Karlsruhe Institute of Technology

Institute for Data Processing and Electronics

NEWTON'S METHOD BASED SELF CALIBRATION FOR A 3D ULTRASOUND COMPUTER TOMOGRAPHY SYSTEM

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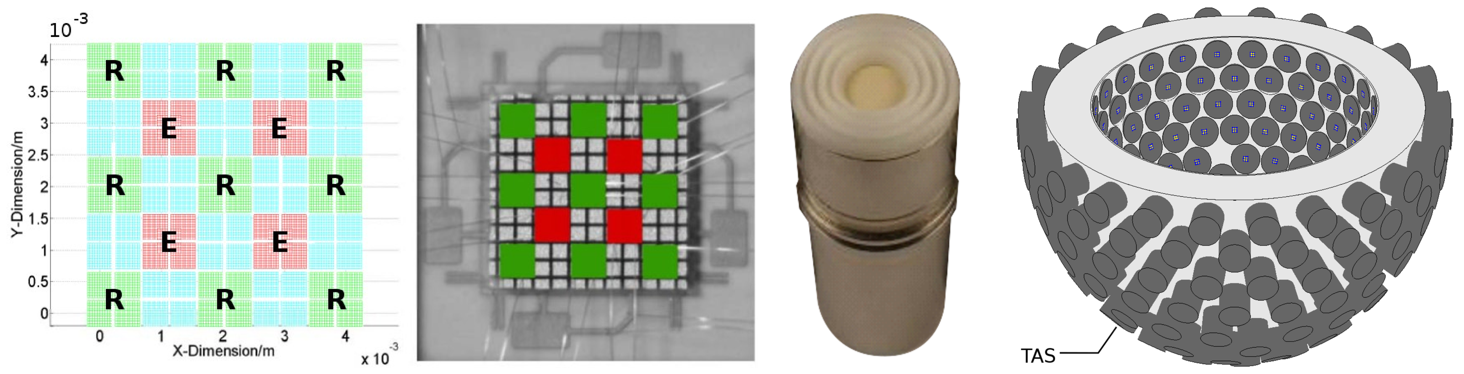
Background and Challenges

3D ultrasound computer tomography (3D USCT) at KIT:

- 2041 transducers in 157 transducer arrays (TAS) with 2.5 MHz resonance frequency
- Semi-ellipsoidal aperture for nearly isotropic 3D point spread function (PSF) in region of interest (ROI)
- 3 modalities in single measurement: reflectivity, SOS and attenuation

Challenges:

- **Key to high quality image: accurate system calibration**
- **Time-of-flight (TOF) accuracy required $\lambda/4$ = 0.152 mm**
- **Limitation of simultaneous calibration: 10362 unknowns**



Schematics of single TAS and the USCT aperture

Method

- Calibration based on time-of-flight (TOF) measurements:

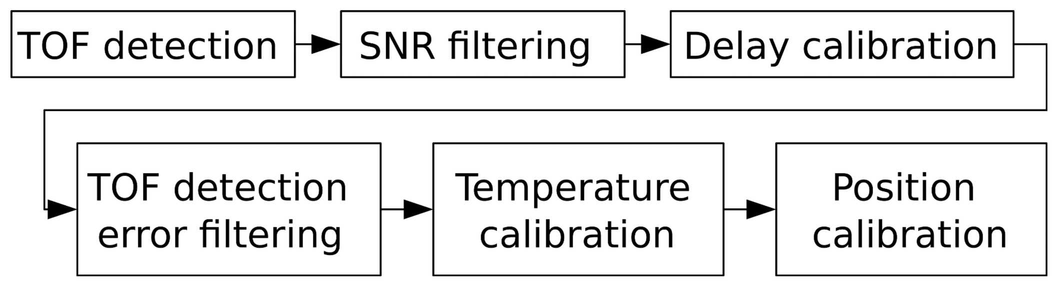
$$\|\mathbf{s}_i - \mathbf{r}_j\| = c_{ij} \cdot \left(t_{ij} - \tau_{s_i} - \tau_{r_j}\right)$$

$\mathbf{s}_i$: i^{th} emitter, $\mathbf{r}_j$: j^{th} receiver, $\mathbf{c}_{ij}$: mean SOS

t_{ij}: TOF, τ_{s_i}: transmission delay, τ_{r_j}: reception delay

- Sequential calibration according to error magnitudes
- Pre-filtering of TOF detection error
- Solving iteratively with Newton's method for unique solution satisfying $\mathbf{x}$: $f(\mathbf{x}) = 0$ by:

$$\mathbf{x}_{n+1} = \mathbf{x}_n - J^{-1}(\mathbf{x}_n) \cdot f(\mathbf{x}_n)$$



Calibration process

Possible Error Sources

Error Source	Error Magnitudes
Machining accuracy	≤ 10 µm
Center deviation in each TAS	x, y ≤ 1 mm, z ≤ 300 µm
Radial offset of each TAS	≤ 10 µm
Rotation of each TAS	≤ 2°
Aperture material coefficient of thermal expansion (POM-C)	$1.1 \cdot 10^{-4}$ K^{-1} ≙ 28.6 µm K^{-1}
Electrical mechanical delay	≈ 1 µs ≙ 1.5 mm at 25 °C
Temperature offset in TAS	≈ 3 °C ≙ 1.5 mm at 25 °C
Temperature error in TAS	≈ 1 °C ≙ 0.5 mm at 25 °C
Potential jitter of electronics	20 MHz ≙ 76 µm at 25 °C

1. Delay Calibration and Correction

Assumptions: SOS and position errors negligible

$$f\left(\tau_{s_i}, \tau_{r_j}\right) = \|\mathbf{s}_i - \mathbf{r}_j\| - c_{ij} \cdot \left(t_{ij} - \tau_{s_i} - \tau_{r_j}\right) = 0$$

$$f\left(\tau_{s_i}\right) = \left|\tau_{s_i}\right| - \tau_{s_i} = 0, \quad f\left(\tau_{r_j}\right) = \left|\tau_{r_j}\right| - \tau_{r_j} = 0$$

$$\mathbf{x} = \left[\tau_{s_1}, \ldots, \tau_{s_m}, \tau_{r1}, \ldots, \tau_{r_n}\right]^T$$

Delay correction:

$$\hat{t}_{ij} = t_{ij} - \overline{(\tau_s + \tau_r)}$$

2. Temperature Calibration

Assumptions: delays and position errors negligible

$$f\left(T_{s_i}, T_{r_j}\right) = \|\mathbf{s}_i - \mathbf{r}_j\| - \bar{c}_{ij}(T_{s_i}, T_{r_j}) \cdot \hat{t}_{ij} = 0$$

$$f(\mathbf{x}) = \bar{\mathbf{x}} - T_{cali} = 0$$

$$\mathbf{x} = \left[T_{s_1}, \ldots, T_{s_m}, T_{r1}, \ldots, T_{r_n}\right]^T$$

3. Position Calibration

Assumptions: delays and SOS errors negligible

$$f(\mathbf{s}_i, \mathbf{r}_j) = \|\mathbf{s}_i - \mathbf{r}_j\| - c_{ij} \cdot \hat{t}_{ij} = 0$$

$$\mathbf{s}_i = \left(x_{s_i}, y_{s_i}, z_{s_i}\right), \quad \mathbf{r}_j = \left(x_{r_j}, y_{r_j}, z_{r_j}\right)$$

$$\mathbf{x} = \left[\mathbf{s}_1, \ldots, \mathbf{s}_m, \mathbf{r}_1, \ldots, \mathbf{r}_n\right]^T$$

Results

Simulated 3D USCT with top 114 TASs

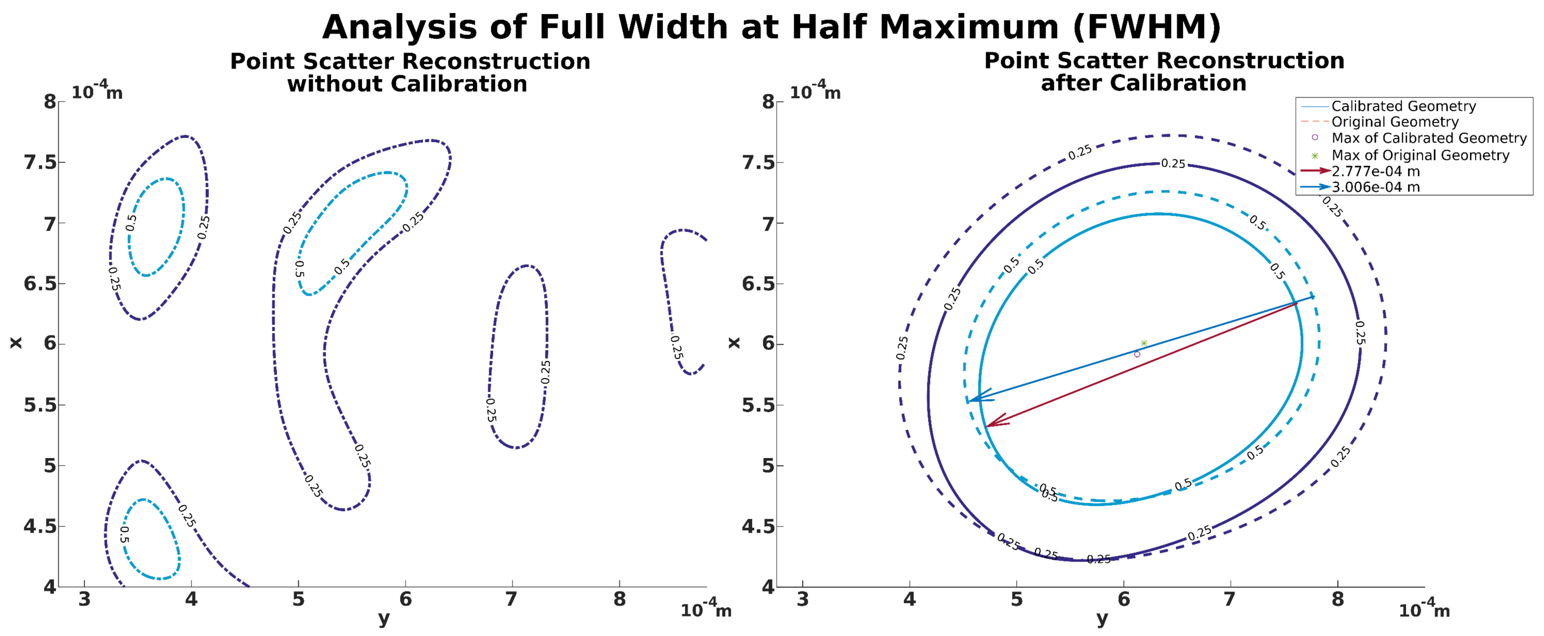
Error Sources	Simulated Error	Calibration Error
Electrical delay	1.2 µs	0.175 µs
Temperature offset	-3 °C	-0.517 °C
Temperature error	1 °C	0.0031 °C
Position error	$\mu \leq 1$ mm	μ = 16.29 µm
		σ = 4.08 µm

Application to real 3D USCT data

Error Sources		Calibration Result	
		Mean	Standard Deviation
Electrical delay		1.2152 µs	0.012 µs
Temperature offset		-2.5955 °C	0.076 °C
Temperature error		0.0662 °C	0.017 °C
Position error	μ	141.53 µm	0.820 µm
	σ	285.84 µm	63.58 µm

Conclusions

- Simulations show ability to quantify and compensate multiple error sources.
- Application to real data has residuum of ≈ $\lambda/4$.
- Reason of larger residuum compared to simulation needs further investigation.



Full width at half maximum analysis of a simulated point scatter