Acoustic Wave Field Modeling for Ultrasound Breast Cancer Detection

3rd Dutch Bio-Medical Engineering Conference 2011

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1. Introduction

Nowadays, many women are diagnosed with breast cancer. Due to lack of signs at early stages, it can be difficult to notice the tumor. However, early detection will significantly increase the survival rate. An efficient screening technique could be ultrasound, which uses high-frequency sound waves to provide real time images of the breast. Unlike other screening methods (Mammography, MRI etc.) ultrasound is painless, harmless, cost-effective and provides good cancer detection rates in dense breasts. In collaboration with Karlsruhe Institute of Technology (KIT), we work on the development of new ultrasound imaging techniques to improve the breast scanning quality by providing detailed characterization of the tissue.

Figure 1: Screening setup of ultrasound scanning system.

2. Theory:

Acoustic wave equation

Time Domain

$$\nabla^2 p(\vec{x},t) - \frac{1}{c^2}\frac{\partial^2 p(\vec{x},t)}{\partial t^2} = -S(\vec{x},t)$$

$$S(\vec{x},t) = \rho\frac{\partial q(\vec{x},t)}{\partial t} - \nabla\cdot\vec{f}(\vec{x},t)$$

Frequency Domain

$$\nabla^2 \hat{p}(\vec{x},\omega) + \frac{\omega^2}{c^2}\hat{p}(\vec{x},\omega) = -\hat{S}(\vec{x},\omega)$$

$$c = \frac{1}{\sqrt{\kappa\rho}} = \text{ speed of sound}$$

Integral equation representation [1]

$$\hat{p}^{tot}(\vec{x}) = \hat{p}^{inc}(\vec{x}) + \hat{p}^{sct}(\vec{x})$$

$$\hat{p}^{inc}(\vec{x}) = \int \hat{G}(\vec{x}-\vec{x}')\hat{S}(\vec{x}')dV$$

$$\hat{p}^{sct}(\vec{x}) = \omega^2\int \hat{G}(\vec{x}-\vec{x}')X(\vec{x}')\hat{p}^{tot}(\vec{x}')dV$$

$\hat{p}^{inc}$: incident pressure field
$\hat{p}^{tot}$: total pressure field
$\hat{p}^{sct}$: scattered pressure field
$\hat{G}$: impulse response of the background medium
$\hat{X} = \frac{1}{c_{obj}^2} - \frac{1}{c_{bg}^2}$: contrast function

$$\hat{p}^{inc}(\vec{x}) = \hat{p}^{tot}(\vec{x}) - \omega^2\int \hat{G}(\vec{x}-\vec{x}')\hat{p}^{tot}(\vec{x}')X(\vec{x}')dV$$

Forward Problem

Situations where the contrast function is known and the total pressure field is unknown are referred to as the forward problem. To test our imaging algorithms, synthetically measured data is obtained by solving the forward problem for various source and receiver locations using Bi-CGstab [2].

Inverse Problem

The inverse problem refer to the situations where the contrast function is unknown and the measured total field is known. In our study, breast ultrasound imaging will be based on solving the inverse problem.

3. Solution of the Inverse Problem

Synthetic Aperture Focusing Technique (SAFT)

Time Domain

$$X^{SAFT}(\vec{x}') = \sum_{\vec{x}^s,\vec{x}^r} p^{sct}(\vec{x}^s,\vec{x}^r,t), \qquad t = \frac{\|\vec{x}^s - \vec{x}'\| + \|\vec{x}^r - \vec{x}'\|}{c_{bg}}$$

Frequency Domain

$$\hat{X}^{SAFT}(\vec{x}') = \sum_{\vec{x}^s,\vec{x}^r}\sum_{\omega} \hat{p}^{sct}(\vec{x}^s,\vec{x}^r)\,\hat{G}^*(\vec{x}^r-\vec{x}')\,\hat{p}^{*inc}(\vec{x}^s,\vec{x}')\|\vec{x}^s-\vec{x}'\|\;\|\vec{x}^r-\vec{x}'\|$$

Back-Propagation

$$\hat{X}^{bp}(\vec{x}') = \sum_{\vec{x}^s,\vec{x}^r}\sum_{\omega}\omega^2\hat{p}^{sct}(\vec{x}^s,\vec{x}^r)\hat{G}^*(\vec{x}^r-\vec{x}')\hat{p}^{*inc}(\vec{x}^s,\vec{x}')$$

Born Inversion

To solve the inverse problem, the Born approximation was applied by replacing the total pressure field with the incident pressure field. The resulting linear equation was solved using a conjugate gradient inversion scheme.

$$\hat{p}^{sct}(\vec{x}^s,\vec{x}^r) = \sum_{\vec{x}'}\omega^2\hat{G}(\vec{x}^r-\vec{x}')\hat{p}^{inc}(\vec{x}^s,\vec{x}')X(\vec{x}')$$

4. Results

First, to test the accuracy of the algorithm, the outcome was compared with an analytical solution for an acoustically penetrable sphere illuminated by a time harmonic plane wave field (Figure 2). Both methods yielded identical results. The efficiency of the imaging algorithms was tested on synthetically measured, noise free data (Figure 3).

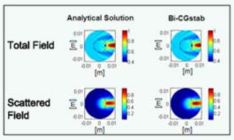

Figure 2: Scattering off a soft sphere.

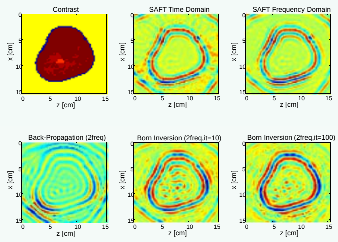

Figure 3: Cross-sections of breast reconstruction.

5. Conclusion and Future Work

Basic imaging algorithms have been developed and tested on synthetic data generated by our forward model. In future, we aim to develop imaging algorithms which work beyond the Born approximation, test the algorithms with measured data, and reduce the computational time.

6. References

[1] J.T. Fokkema and P.M. van den Berg, Seismic Applications of Acoustic Reciprocity. Elsevier, Amsterdam, 1993.

[2] H.A. van der Vorst, "Bi-cgstab: A fast and smoothly converging variant of bi-cg for the solution of nonsymmetric linear systems," SIAM J. Sci. Stat. Comput., vol. 13, no. 2, pp. 631-644, 1992.