w(t)$ (half-substitution), we have

$$y(t) = \rho \cdot G(q^{-1}, \boldsymbol{\theta})u(t) + N^*(w(t), \boldsymbol{\eta}^*) + \varepsilon(t)\,. \tag{2.27}$$

The key term $w(t)$ can be recursively estimated by Eq. (2.23) with the estimated parameters. And Eq. (2.27) could also be written in a linear regression form in which the parameters are in a linear form and without parameter redundancy.

2.3.2.2. Hammerstein system

Considering a Hammerstein system as shown in Fig. 1.3, the variable relationships can be written as

$$w(t) = N(u(t), \boldsymbol{\eta}) \tag{2.28}$$

and

$$y(t) = G(q^{-1}, \boldsymbol{\theta})w(t) + \varepsilon(t)\,. \tag{2.29a}$$

Because of the special model structure in Hammerstein system, Eq. (2.29a) can also be written as an ARMAX model form

$$A(q^{-1})y(t) = B(q^{-1})w(t) + \varepsilon(t). \tag{2.29b}$$

Substituting Eq. (2.28) into Eq. (2.29a) or Eq. (2.29b) directly, the system output is as follows:

$$y(t) = G(q^{-1}, \boldsymbol{\theta})N(u(t), \boldsymbol{\eta}) + \varepsilon(t) \tag{2.30a}$$

or

$$A(q^{-1})y(t) = B(q^{-1})N(u(t), \boldsymbol{\eta}) + \varepsilon(t). \tag{2.30b}$$

If we use the key term separation principle, the linear subsystem function in Eq. (2.29a) and Eq. (2.29b) can be separated as

$$y(t) = \rho \cdot w(t) + G^*(q^{-1}, \boldsymbol{\theta}^*)w(t) + \varepsilon(t) \tag{2.31a}$$

or

$$A(q^{-1})y(t) = \rho \cdot w(t) + B^*(q^{-1})w(t) + \varepsilon(t) \tag{2.31b}$$

where $w(t)$ in $\rho \cdot w(t)$ is the key term. ρ is a constant coefficient.

If the substitution of Eq. (2.28) into Eq. (2.31a) or Eq. (2.31b) is only for the key term $w(t)$ (half-substitution), we have

$$y(t) = \rho \cdot N(u(t), \boldsymbol{\eta}) + G^*(q^{-1}, \boldsymbol{\theta}^*)w(t) + \varepsilon(t) \tag{2.32a}$$

or

$$A(q^{-1})y(t) = \rho \cdot N(u(t), \boldsymbol{\eta}) + B^*(q^{-1})w(t) + \varepsilon(t) \tag{2.32b}$$

The key term $w(t)$ can be recursively estimated by Eq. (2.28). Eq. (2.32a) and Eq. (2.32b) could also be written in a linear regression form in which the parameters are in a linear form and without parameter redundancy.

In the same way, other Wiener and Hammerstein systems can also be described by using recursively estimated intermediate variables, i.e., key terms to form RPLR estimates.

2.3.3. Concept of the Identification method

In the following we describe the single steps of the identification method in a more detail.

- ***Initial values of parameters***

A good choice of initial values of parameters can make the identification algorithm stable and can give convergence to the global minimum. Some suggestions on how to initialize the search algorithm for nonlinear black-box models can be found in Sjöberg (1997). The result from one identification method can be used as initial value of other methods. To illustrate the efficiency of the new identification method we set all the initial values of parameters to zero.

- ***Smoothing techniques to estimate intermediate variables***

In a RPLR identification the parameter variation will strongly affect identification quality. The estimation of the intermediate variables depends on the corresponding estimated parameters of the front submodels from last time instant. Because of these unmeasurable intermediate variables or their combinations, the constructed regressive nonlinear dynamic models are especially sensitive to the estimated intermediate variables and could not converge to a good minimum. Therefore smoothing and filtering in the estimation of these intermediate variables are necessary.

One can smooth and filter the historical values of intermediate variables directly or the historical values of corresponding submodel parameters which are used to calculate the intermediate variables. Various data smoothing techniques provide possibility to mitigate the estimate errors of intermediate variables and to avoid the possible oscillations to achieve better convergence.

We use the moving average parameter smoothing technique (Brown, 1963) which means exponentially averaging the parameter vector ϑ_t with a fixed moving window length mov

$$\vartheta_t^* = \alpha\vartheta_t + \alpha(1-\alpha)\vartheta_{t-1} + \alpha(1-\alpha)^2\vartheta_{t-2} + \cdots + \alpha(1-\alpha)^{mov-2}\vartheta_{t-mov+2} + (1-\alpha)^{mov-1}\vartheta_{t-mov+1} \quad (2.33)$$

where α is a constant value between zero and one and the sum of the weight coefficients is 1.

$$\alpha + \alpha(1-\alpha) + \alpha(1-\alpha)^2 + \cdots + \alpha(1-\alpha)^{mov-2} + (1-\alpha)^{mov-1} = 1. \quad (2.34)$$

In further, the double exponential smoothing (Brown, 1963) can be used.

$$\vartheta_t^{**} = \alpha\vartheta_t^* + (1-\alpha)\vartheta_{t-1}^{**} \quad (2.35a)$$

$$\overline{\vartheta}_t = \alpha\vartheta_t^{**} + (1-\alpha)\overline{\vartheta}_{t-1} \quad (2.35b)$$

The smoothed parameter vector $\overline{\vartheta}_t$ can be used to estimate the intermediate variables. A serious problem is the choice of the smoothing parameter α. The role of such a parameter is to damp out random fluctuations. It must be specified to minimize the smoothing errors. This can be done by trial and error method over a past time interval. In general, small values have a stronger smoothing effect than large values.

In order to facilitate specification of α and to improve alertness ability of the smoothing, another alternative adaptive smoothing method (Trigg and Leach, 1967) can be used.

$$\vartheta_{t+1} = \alpha_t\vartheta_t^* + (1-\alpha_t)\vartheta_t \quad (2.36a)$$

$$\kappa_t = \vartheta_t^* - \vartheta_t \quad (2.36b)$$

$$E_t = \rho\kappa_t + (1-\rho)E_{t-1} \quad (2.36c)$$

$$M_t = \rho|\kappa_t| + (1-\rho)M_{t-1} \quad (2.36d)$$

$$\alpha_{t+1} = |E_t / M_t| \quad (2.36e)$$

where ρ is usually set at 0.1 or 0.2. Finally, α_{t+1} is computed instead of α_t to allow the system to "settle" a little by not being too responsive to changes.

Other smoothing methods are a piecewise line of polynomial fitting approach using a moving window with another fixed window length, Kalman filter, fuzzy sets and so on.

- ***Adaptive identification method***

Adaptive identification methods are generally for time-variant dynamic systems, especially under the white or colored measurement noise. In our new recursive identification algorithm, because there are intermediate variables estimation in every time instant, the reconstructed identification model can be regarded as a linear time-variant dynamic model. In order to guarantee and accelerate the convergence, we use the forgetting factor approach with a variant forgetting factor, $\lambda(t) = \lambda(t-1) + (1-\lambda(t-1)) \cdot \Delta\lambda$ to adaptation and to identify the parameters.

The general adaptive recursive algorithm (Ljung, 1987) is given by

$$\vartheta(t) = \vartheta(t-1) + \mathbf{K}(t)(y(t) - \hat{y}(t)) \tag{2.37}$$

$$\hat{y}(t) = \boldsymbol{\varphi}^{\mathrm{T}}(t)\vartheta(t-1) \tag{2.38}$$

$$\mathbf{K}(t) = \mathbf{Q}(t)\boldsymbol{\varphi}(t) \tag{2.39}$$

$$\mathbf{Q}(t) = \frac{\mathbf{P}(t-1)}{\lambda(t) + \boldsymbol{\varphi}^{\mathrm{T}}(t)\mathbf{P}(t-1)\boldsymbol{\varphi}(t)} \tag{2.40}$$

$$\mathbf{P}(t) = \left(\mathbf{P}(t-1) - \frac{\mathbf{P}(t-1)\boldsymbol{\varphi}(t)\boldsymbol{\varphi}^{\mathrm{T}}(\mathbf{t})\mathbf{P}(t-1)}{\lambda(t) + \boldsymbol{\varphi}^{\mathrm{T}}(t)\mathbf{P}(t-1)\boldsymbol{\varphi}(t)} \right) / \lambda(t). \tag{2.41}$$

Therefore, concept of the new identification method is the following:

1. Suppose the system input $u(t)$ is persistently excited and the Wiener and Hammerstein system is stable.

2. Select the linear and nonlinear submodel structures and use the key term separation principle to extract the key terms and to define the intermediate variables or their combinations as pseudo multiple inputs.

3. Transform the Wiener and Hammerstein system into a pseudo-linear MISO system with all explicit system parameters and form a prediction error model.

4. Fix some parameters to obtain a unique parameterization.

5. Identify the original parameters of the Wiener and Hammerstein system by using smoothing techniques and the adaptive recursive pseudo-linear regression (RPLR) method.

This new concept will be applied now to different cascade and block-oriented structures of Wiener and Hammerstein systems.

3. Identification of SISO and MISO Wiener and Hammerstein systems

3.1. SISO Wiener and Hammerstein systems

In this section, we apply the new identification method described in section 2.3.3 to identify:

- a Wiener system (Fig. 3.1),
- a Hammerstein system (Fig. 3.2),
- a Feedback-Wiener system (Fig. 3.3) and
- a Feedback-Hammerstein system (Fig. 3.4).

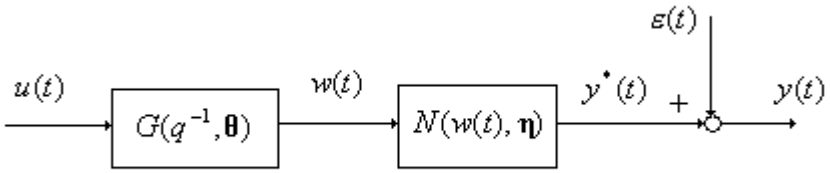


Fig. 3.1 Wiener system

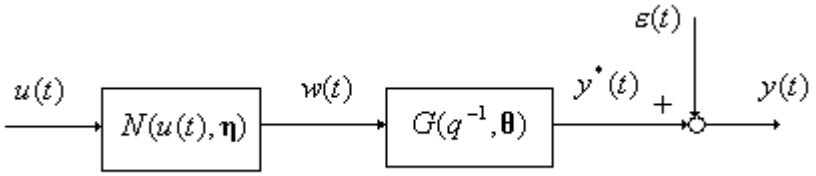


Fig. 3.2 Hammerstein system

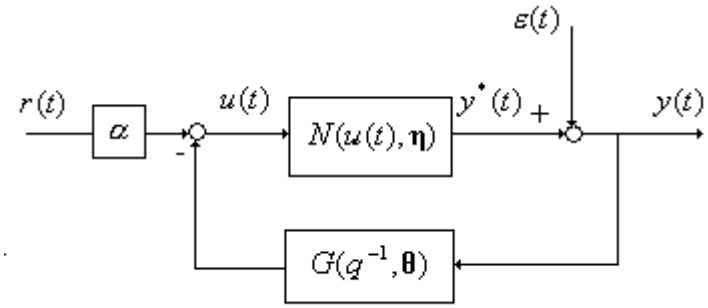


Fig. 3.3 Feedback-Wiener system

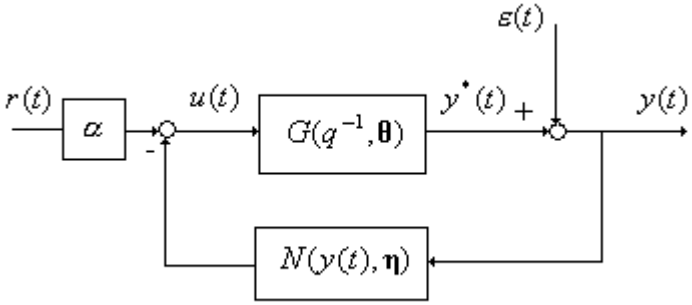


Fig. 3.4 Feedback-Hammerstein system

In the above Figures $r(t)$ is the reference signal, $u(t)$ is the system input, $y(t)$ is the system output, and $y^*(t)$ is the unmeasurable system output without measurement noise. In Fig. 3.1 and Fig. 3.2, $w(t)$ is an unmeasurable intermediate variable. In Fig. 3.3 and Fig. 3.4, $u(t)$ is also an unmeasurable intermediate variable and α is an unknown constant gain which is used to distribute the whole system gain. $\boldsymbol{\theta}$ is a parameter vector determining linear dynamic subsystem $G(q^{-1},\boldsymbol{\theta})$. $\boldsymbol{\eta}$ is a parameter vector determining nonlinear static subsystem $N(\cdot,\boldsymbol{\eta})$. $\varepsilon(t)$ is the colored measurement noise which is the result of a white noise $e(t)$ through a linear filter $H(q^{-1},\boldsymbol{\xi})$. All functions are defined as before.

3.1.1. Wiener system

In Fig. 3.1, the linear dynamic block $G(q^{-1},\boldsymbol{\theta})$ is a RTF model of order n

$$w(t)=\frac{B(q^{-1})}{F(q^{-1})}u(t). \tag{3.1}$$

The nonlinear static block $N(w(t),\boldsymbol{\eta})$ is described by a polynomial of order m

$$\begin{aligned} y^*(t) &= \sum_{k=1}^{m}\beta_k\cdot w^k(t) \\ &= \beta_1\cdot w(t)+\sum_{k=2}^{m}\beta_k\cdot w^k(t) \end{aligned} \tag{3.2}$$

and $w(t)$ in the first part of Eq. (3.2) is the key term.

Half-substituting Eq. (3.1) into the key term $w(t)$ of Eq. (3.2), the system output of a Wiener system can be written as

$$\begin{aligned} y(t) &= y^*(t)+\varepsilon(t) \\ &= \frac{\beta_1\cdot B(q^{-1})}{F(q^{-1})}u(t)+\sum_{k=2}^{m}\beta_k\cdot w^k(t)+\frac{C(q^{-1})}{D(q^{-1})}e(t). \end{aligned} \tag{3.3}$$

To avoid the over-parameterization problem, one parameter of $\beta_1,b_0,b_1,\cdots,b_n$ in Eq. (3.3) should be fixed. It is shown that a Wiener system can be transformed approximately into a pseudo-linear MISO system which has m independent inputs: the system input $u(t)$ and the recursively estimated intermediate variables $w^k(t)$ for $k=2,3,\cdots,m$ according to Eq. (3.1). All the parameters in this pseudo-linear MISO system Eq. (3.3) are explicitly given.

3.1.2. Hammerstein system

Considering Hammerstein system as shown in Fig. 3.2. The nonlinear static block $N(u(t),\boldsymbol{\eta})$ is again a polynomial of order m

$$w(t) = \sum_{k=1}^{m} \beta_k \cdot u^k(t). \tag{3.4}$$

The linear dynamic block $G(q^{-1}, \boldsymbol{\theta})$ is a RTF model of order n

$$\begin{aligned} y^*(t) &= \frac{B(q^{-1})}{F(q^{-1})} w(t) \\ &= b_0 \cdot w(t) + \frac{B^*(q^{-1})}{F(q^{-1})} w(t) \end{aligned} \tag{3.5a}$$

where we separate a part $b_0 \cdot w(t)$ from the transfer function $\frac{B(q^{-1})}{F(q^{-1})} w(t)$. According to the function definitions before, $B^*(q^{-1})$ in the second part $\frac{B^*(q^{-1})}{F(q^{-1})}$ is

$$B^*(q^{-1}) = b_1^* q^{-1} + b_2^* q^{-2} + \cdots + b_n^* q^{-n} \tag{3.5b}$$

with

$$b_j^* = b_j - b_0 \cdot f_j \tag{3.5c}$$

for $j = 1,2,\cdots,n$ and $w(t)$ in the first part of Eq. (3.5a) is the key term.

Half-substituting Eq. (3.4) into the key term $w(t)$ of Eq. (3.5a), then the system output of a Hammerstein system can be written as

$$\begin{aligned} y(t) &= y^*(t) + \varepsilon(t) \\ &= \sum_{k=1}^{m} b_0 \cdot \beta_k \cdot u^k(t) + \frac{B^*(q^{-1})}{F(q^{-1})} w(t) + \frac{C(q^{-1})}{D(q^{-1})} e(t). \end{aligned} \tag{3.6}$$

We can also get another alternative description. Because of the special structure of Hammerstein system, it is also convenient to use an ARMAX model of order n to describe the linear block $G(q^{-1}, \boldsymbol{\theta})$, that is

$$\begin{aligned} A(q^{-1}) y(t) &= B(q^{-1}) w(t) + C(q^{-1}) e(t) \\ &= b_0 \cdot w(t) + B^*(q^{-1}) w(t) + C(q^{-1}) e(t) \end{aligned} \tag{3.7}$$

where we also separate a part $b_0 \cdot w(t)$ from $B(q^{-1}) w(t)$. According to the function definitions before, $B^*(q^{-1})$ in the second part is $B^*(q^{-1}) = b_1 q^{-1} + b_2 q^{-2} + \cdots + b_n q^{-n}$ which is not the same as in Eq. (3.5b). Again $w(t)$ in the first part of Eq. (3.7) is the key term.

Half-substituting Eq. (3.4) into the key term $w(t)$ of Eq. (3.7), the system output of a Hammerstein system can also be written as

$$A(q^{-1})y(t) = \sum_{k=1}^{m} b_0 \cdot \beta_k \cdot u^k(t) + B^*(q^{-1})w(t) + C(q^{-1})e(t)\,. \tag{3.8}$$

Both Eq. (3.6) and Eq. (3.8) can be used to identify Hammerstein systems and it can be expected that the model in Eq. (3.8) is simpler than the model in Eq. (3.6). To avoid the over-parameterization problem, one parameter of $b_0, \beta_1, \beta_2, \cdots, \beta_m$ in Eq. (3.6) and Eq. (3.8) should be fixed. It is shown that a Hammerstein system can be transformed approximately into a pseudo-linear MISO system which has $m+1$ independent inputs: $u^k(t)$ for $k = 1,2,\cdots,m$ and the recursively estimated $w(t)$ according to Eq. (3.4). The parameters in Eq. (3.6) and Eq. (3.8) can be identified without redundancy and the original nominator parameters of the linear block in Eq. (3.5a) can be recalculated according to Eq. (3.5c).

3.1.3. Feedback-Wiener system

Considering a stable Feedback-Wiener system as shown in Fig. 3.3. We make use of a RTF model of order n to describe the linear block $G(q^{-1}, \boldsymbol{\theta})$. Because of its feedback characteristics, the linear block $G(q^{-1}, \boldsymbol{\theta})$ should contain a time delay, that is, $B(q^{-1}) = b_1 q^{-1} + b_2 q^{-2} + \cdots + b_n q^{-n}$, so that the Feedback-Wiener system is well defined.

Then the input of the nonlinear static block can be derived as

$$u(t) = \alpha \cdot r(t) - \frac{B(q^{-1})}{F(q^{-1})} y(t)\,. \tag{3.9}$$

The nonlinear static block $N(u(t), \boldsymbol{\eta})$ is described by a polynomial of order m

$$\begin{aligned} y^*(t) &= \sum_{k=1}^{m} \beta_k \cdot u^k(t) \\ &= \beta_1 \cdot u(t) + \sum_{k=2}^{m} \beta_k \cdot u^k(t) \end{aligned} \tag{3.10}$$

where $u(t)$ in the first part is the key term.

Half-substituting Eq. (3.9) into the key term $u(t)$ of Eq. (3.10), we get the system output of a Feedback-Wiener system

$$\begin{aligned} y(t) &= y^*(t) + \varepsilon(t) \\ &= \beta_1 \cdot \alpha \cdot r(t) - \frac{\beta_1 \cdot B(q^{-1})}{F(q^{-1})} y(t) + \sum_{k=2}^{m} \beta_k \cdot u^k(t) + \frac{C(q^{-1})}{D(q^{-1})} e(t). \end{aligned} \tag{3.11}$$

To avoid the over-parameterization problem, one parameter in $\beta_1, \alpha, b_1, b_2, \cdots, b_n$ should be fixed. Eq. (3.1) shows that a Feedback-Wiener system can be transformed approximately into an open-loop pseudo-linear MISO system which has $m+1$ independent pseudo-inputs: the reference signal $r(t)$, the system output $-y(t)$ and the recursively estimated intermediate variables $u^k(t)$ for $k = 2,3,\cdots,m$ according to Eq. (3.9). All the parameters in Eq. (3.11) are explicitly given.

3.1.4. Feedback-Hammerstein system

Considering a stable Feedback-Hammerstein system as shown in Fig. 3.4. We use a polynomial of order m to describe the feedback nonlinear static block $N(y(t),\boldsymbol{\eta})$. Therefore, the input of the linear dynamic block can be derived as

$$u(t) = \alpha \cdot r(t) - \sum_{k=1}^{m} \beta_k \cdot y^k(t). \tag{3.12}$$

Because of the structure of Hammerstein-type system and in order to simplify the derivation, we use an ARMAX model of order n to describe the linear dynamic block $G(q^{-1},\boldsymbol{\theta})$

$$A(q^{-1})y(t) = B(q^{-1})u(t) + C(q^{-1})e(t). \tag{3.13}$$

And because of its feedback system characteristics, the linear block $G(q^{-1},\boldsymbol{\theta})$ should contain a time delay, that is, $B(q^{-1}) = b_1 q^{-1} + b_2 q^{-2} + \cdots + b_n q^{-n}$, so that the Feedback-Hammerstein system is well defined.

Substituting Eq. (3.12) into Eq. (3.13), we get

$$A(q^{-1})y(t) = B(q^{-1})\left[\alpha \cdot r(t) - \sum_{k=1}^{m} \beta_k \cdot y^k(t)\right] + C(q^{-1})e(t). \tag{3.14a}$$

From Eq. (3.14a) we find

$$[A(q^{-1}) + \beta_1 \cdot B(q^{-1})]y(t) = B(q^{-1})\left[\alpha \cdot r(t) - \sum_{k=2}^{m} \beta_k \cdot y^k(t)\right] + C(q^{-1})e(t). \tag{3.14b}$$

Now we define two equivalent expressions:

$$\begin{aligned} &B(q^{-1}) = b_1 q^{-1} + b_2 q^{-2} + \cdots + b_n q^{-n} \\ &= b_1 q^{-1} + B^*(q^{-1}) \end{aligned} \tag{3.15}$$

with $B^*(q^{-1}) = b_2 q^{-2} + b_3 q^{-3} + \cdots + b_n q^{-n}$ and

$$\begin{aligned} &A^*(q^{-1}) = A(q^{-1}) + \beta_1 \cdot B(q^{-1}) \\ &= 1 + a_1^* q^{-1} + a_2^* q^{-2} + \cdots + a_n^* q^{-n} \end{aligned} \tag{3.16a}$$

with

$$a_k^* = a_k + \beta_1 \cdot b_k \tag{3.16b}$$

for $k = 1,2,\cdots,n$.

And we define a new combined intermediate variable in Eq. (3.14b)

$$\tilde{y}(t) = b_1 \cdot \left[\alpha \cdot r(t) - \sum_{k=2}^{m} \beta_k \cdot y^k(t) \right]. \tag{3.17}$$

Then Eq. (3.14b) can be rewritten as

$$\begin{aligned} A^*(q^{-1})y(t) &= B(q^{-1})\tilde{y}(t) + C(q^{-1})e(t) \\ &= \tilde{y}(t-1) + \frac{B^*(q^{-1})}{b_1}\tilde{y}(t) + C(q^{-1})e(t) \end{aligned} \tag{3.18}$$

where the separated part $\tilde{y}(t-1)$ is the key term.

Half-substituting Eq. (3.17) into the key term $\tilde{y}(t-1)$ in Eq. (3.18), we get the system output of Feedback-Hammerstein system

$$A^*(q^{-1})y(t) = b_1 \cdot \alpha \cdot r(t-1) - \sum_{k=2}^{m} b_1 \cdot \beta_k \cdot y^k(t-1) + \frac{B^*(q^{-1})}{b_1}\tilde{y}(t) + C(q^{-1})e(t). \tag{3.19}$$

To avoid the over-parameterization problem, one parameter in $b_1, \alpha, \beta_2, \beta_3, \cdots, \beta_m$ should be fixed. Eq. (3.19) shows that a Feedback-Hammerstein system can be transformed approximately into an open-loop pseudo-linear MISO system of $m+1$ independent pseudo-inputs: the reference signal $r(t-1)$, $-y^k(t-1)$ for $k = 2, \cdots, m$ and the combined intermediate variable $\tilde{y}(t)$ which can be recursively estimated with Eq. (3.17). It should be noted that only a_k^* for $k = 1,2\cdots, n$ in Eq. (3.19) are identifiable because of the inherent dependence. Actually, the term $\beta_1 y(t)$ in the nonlinear static block $N(y(t), \boldsymbol{\eta})$ in the feedback path will cause a linear feedback identification problem. And only under some special assumptions, for example, if β_1 is a known or fixed, the original parameters a_k for $k = 1,2\cdots, n$ in Eq. (3.14a) can be uniquely determined by Eq. (3.16b).

3.1.5. Simulation results

We now consider the application of the new identification method to the following four special SISO Wiener and Hammerstein systems.

For the Wiener system and the Hammerstein system, two standard examples are used (Kortmann, 1989). In order to compare the identification results with Kortmann (1989), we use the same identification conditions and signals. A uniformly distributed random signal with an amplitude ± 0.5 is used as input $u(t)$. Another independent random numbers is as zero mean white noise $e(t)$. $\mathbf{N} = 1000$ data points are collected.

The linear dynamic subsystem is described by

$$G(q^{-1}, \boldsymbol{\theta}) = \frac{0.1333q^{-1} + 0.0667q^{-2}}{1 - 1.5q^{-1} + 0.7q^{-2}} \tag{3.20}$$

and the nonlinear static subsystem by

$$\begin{aligned}\gamma(t) &= N(\chi(t), \boldsymbol{\eta}) \\ &= \chi(t) + 3\chi^2(t) + 1.5\chi^3(t)\end{aligned} \tag{3.21}$$

where $\chi(t)$ is the input of the nonlinear block and $\gamma(t)$ is the output of the nonlinear block. We fix the first power parameter in nonlinear subsystem $\beta_1 = 1$.

For the Feedback-Wiener system, the linear and nonlinear blocks are also described by the Eqs. (3.20-3.21). The original constant gain a in the Feedback-Wiener system is "$rf\alpha = 0.1$".

For the Feedback-Hammerstein system, we assume it consists of an ARMAX model

$$\begin{aligned}&(1 - 1.5q^{-1} + 0.7q^{-2})y(t) \\ &= (0.1333q^{-1} + 0.0667q^{-2})u(t) + (1 + 0.2q^{-1} + 0.1q^{-2})e(t)\end{aligned} \tag{3.22}$$

and a nonlinear static feedback block Eq. (3.21). The original constant gain a is "$rf\alpha = 1$". According to the derivation, an equivalent Feedback-Hammerstein system can be formed as

$$\begin{aligned}&(1 - 1.3667q^{-1} + 0.7667q^{-2})y(t) \\ &= 0.1333r(t-1) - 0.3999y^2(t-1) - 0.1999y^3(t-1) + 0.5004\tilde{y}(t-2) \\ &+ (1 + 0.2q^{-1} + 0.1q^{-2})e(t).\end{aligned} \tag{3.23}$$

A standard random numbers are used as the reference signal $r(t)$ for the Feedback-Wiener system and the Feedback-Hammerstein system. Another independent random numbers is as zero mean white noise $e(t)$. $\mathbf{N} = 2000$ data points are collected.

For the Wiener system, Hammerstein system and Feedback-Wiener system, the linear filter for a colored measurement noise is given by

$$H(q^{-1}, \xi) = \frac{1 + 0.2q^{-1} + 0.1q^{-2}}{1 - 0.9q^{-1} + 0.85q^{-2}}. \tag{3.25}$$

The Noise-Signal ratio is defined as

$$N./S. = \frac{Var[y(t)]_{u(t)=0}}{Var[y(t)]_{e(t)=0}}. \tag{3.24}$$

We consider different measurement noise levels for the four different block-oriented structures. Besides the non-noisy case, $N./S. = 5\%$, 10% and 20% white and 10% colored measurement noise are added to the system output, respectively.

In order to calculate the unmeasurable intermediate variable $w(t)$ in Wiener system and Hammerstein system and the unmeasurable intermediate variable $u(t)$ in Feedback-Wiener system and Feedback-Hammerstein system, we use the adaptive exponentially moving average smoothing technique to filter the estimated parameters. The moving window with a fixed length is $Mov = 4$. Then the standard recursive prediction errors method (RPEM) function for linear MISO system with various forgetting factors $\lambda(0) = 0.7$ and $\Delta\lambda = 0.01$ in MATLAB is applied. The initial values of all the unknown parameters are taken as zero.

Identification results with different measurement noises are shown in Tables 3.1A-3.4A. The identification processes with a N./S.=10% colored measurement noise are shown in Figs. 3.5-3.8. The red lines in Figs. 3.5-3.8 are the real values of parameters. We calculate single parameter identification error Δp and the avarage identification error $|\Delta|$ of each structure according to: $|\Delta p| = |estimated - true|$ and $|\Delta| = \frac{\sum |\Delta p|}{\zeta}$. ζ is the parameter number. Parameter identification errors with different measurement noises are shown in Tables 3.1B-3.4B. Comparison of avarage parameter identification errors $|\Delta|$ of the four structures is shown in Table 3.5.

Table 3.1A Identification results of the Wiener system

$\mathbf{N} = 1000$	b_1	b_2	f_1	f_2	β_2	β_3
10% C. N.	0.1314	0.0623	-1.5119	0.7111	2.9940	1.7155
20% W. N.	0.1332	0.0723	-1.5046	0.7015	2.9175	1.1322
10% W. N.	0.1323	0.0659	-1.5085	0.7048	2.9974	1.4243
5% (Kortm.)	0.1453	0.0489	-1.5043	0.6986	3.0161	1.4477
5% W. N.	0.1315	0.0675	-1.5050	0.7033	3.0711	1.5087
Non-noise	0.1333	0.0667	-1.5000	0.7000	3.0000	1.5000
True values	0.1333	0.0667	-1.5000	0.7000	3.0000	1.5000
$\mathbf{N} = 1000$	c_1	c_2	d_1	d_2	-	-
10% C. N.	0.1774	0.0832	-0.8880	0.8490	-	-
True values	0.2000	0.1000	-0.9000	0.8500	-	-

Table 3.1B Parameter identification errors of Wiener system

$\mathbf{N} = 1000$	$\|\Delta b_1\|$	$\|\Delta b_2\|$	$\|\Delta f_1\|$	$\|\Delta f_2\|$	$\|\Delta \beta_2\|$	$\|\Delta \beta_3\|$	$\|\Delta\|$
10% C. N.	0.0019	0.0044	0.0119	0.0111	0.0060	0.2155	0.0418
20% W. N.	1E-04	0.0056	0.0046	0.0015	0.0825	0.3678	0.0770
10% W. N.	0.0010	0.0008	0.0085	0.0048	0.0026	0.0757	0.0155
5% (Kort.)	0.0120	0.0178	0.0043	0.0014	0.0161	0.0523	0.0173
5% W. N.	0.0018	0.0008	0.0050	0.0033	0.0711	0.0087	0.0151

Table 3.2A Identification results of the Hammerstein system

$\mathbf{N} = 1000$	β_2	β_3	b_1	b_2	f_1	f_2
10% C. N.	3.1206	1.3557	0.0954	0.1407	-1.4964	0.7047
20% W. N.	3.2751	2.2686	0.1250	0.0236	-1.4225	0.6539
10% W. N.	3.0861	1.6330	0.1341	0.0311	-1.5010	0.7034
5% (Kortm.)	3.0240	1.5191	0.1414	0.0526	-1.5067	0.7385
5% W. N.	3.0315	1.5051	0.1373	0.0469	-1.5016	0.6983
Non-noise	2.9974	1.4913	0.1334	0.0671	-1.4999	0.7000
True values	3.0000	1.5000	0.1333	0.0667	-1.5000	0.7000
$\mathbf{N} = 1000$	c_1	c_2	d_1	d_2	-	-
10% C. N.	0.2738	0.3924	-0.8619	0.7379	-	-
True values	0.2000	0.1000	-0.9000	0.8500	-	-

Table 3.2B Parameter identification errors of Hammerstein system

N = 1000	$\lvert\Delta\beta_2\rvert$	$\lvert\Delta\beta_3\rvert$	$\lvert\Delta f_1\rvert$	$\lvert\Delta f_2\rvert$	$\lvert\Delta b_1\rvert$	$\lvert\Delta b_2\rvert$	$\lvert\Delta\rvert$
10% C. N.	0.1206	0.1443	0.0379	0.074	0.0036	0.0047	0.0641
20% W. N.	0.2751	0.7686	0.0083	0.0431	0.0775	0.0461	0.2031
10% W. N.	0.0861	0.1330	0.0008	0.0356	0.0010	0.0034	0.0433
5% (Kort.)	0.0240	0.0191	0.0081	0.0141	0.0067	0.0385	0.0184
5% W. N.	0.0315	0.0051	0.004	0.0198	0.0016	0.0017	0.0106

Table 3.3A Identification results of the Feedback-Wiener system

N = 2000	*rfa*	b_1	b_2	f_1	f_2	β_2	β_3
10% C. N.	0.1005	0.1323	0.0334	-1.5297	0.7446	3.0081	0.2417
20% W. N.	0.0997	0.0562	0.0757	-1.4984	0.7671	2.9780	2.6249
10% W. N.	0.0997	0.0802	0.0754	-1.4957	0.7411	3.0027	2.4885
5% W. N.	0.0998	0.0997	0.0729	-1.4962	0.7243	3.0222	2.3083
Non-noise	0.1000	0.1333	0.0667	-1.5000	0.7000	3.0001	1.5007
True values	0.1000	0.1333	0.0667	-1.5000	0.7000	3.0000	1.5000
N = 2000	c_1	c_2	d_1	d_2	-	-	-
10% C. N.	0.3564	0.3152	-0.7950	0.7171	-	-	-
True values	0.2000	0.1000	-0.9000	0.8500	-	-	-

Table 3.3B Parameter identification errors of Feedback-Wiener system

N = 2000	$\lvert\Delta\alpha\rvert$	$\lvert\Delta b_1\rvert$	$\lvert\Delta b_2\rvert$	$\lvert\Delta f_1\rvert$	$\lvert\Delta f_2\rvert$	$\lvert\Delta\beta_2\rvert$	$\lvert\Delta\beta_3\rvert$	$\lvert\Delta\rvert$
10% C.	0.0005	0.001	0.0333	0.0297	0.0446	0.008	1.2583	0.1953
20% W.	0.0003	0.0771	0.0090	0.0016	0.0671	0.0221	1.1249	0.1828
10% W.	0.0003	0.0531	0.0087	0.0043	0.0411	0.0026	0.9885	0.1565
5% W.	0.0002	0.0336	0.0062	0.0038	0.0243	0.0221	0.8083	0.1252

Table 3.4A Identification results of the Feedback-Hammerstein system

N = 2000	a_1^*	a_2^*	*rfa*	b_2	β_2	β_3	β^*
10% C. N.	-1.3617	0.7489	0.1348	0.0680	0.3122	0.1124	0.4556
20% W. N.	-1.3732	0.7520	0.1353	0.0666	0.2750	0.0757	0.3664
10% W. N.	-1.3719	0.7559	0.1347	0.0666	0.3208	0.1257	0.4456
5% W. N.	-1.3700	0.7592	0.1343	0.0667	0.3506	0.1540	0.4545
Non-noise	-1.3667	0.7667	0.1333	0.0667	0.3999	0.1998	0.4999
True values	-1.3667	0.7667	0.1333	0.0667	0.3999	0.1999	0.5004
N = 2000	c_1	c_2	-	-	-	-	-
10% C. N.	0.3004	0.1649	-	-	-	-	-
True values	0.2000	0.1000	-	-	-	-	-

Table 3.4B Parameter identification errors of Feedback-Hammerstein system

N = 2000	$\lvert\Delta a_1^*\rvert$	$\lvert\Delta a_2^*\rvert$	$\lvert\Delta\alpha\rvert$	$\lvert\Delta b_2\rvert$	$\lvert\Delta\beta_2\rvert$	$\lvert\Delta\beta_3\rvert$	$\lvert\Delta\beta^*\rvert$	$\lvert\Delta\rvert$
10% C.	0.0050	0.0178	0.0015	0.0013	0.0877	0.0875	0.0448	0.0225
20% W.	0.0065	0.0147	0.0020	1E-04	0.1249	0.1242	0.134	0.0403
10% W.	0.0052	0.0108	0.0014	1E-04	0.0791	0.0742	0.0548	0.0216
5% W.	0.0033	0.0075	0.0010	0.0000	0.0493	0.0459	0.0459	0.0152

Table 3.5 Comparison of avarage parameter identification errors $\lvert\Delta\rvert$ of the four structures

N = 2000	Wiener	Hammerstein	Feedback-Wiener	Feedback-Hammerstein	Best accuracy
10% C.	0.0418	0.0641	0.1953	0.0225	Feedback-Ha.
20% W.	0.0770	0.2031	0.1828	0.0403	Feedback-Ha.
10% W.	0.0155	0.0433	0.1565	0.0216	Wiener
5% W.	0.0151	0.0106	0.1252	0.0152	Hammerstein

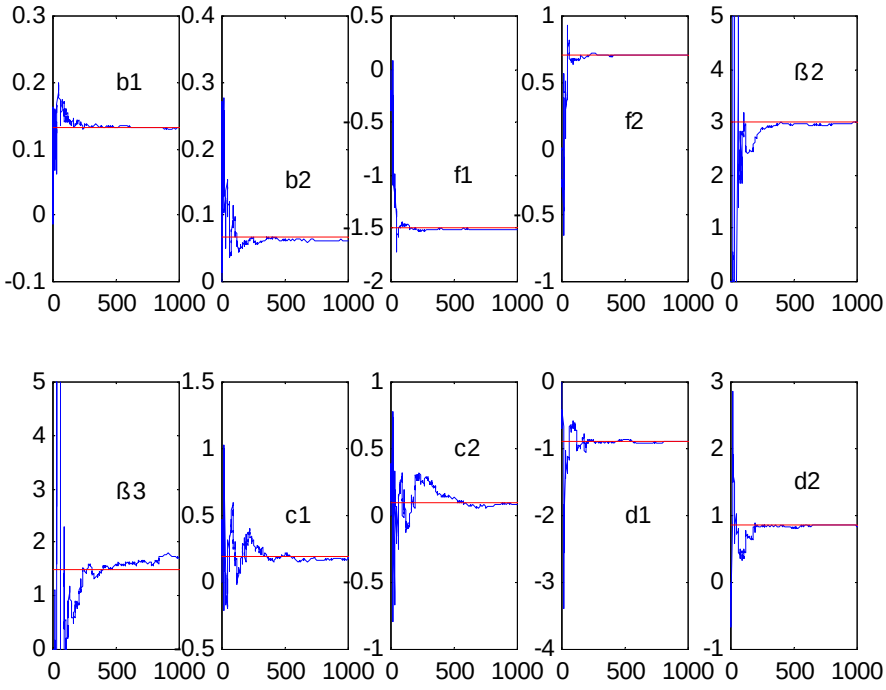


Fig. 3.5 The Wiener system identification process
with a N./S.=10% colored measurement noise

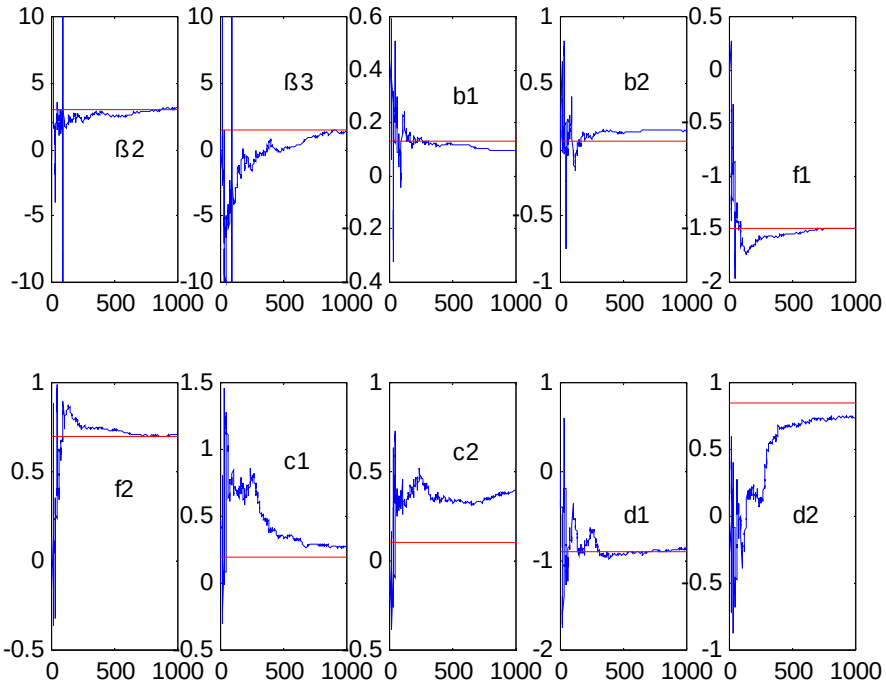


Fig. 3.6 The Hammerstein system identification process with a N./S.=10% colored measurement noise

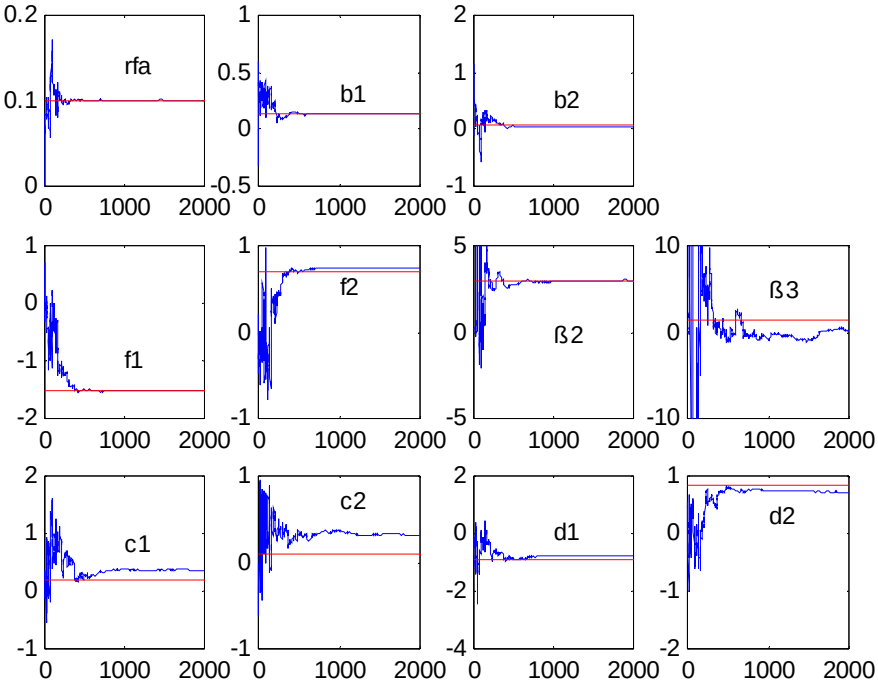


Fig. 3.7 The Feedback-Wiener system identification process with a N./S.=10% colored measurement noise

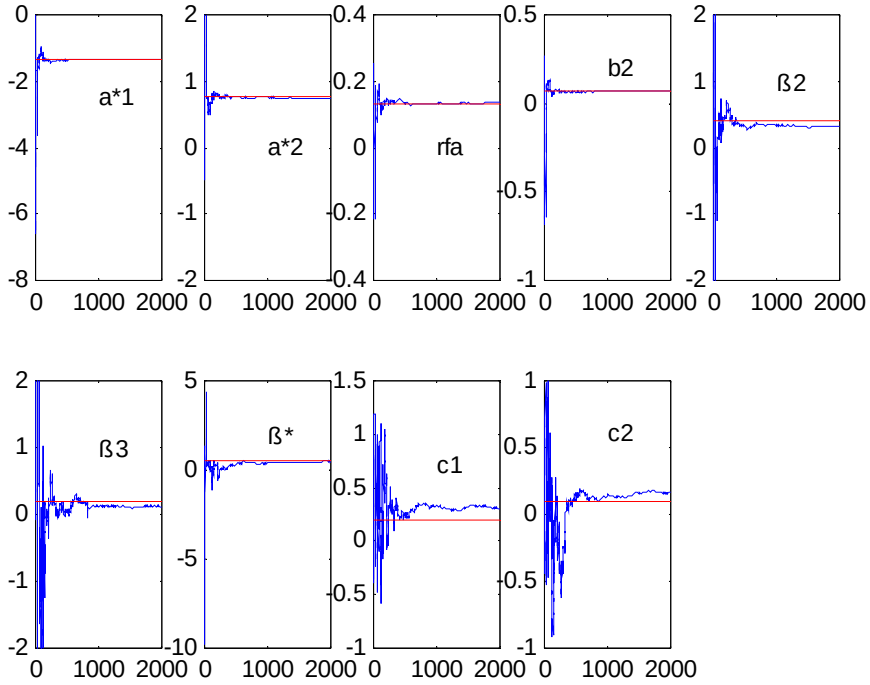


Fig. 3.8 The Feedback-Hammerstein system identification process with a N./S.=10% colored measurement noise

The simulation results show that:

- The new identification method gives good results for all the four structures.
- The accuracy for the two standard examples of the Wiener system and the Hammerstein system is better than in Kortmann (1989).
- From Table 3.5, we can see that which one of the four structures has the best accuracy results under different measurement noises.

3.2. MISO Wiener and Hammerstein systems

Some known identification methods for MISO Wiener and Hammerstein systems were introduced in literature. For example, in Boutayeb and Darouach (1994), two recursive identification methods were extended to a MISO Hammerstein system along the lines of the basic Kalman filter. Kortmann (1987) extended the identification method for a SISO Wiener system and a Hammerstein system to MISO cases with a second procedure based on the recursive prediction error method. In this section, the new identification method is used to identify:

- a MISO Wiener system (Fig. 3.9),
- a MISO Hammerstein system (Fig. 3.10) and
- a MISO mixed Wiener and Hammerstein system (Fig. 3.11).

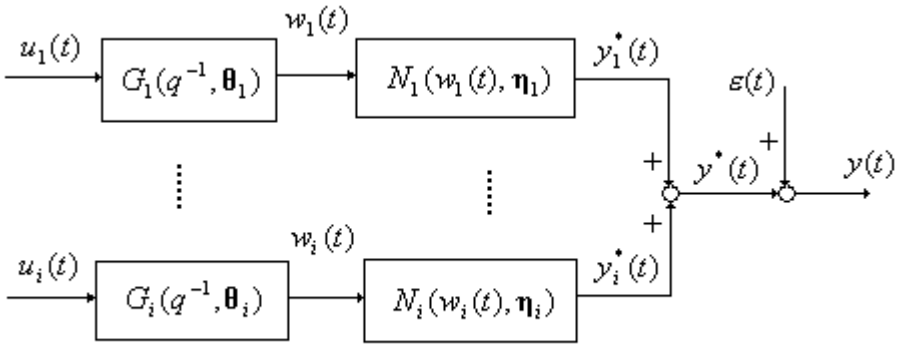


Fig. 3.9 MISO Wiener system

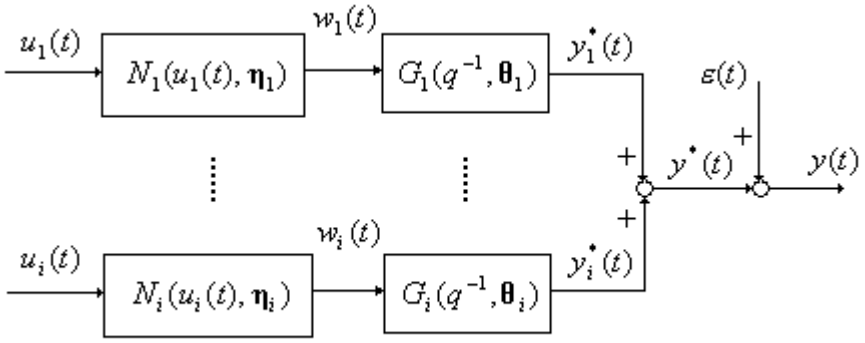


Fig. 3.10 MISO Hammerstein system

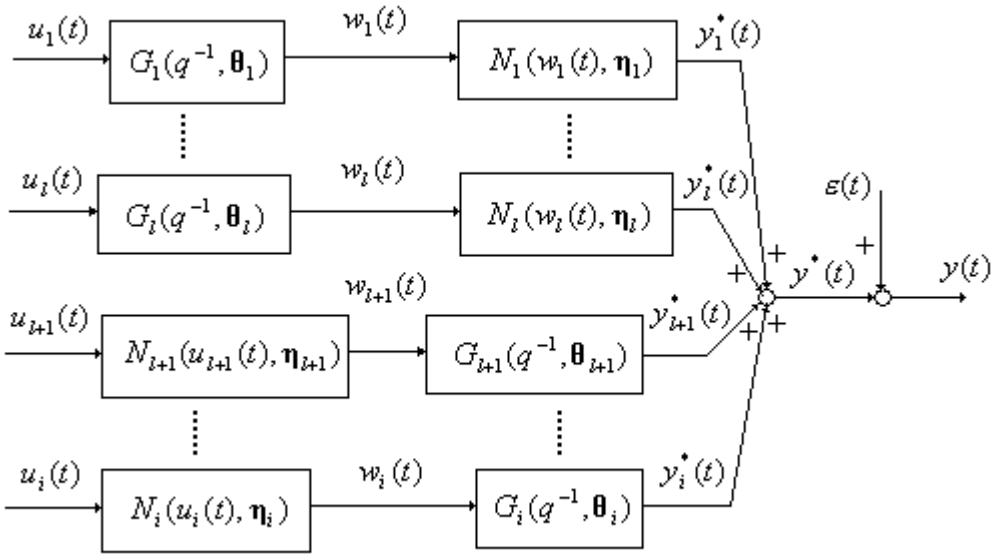


Fig. 3.11 MISO mixed Wiener and Hammerstein system

In the above Figures, we make use of the following notations: $u_1(t),\cdots,u_l(t),u_{l+1}(t),\cdots,u_i(t)$ are i system inputs, $y(t)$ is the single system output, $y^*(t)$ is the unmeasurable system output without measurement noise. $y_1^*(t),\cdots,y_l^*(t)$, $y_{l+1}^*(t),\cdots,y_i^*(t)$ are i unmeasurable outputs of the branches.

The intermediate variables $w_1(t),\cdots,w_l(t)$, $w_{l+1}(t),\cdots,w_i(t)$ can not be measured. $\boldsymbol{\theta}_1,\cdots,\boldsymbol{\theta}_l,\boldsymbol{\theta}_{l+1},\cdots,\boldsymbol{\theta}_i$ are parameter vectors determining linear dynamic subsystems, $G_1(q^{-1},\boldsymbol{\theta}_1),\cdots, G_l(q^{-1},\boldsymbol{\theta}_l), G_{l+1}(q^{-1},\boldsymbol{\theta}_{l+1}),\cdots, G_i(q^{-1},\boldsymbol{\theta}_i)$, respectively. $\boldsymbol{\eta}_1,\cdots,\boldsymbol{\eta}_l,\boldsymbol{\eta}_{l+1},\cdots,\boldsymbol{\eta}_i$ are parameter vectors determining nonlinear static subsystems, $N_1(\cdot,\boldsymbol{\eta}_1),\cdots,N_l(\cdot,\boldsymbol{\eta}_l), N_{l+1}(\cdot,\boldsymbol{\eta}_{l+1}),\cdots,N_i(\cdot,\boldsymbol{\eta}_i)$, respectively. $\varepsilon(t)$ is the colored measurement noise which is a white noise $e(t)$ through a linear filter $H(q^{-1},\boldsymbol{\xi})$. In MISO mixed Wiener and Hammerstein system (Fig. 3.11), there are l-inputs for Wiener branches and s-inputs for Hammerstein branches, and $l+s=i$.

3.2.1. MISO Wiener system

Considering the $j-th$ branch, $j=1,\cdots,i$, of a MISO Wiener system which consists of i single Wiener branches as shown in Fig. 3.9. The linear dynamic subsystem $G_j(q^{-1},\boldsymbol{\theta}_j)$ can be described by using a RTF model of order n_j

$$w_j(t)=\frac{B_j(q^{-1})}{F_j(q^{-1})}u_j(t) \tag{3.26a}$$

where $B_j(q^{-1})$ and $F_j(q^{-1})$ for $j=1,\cdots,i$ are coprime and

$$B_j(q^{-1})=b_{j0}+b_{j1}q^{-1}+\cdots+b_{jn_j}q^{-n_j} \tag{3.26b}$$

$$F_j(q^{-1})=1+f_{j1}q^{-1}+\cdots+f_{jn_j}q^{-n_j}. \tag{3.26c}$$

The static nonlinear subsystem $N_j(w_j(t),\boldsymbol{\eta}_j)$ is assumed to be a polynomial of order m_j

$$\begin{aligned} y_j^*(t)&=\sum_{k=1}^{m_j}\beta_{jk}\cdot w_j^k(t)\\ &=\beta_{j1}\cdot w_j(t)+\sum_{k=2}^{m_j}\beta_{jk}\cdot w_j^k(t)\end{aligned} \tag{3.27}$$

and $w_j(t)$ in the first term of Eq. (3.27) is the key term.

Half-substituting Eq. (3.26a) into the key term $w_j(t)$ of Eq. (3.27), we get the whole system output

$$\begin{aligned} y(t)&=\sum_{j=1}^{i}y_j^*(t)+\varepsilon(t)\\ &=\sum_{j=1}^{i}\left[\frac{\beta_{j1}\cdot B_j(q^{-1})}{F_j(q^{-1})}u_j(t)+\sum_{k=2}^{m_j}\beta_{jk}\cdot w_j^k(t)\right]+\frac{C(q^{-1})}{D(q^{-1})}e(t).\end{aligned} \tag{3.28}$$

To avoid the over-parameterization problem, one parameter in $\beta_{j1},b_{j1},b_{j2},\cdots,b_{jn}$ for $j=1,\cdots,i$, that is, i parameters in Eq. (3.28) should be fixed. From Eq. (3.28) it can be concluded that a MISO

Wiener system can be transformed approximately into a pseudo-linear MISO system which has $\sum_{j=1}^{i} m_j$ independent inputs: $u_j(t)$ and the recursively estimated intermediate variables $w_j^2(t), w_j^3(t), \cdots, w_j^{m_j}(t)$ according to Eq. (3.26a) for $j = 1, \cdots, i$. All the unknown parameters of the above pseudo-linear MISO system are explicitly given.

3.2.2. MISO Hammerstein system

Considering the $j-th$ branch, $j = 1, \cdots, i$, of a MISO Hammerstein system which consists of i single Hammerstein branches as shown in Fig. 3.10. The nonlinear static subsystem $N_j(u_j(t), \boldsymbol{\eta}_j)$ is assumed to be a polynomial of order m_j

$$w_j(t) = \sum_{k=1}^{m_j} \beta_{jk} \cdot u_j^k(t). \tag{3.29}$$

Using a RTF model of order n_j for the dynamic linear subsystem $G_j(q^{-1}, \boldsymbol{\theta}_j)$, we find

$$\begin{aligned} y_j^*(t) &= \frac{B_j(q^{-1})}{F_j(q^{-1})} w_j(t) \\ &= b_{j0} \cdot w_j(t) + \frac{B_j^*(q^{-1})}{F_j(q^{-1})} w_j(t). \end{aligned} \tag{3.30a}$$

As in the SISO case, here we separate a part $b_{j0} \cdot w_j(t)$ from $\frac{B_j(q^{-1})}{F_j(q^{-1})} w_j(t)$. According to the function definitions before, $B_j^*(q^{-1})$ in the second part is

$$B_j^*(q^{-1}) = b_{j1}^* q^{-1} + b_{j2}^* q^{-2} + \cdots + b_{jn_j}^* q^{-n_j} \tag{3.30b}$$

with

$$b_{jk}^* = b_{jk} - b_{j0} \cdot f_{jk} \tag{3.30c}$$

for $k = 1, 2, \cdots, n_j$. The first term $w_j(t)$ of Eq. (3.30a) is the $j-th$ key term.

Half-substituting Eq. (3.29) into the key term $w_j(t)$ of Eq. (3.30a), then the output of MISO Hammerstein system can be written as

$$\begin{aligned} y(t) &= \sum_{j=1}^{i} y_j^*(t) + \varepsilon(t) \\ &= \sum_{j=1}^{i} \left[\sum_{k=1}^{m_j} b_{j0} \cdot \beta_{jk} \cdot u_j^k(t) + \frac{B_j^*(q^{-1})}{F_j(q^{-1})} w_j(t) \right] + \frac{C(q^{-1})}{D(q^{-1})} e(t). \end{aligned} \tag{3.31}$$

We can also get another alternative. Because of the special structure of MISO Hammerstein system, Eq. (3.30a) can be simplified and transformed into an equivalent ARMAX model. Kortmann and Unbehauen (1987) also used such a model to identify MISO Hammerstein systems.

From Eq. (3.30a) the whole system output can be rewritten in an equivalent linear MISO ARMAX model form

$$\begin{aligned}\widetilde{A}(q^{-1})y(t) &= \sum_{j=0}^{i}\widetilde{B}_j(q^{-1})w_j(t) + C(q^{-1})e(t) \\ &= \sum_{j=1}^{i}\widetilde{b}_{j0}\cdot w_j(t) + \sum_{j=1}^{i}\widetilde{B}_j^*(q^{-1})w_j(t) + C(q^{-1})e(t)\end{aligned} \tag{3.32}$$

where the common denominator $A^*(q^{-1})$ can be formed as

$$\begin{aligned}\widetilde{A}(q^{-1}) &= \prod_{k=1}^{i}F_k(q^{-1}) \\ &= 1+\widetilde{a}_1q^{-1}+\widetilde{a}_2q^{-2}+\cdots+\widetilde{a}_{\widetilde{n}}q^{-\widetilde{n}}\end{aligned} \tag{3.33}$$

with the order $\widetilde{n}=\sum_{j=1}^{i}n_j$ and the corresponding extended nominator in the $j-th$ branch is

$$\begin{aligned}\widetilde{B}_j(q^{-1}) &= (b_{j0}+b_{j1}q^{-1}+\cdots+b_{n_j}q^{-n_j})\cdot\prod_{\substack{k=1\\k\neq j}}^{i}F_k(q^{-1}) \\ &= \widetilde{b}_{j0}+\widetilde{b}_{j1}q^{-1}+\cdots+\widetilde{b}_{j\widetilde{n}_j}q^{-\widetilde{n}_j}\end{aligned} \tag{3.34}$$

with the order $\widetilde{n}_j=n_j+\sum_{\substack{k=1\\k\neq j}}^{i}n_k$.

In Eq. (3.32), a part $\widetilde{b}_{j0}\cdot w_j(t)$ is separated from $\widetilde{B}_j(q^{-1})w_j(t)$ with $\widetilde{B}_j^*(q^{-1})=\widetilde{b}_{j1}q^{-1}+\widetilde{b}_{j2}q^{-2}+\cdots+\widetilde{b}_{j\widetilde{n}_j}q^{-\widetilde{n}_j}$. And the $w_j(t)$ in $\widetilde{b}_{j0}\cdot w_j(t)$ is the key term in the $j-th$ branch.

Half-substituting Eq. (3.29) into the key term $w_j(t)$ of Eq. (3.32), the whole system output of MISO a Hammerstein system can be written as

$$\widetilde{A}(q^{-1})y(t)=\sum_{j=1}^{i}\left[\sum_{k=1}^{m_j}\widetilde{b}_{j0}\cdot\beta_{jk}\cdot u_j^k(t)+\widetilde{B}_j^*(q^{-1})w_j(t)\right]+C(q^{-1})e(t). \tag{3.35}$$

Both Eq. (3.31) and Eq. (3.35) can be used to identify MISO Hammerstein system. But in Eq. (3.35), only the equivalent parameters $\widetilde{a}_k$ for $k=1,\cdots,\widetilde{n}$ and $\widetilde{b}_{jk}$ for $k=1,\cdots,\widetilde{n}_j$, $j=1,\cdots,i$ in linear dynamic block can be directly identified. To avoid the over-parameterization problem, one parameter in $b_{j0},\beta_{j1},\beta_{j2},\cdots,\beta_{jm}$ or in $\widetilde{b}_{j0},\beta_{j1},\beta_{j2},\cdots,\beta_{jm}$ for $j=1,\cdots,i$, that is, i parameters in Eq. (3.31) or Eq. (3.35) should be fixed. From Eq. (3.31) and Eq. (3.35), it follows that a MISO Hammerstein system can be transformed approximately into a pseudo-linear MISO system which has

$\sum_{j=1}^{i}(m_j+1)$ independent inputs: $u_j(t), u_j^2(t), \cdots, u_j^{m_j}(t)$ and recursively estimated unmeasurable intermediate variables $w_j(t)$ according to Eq. (3.29) for $j=1,\cdots,i$. All the unknown parameters of the pseudo-linear MISO system are explicitly given.

3.2.3. MISO mixed Wiener and Hammerstein system

Considering MISO mixed Wiener and Hammerstein system of Fig. 3.11 which consists of a l Wiener branches and s Hammerstein branches, and $l+s=i$. In the $j-th$ branch of the MISO Wiener system for $j=1,\cdots,l$, the linear dynamic subsystem $G_j(q^{-1},\boldsymbol{\theta}_j)$ can be described by using a RTF model of order n_j and the nonlinear static subsystem $N_j(w_j(t),\boldsymbol{\eta}_j)$ is assumed to be a polynomial of order m_j. On the other hand, in the $j-th$ branch of the MISO Hammerstein system for $j=1,\cdots,s$, the nonlinear static subsystem $N_j(u_j(t),\boldsymbol{\eta}_j)$ is assumed to be a polynomial of order m_j and the linear dynamic subsystem $G_j(q^{-1},\boldsymbol{\theta}_j)$ is described by using a RTF model of order n_j.

According to the derivation in section 3.2.1 and 3.2.2, half-substituting Eq. (3.26a) and Eq. (3.29) into the corresponding key terms, the output of a MISO mixed Wiener and Hammerstein system can be written as

$$\begin{aligned} y(t) &= \sum_{j=1}^{l}\left[\beta_{j1}\cdot\frac{B_j(q^{-1})}{F_j(q^{-1})}u_j(t)+\sum_{k=2}^{m_j}\beta_{jk}\cdot w_j^k(t)\right] \\ &+\sum_{j=1}^{s}\left[\sum_{k=1}^{m_j}\tilde{b}_{j0}\cdot\beta_{jk}\cdot u_j^k(t)+\tilde{B}_j^*(q^{-1})w_j(t)\right]+\frac{C(q^{-1})}{D(q^{-1})}e(t). \end{aligned} \tag{3.36}$$

To avoid the over-parameterization problem, one parameter of $\beta_{j1}, b_{j1}, b_{j2}, \cdots, b_{jn}$ for $j=1,\cdots,l$, that is, l parameters in the MISO Wiener systems should be fixed and one parameter of $b_{j0}, \beta_{j1}, \beta_{j2}, \cdots, \beta_{jm}$ for $j=1,\cdots,s$, that is, s parameters in the Hammerstein systems should be fixed. Eq. (3.36) shows that a MISO mixed Wiener and Hammerstein system can be transformed approximately into a pseudo-linear MISO system which has $\sum_{j=1}^{l}m_j+\sum_{j=1}^{s}(m_j+1)$ independent inputs: $u_j(t)$ and $w_j^2(t), w_j^3(t), \cdots, w_j^{m_j}(t)$ for $j=1,\cdots,l$; $u_j(t), u_j^2(t), \cdots, u_j^{m_j}(t)$ and $w_j(t)$ for $j=1,\cdots,s$. The unknown intermediate variables can be estimated recursively according to the corresponding equations Eq. (3.26a) and Eq. (3.29). All the unknown parameters of the pseudo-linear MISO system Eq. (3.36) are explicitly given and can be identified without parameter redundancy.

3.2.4. Simulation results

In this section the simulation results for three simple examples will be given, namely for,

- a MISO Wiener system of the following two branches (we fix $\beta_{11}=1$ and $\beta_{21}=1$):

$$\begin{cases} w_1(t)=\dfrac{0.04308q^{-1}+0.0315q^{-2}}{1-1.3139q^{-1}+0.3886q^{-2}}u_1(t) \\ y_1^*(t)=w_1(t)+4w_1^2(t)+2w_1^3(t) \end{cases} \tag{3.37}$$

and

$$\begin{cases} w_2(t)=\dfrac{0.0305q^{-1}+0.0254q^{-2}}{1-1.5218q^{-1}+0.5778q^{-2}}u_2(t) \\ y_2^*(t)=w_2(t)+3w_2^2(t)+2w_2^3(t) \end{cases} \tag{3.38}$$

- a MISO Hammerstein system of the following two branches (we fix $b_{10}=1$ and $b_{20}=1$):

$$\begin{cases} w_1(t)=u_1(t)+3u_1^2(t)+1.5u_1^3(t) \\ y_1^*(t)=\dfrac{1+0.1333q^{-1}+0.0667q^{-2}}{1-1.5q^{-1}+0.7q^{-2}}w_1(t) \end{cases} \tag{3.39}$$

and

$$\begin{cases} w_2(t)=u_2(t)+4u_2^2(t)+2u_2^3(t) \\ y_2^*(t)=\dfrac{1+0.4q^{-1}+0.3q^{-2}}{1-0.9q^{-1}+0.6q^{-2}}w_2(t) \end{cases} \tag{3.40}$$

- a MISO mixed Wiener and Hammerstein of the following two branches (we fix $\beta_{11}=1$ and $b_{20}=1$):

$$\begin{cases} w_1(t)=\dfrac{0.4q^{-1}+0.3q^{-2}}{1-0.9q^{-1}+0.6q^{-2}}u_1(t) \\ y_1^*(t)=w_1(t)+4w_1^2(t)+2w_1^3(t) \end{cases} \tag{3.41}$$

and

$$\begin{cases} w_2(t) = u_2(t) + 3u_2^2(t) + 1.5u_2^3(t) \\ y_2^*(t) = \dfrac{1 + 0.1333q^{-1} + 0.0667q^{-2}}{1 - 1.5q^{-1} + 0.7q^{-2}} w_2(t). \end{cases} \tag{3.42}$$

For the all three MISO systems, two random numbers of zero mean are used as system inputs $u_1(t)$ and $u_2(t)$. The third independent random numbers is used as white measurement noise $e(t)$. $\mathbf{N} = 2000$ data points are collected for each case.

An average smoother using a moving window with fixed length $Mov = 4$ will be used in order to filter the corresponding estimated parameters to calculate the unmeasurable intermediate variables $w_1(t)$ and $w_2(t)$.

Then the standard recursive prediction errors method (RPEM) function for linear MISO system with forgetting factor algorithms in MATLAB will be applied. The algorithm variable settings are $\lambda(0) = 0.7$, $\Delta\lambda = 0.01$. The initial estimates of the unknown parameters are taken as zero.

The linear filter for a colored measurement noise is given by

$$H(q^{-1}, \xi) = \frac{1 + 0.2q^{-1} + 0.1q^{-2}}{1 - 0.9q^{-1} + 0.85q^{-2}}. \tag{3.43}$$

We consider different measurement noise levels for the Three MISO Wiener and Hammerstein systems. Besides the non-noisy case, $N./S. = 5\%$, 10% and 20% white and 10% colored measurement noise, are added to the system output, respectively.

Identification results with different measurement noises are shown in Tables 3.6A-3.8A. The identification processes with a N./S.=10% colored measurement noise are shown in Figs. 3.12-3.14. The red lines in Figs. 3.12-3.14 are the real values of parameters.

Tables 3.6B-3.8B show:

- every parameter identification error Δp,
- the average parameter identification error $|\Delta_1| = \dfrac{\sum |\Delta p_I|}{\zeta_I}$ for the first branch, ζ_I is the parameter number of the first branch,
- the average parameter identification error $|\Delta_2| = \dfrac{\sum |\Delta p_{II}|}{\zeta_{II}}$ for the second branch, ζ_{II} is the parameter number of the second branch,
- the average parameter identification error $|\Delta| = \dfrac{\sum |\Delta p|}{\zeta}$ of each structure with different measurement noises, ζ is the whole parameter number.

Comparison of the average parameter identification errors $|\Delta|$ of the three structures is shown in Table 3.9.

Table 3.6A Identification results of the MISO Wiener system

N = 2000	b_{11}	b_{12}	f_{11}	f_{12}	β_{12}	β_{13}
10% C. N.	0.0450	0.0237	-1.3725	0.4420	4.3116	2.2480
20% W. N.	0.0466	0.0206	-1.3946	0.4627	4.2612	0.8760
10% W. N.	0.0453	0.0239	-1.3700	0.4401	4.3001	1.8561
5% W. N.	0.0450	0.0249	-1.3614	0.4321	4.3113	2.1100
Non-noise	0.0431	0.0315	-1.3139	0.3886	4.0000	1.9999
True values	0.0431	0.0315	-1.3139	0.3886	4.0000	2.0000
N = 2000	b_{21}	b_{22}	f_{21}	f_{22}	β_{22}	β_{23}
10% C. N.	0.0256	0.0305	-1.5228	0.5789	2.8777	2.1427
20% W. N.	0.0340	0.0168	-1.5704	0.6203	2.8423	2.0197
10% W. N.	0.0323	0.0210	-1.5484	0.6008	2.9889	2.0986
5% W. N.	0.0316	0.0222	-1.5419	0.5952	2.9861	2.0621
Non-noise	0.0305	0.0254	-1.5218	0.5778	3.0000	2.0000
True values	0.0305	0.0254	-1.5218	0.5778	3.0000	2.0000
N = 2000	c_1	c_2	d_1	d_2	-	-
10% C. N.	0.2568	0.1943	-0.8703	0.8237	-	-
True values	0.2000	0.1000	-0.9000	0.8500	-	-

Table 3.6B Parameter identification errors of the MISO Wiener system

N = 2000	$\lvert\Delta b_{11}\rvert$	$\lvert\Delta b_{12}\rvert$	$\lvert\Delta f_{11}\rvert$	$\lvert\Delta f_{12}\rvert$	$\lvert\Delta\beta_{12}\rvert$	$\lvert\Delta\beta_{13}\rvert$	$\lvert\Delta_1\rvert$	$\lvert\Delta\rvert$
10% C.	0.0019	0.0078	0.0586	0.0534	0.3116	0.2480	0.1135	0.0798
20% W.	0.0035	0.0109	0.0807	0.0741	0.2612	1.1240	0.2590	0.1529
10% W.	0.0022	0.0076	0.0561	0.0515	0.3001	0.1439	0.0935	0.0605
5% W.	0.0019	0.0066	0.0475	0.0435	0.3113	0.1100	0.0868	0.0532
N = 2000	$\lvert\Delta b_{21}\rvert$	$\lvert\Delta b_{22}\rvert$	$\lvert\Delta f_{21}\rvert$	$\lvert\Delta f_{22}\rvert$	$\lvert\Delta\beta_{22}\rvert$	$\lvert\Delta\beta_{23}\rvert$	$\lvert\Delta_2\rvert$	-
10% C.	0.0049	0.0051	0.001	0.0011	0.1223	0.1427	0.0461	-
20% W.	0.0035	0.0086	0.0486	0.0425	0.1577	0.0197	0.0467	-
10% W.	0.0018	0.0044	0.0266	0.0230	0.0111	0.0986	0.0278	-
5% W.	0.0011	0.0032	0.0201	0.0174	0.0139	0.0621	0.0196	-

Table 3.7A Identification results of the MISO Hammerstein system

N = 2000	β_{11}	β_{12}	β_{13}	b_{11}	b_{12}	f_{11}	f_{12}
10% C. N.	0.7637	3.1767	1.8290	0.1144	0.0690	-1.4991	0.7010
20% W. N.	1.2362	3.2883	0.9522	0.1628	0.1087	-1.4951	0.7003
10% W. N.	1.1684	3.1985	1.1184	0.1447	0.0968	-1.4974	0.7010
5% W. N.	1.1187	3.1384	1.2280	0.1418	0.0760	-1.5014	0.7028
Non-noise	1.0005	3.0005	1.4995	0.1321	0.0671	-1.5001	0.7000
True values	1.0000	3.0000	1.5000	0.1333	0.0667	-1.5000	0.7000
N = 2000	β_{21}	β_{22}	β_{23}	b_{21}	b_{22}	f_{21}	f_{22}
10% C. N.	0.4765	4.0301	3.2280	0.3677	0.2911	-0.9058	0.6028
20% W. N.	0.7458	4.1500	2.1288	0.3591	0.3564	-0.8837	0.6180
10% W. N.	0.8235	4.0770	2.0893	0.3972	0.3075	-0.8945	0.6088
5% W. N.	0.8803	4.0327	2.0403	0.4013	0.2781	-0.9052	0.6077
Non-noise	0.9996	4.0006	2.0011	0.4002	0.2999	-0.9000	0.6000
True values	1.0000	4.0000	2.0000	0.4000	0.3000	-0.9000	0.6000
N = 2000	c_1	c_2	d_1	d_2	-	-	-
10% C. N.	0.2153	0.0901	-0.8484	0.0035	-	-	-
True values	0.2000	0.1000	-0.9000	0.8500	-	-	-

Table 3.7B Parameter identification errors of the MISO Hammerstein system

N = 2000	$\lvert\Delta\beta_{11}\rvert$	$\lvert\Delta\beta_{12}\rvert$	$\lvert\Delta\beta_{13}\rvert$	$\lvert\Delta b_{11}\rvert$	$\lvert\Delta b_{12}\rvert$	$\lvert\Delta f_{11}\rvert$	$\lvert\Delta f_{12}\rvert$	$\lvert\Delta_1\rvert$	$\lvert\Delta\rvert$
10% C.	0.2363	0.1767	0.329	0.0189	0.0023	0.0009	0.0010	0.1091	0.1851
20% W.	0.2362	0.2883	0.5478	0.0295	0.042	0.0049	0.0003	0.1641	0.1282
10% W.	0.1684	0.1985	0.3816	0.0114	0.0301	0.0026	0.0010	0.1132	0.0822
5% W.	0.1187	0.1384	0.272	0.0085	0.0093	0.0014	0.0028	0.0783	0.0549
N = 2000	$\lvert\Delta\beta_{21}\rvert$	$\lvert\Delta\beta_{22}\rvert$	$\lvert\Delta\beta_{23}\rvert$	$\lvert\Delta b_{21}\rvert$	$\lvert\Delta b_{22}\rvert$	$\lvert\Delta f_{21}\rvert$	$\lvert\Delta f_{22}\rvert$	$\lvert\Delta_2\rvert$	-
10% C.	0.5235	0.0301	1.2280	0.0323	0.0089	0.0058	0.0028	0.2612	-
20% W.	0.2542	0.1500	0.1288	0.0409	0.0564	0.0163	0.0180	0.0923	-
10% W.	0.1765	0.077	0.0893	0.0028	0.0075	0.0055	0.0088	0.0512	-
5% W.	0.1197	0.0327	0.0403	0.0013	0.0219	0.0052	0.0077	0.0315	-

Table 3.8A Identification results of the MISO mixed Wiener and Hammerstein system

N = 2000	b_{11}	b_{12}	f_{11}	f_{12}	β_{12}	β_{13}
10% C. N.	0.3688	0.3642	-0.8977	0.5340	3.6810	1.2669
20% W. N.	0.3686	0.4144	-0.8300	0.5399	4.2654	1.9924
10% W. N.	0.3664	0.3973	-0.8385	0.5617	4.2437	2.0838
5% W. N.	0.3731	0.3710	-0.8505	0.5731	4.2533	2.1117
Non-noise	0.4054	0.2927	-0.9082	0.6023	3.9122	1.9183
True values	0.4000	0.3000	-0.9000	0.6000	4.0000	2.0000

N = 2000	β_{21}	β_{22}	β_{23}	b_{21}	b_{22}	f_{21}	f_{22}
10% C. N.	1.1164	2.6320	1.3696	0.2478	0.1195	-1.4665	0.6675
20% W. N.	0.8167	3.1847	1.7072	0.0456	0.0535	-1.5219	0.7143
10% W. N.	0.8603	3.1397	1.6690	0.0754	0.0656	-1.5135	0.7098
5% W. N.	0.8998	3.1046	1.5872	0.0489	0.0708	-1.5165	0.7117
Non-noise	1.0063	3.0335	1.5238	0.0874	0.0926	-1.4995	0.7000
True values	1.0000	3.0000	1.5000	0.1333	0.0667	-1.5000	0.7000
N = 2000	c_1	c_2	d_1	d_2	-	-	-
10% C. N.	0.2883	0.0841	-0.8763	0.6439	-	-	-
True values	0.2000	0.1000	-0.9000	0.8500	-	-	-

Table 3.8B Parameter identification errors of the MISO mixed Wiener and Hammerstein system

N = 2000	$\lvert\Delta b_{11}\rvert$	$\lvert\Delta b_{12}\rvert$	$\lvert\Delta f_{11}\rvert$	$\lvert\Delta f_{12}\rvert$	$\lvert\Delta\beta_{12}\rvert$	$\lvert\Delta\beta_{13}\rvert$	$\lvert\Delta_1\rvert$	$\lvert\Delta\rvert$
10% C.	0.0312	0.0642	0.0023	0.0660	0.3190	0.7331	0.2026	0.1618
20% W.	0.0314	0.1144	0.0700	0.0601	0.2654	0.0076	0.0914	0.0966
10% W.	0.0336	0.0973	0.0615	0.0383	0.2437	0.0838	0.0930	0.0844
5% W.	0.0269	0.0710	0.0495	0.0269	0.2533	0.1117	0.0898	0.0741

N = 2000	$\lvert\Delta\beta_{21}\rvert$	$\lvert\Delta\beta_{22}\rvert$	$\lvert\Delta\beta_{23}\rvert$	$\lvert\Delta b_{21}\rvert$	$\lvert\Delta b_{22}\rvert$	$\lvert\Delta f_{21}\rvert$	$\lvert\Delta f_{22}\rvert$	$\lvert\Delta_2\rvert$	-
10% C.	0.1164	0.368	0.1304	0.1145	0.0528	0.0335	0.0325	0.1211	-
20% W.	0.1833	0.1847	0.2072	0.0877	0.0132	0.0219	0.0143	0.1017	-
10% W.	0.1397	0.1397	0.1690	0.0579	0.0011	0.0135	0.0098	0.0758	-
5% W.	0.1002	0.1046	0.0872	0.0844	0.0041	0.0165	0.0117	0.0583	-

Table 3.9 Comparison of the average parameter identification errors $\lvert\Delta\rvert$ of the three structures

N = 2000	MISO Wiener	MISO Hammerstein	Mixed MISO W. H.	Best accuracy
10% C. N.	0.0798	0.1851	0.1618	MISO Wiener
20% W. N.	0.1529	0.1282	0.0966	MixedMISO W.H.
10% W. N.	0.0605	0.0822	0.0844	MISO Wiener
5% W. N.	0.0532	0.0549	0.0741	MISO Wiener

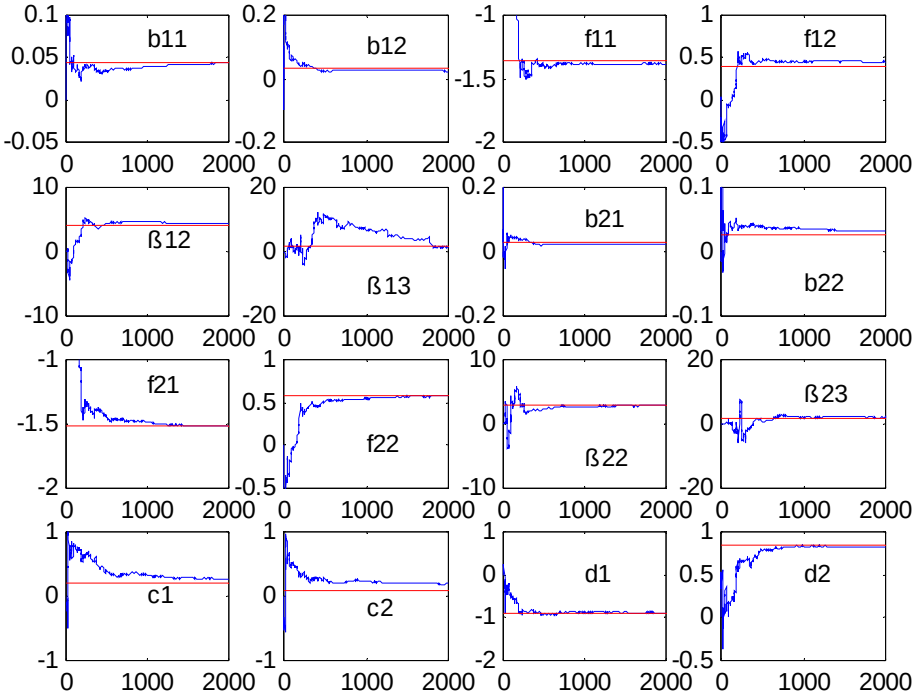


Fig. 3.12 The MISO Wiener system identification process with a N./S.=10% colored measurement noise

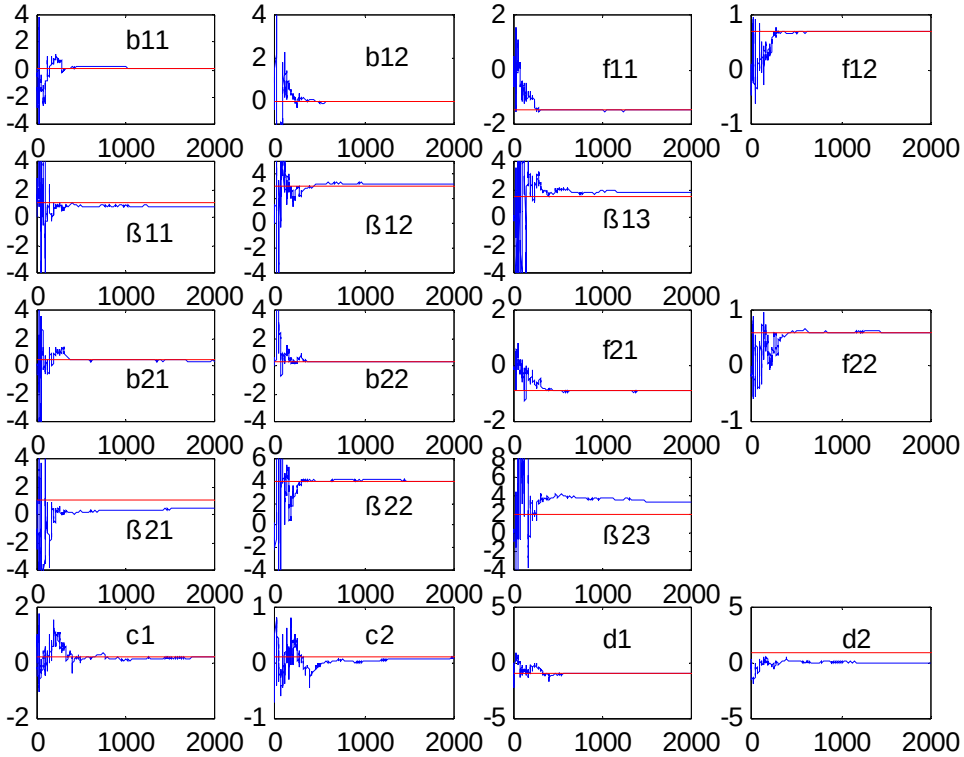


Fig. 3.13 The MISO Hammerstein system identification process with a N./S.=10% colored measurement noise

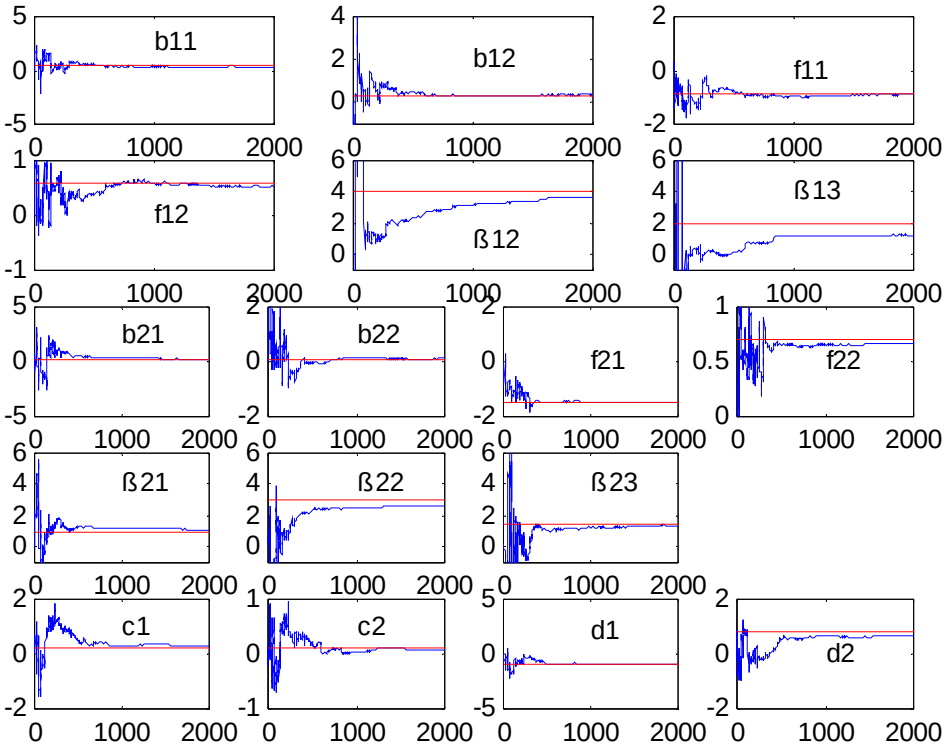


Fig. 3.14 The MISO mixed Wiener and Hammerstein system identification process with a N./S.=10% colored measurement noise

The simulation results show that:

- The new identification method gives good results for all the three structures.
- From Table 3.9, we can see that which one of the three structures has the best accuracy results under different measurement noises.

4. Identification of a Wiener system with different discontinuous nonlinearity

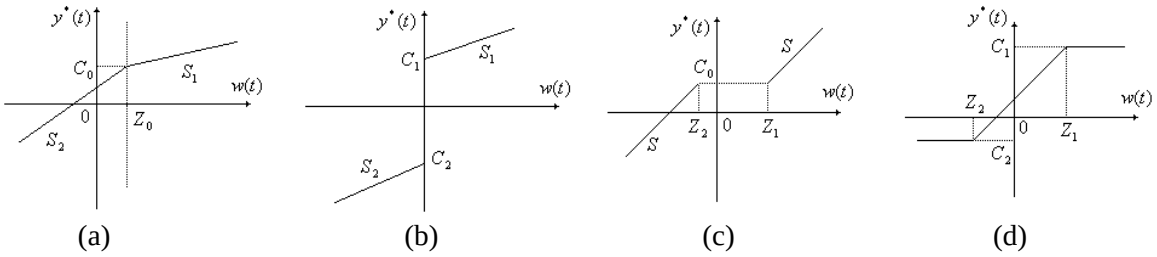
It is usually desirable to find a model as simple as possible for the data. Actually, there is an implicit (sometimes explicit) tradeoff between the acceptable model complexity and how well it matches the data. It is clear that some complicated continuous polynomials can also be used to approximate some discontinuous nonlinearities. But direct identification of these discontinuous nonlinearities with parameters of linear dynamic block together is no doubt more efficient.

Several researchers mainly studied identification methods for Hammerstein system with some simple discontinuous nonlinearities. Vörös (1997) identified directly some discontinuous nonlinearities in a Hammerstein system. Bai (2002) used also the same identification principle to identify the simple discontinuous nonlinearities in a Hammerstein system. Zeng (1999) identified some simple discontinuous nonlinearities in a Hammerstein and a Wiener system but with the inverse of the nonlinearities. The key technique is to definite some switch functions and intermediate variables to write the discontinuous nonlinearities in a continuous form.

In this chapter, we use the new identification method to identify the parameters of a Wiener system (Figure 3.1) with different discontinuous nonlinearities as the nonlinear static subsystem $N(w(t), \boldsymbol{\eta})$, respectively. For it the following discontinuous nonlinearities are taken into consideration:

- Direction-dependent nonlinearity (Figure 4.1 (a)),
- Preload nonlinearity (Figure 4.1 (b)),
- Dead-zone nonlinearity (Figure 4.1 (c)) and
- Saturation nonlinearity (Figure 4.1 (d)).

The other discontinuous nonlinearities or their combinations can also be considered in the same principle.



(a) (b) (c) (d)

Fig. 4.1 Discontinuous nonlinearities
(a) Direction-dependent nonlinearity (b) Preload nonlinearity
(c) Dead-zone nonlinearity (d) Saturation nonlinearity

The corresponding nonlinear relationships in Fig. 4.1 are as:

- Direction-dependent nonlinearity

$$y^*(t) = \begin{cases} S_1 \cdot [w(t) - Z_0] + C_0 & w(t) \geq Z_0 \\ S_2 \cdot [w(t) - Z_0] + C_0 & w(t) < Z_0. \end{cases} \tag{4.1}$$

Here, the slopes S_1, S_2 and the cross-point (Z_0, C_0) are parameters to be identified. It is noted that S_1, S_2, Z_0, C_0 can be positive or negative. We assume that S_1 and S_2 are nonzero.

- Preload nonlinearity

$$y^*(t) = \begin{cases} S_1 \cdot w(t) + C_1 & w(t) > 0 \\ 0 & w(t) = 0 \\ S_2 \cdot w(t) + C_2 & w(t) < 0. \end{cases} \tag{4.2}$$

Here, the slopes S_1, S_2 and the cross-point values C_1, C_2 are parameters to be identified. It is noted that S_1, S_2, C_1, C_2 can also be positive or negative. We assume that S_1 and S_2 are nonzero.

- Dead-zone nonlinearity

$$y^*(t) = \begin{cases} S \cdot [w(t) - Z_1] + C_0 & w(t) > Z_1 \\ C_0 & Z_2 \le w(t) \le Z_1 \\ S \cdot [w(t) - Z_2] + C_0 & w(t) < Z_2. \end{cases} \tag{4.3}$$

Here, the slope S and the cross-point values C_0, Z_1, Z_2 are parameters to be identified. It is noted that S, C_0, Z_1, Z_2 can be positive or negative. We assume that S is nonzero and $Z_1 > Z_2$.

- Saturation nonlinearity

$$y^*(t) = \begin{cases} C_1 & w(t) > Z_1 \\ \dfrac{C_1 - C_2}{Z_1 - Z_2} \cdot (w(t) - Z_1) + C_1 & Z_2 \le w(t) \le Z_1 \\ C_2 & w(t) < Z_2. \end{cases} \tag{4.4}$$

Here, the cross-point values Z_1, Z_2, C_1, C_2 are parameters to be identified. They can be positive or negative. We assume that $Z_1 > Z_2$.

4.1. Wiener system

As shown in Fig. 3.1, the linear dynamic block $G(q^{-1}, \boldsymbol{\theta})$ is described by a RTF model of order n

$$w(t) = \frac{B(q^{-1})}{F(q^{-1})} u(t). \tag{4.5}$$

The nonlinear static block $N(w(t), \boldsymbol{\eta})$ is one of the above discontinuous nonlinearities (Fig. 4.1(a)-(d)), respectively. The resulting system for each single nonlinearity will be described now.

4.1.1. Direction-dependent nonlinearity

We define a switching sequence

$$h_1(t) = \begin{cases} 0 & w(t) \ge Z_0 \\ 1 & w(t) < Z_0. \end{cases} \tag{4.6}$$

Then the direction-dependent nonlinearity Eq. (4.1) can be rewritten as

$$y^*(t) = S_1 \cdot w(t) + (S_2 - S_1) \cdot w(t)h_1(t) + (S_1 - S_2) \cdot Z_0 \cdot h_1(t) + (C_0 - S_1 \cdot Z_0) \tag{4.7}$$

where $w(t)$ in the first term $S_1 \cdot w(t)$ is the key term.

Half-substituting Eq. (4.5) into the key term $w(t)$ in Eq. (4.7) , then the output $y(t)$ of a Wiener system with direction-dependent nonlinearity is given by

$$\begin{aligned} y(t) &= y^*(t) + \varepsilon(t) \\ &= \frac{S_1 \cdot B(q^{-1})}{F(q^{-1})}u(t) + (S_2 - S_1) \cdot w(t)h_1(t) + (S_1 - S_2) \cdot Z_0 \cdot h_1(t) + (C_0 - S_1 \cdot Z_0) \\ &+ \frac{C(q^{-1})}{D(q^{-1})}e(t). \end{aligned} \tag{4.8}$$

To avoid the over-parameterization problem, one parameter in $S_1, b_0, b_1, \cdots, b_n$ should be fixed. Eq. (4.8) shows that a Wiener system with direction-dependent nonlinearity can be transformed into a pseudo-linear MISO system with four independent inputs, $u(t)$, $w(t)h_1(t), h_1(t)$ and 1. The unmeasurable intermediate variable $w(t)$ can be recursively estimated according to Eq. (4.5). All the parameters in this pseudo-linear MISO system are explicitly given. Although some parameters of direction-dependent nonlinearity in Eq. (4.8) are combined, the original parameters can be easily solely recalculated.

4.1.2. Preload nonlinearity

We define another switching function

$$h_2(t) = \begin{cases} 0 & w(t) \geq 0 \\ 1 & w(t) < 0. \end{cases} \tag{4.9}$$

Then the preload nonlinearity Eq. (4.2) can be rewritten as

$$y^*(t) = S_1 \cdot w(t) + (S_2 - S_1) \cdot w(t)h_2(t) + (C_2 - C_1) \cdot h_2(t) + C_1 \tag{4.10}$$

where $w(t)$ in the first term $S_1 \cdot w(t)$ is the key term.

Half-substituting Eq. (4.5) into the key term $w(t)$ in Eq. (4.10) , the output $y(t)$ of a Wiener system with preload nonlinearity is given by

$$\begin{aligned} y(t) &= y^*(t) + \varepsilon(t) \\ &= \frac{S_1 \cdot B(q^{-1})}{F(q^{-1})}u(t) + (S_2 - S_1) \cdot w(t)h_2(t) + (C_2 - C_1) \cdot h_2(t) + C_1 + \frac{C(q^{-1})}{D(q^{-1})}e(t). \end{aligned} \tag{4.11}$$

To avoid the over-parameterization problem, one parameter in $S_1, b_0, b_1, \cdots, b_n$ should be fixed. Eq. (4.11) shows that a Wiener system with preload nonlinearity can be transformed into a pseudo-linear MISO system with four independent inputs, $u(t)$, $w(t)h_2(t)$, $h_2(t)$ and 1. The unmeasurable

intermediate variable $w(t)$ can be recursively estimated according to Eq. (4.5). All the parameters in this pseudo-linear MISO system are explicitly given. Although some parameters of preload nonlinearity in Eq. (4.11) are also combined, the original parameters can be solely recalculated.

4.1.3. Dead-zone nonlinearity

In order to simplify the derivation, here we use the dead-zone width B_Z which is $B_Z = Z_1 - Z_2$ and the center point of the dead-zone Z_0 with $Z_0 = \frac{Z_1 + Z_2}{2}$.

We make use of the standard sign function $\mathrm{sgn}(\cdot)$, then the dead-zone nonlinearity Eq. (4.3) can be rewritten as

$$\begin{aligned} y^*(t) = S \cdot w(t) - S \cdot \frac{1 + \mathrm{sgn}(\frac{B_Z}{2} - |w(t) - Z_0|)}{2} w(t) - S \cdot Z_0 \cdot \frac{1 - \mathrm{sgn}(\frac{B_Z}{2} - |w(t) - Z_0|)}{2} \\ + S \cdot \frac{B_Z}{2} \cdot \frac{\mathrm{sgn}(\frac{B_Z}{2} - |w(t) - Z_0|) - 1}{2} \mathrm{sgn}(w(t) - Z_0) + C_0. \end{aligned} \tag{4.12a}$$

We define the following intermediate variables:

$$g_1(t) = -\frac{1 + \mathrm{sgn}(\frac{B_Z}{2} - |w(t) - Z_0|)}{2} w(t) \tag{4.12b}$$

$$g_2(t) = -\frac{1 - \mathrm{sgn}(\frac{B_Z}{2} - |w(t) - Z_0|)}{2} \tag{4.12c}$$

$$g_3(t) = \frac{\mathrm{sgn}(\frac{B_Z}{2} - |w(t) - Z_0|) - 1}{4} \mathrm{sgn}(w(t) - Z_0). \tag{4.12d}$$

Then Eq. (4.12a) can be simplified as

$$y^*(t) = S \cdot w(t) + S \cdot g_1(t) + S \cdot Z_0 \cdot g_2(t) + S \cdot B_Z \cdot g_3(t) + C_0 \tag{4.13}$$

where $w(t)$ in the first term $S_1 \cdot w(t)$ is the key term.

Half-substituting Eq. (4.5) into the key term $w(t)$ in Eq. (4.13), the Wiener system output $y(t)$ is described in the following form,

$$y(t) = \frac{S \cdot B(q^{-1})}{F(q^{-1})} u(t) + S \cdot g_1(t) + S \cdot Z_0 \cdot g_2(t) + S \cdot B_Z \cdot g_3(t) + C_0 + \frac{C(q^{-1})}{D(q^{-1})} e(t). \tag{4.14}$$

Eq. (4.14) shows that a Wiener system with a dead-zone nonlinearity can be transformed approximately into a pseudo-linear MISO system with five independent inputs, $u(t)$, $g_1(t)$, $g_2(t)$,

$g_3(t)$ and 1. The unmeasurable intermediate variables $w(t)$, $g_1(t)$, $g_2(t)$ and $g_3(t)$ can be recursively estimated according to Eq. (4.5) and Eqs. (4.12b)-(4.12d). All the parameters in this pseudo-linear MISO system are explicitly given and there is no over-parameterization problem. Although some parameters of the dead-zone nonlinearity in Eq. (4.14) are combined and replaced by their equivalent values B_Z and Z_0, the original parameters Z_1 and Z_2 can be solely recalculated according to $B_Z = Z_1 - Z_2$ and $Z_0 = \frac{Z_1 + Z_2}{2}$.

4.1.4. Saturation nonlinearity

In order to simplify the derivation, here we use the two widths B_Z and B_C between the two saturation boundaries which are $B_Z = Z_1 - Z_2$ and $B_C = C_1 - C_2$. We also use the center point of the linear part (Z_0, C_0) with $Z_0 = \frac{Z_1 + Z_2}{2}$ and $C_0 = \frac{C_1 + C_2}{2}$.

We make use of the standard sign function $\mathrm{sgn}(\cdot)$. Then the saturation nonlinearity Eq. (4.4) can be rewritten as

$$\begin{aligned} y^*(t) &= \frac{B_C}{2B_Z} \cdot w(t) + \frac{B_C}{2B_Z} \cdot \mathrm{sgn}\left(\frac{B_Z}{2} - |w(t) - Z_0|\right) w(t) \\ &- \frac{B_C}{B_Z} \cdot Z_0 \cdot \frac{1 + \mathrm{sgn}\left(\frac{B_Z}{2} - |w(t) - Z_0|\right)}{2} \\ &+ B_C \cdot \frac{1 + \mathrm{sgn}\left(|w(t) - Z_0| - \frac{B_Z}{2}\right)}{4} \mathrm{sgn}(w(t) - Z_0) + C_0. \end{aligned} \tag{4.15a}$$

We define the following intermediate variables:

$$g_1(t) = \frac{\mathrm{sgn}\left(\frac{B_Z}{2} - |w(t) - Z_0|\right) w(t)}{2} \tag{4.15b}$$

$$g_2(t) = \frac{-1 - \mathrm{sgn}\left(\frac{B_Z}{2} - |w(t) - Z_0|\right)}{2} \tag{4.15c}$$

$$g_3(t) = \frac{1 + \mathrm{sgn}\left(|w(t) - Z_0| - \frac{B_Z}{2}\right)}{4} \mathrm{sgn}(w(t) - Z_0). \tag{4.15d}$$

Then Eq. (4.15a) can be simplified as

$$y^*(t) = \frac{B_C}{2B_Z} \cdot w(t) + \frac{B_C}{B_Z} \cdot g_1(t) + \frac{B_C}{B_Z} \cdot Z_0 \cdot g_2(t) + B_C \cdot g_3(t) + C_0 \tag{4.16}$$

where $w(t)$ in the first term $\frac{B_C}{2B_Z} \cdot w(t)$ is the key term.

Half-substituting Eq. (4.5) into the key term $w(t)$ in Eq. (4.16), a Wiener system output $y(t)$ is given by

$$y(t) = \frac{B_C}{2B_Z} \cdot \frac{B(q^{-1})}{F(q^{-1})} u(t) + \frac{B_C}{B_Z} \cdot g_1(t) + \frac{B_C}{B_Z} \cdot Z_0 \cdot g_2(t) + B_C \cdot g_3(t) + C_0 + \frac{C(q^{-1})}{D(q^{-1})} e(t). \qquad (4.17)$$

Eqs. (4.17) shows that a Wiener system with a saturation nonlinearity can be transformed approximately into a pseudo-linear MISO system with five independent inputs, $u(t)$, $g_1(t)$, $g_2(t)$, $g_3(t)$ and 1. The unmeasurable intermediate variables $w(t)$, $g_1(t)$, $g_2(t)$ and $g_3(t)$ can be recursively estimated according to Eq. (4.5) and Eqs. (4.15b)-(4.15d). All the parameters in this pseudo-linear MISO system are explicitly given and there is also no over-parameterization problem. Although some parameters of saturation nonlinearity in Eq. (4.17) are combined and replaced by their equivalent values B_Z, Z_0, B_C and C_0, the original parameters Z_1, Z_2, C_1 and C_2 can be solely recalculated according to $B_Z = Z_1 - Z_2$, $B_C = C_1 - C_2$, $Z_0 = \frac{Z_1 + Z_2}{2}$ and $C_0 = \frac{C_1 + C_2}{2}$.

4.2. Simulation results

In this section some simulation results will be given for a Wiener system with the four different discontinuous nonlinearities.

The linear dynamic block in the Wiener system is described by

$$G(q^{-1}, \boldsymbol{\theta}) = \frac{q^{-1} + 0.5004 q^{-2}}{1 - 1.5 q^{-1} + 0.7 q^{-2}} \qquad (4.18)$$

where we fix $b_1 = 1$ for the cases of direction-dependent and preload nonlinearities. The nonlinear static block is one of the following discontinuous nonlinearities, respectively.

- Direction-dependent nonlinearity:

$$y^*(t) = \begin{cases} 0.45[w(t) - 2] + 1 & w(t) \geq 2 \\ 1.2[w(t) - 2] + 1 & w(t) < 2 \end{cases} \qquad (4.19)$$

with the parameters: $S_1 = 0.45$, $S_2 = 1.2$, $Z_0 = 2$ and $C_0 = 1$.

- Preload nonlinearity

$$y^*(t) = \begin{cases} 0.45 w(t) + 1.55 & w(t) > 0 \\ 0 & w(t) = 0 \\ 1.2 w(t) - 2.7 & w(t) < 0 \end{cases} \qquad (4.20)$$

with the parameters: $S_1 = 0.45$, $S_2 = 1.2$, $C_1 = 1.55$ and $C_2 = -2.7$.

- Dead-zone nonlinearity

$$y^*(t)=\begin{cases}[w(t)-3]+1 & w(t)>3\\ 1 & -1\le w(t)\le 3\\ [w(t)+1]+1 & w(t)<-1\end{cases} \tag{4.21}$$

with the parameters: $S=1$, $Z_1=3$, $Z_2=-1$ and $C_0=1$.

- Saturation nonlinearity

$$y^*(t)=\begin{cases}7 & w(t)>5\\ w(t) & -7\le w(t)\le 5\\ -5 & w(t)<-7\end{cases} \tag{4.22}$$

with the parameters: $Z_1=5$, $Z_2=-7$, $C_1=7$, and $C_2=-5$.

A standard random numbers with zero mean is used as input $u(t)$. Another independent zero mean random numbers is used as white noise $e(t)$. $\mathbf{N}=2000$ data points are collected.

Besides the non-noisy case, $N./S.=5\%$, 10% and 20% white and 10% colored measurement noise, are added to the system output, respectively. The colored measurement noise filter is given by

$$H(q^{-1},\xi)=\frac{1+0.2q^{-1}+0.1q^{-2}}{1-1.2q^{-1}+0.5q^{-2}}. \tag{4.23}$$

A exponentially average smoother using a moving window with fixed length $Mov=4$ will be used to filter the estimated parameters to calculate the constructed intermediate variables.

Then apply the standard RPEM function in MATLAB with forgetting factor algorithm for linear MISO system with the algorithm variable settings $\lambda(0)=0.7$ and $\Delta\lambda=0.01$. The initial estimates of the unknown parameters are also taken as zero.

Identification results with different measurement noises are shown in Tables 4.1A-4.4A. The identification processes with a N./S.=10% colored measurement noise are shown in Figs. 4.2-4.5. The red lines in Figs. 4.2-4.5 are the real values of parameters. The single parameter identification error Δp and the average parameter identification error $|\Delta|$ of each case with different measurement noises are calculated and shown in Tables 4.1B-4.4B. Comparison of average parameter identification errors $|\Delta|$ of the four cases is shown in Table 4.5.

Table 4.1A Identification results with direction-dependent nonlinearity

N = 2000	b_2	f_1	f_2	S_1	S_2	Z_0	C_0
10% C. N.	0.5485	-1.5190	0.7262	0.5331	1.1607	2.1309	1.1159
20% W. N.	0.5739	-1.4958	0.7002	0.4811	1.1764	2.0048	1.0269
10% W. N.	0.5528	-1.4971	0.7004	0.4685	1.1903	1.9459	0.9864
5% W. N.	0.5378	-1.4982	0.7006	0.4468	1.1910	2.0301	1.0612
Non-noise	0.5005	-1.4999	0.6999	0.4561	1.2004	1.9737	0.9694
True values	0.5004	-1.5000	0.7000	0.4500	1.2000	2.0000	1.0000
N = 2000	c_1	c_2	d_1	d_2	-	-	-
10% C. N.	0.2728	0.0988	-1.1200	0.4371	-	-	-
True values	0.2000	0.1000	-1.2000	0.5000	-	-	-

Table 4.1B Parameter identification errors with direction-dependent nonlinearity

N = 2000	$\lvert\Delta b_2\rvert$	$\lvert\Delta f_1\rvert$	$\lvert\Delta f_2\rvert$	$\lvert\Delta S_1\rvert$	$\lvert\Delta S_2\rvert$	$\lvert\Delta Z_0\rvert$	$\lvert\Delta C_0\rvert$	$\sum\lvert\Delta\rvert$
10% C.	0.0481	0.0190	0.0262	0.0831	0.0393	0.1309	0.1159	0.0660
20% W.	0.0735	0.0042	0.0002	0.0311	0.0236	0.0048	0.0269	0.0234
10% W.	0.0524	0.0029	0.0004	0.0185	0.0097	0.0541	0.0136	0.0216
5% W.	0.0374	0.0018	0.0006	0.0032	0.009	0.0301	0.0612	0.0204

Table 4.2A Identification results with preload nonlinearity

N = 2000	b_2	f_1	f_2	S_1	S_2	C_1	C_2
10% C. N.	0.4914	-1.5090	0.7024	0.4989	1.2480	1.1628	-2.2870
20% W. N.	0.6183	-1.4914	0.6982	0.4841	1.1803	1.5344	-2.4335
10% W. N.	0.5868	-1.4944	0.6995	0.4575	1.1762	1.6201	-2.5299
5% W. N.	0.5603	-1.4962	0.6998	0.4568	1.1862	1.5910	-2.5513
Non-noise	0.4993	-1.5002	0.7002	0.4504	1.2029	1.5483	-2.6821
True values	0.5004	-1.5000	0.7000	0.4500	1.2000	1.5500	-2.7000
N = 2000	c_1	c_2	d_1	d_2	-	-	-
10% C. N.	-0.2650	0.2086	-1.1270	0.4777	-	-	-
True values	0.2000	0.1000	-1.2000	0.5000	-	-	-

Table 4.2B Parameter identification errors with preload nonlinearity

N = 2000	$\lvert\Delta b_2\rvert$	$\lvert\Delta f_1\rvert$	$\lvert\Delta f_2\rvert$	$\lvert\Delta S_1\rvert$	$\lvert\Delta S_2\rvert$	$\lvert\Delta C_1\rvert$	$\lvert\Delta C_2\rvert$	$\sum\lvert\Delta\rvert$
10% C.	0.0090	0.0090	0.0024	0.0489	0.0480	0.3872	0.4130	0.1310
20% W.	0.1179	0.0086	0.0018	0.0341	0.0197	0.0156	0.2665	0.0663
10% W.	0.0864	0.0056	0.0005	0.0075	0.0238	0.0701	0.1701	0.0520
5% W.	0.0599	0.0038	0.0002	0.0068	0.0138	0.0410	0.1487	0.0391

Table 4.3A Identification results with dead-zone nonlinearity

N = 2000	b_1	b_2	f_1	f_2
10% C. N.	0.9524	0.4941	-1.5190	0.7163
20% W. N.	0.9551	0.5844	-1.4944	0.6994
10% W. N.	0.9807	0.5411	-1.4978	0.7008
5% W. N.	0.9805	0.5440	-1.4969	0.6994
Non-noise	0.9999	0.5005	-1.5000	0.7000
True values	1.0000	0.5004	-1.5000	0.7000
N = 2000	S	Z_1	Z_2	C_0
10% C. N.	0.9613	3.0278	-0.9390	1.1288
20% W. N.	0.9530	2.9547	-0.7995	1.1232
10% W. N.	0.9671	2.9954	-0.9093	1.0693
5% W. N.	0.9619	3.0036	-0.9293	1.0419
Non-noise	0.9998	3.0002	-0.9998	1.0000
True values	1.0000	3.0000	-1.0000	1.0000
N = 2000	c_1	c_2	d_1	d_2
10% C. N.	0.2252	0.1556	-1.1390	0.4378
True values	0.2000	0.1000	-1.2000	0.5000

Table 4.3B Parameter identification errors with dead-zone system

N = 2000	$\lvert\Delta b_1\rvert$	$\lvert\Delta b_2\rvert$	$\lvert\Delta f_1\rvert$	$\lvert\Delta f_2\rvert$	$\lvert\Delta S\rvert$	$\lvert\Delta Z_1\rvert$	$\lvert\Delta Z_2\rvert$	$\lvert\Delta C_0\rvert$	$\sum\lvert\Delta\rvert$
10% C.	0.0476	0.0063	0.0190	0.0163	0.0387	0.0278	0.0610	0.1288	0.0431
20% W.	0.0449	0.0840	0.0056	0.0006	0.0470	0.0453	0.2005	0.1232	0.0688
10% W.	0.0193	0.0407	0.0022	0.0008	0.0329	0.0046	0.0907	0.0693	0.0325
5% W.	0.0195	0.0436	0.0031	0.0006	0.0381	0.0036	0.0002	0.0419	0.0273

Table 4.4A Identification results with saturation nonlinearity

N = 2000	b_1	b_2	f_1	f_2
10% C. N.	1.1999	0.6661	-1.5150	0.7232
20% W. N.	0.8575	0.6192	-1.4900	0.6974
10% W. N.	1.4182	0.5744	-1.4924	0.6969
5% W. N.	1.0591	0.5279	-1.4979	0.7001
Non-noise	0.9996	0.5004	-1.5000	0.7000
True values	1.0000	0.5004	-1.5000	0.7000
N = 2000	Z_1	Z_2	C_1	C_2
10% C. N.	4.8750	-1.7094	5.7951	-7.4845
20% W. N.	7.2212	-51.4095	5.9162	-62.1279
10% W. N.	3.8340	-1.0738	3.0517	-5.5874
5% W. N.	6.6062	-4.3157	4.8726	-6.8159
Non-noise	5.0001	-7.0021	7.0002	-5.0027
True values	5.0000	-7.0000	7.0000	-5.0000
N = 2000	c_1	c_2	d_1	d_2
10% C. N.	0.2009	0.1071	-1.0587	0.4111
True values	0.2000	0.1000	-1.2000	0.5000

Table 4.4B Parameter identification errors with saturation system

N = 2000	$\lvert\Delta b_1\rvert$	$\lvert\Delta b_2\rvert$	$\lvert\Delta f_1\rvert$	$\lvert\Delta f_2\rvert$	$\lvert\Delta Z_1\rvert$	$\lvert\Delta Z_2\rvert$	$\lvert\Delta C_1\rvert$	$\lvert\Delta C_2\rvert$	$\sum\lvert\Delta\rvert$
10% C.	0.1999	0.1657	0.0150	0.0232	0.1250	5.2906	1.2049	2.4845	1.1886
20% W.	0.1425	0.1188	0.0100	0.0026	2.2212	44.4090	1.0838	57.1270	13.1370
10% W.	0.4182	0.0740	0.0076	0.0031	1.1660	5.9262	3.9483	0.5874	1.5162
5% W.	0.0591	0.0275	0.0021	1E-04	1.6062	2.6843	2.1274	1.8159	1.0403

Table 4.5 Comparison of average parameter identification errors $\lvert\Delta\rvert$ of the four cases

N = 2000	Direction dependent	Preload	Dead-zone	Saturation	Best accuracy
10% C.	0.0660	0.1310	0.0431	1.1886	Dead-zone
20% W.	0.0234	0.0663	0.0688	13.137	Direction dependent
10% W.	0.0216	0.0520	0.0325	1.5162	Direction dependent
5% W.	0.0204	0.0391	0.0273	1.0403	Direction dependent

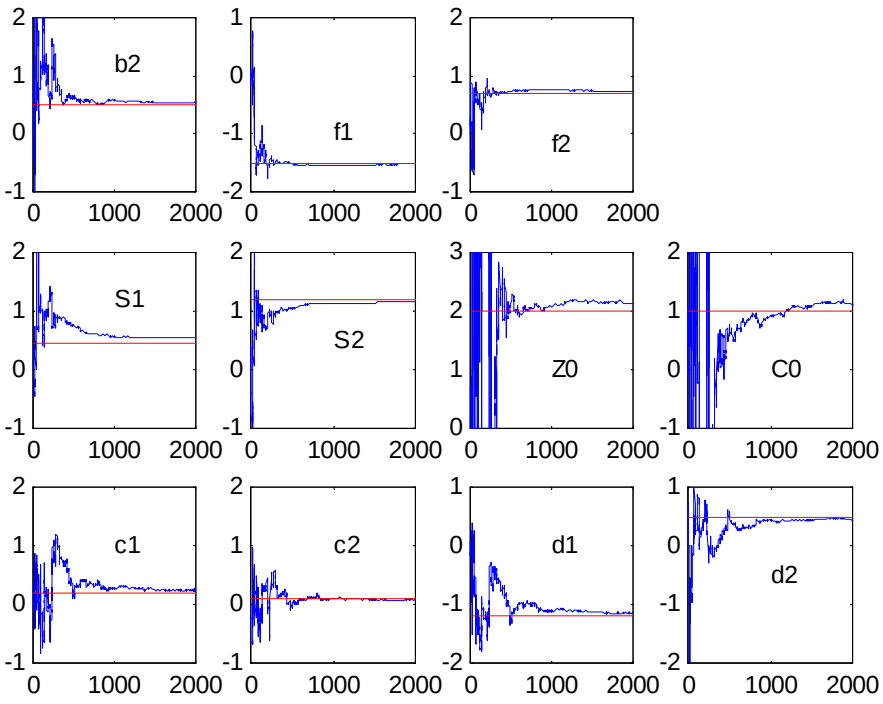


Fig. 4.2 A Wiener system identification process with direction dependent nonlinearity with a N./S.=10% colored measurement noise

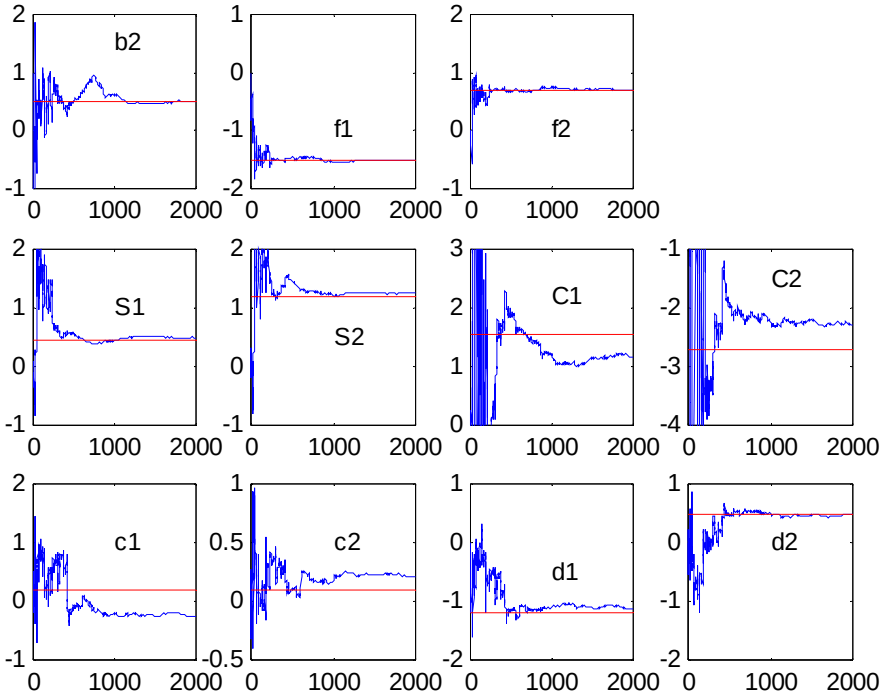


Fig. 4.3 A Wiener system identification process with preload nonlinearity with a N./S.=10% colored measurement noise

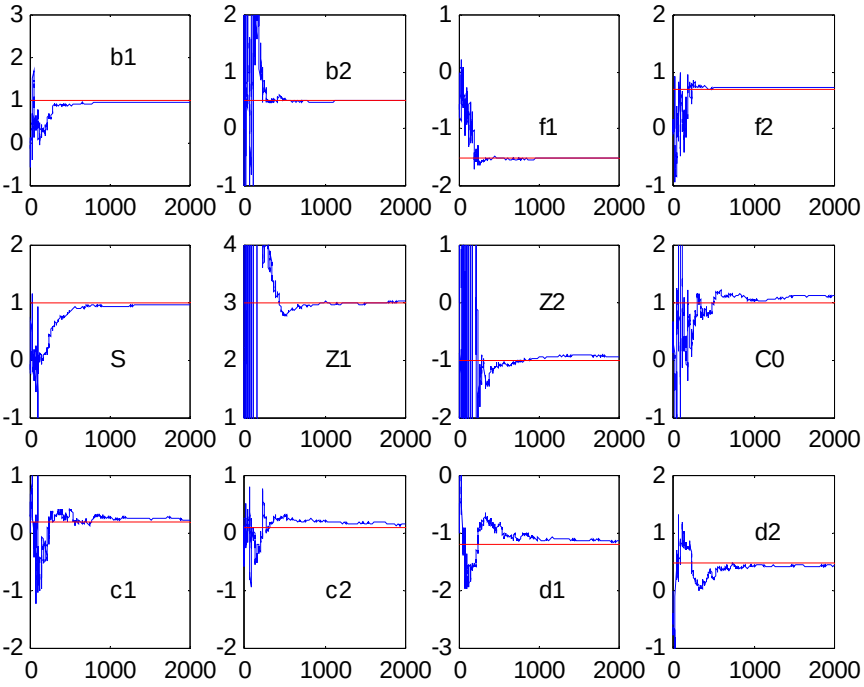


Fig. 4.4 A Wiener system identification process with dead-zone nonlinearity with a N./S.=10% colored measurement noise

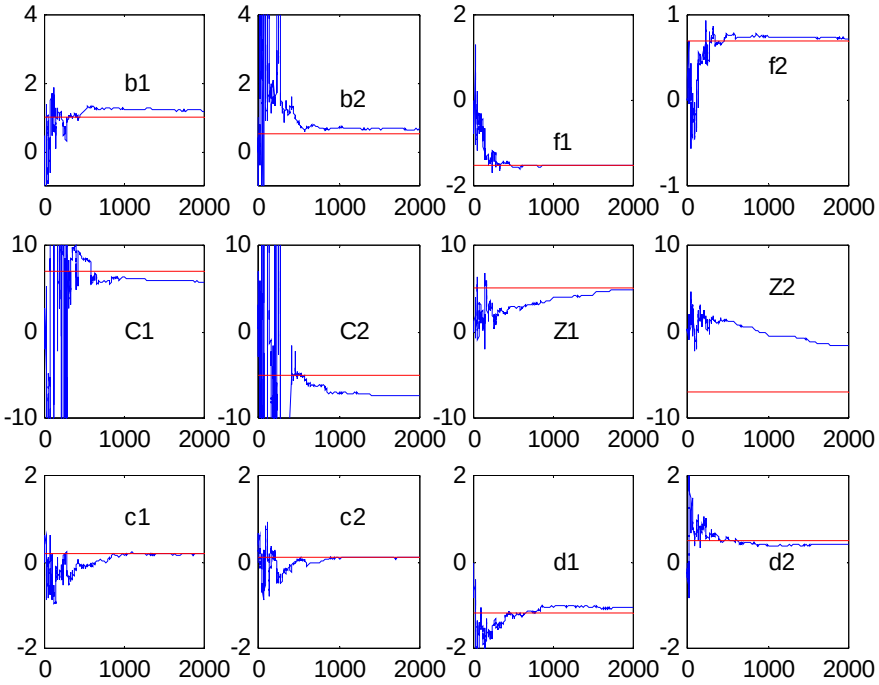


Fig. 4.5 A Wiener system identification process with saturation nonlinearity with a N./S.=10% colored measurement noise

The simulation results show that:

- The new identification method gives good results for a Wiener system with all the four discontinuous nonlinearities, respectively.
- From Table 4.5, we can see that which one of the four cases has the best accuracy results under different measurement noises.

5. Identification of cascade Wiener and Hammerstein systems

A Wiener-Hammerstein system (LNL, Fig. 5.1) is defined as a linear dynamic subsystem in cascade with a nonlinear static subsystem followed by another linear dynamic subsystem and A Hammerstein-Wiener system (NLN, Fig. 5.2) is defined as a nonlinear static subsystem in cascade with a linear dynamic subsystem followed by another nonlinear static subsystem.

Billings *et al.* (1982, 1997) proposed an identification algorithm for Wiener-Hammerstein system based on correlation analysis. However, their algorithm suffered from some restrictive assumptions for the input sequences to preserve the separability principle and computational requirements. Yoshine *et al.* (1992) suggested another approach for identification of the LNL system which consists of estimating impulse responses of the linear subsystems and the parameters of the nonlinear element. Based on a formulated model, Boutayeb (1995) developed a recursive method to separately estimate parameters of the linear and nonlinear parts of the LNL system. Bai (1998) introduced an optimal two-stage identification algorithm for Hammerstein-Wiener system.

In this chapter, the new identification method is used to identify

- a Wiener-Hammerstein system and
- a Hammerstein-Wiener system

with continuous and discontinuous nonlinearities, respectively.

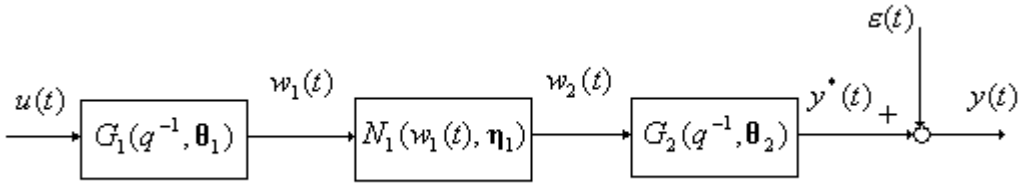


Fig. 5.1 Wiener-Hammerstein system (LNL)

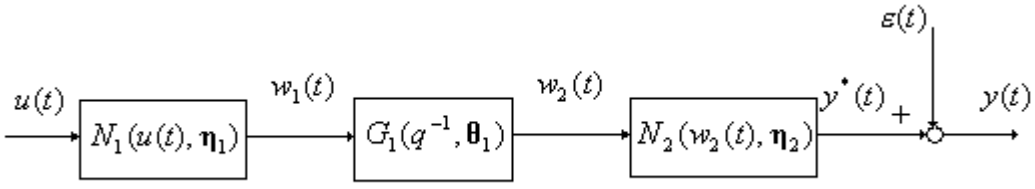


Fig. 5.2 Hammerstein-Wiener system (NLN)

In the above Figures, $u(t)$ is the system input, $y(t)$ is the system output, $y^*(t)$ is the unmeasurable system output without measurement noise. $w_1(t)$ and $w_2(t)$ are unmeasurable intermediate variables. $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ are parameter vectors determining the linear dynamic subsystems $G_1(q^{-1},\boldsymbol{\theta}_1)$ and $G_2(q^{-1},\boldsymbol{\theta}_2)$, respectively. $\boldsymbol{\eta}_1$ and $\boldsymbol{\eta}_2$ are parameter vectors determining the nonlinear subsystems $N_1(\cdot,\boldsymbol{\eta}_1)$ and $N_2(\cdot,\boldsymbol{\eta}_2)$, respectively. $\varepsilon(t)$ is the colored measurement noise which is a white noise $e(t)$ through a linear filter $H(q^{-1},\boldsymbol{\xi})$.

A rational transfer function of order n_1 will be used as the linear block $G_1(q^{-1},\boldsymbol{\theta}_1)$. For Wiener-Hammerstein system (LNL), because it is a Hammerstein-type system we use an ARMAX model of order n_2 to describe the linear block $G_2(q^{-1},\boldsymbol{\theta}_2)$.

If the nonlinearities in both Wiener-Hammerstein and Hammerstein-Wiener systems are continuous, a polynomial function of order m_j will be used to describe the nonlinear static subsystem $N_j(\chi_j(t),\boldsymbol{\eta}_j)$. For the output we get

$$\gamma_j(t)=\sum_{k=1}^{m_j}\beta_{jk}\cdot\chi_j^k(t) \tag{5.1}$$

where $\chi_j(t)$ is the input of the nonlinear static block and $\gamma_j(t)$ is the output of the nonlinear static block for $j=1.2$.

In the case of discontinuous nonlinearities, here only a typical symmetrical dead-zone nonlinearity (Figure 5.3 (a)) and a symmetrical saturation nonlinearity (Figure 5.3 (b)) are considered.

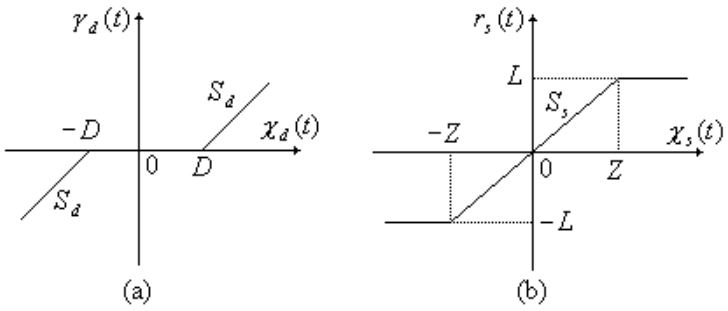


Fig. 5.3 (a). Dead-zone (b). Saturation

In the above Figures, $\chi_d(t)$ is the input of the dead-zone nonlinearity, $\gamma_d(t)$ is the output of the dead-zone nonlinearity; $\chi_s(t)$ is the input of the saturation nonlinearity and $\gamma_s(t)$ is the output of the saturation nonlinearity.

The slope S_d and the cross-point values D, Z and L are nonzero parameters to be identified. For convenience, the slope S_s is also used with $S_s=\dfrac{L}{Z}$. In order to simplify the derivation, we assume D, Z and L are positive and S_d, S_s can be positive or negative. More general descriptions of a dead-zone and a saturation or other discontinuous nonlinearities can be defined and derived in the same principle.

The dead-zone relationship is given by

$$\gamma_d(t)=\begin{cases}S_d\cdot[\chi_d(t)-D] & \chi_d(t)>D\\ 0 & -D\le\chi_d(t)\le D\\ S_d\cdot[\chi_d(t)+D] & \chi_d(t)<-D.\end{cases} \tag{5.2}$$

The saturation nonlinear relationship is given by

$$\gamma_s(t) = \begin{cases} L & \chi_s(t) > Z \\ S_s \cdot \chi_s(t) & -Z \le \chi_s(t) \le Z \\ -L & \chi_s(t) < -Z. \end{cases} \tag{5.3}$$

By introducing the standard sign function $\mathrm{sgn}(\cdot)$, Eq. (5.2) can be rewritten as

$$\begin{aligned} \gamma_d(t) &= S_d \cdot \chi_d(t) - S_d \cdot \frac{1+\mathrm{sgn}(D-|\chi_d(t)|)}{2}\chi_d(t) \\ &+ S_d \cdot D \cdot \frac{\mathrm{sgn}(D-|\chi_d(t)|)-1}{2}\mathrm{sgn}(\chi_d(t)) \end{aligned} \tag{5.4}$$

and Eq.(5.3) can be rewritten as

$$\gamma_s(t) = \frac{S_s}{2} \cdot \chi_s(t) + S_s \cdot \frac{\mathrm{sgn}(Z-|\chi_s(t)|)}{2}\chi_s(t) + L \cdot \frac{1+\mathrm{sgn}(|\chi_s(t)|-Z)}{2}\mathrm{sgn}(\chi_s(t)). \tag{5.5}$$

We define the following intermediate variables:

$$w_d(t) = -\frac{1+\mathrm{sgn}(D-|\chi_d(t)|)}{2}\chi_d(t) \tag{5.6}$$

$$v_d(t) = \frac{\mathrm{sgn}(D-|\chi_d(t)|)-1}{2}\mathrm{sgn}(\chi_d(t)) \tag{5.7}$$

$$w_s(t) = \frac{\mathrm{sgn}(Z-|\chi_s(t)|)}{2}\chi_s(t) \tag{5.8}$$

$$v_s(t) = \frac{1+\mathrm{sgn}(|\chi_s(t)|-Z)}{2}\mathrm{sgn}(\chi_s(t)). \tag{5.9}$$

Then, Eqs. (5.4) and (5.5) can be simplified as

$$\gamma_d(t) = S_d \cdot \chi_d(t) + S_d \cdot w_d(t) + S_d \cdot D \cdot v_d(t) \tag{5.10}$$

$$\gamma_s(t) = \frac{S_s}{2} \cdot \chi_s(t) + S_s \cdot w_s(t) + L \cdot v_s(t). \tag{5.11}$$

These equations for the dead-zone and the saturation nonlinearities will be used in the next two sections.

5.1. Identification of a Wiener-Hammerstein system (LNL)

As shown in Fig. 5.1, the first linear dynamic subsystem $G_1(q^{-1}, \boldsymbol{\theta}_1)$ of order n_1 is given by

$$w_1(t) = \frac{B_1(q^{-1})}{F_1(q^{-1})} u(t) \tag{5.12a}$$

with

$$B_1(q^{-1}) = b_{10} + b_{11}q^{-1} + b_{12}q^{-2} + \cdots + b_{1n_1}q^{-n_1} \tag{5.12b}$$

$$F_1(q^{-1}) = 1 + f_{11}q^{-1} + \cdots + f_{1n_1}q^{-n_1} \tag{5.12c}$$

and, in addition, $B_1(q^{-1})$ and $F_1(q^{-1})$ are coprime.

The second linear dynamic block $G_2(q^{-1}, \boldsymbol{\theta}_2)$ of order n_2 is given by

$$\begin{aligned} &A(q^{-1})y(t) = B_2(q^{-1})w_2(t) + C(q^{-1})e(t) \\ &= b_{20} \cdot w_2(t) + B_2^*(q^{-1})w_2(t) + C(q^{-1})e(t) \end{aligned} \tag{5.13a}$$

with the following notations:

$$A(q^{-1}) = 1 + a_1q^{-1} + \cdots + a_{n_a}q^{-n_a} \tag{5.13b}$$

$$B_2(q^{-1}) = b_{20} + b_{21}q^{-1} + \cdots + b_{2n_2}q^{-n_2} \tag{5.13c}$$

$$B_2^*(q^{-1}) = b_{21}q^{-1} + b_{22}q^{-1} + \cdots + b_{2n_2}q^{-n_2} \tag{5.13d}$$

$$C(q^{-1}) = 1 + c_1q^{-1} + \cdots + c_{n_c}q^{-n_c} \tag{5.13e}$$

We separate a term $b_{20} \cdot w_2(t)$ in Eq. (5.13a). Then $w_2(t)$ in the first term $b_{20} \cdot w_2(t)$ of Eq. (5.13a) is one key term. To avoid the over-parameterization problem, two parameters in the Wiener-Hammerstein system should be fixed. Without loss of generality, let $b_{20} = 1$ and the other one will be fixed later.

5.1.1. Continuous nonlinearity

If the nonlinear block $N_1(w_1(t), \boldsymbol{\eta}_1)$ between the two linear blocks is a continuous nonlinearity, that is, a polynomial of order m_1, then we have

$$\begin{aligned} &w_2(t) = \sum_{k=1}^{m_1} \beta_k \cdot w_1^k(t) \\ &= \beta_1 \cdot w_1(t) + \sum_{k=2}^{m_1} \beta_k \cdot w_1^k(t) \end{aligned} \tag{5.14}$$

where $w_1(t)$ in the separated term $\beta \cdot w_1(t)$ of Eq. (5.14) is the other key term. Without loss of generality, let $\beta_1 = 1$.

Half-substituting Eq. (5.12a) into the key term $w_1(t)$ in Eq. (5.14) and then half-substituting Eq. (5.14) into the key term $w_2(t)$ in Eq. (5.13a) respectively, the system output of a Wiener-Hammerstein system with a continuous nonlinearity can be given by

$$A(q^{-1})y(t) = \frac{B_1(q^{-1})}{F_1(q^{-1})}u(t) + \sum_{k=2}^{m_1} \beta_k \cdot w_1^k(t) + B_2^*(q^{-1})w_2(t) + C(q^{-1})e(t). \tag{5.15}$$

Eq. (5.15) shows that the Wiener-Hammerstein system with a continuous nonlinearity can be approximately transformed into a pseudo-linear MISO system which has $m_1 + 1$ independent pseudo-inputs: $u(t)$, $w_1^2(t), \cdots, w_1^{m_1}(t)$ and $w_2(t)$. The unmeasurable intermediate variables $w_1(t)$ and $w_2(t)$ can be recursively estimated respectively, according to Eq. (5.12a) and Eq. (5.14). All the parameters in this pseudo-linear MISO system are explicitly given.

5.1.2. Discontinuous nonlinearity

In the same way, we consider the Wiener-Hammerstein system with a discontinuous nonlinearity, e.g., with a dead-zone.

Using Eqs (5.6) and (5.7) in the form:

$$w_d(t) = -\frac{1 + \mathrm{sgn}(D - |w_1(t)|)}{2} w_1(t) \tag{5.16}$$

$$v_d(t) = \frac{\mathrm{sgn}(D - |w_1(t)|) - 1}{2} \mathrm{sgn}(w_1(t)) \tag{5.17}$$

and with Eq. (5.10) we find

$$w_2(t) = S_d \cdot w_1(t) + S_d \cdot w_d(t) + S_d \cdot D \cdot v_d(t) \tag{5.18}$$

where $w_1(t)$ in the first term $S_d \cdot w_1(t)$ in Eq. (5.18) is the other key term. Without loss of generality, let $S_d = 1$.

Half-substituting Eq. (5.12a) into the key term $w_1(t)$ in Eq. (5.18) and then half-substituting Eq. (5.18) into the key term $w_2(t)$ in Eq. (5.13a) respectively, the system output of a Wiener-Hammerstein system with a dead-zone can be given by

$$A(q^{-1})y(t) = \frac{B_1(q^{-1})}{F_1(q^{-1})}u(t) + w_d(t) + D \cdot v_d(t) + B_2^*(q^{-1})w_2(t) + C(q^{-1})e(t). \tag{5.19}$$

Eq. (5.19) shows that the Wiener-Hammerstein system with a dead-zone can be approximately transformed into a pseudo-linear MISO system with four independent corresponding pseudo-inputs: $u(t)$, $w_d(t)$, $v_d(t)$ and $w_2(t)$. The unmeasurable intermediate variables $w_1(t)$, $w_d(t)$, $v_d(t)$ and

$w_2(t)$ above can be recursively estimated according to Eq. (5.12a) and Eqs. (5.16)-(5.18), respectively. All the parameters in this pseudo-linear MISO system are explicitly given. Other discontinuous nonlinearities can also be considered in the same way.

5.2. Identification of a Hammerstein-Wiener system (NLN)

As shown in Fig. 5.2, the linear block $G_1(q^{-1}, \boldsymbol{\theta}_1)$ of order n_1 is given by

$$\begin{aligned} w_2(t) &= \frac{B_1(q^{-1})}{F_1(q^{-1})} w_1(t) \\ &= b_{10} \cdot w_1(t) + \frac{B_1^*(q^{-1})}{F_1(q^{-1})} w_1(t) \end{aligned} \tag{5.20a}$$

where

$$B_1(q^{-1}) = b_{10} + b_{11}q^{-1} + \cdots + b_{1n_1}q^{-n_1} \tag{5.20b}$$

$$F_1(q^{-1}) = 1 + f_1 q^{-1} + \cdots + f_{n_1} q^{-n_1} \tag{5.20c}$$

$$B_1^*(q^{-1}) = b_{11}^* q^{-1} + b_{12}^* q^{-2} + \cdots + b_{1n_1}^* q^{-n_1} \tag{5.20d}$$

with

$$b_{1k}^* = b_{1k} - b_{10} \cdot f_{1k} \tag{5.20e}$$

for $k = 1,2,\cdots,n_1$.

In Eq. (5.20a), we separate a term $b_{10} \cdot w_1(t)$, then $w_1(t)$ in this term is one key term. To avoid the over-parameterization problem, two parameters in the Hammerstein-Wiener system should be fixed. Without loss of generality, let $b_{10} = 1$ and the other one will be fixed later.

5.2.1. Two continuous nonlinearities

If there are two continuous nonlinearities in Hammerstein-Wiener system, we use a polynomial model of order m_1 to describe the first nonlinear block $N_1(u(t), \boldsymbol{\eta}_1)$

$$w_1(t) = \sum_{k=1}^{m_1} \beta_{1k} \cdot u^k(t) \tag{5.21}$$

and use another polynomial model of order m_2 to describe the second nonlinear block $N_2(w_2(t), \boldsymbol{\eta}_2)$

$$\begin{aligned} y^*(t) &= \sum_{k=1}^{m_2} \beta_{2k} \cdot w_2^k(t) \\ &= \beta_{21} \cdot w_2(t) + \sum_{k=2}^{m_2} \beta_{2k} \cdot w_2^k(t). \end{aligned} \tag{5.22}$$

Then $w_2(t)$ in the separated term $\beta_{21} \cdot w_2(t)$ in Eq. (5.22) is the other key term. Without loss of generality, let $\beta_{21} = 1$.

Half-substituting Eq. (5.21) into the key term $w_1(t)$ in Eq. (5.20a) and then half-substituting Eq. (5.20a) into the key term $w_2(t)$ in Eq. (5.22) respectively, the system output of a Hammerstein-Wiener system with two continuous nonlinearities can be given by

$$\begin{aligned} y(t) &= y^*(t) + \varepsilon(t) \\ &= \sum_{k=1}^{m_1} \beta_{1k} \cdot u^k(t) + \frac{B_1^*(q^{-1})}{F_1(q^{-1})} w_1(t) + \sum_{k=2}^{m_2} \beta_{2k} \cdot w_2^k(t) + \frac{C(q^{-1})}{D(q^{-1})} e(t). \end{aligned} \tag{5.23}$$

Eq. (5.23) shows that the Hammerstein-Wiener system with two continuous nonlinearities can be approximately transformed into a pseudo-linear MISO system which has $m_1 + m_2$ independent pseudo-inputs: $u(t), u^2(t), \cdots, u^{m_1}(t)$, and $w_1(t), w_2^2(t), w_2^3(t), \cdots, w_2^{m_2}(t)$. The unmeasurable intermediate variables $w_1(t)$ and $w_2(t)$ can be recursively estimated with Eq. (5.21) and Eq. (5.20a), respectively. All the parameters in this pseudo-linear MISO system are explicitly given.

5.2.2. Two discontinuous nonlinearities

Here, we consider two cases:

- Deadzone-Linear-Saturation system and
- Saturation-Linear-Deadzone system.

The corresponding equations will be derived now.

- Deadzone-Linear-Saturation system

According to the descriptions of a dead-zone and a saturation nonlinearities in Eqs. (5.6)-(5.11), the variable relationships for the two nonlinear blocks in a Deadzone-Linear-Saturation system can be described by the following equations:

$$w_d(t) = -\frac{1 + \mathrm{sgn}(D - |u(t)|)}{2} u(t) \tag{5.24}$$

$$v_d(t) = \frac{\mathrm{sgn}(D - |u(t)|) - 1}{2} \mathrm{sgn}(u(t)) \tag{5.25}$$

$$w_s(t) = \frac{\mathrm{sgn}(Z - |w_2(t)|)}{2} w_2(t) \tag{5.26}$$

$$v_s(t) = \frac{1+\text{sgn}(|w_2(t)| - Z)}{2}\text{sgn}(w_2(t)) \tag{5.27}$$

and

$$w_1(t) = S_d \cdot u(t) + S_d \cdot w_d(t) + S_d \cdot D \cdot v_d(t) \tag{5.28}$$

$$y^*(t) = \frac{S_s}{2} \cdot w_2(t) + S_s \cdot w_s(t) + L \cdot v_s(t) \tag{5.29}$$

where $w_2(t)$ in the first term $\frac{S_s}{2} \cdot w_2(t)$ is the other key term. Without loss of generality, let $S_s = 2$.

Half-substituting Eq. (5.28) into the key term $w_1(t)$ in Eq. (5.20a) and then half-substituting Eq. (5.20a) into the key term $w_2(t)$ in Eq. (5.29) respectively, the system output of Deadzone-Linear-Saturation system can be given by

$$\begin{aligned} y(t) &= y^*(t) + \varepsilon(t) \\ &= S_d \cdot u(t) + S_d \cdot w_d(t) + S_d \cdot D \cdot v_d(t) + \frac{B_1^*(q^{-1})}{F_1(q^{-1})} w_1(t) + 2 \cdot w_s(t) + L \cdot v_s(t) + \frac{C(q^{-1})}{D(q^{-1})} e(t). \end{aligned} \tag{5.30}$$

- Saturation-Linear-Deadzone system

According to the descriptions of dead-zone and saturation in Eqs. (5.6)-(5.11). The variable relationships for the two nonlinear blocks in Saturation-Linear-Deadzone system can be described by the following equations:

$$w_s(t) = \frac{\text{sgn}(Z - |u(t)|)}{2} u(t) \tag{5.31}$$

$$v_s(t) = \frac{1+\text{sgn}(|u(t)| - Z)}{2}\text{sgn}(u(t)) \tag{5.32}$$

$$w_d(t) = -\frac{1+\text{sgn}(D - |w_2(t)|)}{2} w_2(t) \tag{5.33}$$

$$v_d(t) = \frac{\text{sgn}(D - |w_2(t)|) - 1}{2}\text{sgn}(w_2(t)) \tag{5.34}$$

and

$$w_1(t) = \frac{S_s}{2} \cdot u(t) + S_s \cdot w_s(t) + L \cdot v_s(t) \tag{5.35}$$

$$y^*(t) = S_d \cdot w_2(t) + S_d \cdot w_d(t) + S_d \cdot D \cdot v_d(t). \tag{5.36}$$

where $w_2(t)$ in the first term $S_d \cdot w_2(t)$ is the other key term. Without loss of generality, let $S_d = 1$.

Half-substituting Eq. (5.35) into the key term $w_1(t)$ in Eq. (5.20a) and then half-substituting Eq. (5.20a) into the key term $w_2(t)$ in Eq. (5.36) respectively, the system output of a Saturation-Linear-Deadzone system can be given by

$$\begin{aligned} y(t) &= y^*(t) + \varepsilon(t) \\ &= \frac{S_s}{2} \cdot u(t) + S_s \cdot w_s(t) + L \cdot v_s(t) + \frac{B_1^*(q^{-1})}{F_1(q^{-1})} w_1(t) + w_d(t) + D \cdot v_d(t) + \frac{C(q^{-1})}{D(q^{-1})} e(t). \end{aligned} \tag{5.37}$$

Eq. (5.30) and Eq. (5.37) show that the Hammerstein-Wiener system with two discontinuous nonlinearities (Deadzone-Linear-Saturation system or Saturation-Linear-Deadzone system) can also be approximately transformed into a pseudo-linear MISO system which has six independent corresponding pseudo-inputs, $u(t)$, $w_d(t)$, $v_d(t)$, $w_s(t)$, $v_s(t)$ and $w_1(t)$. The unmeasurable intermediate variables $w_1(t)$, $w_d(t)$, $v_d(t)$, $w_s(t)$, $v_s(t)$ and $w_2(t)$ can be recursively estimated according to the corresponding equations. All the parameters in this pseudo-linear MISO system are explicitly given.

From the derivations above we can conclude that in Wiener-Hammerstein or Hammerstein-Wiener systems any continuous and discontinuous nonlinearities or their combinations can be considered and identified in the same principle. The important problem is to write the nonlinearities in continuous forms and according to the separated key term principle to form a pseudo-linear MISO prediction error model.

5.3. Simulation results

In the following we consider five test systems, namely

- a Wiener-Hammerstein system with a continuous nonlinearity,
- a Wiener-Hammerstein system with a dead-zone,
- a Hammerstein-Wiener system with two continuous nonlinearities,
- a Deadzone-Linear-Saturation system and
- a Saturation-Linear-Deadzone system.

The Wiener-Hammerstein system with a continuous nonlinearity consists of the following linear and nonlinear blocks:

$$w_1(t) = \frac{0.1333q^{-1} + 0.0667q^{-2}}{1 - 1.5q^{-1} + 0.7q^{-2}} u(t) \tag{5.38}$$

$$w_2(t) = w_1(t) + 3w_1^2(t) + 1.5w_1^3(t) \tag{5.39}$$

$$(1 - 0.9q^{-1} + 0.85q^{-2}) y(t) = (1 + 0.1333q^{-1} + 0.0667q^{-2}) w_2(t) + (1 + 0.2q^{-1} + 0.1q^{-2}) e(t) \tag{5.40}$$

The Hammerstein-Wiener system with two continuous nonlinearities is given by:

$$w_1(t) = 0.3u(t) + 0.4u^2(t) + 0.5u^3(t) \tag{5.41}$$

$$w_2(t) = \frac{1 - 0.7q^{-1} + 0.12q^{-2}}{1 - 1.5q^{-1} + 0.7q^{-2}} w_1(t) \tag{5.42}$$

$$y(t) = w_2(t) + 0.8w_2^2(t) + 0.9w_2^3(t) + \frac{1+0.2q^{-1}+0.1q^{-2}}{1-1.2q^{-1}+0.5q^{-2}}e(t) \tag{5.43}$$

The discontinuous dead-zone and saturation nonlinearities are:

$$\gamma_d(t) = \begin{cases} \chi_d(t) - 0.5 & \chi_d(t) > 0.5 \\ 0 & -0.5 \le \chi_d(t) \le 0.5 \\ \chi_d(t) + 0.5 & \chi_d(t) < -0.5 \end{cases} \tag{5.44}$$

$$\gamma_s(t) = \begin{cases} 1 & \chi_s(t) > 1 \\ \chi_s(t) & -1 \le \chi_s(t) \le 1 \\ -1 & \chi_s(t) < -1 \end{cases} \tag{5.45}$$

A random numbers of zero mean is used as the system input $u(t)$. Another independent random numbers as white measurement noise $e(t)$. $\mathbf{N} = 2000$ data points are collected for each case.

Because the relatively simple examples above are used, here we consider only for each cascade Wiener and Hammerstein system with a non-noise, a N./S.=5% white measurement noise and a N./S.=5% colored measurement noise, respectively.

An exponentially average smoother using a moving window with fixed length $Mov = 4$ will be used to filter the estimated parameters to calculate the intermediate variables.

Then apply the standard RPEM function in MATLAB with forgetting factor algorithm for linear MISO system with the algorithm variable settings $\lambda(0) = 0.7$ and $\Delta\lambda = 0.01$. The initial estimates of the unknown parameters are also taken as zero.

Identification results with different measurement noises are shown in Tables 5.1A-5.5A. The identification processes with a N./S.=5% colored measurement noise are shown in Figs. 5.4-5.8. The red lines in Figs. 5.4-5.8 are the real values of parameters. The single parameter identification error Δp and the average parameter identification error $|\Delta|$ of each case with different measurement noises are calculated and shown in Tables 5.1B-5.5B. Comparison of average parameter identification errors $|\Delta|$ of the all five cases is shown in Table 5.6. Comparison of average parameter identification errors $|\Delta|$ of the two systems with continuous nonlinearities is shown in Table 5.7. Comparison of average parameter identification errors $|\Delta|$ of the two systems with discontinuous nonlinearities is shown in Table 5.8.

Table 5.1A Identification results of the Wiener-Hammerstein system with a continuous nonlinearity

N = 2000	b_{11}	b_{12}	f_{11}	f_{12}	β_2	β_3
5% C. N.	0.1348	0.0626	-1.4941	0.6990	2.9655	1.4219
5% W. N.	0.1327	0.0723	-1.4932	0.6963	2.9940	1.5419
Non-noise	0.1333	0.0665	-1.5000	0.7000	3.0023	1.5021
True values	0.1333	0.0667	-1.5000	0.7000	3.0000	1.5000
N = 2000	a_1	a_2	b_{21}	b_{22}	c_1	c_2
5% C. N.	-0.8879	0.8380	0.1843	0.0479	0.2175	0.0613
5% W. N.	-0.8961	0.8404	0.1281	0.0680	-	-
Non-noise	-0.9000	0.8500	0.1344	0.0665	-	-
True values	-0.9000	0.8500	0.1333	0.0667	0.2000	0.1000

Table 5.1B Parameter identification errors of the Wiener-Hammerstein system with a continuous nonlinearity

N = 2000	$\lvert\Delta b_{11}\rvert$	$\lvert\Delta b_{12}\rvert$	$\lvert\Delta f_{11}\rvert$	$\lvert\Delta f_{12}\rvert$	$\lvert\Delta\beta_2\rvert$	$\lvert\Delta\rvert$
5% C. N.	0.0015	0.0041	0.0059	0.0010	0.0345	0.0219
5% W. N.	0.0006	0.0056	0.0068	0.0037	0.0060	0.0085
N = 2000	$\lvert\Delta a_1\rvert$	$\lvert\Delta a_2\rvert$	$\lvert\Delta b_{21}\rvert$	$\lvert\Delta b_{22}\rvert$	$\lvert\Delta\beta_3\rvert$	-
5% C. N.	0.0781	0.0121	0.0120	0.0510	0.0188	-
5% W. N.	0.0419	0.0039	0.0096	0.0052	0.0013	-

Table 5.2A Identification results of the Wiener-Hammerstein system with a dead-zone

N = 2000	b_{11}	b_{12}	f_{11}	f_{12}	D	-
5% C. N.	0.1295	0.0743	-1.4974	0.7005	0.4957	-
5% W. N.	0.1294	0.0757	-1.4985	0.6998	0.4980	-
Non-noise	0.1333	0.0667	-1.5000	0.7000	0.5000	-
True values	0.1333	0.0667	-1.5000	0.7000	0.5000	-
N = 2000	a_1	a_2	b_{21}	b_{22}	c_1	c_2
5% C. N.	-0.8917	0.8464	0.1265	0.0635	0.2361	0.1446
5% W. N.	-0.8944	0.8434	0.1226	0.0621	-	-
Non-noise	-0.9000	0.8500	0.1333	0.0667	-	-
True values	-0.9000	0.8500	0.1333	0.0667	0.2000	0.1000

Table 5.2A Parameter identification errors of the Wiener-Hammerstein system with a dead-zone

N = 2000	$\lvert\Delta b_{11}\rvert$	$\lvert\Delta b_{12}\rvert$	$\lvert\Delta f_{11}\rvert$	$\lvert\Delta f_{12}\rvert$	$\lvert\Delta D\rvert$	$\lvert\Delta\rvert$
5% C. N.	0.0038	0.0076	0.0026	0.0005	0.0043	0.0005
5% W. N.	0.0039	0.0090	0.0015	0.0002	0.0020	0.0005
N = 2000	$\lvert\Delta a_1\rvert$	$\lvert\Delta a_2\rvert$	$\lvert\Delta b_{21}\rvert$	$\lvert\Delta b_{22}\rvert$	-	
5% C. N.	0.0083	0.0036	0.0068	0.0032	-	-
5% W. N.	0.0056	0.0066	0.0107	0.0046	-	-

Table 5.3A Identification results of the Hammerstein-Wiener system with two continuous nonlinearities

N = 2000	b_1	b_2	f_1	f_2
5% C. N.	-0.6742	0.0877	-1.4952	0.6910
5% W. N.	-0.6964	0.1296	-1.4936	0.6956
Non-noise	-0.6988	0.1207	-1.4990	0.6995
True values	-0.7000	0.1200	-1.5000	0.7000
β_{11}	β_{12}	β_{13}	β_{22}	β_{23}
0.2506	0.3946	0.4208	0.7219	0.8052
0.3055	0.4044	0.4771	0.6573	0.8381
0.3014	0.4013	0.4920	0.7810	0.9135
0.3000	0.4000	0.5000	0.8000	0.9000
N = 2000	c_1	c_2	d_1	d_2
5% C. N.	-0.1289	0.3297	-0.8471	0.7871
True values	0.2000	0.1000	-1.2000	0.5000

Table 5.3B Parameter errors of the Hammerstein-Wiener system with two continuous nonlinearities

N = 2000	$\lvert\Delta b_1\rvert$	$\lvert\Delta b_2\rvert$	$\lvert\Delta f_1\rvert$	$\lvert\Delta f_2\rvert$	$\lvert\Delta\beta_{11}\rvert$	$\lvert\Delta\rvert$
5% C. N.	0.0258	0.0323	0.0048	0.0090	0.0494	0.0421
5% W. N.	0.0036	0.0096	0.0064	0.0044	0.0055	0.0291
N = 2000	$\lvert\Delta\beta_{12}\rvert$	$\lvert\Delta\beta_{13}\rvert$	$\lvert\Delta\beta_{22}\rvert$	$\lvert\Delta\beta_{23}\rvert$	-	-
5% C. N.	0.0054	0.0792	0.0781	0.0948	-	-
5% W. N.	0.0044	0.0229	0.1427	0.0619	-	-

Table 5.4A Identification results of the Saturation-Linear-Deadzone system

N = 2000	b_1	b_2	f_1	f_2
5% C. N.	-0.7191	0.1327	-1.5188	0.7131
5% W. N.	-0.6863	0.1055	-1.5069	0.7054
Non-noise	-0.7001	0.1197	-1.5000	0.6999
True values	-0.7000	0.1200	-1.5000	0.7000
N = 2000	L	Z	D	-
5% C. N.	1.0050	0.9803	0.4990	-
5% W. N.	0.9660	1.0083	0.4929	-
Non-noise	1.0002	0.9999	0.5000	-
Real	1.0000	1.0000	0.5000	-
N = 2000	c_1	c_2	d_1	d_2
5% C. N.	0.2682	0.2811	-0.9682	0.3015
True values	0.2000	0.1000	-1.2000	0.5000

Table 5.4B Parameter identification errors of the Saturation-Linear-Deadzone system

N = 2000	$\lvert\Delta b_1\rvert$	$\lvert\Delta b_2\rvert$	$\lvert\Delta f_1\rvert$	$\lvert\Delta f_2\rvert$	$\lvert\Delta\rvert$
5% C. N.	0.0191	0.0127	0.0188	0.0131	0.0128
5% W. N.	0.0137	0.0145	0.0069	0.0054	0.0128
N = 2000	$\lvert\Delta L\rvert$	$\lvert\Delta Z\rvert$	$\lvert\Delta D\rvert$	-	-
5% C. N.	0.0050	0.0197	0.0010	-	-
5% W. N.	0.0340	0.0083	0.0071	-	-

Table 5.5A Identification results of the Deadzone-Linear-Saturation system

N = 2000	b_1	b_2	f_1	f_2
5% C. N.	-0.6870	0.1306	-1.4954	0.7000
5% W. N.	-0.6918	0.1219	-1.4998	0.7032
Non-noise	-0.7007	0.1205	-1.5000	0.7000
True values	-0.7000	0.1200	-1.5000	0.7000
2000	S_d	D	Z	-
5% C. N.	0.9843	0.4748	1.0235	-
5% W. N.	1.0469	0.5189	0.9987	-
Non-noise	1.0010	0.5004	0.9998	-
True values	1.0000	0.5000	1.0000	-
N = 2000	c_1	c_2	d_1	d_2
5% C. N.	0.2467	0.1717	-1.1220	0.4273
True values	0.2000	0.1000	-1.2000	0.5000

Table 5.5B Parameter errors of the Deadzone-Linear-Saturation system

N = 2000	$\lvert\Delta b_1\rvert$	$\lvert\Delta b_2\rvert$	$\lvert\Delta f_1\rvert$	$\lvert\Delta f_2\rvert$	$\lvert\Delta\rvert$
5% C. N.	0.013	0.0106	0.0046	0.0000	0.0132
5% W. N.	0.0082	0.0019	0.0002	0.0032	0.0115
N = 2000	$\lvert\Delta S_d\rvert$	$\lvert\Delta D\rvert$	$\lvert\Delta Z\rvert$	-	-
5% C. N.	0.0157	0.0252	0.0235	-	-
5% W. N.	0.0469	0.0189	0.0013	-	-

Table 5.6 Comparison of average parameter identification errors $\lvert\Delta\rvert$ of the five cases

N = 2000	L-N-L	L-D-L	N-L-N	S-L-D	D-L-S	Best accur.
5% C. N.	0.0219	0.0005	0.0421	0.0128	0.0132	L-D-L
5% W. N.	0.0085	0.0005	0.0291	0.0128	0.0115	L-D-L

Table 5.7 Comparison of average parameter identification errors $\lvert\Delta\rvert$ of the two systems with continuous nonlinearities

N = 2000	L-N-L	N-L-N	Best accuracy
5% C. N.	0.0219	0.0421	L-N-L
5% W. N.	0.0085	0.0291	L-N-L

Table 5.8 Comparison of average parameter identification errors $\lvert\Delta\rvert$ of the two systems with discontinuous nonlinearities

N = 2000	L-D-L	S-L-D	D-L-S	Best accuracy
5% C. N.	0.0005	0.0128	0.0132	L-D-L
5% W. N.	0.0005	0.0128	0.0115	L-D-L

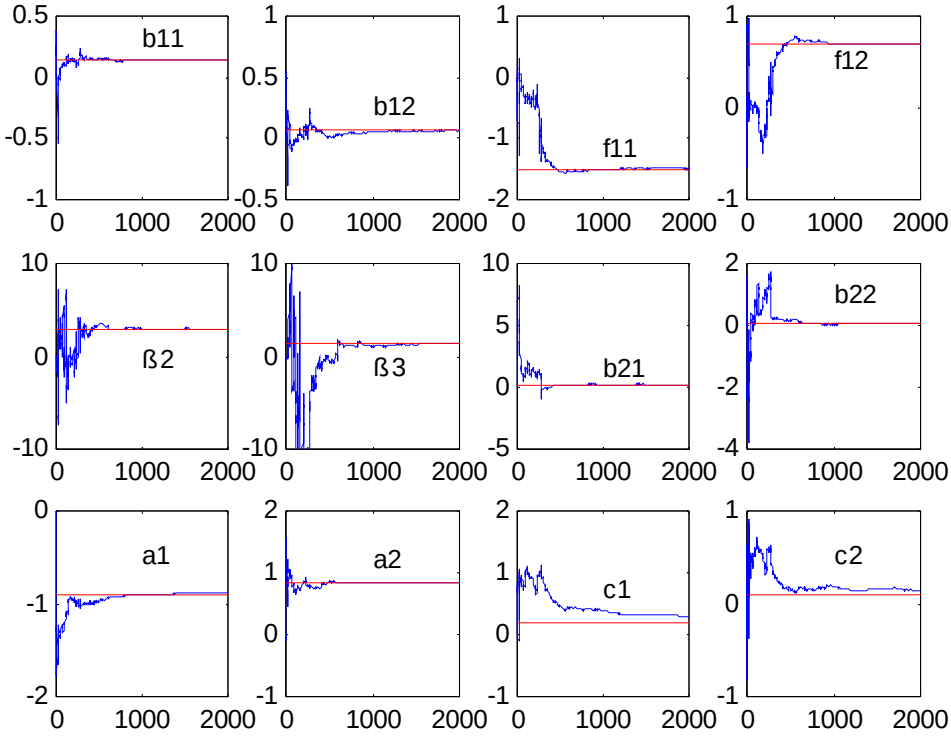


Fig. 5.4 The Wiener-Hammerstein system identification process
with continuous nonlinearity with a N./S.=5% colored measurement noise

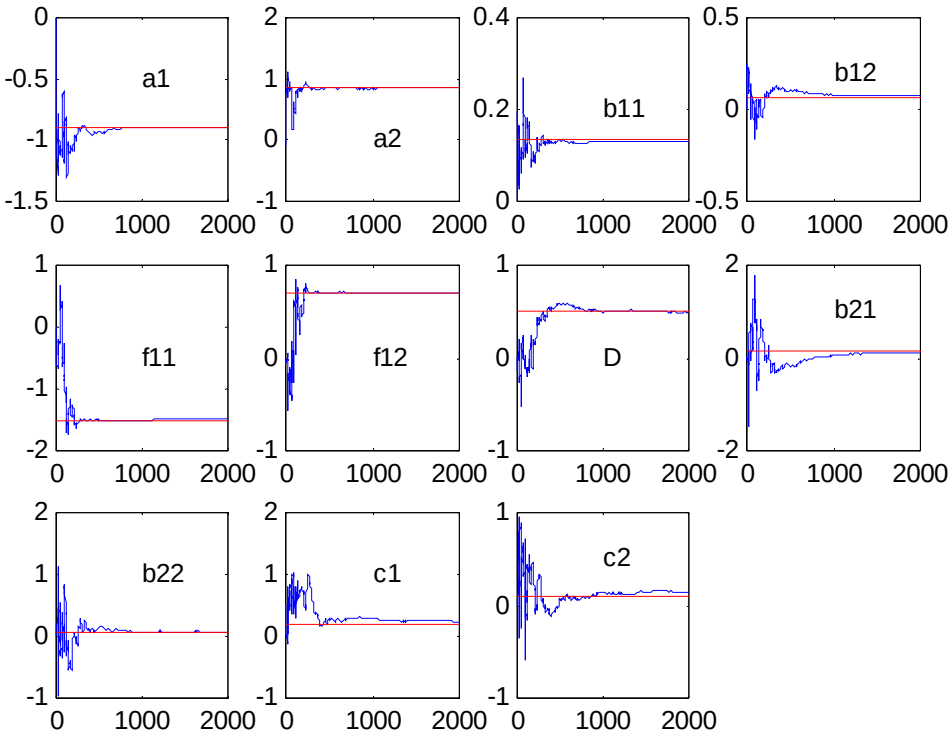


Fig. 5.5 Wiener-Hammerstein system identification process
with a dead-zone with a N./S.=5% colored measurement noise

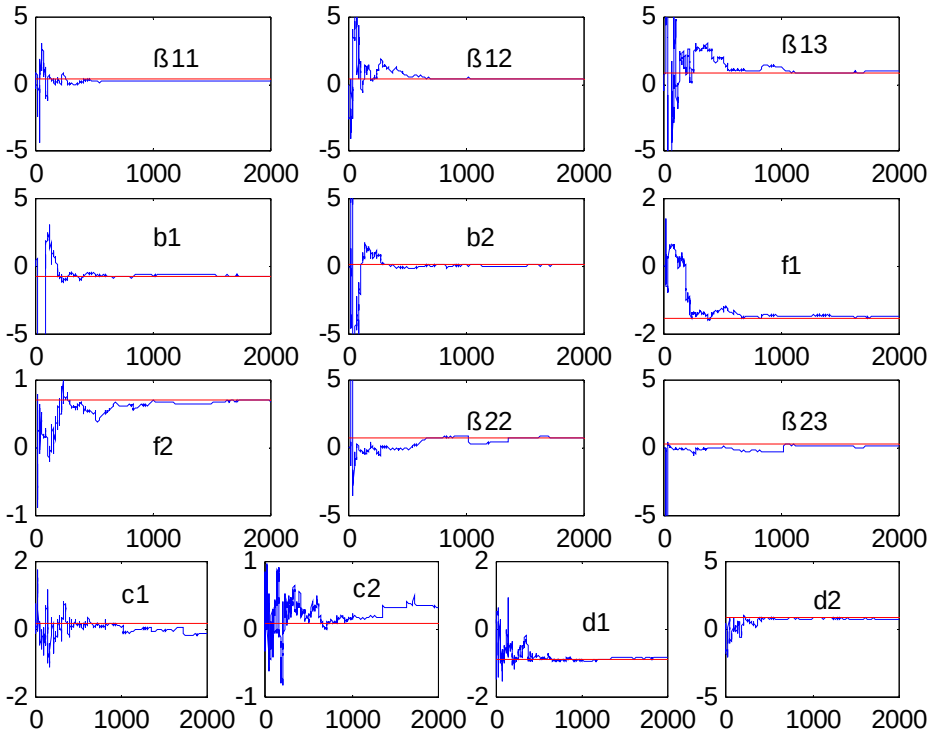


Fig. 5.6 Hammerstein-Wiener system identification process
with two continuous nonlinearities with a N./S.=5% colored measurement noise

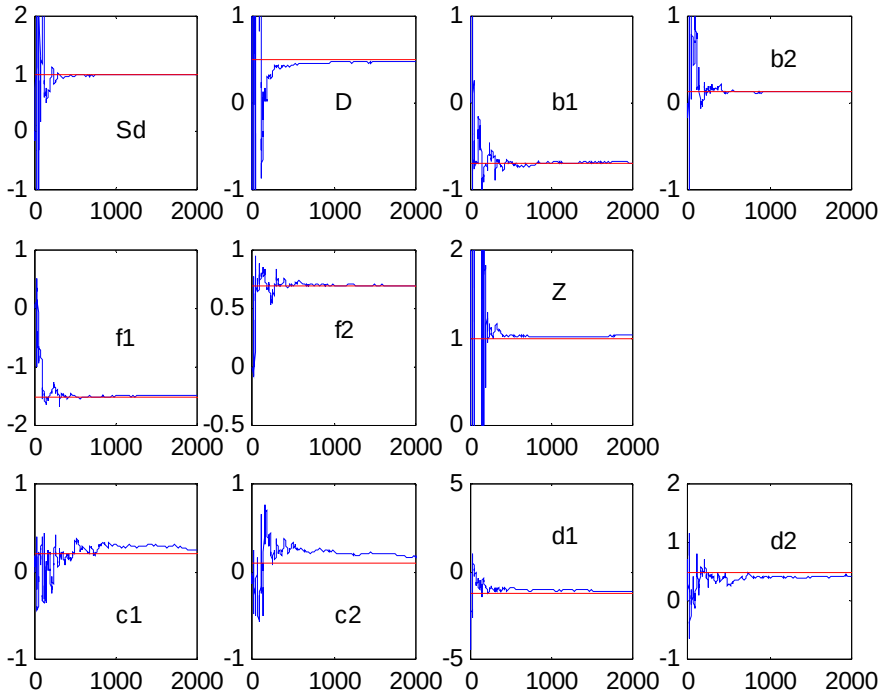


Fig. 5.7 Deadzone-Linear-Saturation system identification process
with a N./S.=5% colored measurement noise

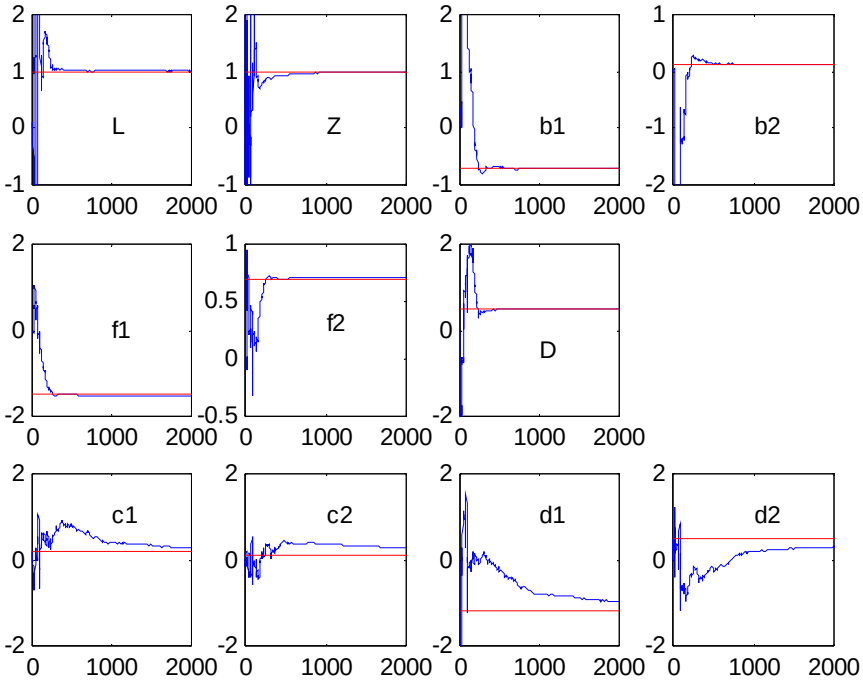


Fig. 5.8 Saturation-Linear-Deadzone system identification process with a N./S.=5% colored measurement noise

The simulation results show that:

- The new identification method gives good results for all two cascade Wiener and Hammerstein systems with continuous and discontinuous nonlinearities, respectively.

- From Tables 5.6-5.8, we can see that which one of the five cases has the best accuracy results under different measurement noises, respectively.

6. Identification of generalized Wiener and Hammerstein systems

In this chapter, the new identification method for Wiener and Hammerstein systems will be extended to identify a more general nonlinear dynamic system. The relationship among Wiener and Hammerstein systems and parametric Volterra-series will also be highlighted. And so-called generalized Wiener and Hammerstein systems will be proposed and identified.

It is known that the parametric Volterra-series provide an important general representation for a time-invariant stable nonlinear dynamic system. It is, roughly speaking, always valid for nonlinear dynamic systems with analytic nonlinearities. A discrete parametric Volterra-series is described as

$$\begin{aligned} y(t) &= y_0 + \sum_{j=1}^{\infty} a_j y(t-j) + \sum_{j=0}^{\infty} b_j u(t-j) + \sum_{j_1=0}^{\infty}\sum_{j_2=j_1}^{\infty} b_{j_1 j_2} u(t-j_1)u(t-j_2) \\ &+ \sum_{j_1=0}^{\infty}\sum_{j_2=j_1}^{\infty}\sum_{j_3=j_2}^{\infty} b_{j_1 j_2 j_3} u(t-j_1)u(t-j_2)u(t-j_3) + \cdots \end{aligned} \tag{6.1}$$

with $u(t)$ as the system input, $y(t)$ a the system output and y_0 as a mean value. a_j are the autoregressive parameters of the system and a_j, b_j, $b_{j_1 j_2}$, $b_{j_1 j_2 j_3}, \cdots$ are the 0-th, first, second, third $, \cdots$ Volterra kernels which will trend towards zero as $j \to \infty$, $j_1 \to \infty$, $j_2 \to \infty$, $j_3 \to \infty, \cdots$, (Kurth, 1996).

This class of systems is very broad, but to be practically useful, the sums must be truncated to some finite upper limit n and the number of sums included must also be made finite m. Therefore, Eq. (6.1) can be approximated by

$$\begin{aligned} y(t) &= y_0 + \sum_{j=1}^{n} a_j y(t-j) + \sum_{j=0}^{n} b_j u(t-j) + \sum_{j_1=0}^{n}\sum_{j_2=j_1}^{n} b_{j_1 j_2} u(t-j_1)u(t-j_2) \\ &+ \cdots + \sum_{j_1=0}^{n}\sum_{j_2=j_1}^{n}\cdots\sum_{j_m=j_{m-1}}^{n} b_{j_1 j_2 j_3 \cdots j_m} u(t-j_1)u(t-j_2)\cdots u(t-j_m) + \varepsilon(t) \end{aligned} \tag{6.2}$$

where $\varepsilon(t)$ is the sum of cutting error and the measurement noise.

Direct identification of parametric Volterra-series results to estimate the parametric Volterra kernels on the basis of input-output data sequences $u(t)$ and $y(t)$. In principle, Eq. (6.2) can be identified using least squares method with the help of various types of deterministic as well as stochastic inputs. But in the normal case there are too many parameters (>10000) to be identified (Kurth, 1996). This has been proved to be a nontrivial task. Therefore, a particular problem is to deal with the large number of potentially necessary parameters. Kurth (1996) developed a method for identification of modified compressed discrete Volterra-series by introducing basis functions and structure selection.

Here, by using shifting operators $(q^{-1}, q_1^{-1}, q_2^{-1}, \cdots, q_m^{-1})$ and according to the definition of the generalized transfer function, we write Eq. (6.2) in the form

$$\begin{aligned} y(t) &= y_0 + G_0(q^{-1})y(t-1) + G_1(q^{-1})u(t) + G_2(q_1^{-1}, q_2^{-1})u^2(t) \\ &+ \cdots + G_m(q_1^{-1}, q_2^{-1}, \cdots, q_m^{-1})u^m(t) + \varepsilon(t) \end{aligned} \tag{6.3}$$

with $n_0, n_1, n_2, \cdots, n_m$ as the orders of the corresponding generalized transfer functions $G_0(q^{-1}), G_1(q^{-1}), G_2(q_1^{-1}, q_2^{-1}), \cdots, G_m(q_1^{-1}, q_2^{-1}, \cdots, q_m^{-1})$, respectively.

Eq. (6.3) shows that the nonlinear dynamic system output $y(t)$ consists of the mean value y_0, the autoregressive part $G_0(q^{-1})y(t-1)$, the linear dynamic part $G_1(q^{-1})u(t)$ and different nonlinear dynamic parts from different inputs $u^2(t), \cdots, u^m(t)$.

In Eq. (6.3), because $y(t-1)$, $u(t)$, $u^2(t), \cdots, u^m(t)$ are independent, they can be regarded as pseudo inputs which specify particular nonlinearities and drive their corresponding dynamic subsystems $G_0(q^{-1}), G_1(q^{-1}), G_2(q_1^{-1}, q_2^{-1}), \cdots, G_m(q_1^{-1}, q_2^{-1}, \cdots, q_m^{-1})$ to the single output $y(t)$. This is illustrated in Fig. 6.1.

In this sense we transform a nonlinear dynamic system approximately into a pseudo-linear MISO system with the pseudo inputs: $y(t-1)$, $u(t)$, $u^2(t), \cdots, u^m(t)$. On the one hand they are pseudo inputs of the nonlinear dynamic system and on the other hand they illustrate also the nonlinear couplings between the pseudo multiple inputs.

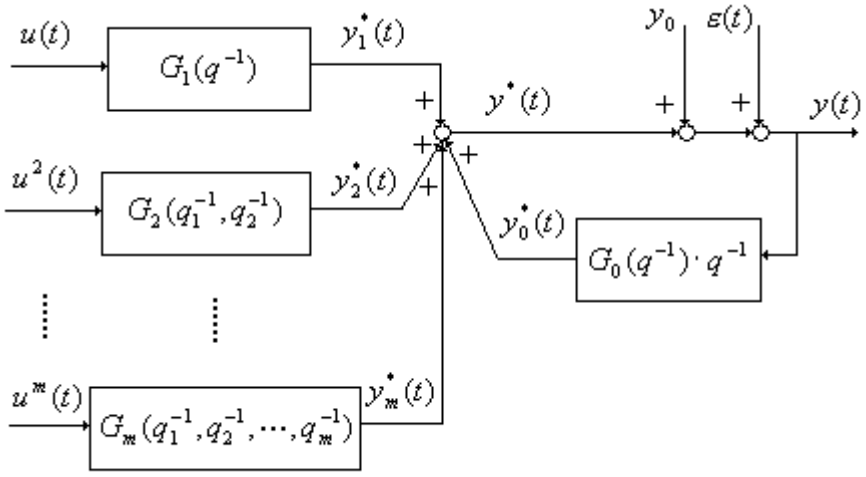


Fig. 6.1 Parametric Volterra-series model

6.1. Generalized Wiener and Hammerstein systems

A model is always a system description in some approximation level. Different approximations to a nonlinear dynamic system can be got by regularization, shrinking, pruning and partly selecting of different terms of the parametric Volterra-series Eq. (6.3). For example, the linear dynamic model is just the simplest approximation. Actually, the block-oriented systems as well as the Wiener and Hammerstein systems are all approximations of the parametric Volterra-series. This is handled by making the number of 'used' parameters considerably less than the number of 'offered' parameters.

From Eq. (6.3) we can derive the following different approximations of a nonlinear dynamic system:

- Linear model

$$y(t) = y_0 + G_0(q^{-1})y(t-1) + G_1(q^{-1})u(t) + \varepsilon(t). \tag{6.4}$$

It is also a linear predication error model as shown in Ljung (1987).

- Hammerstein model

$$y(t) = y_0 + G_0(q^{-1})y(t-1) + G_1(q^{-1})u(t) + [\beta_2 \cdot u^2(t) + \cdots + \beta_m \cdot u^m(t)]G_1(q^{-1}) + \varepsilon(t). \quad (6.5a)$$

Eq. (6.5a) shows that a Hammerstein model is an approximation of the parametric Volterra-series with the following assumptions:

$$\begin{aligned} &G_2(q_1^{-1}, q_2^{-1}) = \beta_2 \cdot G_1(q^{-1}) \\ &\vdots \qquad\qquad \vdots \end{aligned} \quad (6.5b)$$

$$G_m(q_1^{-1}, q_2^{-1}, \cdots, q_m^{-1}) = \beta_m \cdot G_1(q_1^{-1}) \quad (6.5c)$$

where β_j for $j = 2,3,\cdots,m$ are defined as constant parameters.

- Wiener model

$$y(t) = y_0 + G_0(q^{-1})y(t-1) + G_1(q^{-1})u(t) + \beta_2 \cdot w^2(t) + \cdots + \beta_m \cdot w^m(t) + \varepsilon(t). \quad (6.6a)$$

Eq. (6.6a) shows that a Wiener model is an approximation of the parametric Volterra-series with the following assumptions:

$$\begin{aligned} &G_2(q_1^{-1}, q_2^{-1}) = \beta_2 \cdot G_1(q^{-1}) \cdot G_1(q^{-1}) \\ &\vdots \qquad\qquad \vdots \end{aligned} \quad (6.6b)$$

$$G_m(q_1^{-1}, q_2^{-1}, \cdots, q_m^{-1}) = \beta_m \cdot \underbrace{G_1(q_1^{-1}) \cdot \cdots \cdot G_1(q_1^{-1})}_{m \quad times}. \quad (6.6c)$$

The unmeasurable intermediate variable $w(t)$ in Eq. (6.6a) is defined as

$$w(t) = G_1(q^{-1})u(t) \quad (6.7)$$

and β_j for $j = 2,3,\cdots,m$ are defined as constant parameters.

- Wiener-Hammerstein model

$$y(t) = y_0 + G_0(q^{-1})y(t-1) + G_1(q^{-1})u(t) + [\beta_2 \cdot w^2(t) + \cdots + \beta_m \cdot w^m(t)]G_2(q^{-1}) + \varepsilon(t) \quad (6.8a)$$

Eq. (6.8a) shows that a Wiener-Hammerstein model is an approximation of the parametric Volterra-series with the following assumptions:

$$\begin{aligned} &G_2(q_1^{-1}, q_2^{-1}) = \beta_2 \cdot G_1(q^{-1}) \cdot G_1(q^{-1}) \cdot G_2(q^{-1}) \\ &\vdots \qquad\qquad \vdots \end{aligned} \quad (6.8b)$$

$$G_m(q_1^{-1}, q_2^{-1}, \cdots, q_m^{-1}) = \beta_m \cdot \underbrace{G_1(q_1^{-1}) \cdot \cdots \cdot G_1(q_1^{-1})}_{m \quad times} \cdot G_2(q^{-1}). \quad (6.8c)$$

The unmeasurable intermediate variable $w(t)$ in Eq. (6.8a) is defined as Eq. (6.7) and β_j for $j = 2,3,\cdots,m$ are constant parameters.

In the same principle, we propose three generalized Wiener and Hammerstein models which are better approximations of the parametric Volterra-series Eq. (6.3).

- Generalized Bilinear model

$$y(t) = y_0 + G_0(q^{-1})y(t-1) + G_1(q^{-1})u(t) + \tilde{G}_0(q^{-1})u^2(t) + \tilde{G}_1(q^{-1})[u(t)u(t-1)] + \cdots + \tilde{G}_m(q^{-1})[u(t)u(t-m)] + \varepsilon(t) \quad (6.9a)$$

In the generalized Bilinear model, only terms to the second power are selected from the parametric Volterra-series. It is an approximation model with the following assumptions:

$$G_2(q_1^{-1}, q_2^{-1}) = G_2(q^{-1}) = \tilde{G}_0(q^{-1}) \quad (6.9b)$$

$$G_3(q_1^{-1}, q_2^{-1}) = G_3(q^{-1}) \cdot q^{-1} = \tilde{G}_1(q^{-1}) \quad (6.9c)$$

$$\vdots \qquad \vdots$$

$$G_m(q_1^{-1}, q_2^{-1}, \cdots, q_m^{-1}) = G_m(q^{-1}) \cdot q^{-m} = \tilde{G}_m(q^{-1}). \quad (6.9d)$$

- Generalized Hammerstein model

$$y(t) = y_0 + G_0(q^{-1})y(t-1) + G_1(q^{-1})u(t) + G_2(q^{-1})u^2(t) + \cdots + G_m(q^{-1})u^m(t) + \varepsilon(t) \quad (6.10a)$$

The generalized Hammerstein model is an approximation of the parametric Volterra-series with the following assumptions:

$$G_2(q_1^{-1}, q_2^{-1}) = G_2(q^{-1}) \quad (6.10b)$$

$$\vdots \qquad \vdots$$

$$G_m(q_1^{-1}, q_2^{-1}, \cdots, q_m^{-1}) = G_m(q^{-1}). \quad (6.10c)$$

- Generalized Wiener model

$$y(t) = y_0 + G_0(q^{-1})y(t-1) + G_1(q^{-1})u(t) + G_2(q^{-1})w^2(t) + \cdots + G_m(q^{-1})w^m(t) + \varepsilon(t). \quad (6.11a)$$

The generalized Wiener model is an approximation of the parametric Volterra-series with the following assumptions:

$$G_2(q_1^{-1}, q_2^{-1}) = G_2(q^{-1}) \cdot G_1(q^{-1}) \cdot G_1(q^{-1}) \quad (6.11b)$$

$$\vdots \qquad \vdots$$

$$G_m(q_1^{-1}, q_2^{-1}, \cdots, q_m^{-1}) = G_m(q^{-1}) \cdot \underbrace{G_1(q_1^{-1}) \cdot \cdots \cdot G_1(q_1^{-1})}_{m \ \ times}. \quad (6.11c)$$

The unmeasurable intermediate variable $w(t)$ in Eq. (6.11a) is defined again as in Eq. (6.7).

It should be noted that the Hammerstein system (Eq. (6.5a)) is a special case of the generalized Hammerstein system (Eq. (6.10a)). The Wiener system (Eq. (6.6a)) and the Wiener-Hammerstein system (Eq. (6.8a)) are just special cases of the generalized Wiener system (Eq. (6.11a)).

In Eqs. (6.4)-(6.11), the rational transfer functions $G_j(q^{-1})$ of order n_j as well as $\tilde{G}_j(q^{-1})$ for $j = 0,1,2,\cdots,m$ are described by

$$G_j(q^{-1}) = \frac{B_j(q^{-1})}{F_j(q^{-1})} \tag{6.12a}$$

with

$$B_j(q^{-1}) = b_{j0} + b_{j1}q^{-1} + \cdots + b_{jn_j}q^{-n_j} \tag{6.12b}$$

$$F_j(q^{-1}) = 1 + f_{j1}q^{-1} + \cdots + f_{jn_j}q^{-n_j}. \tag{6.12c}$$

It is assumed that $B_j(q^{-1})$ and $F_j(q^{-1})$ for $j = 0,1,2,\cdots,m$ are coprime.

It should be noted that the autoregressive part $G_0(q^{-1})y(t-1)$ would help to reduce the parametric Volterra-series order. And in practice, in order to enhance the system performance, it is reasonable to choose a nonlinear dynamic model architecture which also contains a linear dynamic model $G_0(q^{-1})$ as a special case. For the nonlinear dynamic model Eqs. (6.5)-(6.11), it can be advantageously to establish a linear dynamic model $G_0(q^{-1})$ in parallel. So the overall model output is the sum of the linear and the nonlinear model parts. This strategy is very appealing because it ensures that the overall nonlinear model performance is better than that of the linear model.

Now, the new identification method will be used to identify the generalized Hammerstein and generalized Wiener systems as shown in Figs. 6.2 and 6.3.

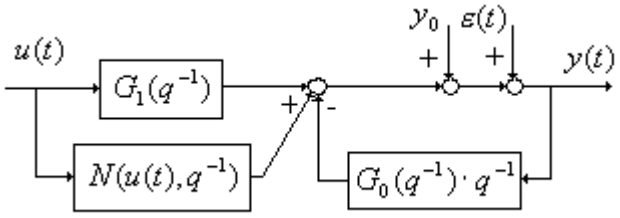


Fig. 6.2 Generalized Hammerstein system

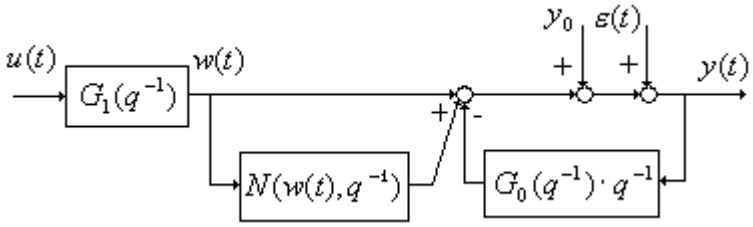


Fig. 6.3 Generalized Wiener system

In these Figures, $N(\chi(t), q^{-1})$ is defined as a nonlinear dynamic polynomial of order m but only with high power terms

$$\begin{aligned}\gamma(t) &= N(\chi(t), q^{-1}) \\ &= \sum_{k=2}^{m} G_k(q^{-1}) \cdot \chi^k(t)\end{aligned} \tag{6.13}$$

where $\chi(t)$ is the input of the nonlinear dynamic block and $\gamma(t)$ is the output of the nonlinear dynamic block. $G_k(q^{-1})$ are the corresponding linear dynamics of $\chi^k(t)$ for $k = 2, \cdots, m$.

From Eq. (6.10a) and Fig. (6.2), we can see that the generalized Hammerstein system can be directly transformed approximately into a pseudo-linear MISO system which has $m+2$ independent pseudo-inputs: 1, $y(t-1)$ and $u^k(t)$ for $k = 1, \cdots, m$. All the parameters are explicitly given without redundancy.

Actually, the generalized Bilinear system Eq. (6.9a) can also be directly transformed approximately into a pseudo-linear MISO system which has $m+4$ independent pseudo-inputs: 1, $y(t-1)$, $u(t)$ and $u(t)u(t-k)$ for $k = 0, \cdots, m$ and the parameters are also explicitly given. In these forms as in Eqs. (6.9a) and (6.10a), both generalized Bilinear and Hammerstein systems can be identified using the identification method for pseudo-linear MISO dynamic system.

Considering the generalized Wiener system Eq. (6.11a) and Fig. (6.3), although there is an unmeasurable intermediate variable $w(t)$, it can be estimated recursively according to Eq. (6.7). Then the generalized Wiener system can be transformed approximately into a pseudo-linear MISO system which has $m+2$ independent pseudo-inputs: 1, $y(t-1)$, $u(t)$ and the recursively estimated intermediate variables $w^k(t)$ for $k = 2, \cdots, m$. All the parameters are also explicitly given and without redundancy.

6.2. Simulation results

In the following the new identification method will be applied to three different special nonlinear systems, namely

- a generalized Bilinear system,
- a generalized Hammerstein system and
- a generalized Wiener system.

They are described as follows:

- Generalized Bilinear system

$$\begin{aligned}y(t) &= y_0 + G_0(q^{-1})y(t-1) + G_1(q^{-1})u(t) \\ &+ G_2(q^{-1})u(t)u(t-1) + G_3(q^{-1})u(t)u(t-2) + \varepsilon(t).\end{aligned} \tag{6.14}$$

- Generalized Hammerstein system

$$\begin{aligned}y(t) &= y_0 + G_0(q^{-1})y(t-1) + G_1(q^{-1})u(t) \\ &+ G_2(q^{-1})u^2(t) + G_3(q^{-1})u^3(t) + \varepsilon(t).\end{aligned} \tag{6.15}$$

- Generalized Wiener system

$$y(t) = y_0 + G_0(q^{-1})y(t-1) + G_1(q^{-1})u(t) + G_2(q^{-1})w^2(t) + G_3(q^{-1})w^3(t) + \varepsilon(t). \tag{6.16}$$

The corresponding linear dynamics in the above equations are as follows:

$$G_0(q^{-1}) = \frac{0.1333q^{-1} + 0.0667q^{-2}}{1 - 1.5q^{-1} + 0.7q^{-2}} \tag{6.17}$$

$$G_1(q^{-1}) = \frac{0.4q^{-1} + 0.3q^{-2}}{1 - 0.9q^{-1} + 0.8q^{-2}} \tag{6.18}$$

$$G_2(q^{-1}) = \frac{0.3q^{-1} + 0.2q^{-2}}{1 - 1.2q^{-1} + 0.5q^{-2}} \tag{6.19}$$

$$G_3(q^{-1}) = \frac{0.2q^{-1} + 0.1q^{-2}}{1 - 1.7q^{-1} + 0.9q^{-2}}. \tag{6.20}$$

For the generalized Wiener system Eq. (6.16), the intermediate variable $w(t)$ is given by

$$w(t) = G_1(q^{-1})u(t) = \frac{0.4q^{-1} + 0.3q^{-2}}{1 - 0.9q^{-1} + 0.8q^{-2}}u(t). \tag{6.21}$$

A random numbers of zero mean are used as system input $u(t)$. An independent random numbers as white measurement noise $\varepsilon(t)$. Because of the increase of the parameter number, 4000 data points are collected for each case. Each system will be tested with non-noisy (N. N.) and with a N./S.=5% white measurement noise (5% W. N.).

By identifying the generalized Wiener system, in order to improve alertness ability, we use the adaptive moving average parameter smoothing (Trigg and Leach, 1967) which means recursively adaptive exponentially averaging and smoothing the estimated parameters of $G_1(q^{-1})$ using a moving window with fixed length $Mov = 5$. Brown's double exponential smoothing application (Brown, 1963) is used to mitigate the estimate of the unmeasurable intermediate variable $w(t)$.

The standard recursive prediction errors method (RPEM) with forgetting factor for linear MISO system in MATLAB will be applied. The algorithm variable settings are $\lambda(0) = 0.7$, $\Delta\lambda = 0.01$. The initial estimates of the unknown parameters are taken as zero.

The identification results are shown in Tables 6.1A-6.3A. The identification processes with a N./S.=5% colored measurement noise are shown in Figs. 6.4-6.6. The red lines in Figs. 6.4-6.6 are the true values of parameters. The single parameter identification error Δp and the average parameter identification error $|\Delta|$ of each case are calculated and shown in Tables 6.1B-6.3B. Comparison of average parameter identification errors $|\Delta|$ of the three cases is shown in Table 6.4.

Table 6.1A Identification results of the generalized Bilinear system

N = 4000	b_{01}	b_{02}	f_{01}	f_{02}	b_{11}	b_{12}	f_{11}	f_{12}
5% W. N.	0.1217	0.0833	-1.4820	0.6854	0.4004	0.2963	-0.9035	0.8066
N. N.	0.1316	0.0691	-1.4975	0.6979	0.4012	0.2993	-0.9005	0.8007
True val.	0.1333	0.0667	-1.5000	0.7000	0.4000	0.3000	-0.9000	0.8000
N = 4000	b_{21}	b_{22}	f_{21}	f_{22}	b_{31}	b_{32}	f_{31}	f_{32}
5% W. N.	0.3373	0.1886	-1.1499	0.4432	0.2052	0.0822	-1.7032	0.9046
N. N.	0.2975	0.2033	-1.1972	0.4980	0.2011	0.0985	-1.6996	0.8995
True val.	0.3000	0.2000	-1.2000	0.5000	0.2000	0.1000	-1.7000	0.9000

Table 6.1B Parameter identification errors of the generalized Bilinear system

N = 4000	$\lvert\Delta b_{01}\rvert$	$\lvert\Delta b_{02}\rvert$	$\lvert\Delta f_{01}\rvert$	$\lvert\Delta f_{02}\rvert$	$\lvert\Delta b_{11}\rvert$	$\lvert\Delta b_{12}\rvert$	$\lvert\Delta f_{11}\rvert$	$\lvert\Delta f_{12}\rvert$	$\lvert\Delta\rvert$
5% W.	0.0116	0.0166	0.018	0.0146	0,0004	0.0037	0.0035	0.0066	0.0163
N = 4000	$\lvert\Delta b_{21}\rvert$	$\lvert\Delta b_{22}\rvert$	$\lvert\Delta f_{21}\rvert$	$\lvert\Delta f_{22}\rvert$	$\lvert\Delta b_{31}\rvert$	$\lvert\Delta b_{32}\rvert$	$\lvert\Delta f_{31}\rvert$	$\lvert\Delta f_{32}\rvert$	
5% W.	0.0373	0.0114	0.0501	0.0568	0.0052	0.0178	0.0032	0.0046	

Table 6.2A Identification results of the generalized Hammerstein system

N = 4000	b_{01}	b_{02}	f_{01}	f_{02}	b_{11}	b_{12}	f_{11}	f_{12}
5% W. N.	0.1216	0.0768	-1.4877	0.6931	0.3979	0.3015	-0.9035	0.8038
N. N.	0.1291	0.0726	-1.4936	0.6948	0.4042	0.2986	-0.9011	0.8010
True val.	0.1333	0.0667	-1.5000	0.7000	0.4000	0.3000	-0.9000	0.8000
N = 4000	b_{21}	b_{22}	f_{21}	f_{22}	b_{31}	b_{32}	f_{31}	f_{32}
5% W. N.	0.2911	0.3095	-1.0938	0.4495	0.1945	0.0974	-1.6973	0.9012
N. N.	0.3037	0.2008	-1.1970	0.4998	0.1904	0.1089	-1.6982	0.8990
True val.	0.3000	0.2000	-1.2000	0.5000	0.2000	0.1000	-1.7000	0.9000

Table 6.2B Parameter identification errors of the generalized Hammerstein system

N = 4000	$\lvert\Delta b_{01}\rvert$	$\lvert\Delta b_{02}\rvert$	$\lvert\Delta f_{01}\rvert$	$\lvert\Delta f_{02}\rvert$	$\lvert\Delta b_{11}\rvert$	$\lvert\Delta b_{12}\rvert$	$\lvert\Delta f_{11}\rvert$	$\lvert\Delta f_{12}\rvert$	$\lvert\Delta\rvert$
5% W.	0.0117	0.0101	0.0123	0.0069	0.0021	0.0015	0.0035	0.0038	0.0212
N = 4000	$\lvert\Delta b_{21}\rvert$	$\lvert\Delta b_{22}\rvert$	$\lvert\Delta f_{21}\rvert$	$\lvert\Delta f_{22}\rvert$	$\lvert\Delta b_{31}\rvert$	$\lvert\Delta b_{32}\rvert$	$\lvert\Delta f_{31}\rvert$	$\lvert\Delta f_{32}\rvert$	-
5% W.	0.0089	0.1095	0.1062	0.0505	0.0055	0.0026	0.0027	0.0012	-

Table 6.3A Identification results of the generalized Wiener system

N = 4000	b_{01}	b_{02}	f_{01}	f_{02}	b_{11}	b_{12}	f_{11}	f_{12}
5% W. N.	0.1058	0.0732	-1.4826	0.7096	0.4002	0.3074	-0.9041	0.8051
N. N.	0.1307	0.0747	-1.4898	0.6884	0.3999	0.2964	-0.9003	0.8012
True val.	0.1333	0.0667	-1.5000	0.7000	0.4000	0.3000	-0.9000	0.8000
N = 4000	b_{21}	b_{22}	f_{21}	f_{22}	b_{31}	b_{32}	f_{31}	f_{32}
5% W. N.	0.3925	0.0614	-1.3228	0.6051	0.0028	0.2720	-1.1816	0.5180
N. N.	0.3172	0.1395	-1.2831	0.5542	0.1440	0.1793	-1.6844	0.8882
True val.	0.3000	0.2000	-1.2000	0.5000	0.2000	0.1000	-1.7000	0.9000

Table 6.3B Parameter identification errors of the generalized Wiener system

N = 4000	$\lvert\Delta b_{01}\rvert$	$\lvert\Delta b_{02}\rvert$	$\lvert\Delta f_{01}\rvert$	$\lvert\Delta f_{02}\rvert$	$\lvert\Delta b_{11}\rvert$	$\lvert\Delta b_{12}\rvert$	$\lvert\Delta f_{11}\rvert$	$\lvert\Delta f_{12}\rvert$	$\lvert\Delta\rvert$
5% W.	0.0275	0.0065	0.0174	0.0096	0.0002	0.0074	0.0041	0.0051	0.1129
N = 4000	$\lvert\Delta b_{21}\rvert$	$\lvert\Delta b_{22}\rvert$	$\lvert\Delta f_{21}\rvert$	$\lvert\Delta f_{22}\rvert$	$\lvert\Delta b_{31}\rvert$	$\lvert\Delta b_{32}\rvert$	$\lvert\Delta f_{31}\rvert$	$\lvert\Delta f_{32}\rvert$	-
5% W.	0.0925	0.1386	0.1228	0.1051	0.1972	0.172	0.5184	0.3822	-

Table 6.4 Comparison of average parameter identification errors $\lvert\Delta\rvert$ of the three systems

N = 4000	Generaliz. Bilinear	Generaliz. Hamm.	Generaliz. Wiener	Best accuracy
5% W.	0.0163	0.0212	0.1129	Generaliz. Bilinear

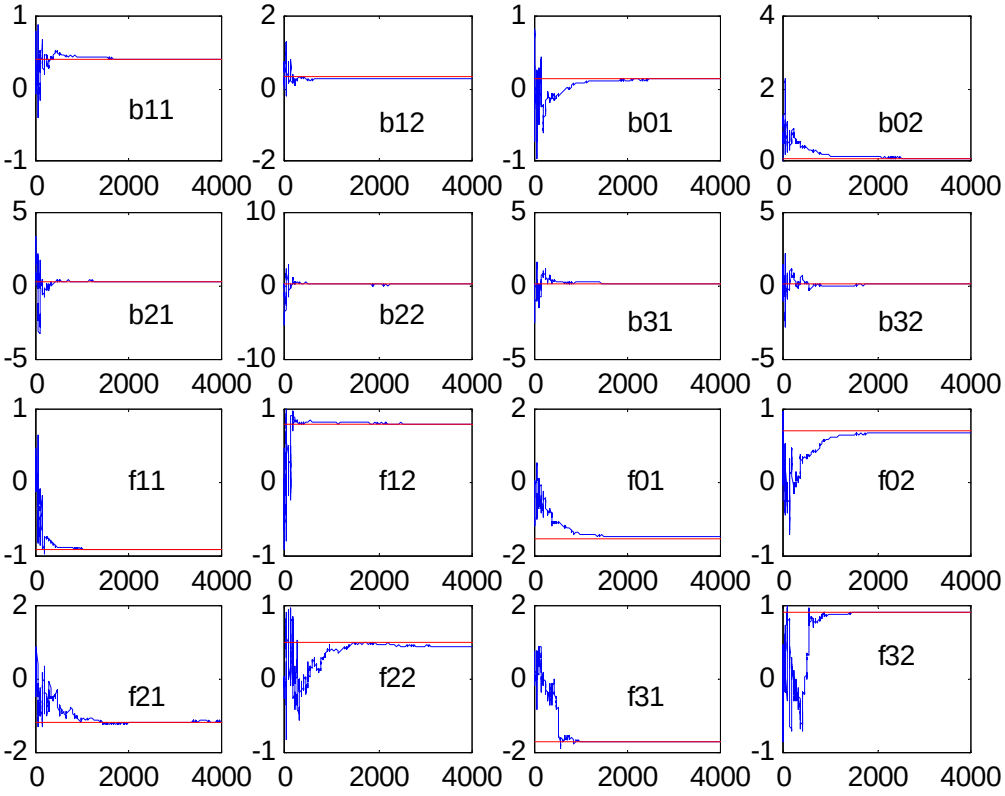


Fig. 6.4 The generalized Bilinear system identification process
with a N./S.=5% colored measurement noise

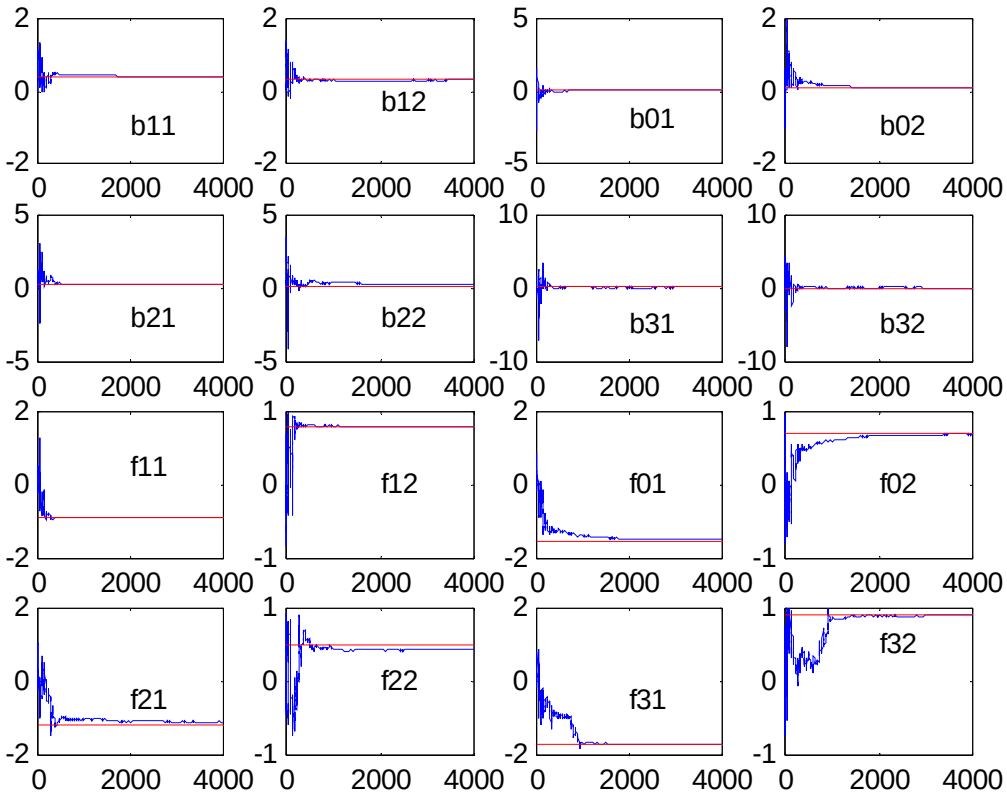


Fig. 6.5 The generalized Hammerstein system identification process with a N./S.=5% colored measurement noise

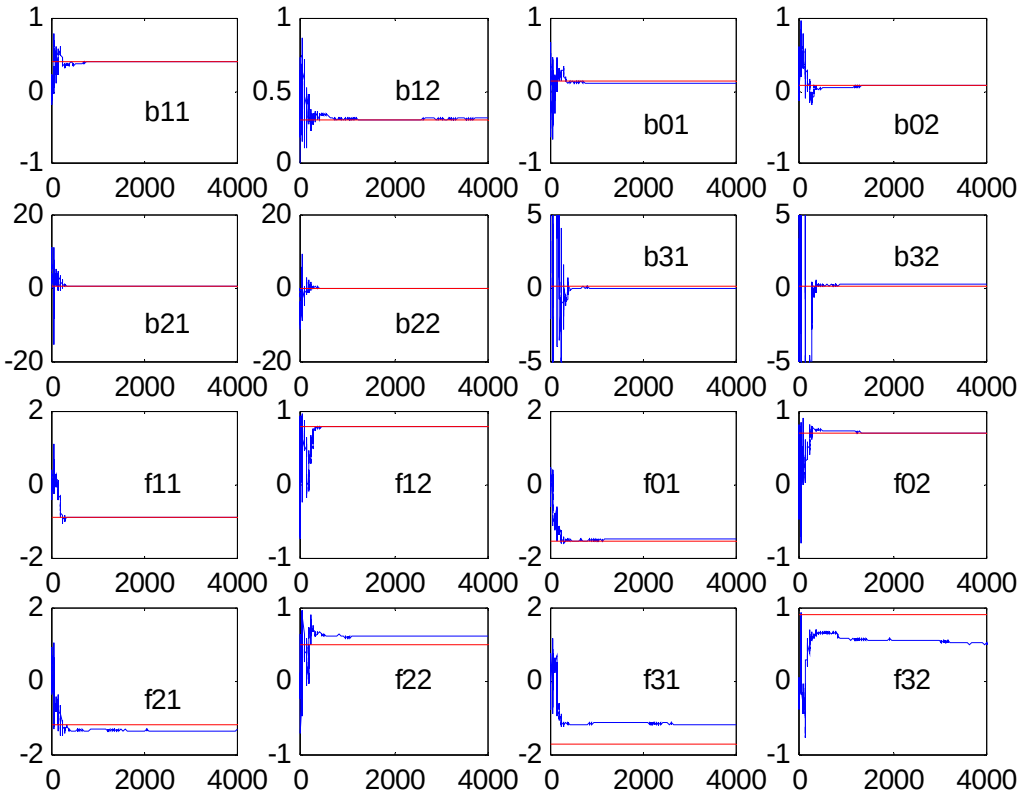


Fig. 6.6 The generalized Wiener system identification process
with a N./S.=5% colored measurement noise

The simulation results show that:

- The new identification method gives good results for all three generalized Wiener and Hammerstein systems.
- From Table 6.4, we can see that the generalized Bilinear system has the best accuracy results.

7. Conclusions

Nonlinear models can provide an accurate description and prediction of physical systems that have a nonlinear behaviour. Modelling nonlinear systems has become an important issue with many practical applications. However, there exist no general valid descriptions for the full class of nonlinear systems and finding the model parameters for given measurements is an open question.

Wiener systems and Hammerstein systems are nonlinear models that are used in many domains for their simplicity and physical meaning. Different nonlinear systems with different nonlinearities should use different Wiener and Hammerstein structures.

In this thesis a new identification method and unified identification concepts for a class of Wiener and Hammerstein systems have been developed. The new identification method is based only on the observed input and output data and the recursively estimated intermediate variables.

For continuous nonlinearities, polynomial functions are applied because they are common used and their key terms can be easily separated. For the typical discontinuous nonlinearities, it is necessary to establish some extra independent intermediate variables as the key terms. We use RTF and ARMAX models to describe the linear subsystems.

After having selected the suitable model structures for each subsystem in a Wiener and Hammerstein system, by constructing intermediate variables and using the key term separation principle, such a Wiener and Hammerstein system can be approximately transformed into a pseudo-linear MISO system. Then we have considered our identification problems based on recursive pseudo-linear regressions (RPLR) in the prediction error and model framework.

The constructed intermediate variables are recursively estimated. In order to deduce oscillations and get better convergence, the parameters are smoothed with smoothing and filtering techniques to estimate the intermediate variables. After that, a general prediction error model is formed and satisfied parameter estimates of the Wiener and Hammerstein system are obtained in the presence of a white or a colored measurement noise without parameter redundancy.

In this thesis we have derived new algorithms for:

- four SISO Wiener and Hammerstein systems with continuous nonlinearities,
- a Wiener system with one of four general discontinuous nonlinearities, respectively,
- three MISO Wiener and Hammerstein systems with continuous nonlinearities,
- two cascade Wiener and Hammerstein systems with continuous and discontinuous nonlinearities, respectively,
- and finally, three generalized Wiener and Hammerstein systems which are simplified from the parametric Volterra-series.

All the derived algorithms have been tested by simulation examples. The new identification method gives good results for all the considered Wiener and Hammerstein systems. From the derivations and the simulation results, it can be concluded that the new developed identification method and identification concepts are clear and efficient. It can be easily extended to other block-oriented nonlinear systems with different nonlinearities.

Compared with the known methods, the major advantage of the new nonlinear identification method is its unity and efficiency. We have built a "bridge" from the known linear modelling techniques, over

the new nonlinear identification method developed in this thesis, to the complex but accurate full blown nonlinear models.

Nonlinear system identification is a new and broad research area. There are still many problems to study. For the future investigation:

- The developed identification method and concepts are expected to be used in practice.
- The difficult convergence problem of PLR in nonlinear data case should also be theoretically concentrated.
- Suitable initial parameter values are no doubt important and necessary. The developed identification method and concepts can also be combined with other identification methods for nonlinear systems to get more better results.
- Develop a corresponding toolbox with graphical user interface (GUI) for a class of Wiener and Hammerstein systems with different nonlinearities.
- The model structures that the system noise disturbs Wiener and Hammerstein systems from different positions should also be studied.

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