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Scaling of the velocity field induced by a bubble rising rectilinearly through liquid under variation of the gas-liquid density ratio

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Motivation

- Difficulties associated with low values of gas-liquid density ratio Γ_ρ in DNS of two-phase:
  - convergence of iterative solvers for pressure Poisson equation
  - computation of gradients of discontinuous quantities at interface
  - large difference in diffusive time scale of both phases
- For reasons of computational efficiency usually a density ratio of $\Gamma_\rho \approx 1/50$ is used instead of 1/1000
- Computational studies indicate that for $\Gamma_\rho \approx 1/50$ influence of Γ_ρ on bubble rise velocity is small while it is notable for $\Gamma_\rho \approx 1/10$
- Does there exist a general scaling so that also simulation results for $\Gamma_\rho = O(0.1)$ can be transferred to $\Gamma_\rho = O(0.001)$?

  <u>Here:</u> Investigation by 3D Volume-of-fluid computations

Similitude analysis (Grace, 1973)

- Single bubble rising steadily in liquid of infinite extent
- Non-dimensional bubble terminal rise velocity:

$$\boxed{\mathrm{Re}_B = f(M, E\ddot{o}_B, \Gamma_\rho, \Gamma_\mu)}$$

- Bubble Reynolds number $Re_B \equiv \rho_l^* d_V^* U_T^* / \mu_l^*$

  Morton number $M \equiv \left(\rho_l^* - \rho_g^*\right) \; g^* \mu_l^{*4} \Big/ \left(\rho_l^{*2} \sigma^{*3}\right)$

  Bubble Eötvös number $E\ddot{o}_B \equiv \left(\rho_l^* - \rho_g^*\right) \; g^* d_V^{*2} \Big/ \sigma^*$

  Gas-liquid density ratio $\Gamma_\rho \equiv \rho_g^* / \rho_l^*$

  Gas-liquid viscosity ratio $\Gamma_\mu \equiv \mu_g^* / \mu_l^*$

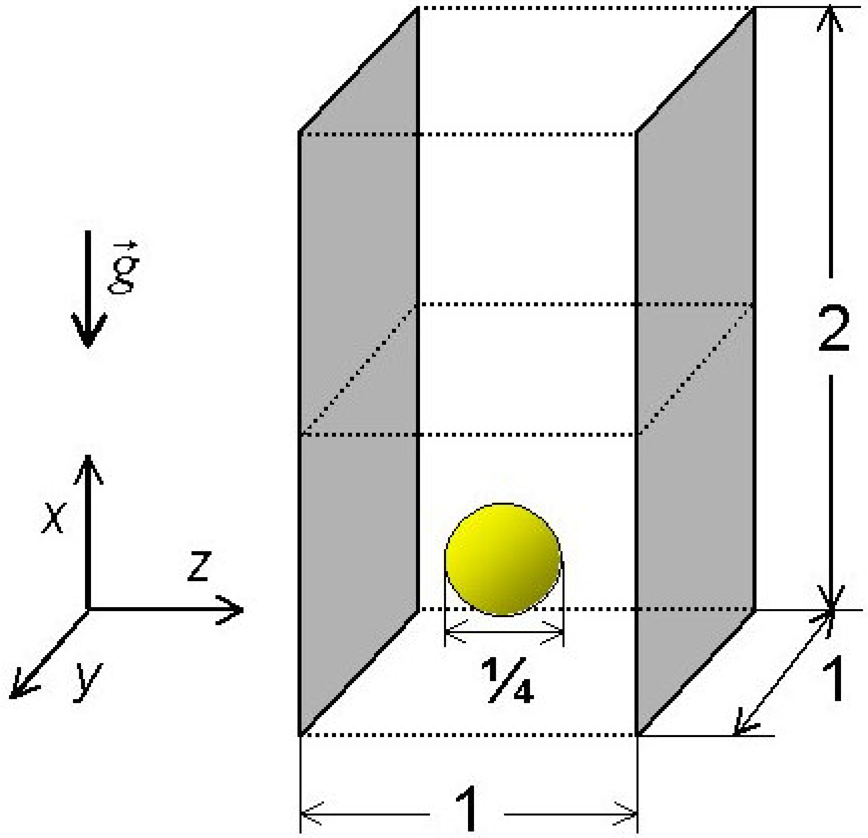
(dimensional variables are marked by superscript *)

Computer code TURBIT-VOF for DNS of two-phase flow

- Volume-of-Fluid method for interface tracking
  - Piecewise-linear interface calculation (EPIRA algorithm)
- Finite volume method on rectangular staggered grid
- Spatial discretization by central differences (2nd O.)
- Explicit Runge-Kutta time integration scheme (3rd O.)
- Conjugate gradient solver for pressure Poisson eq.

Computational set-up

- Domain: 2 x 1 x 1
- Grid: 128 x 64 x 64
- Bubble diameter: 0.25
  (= 16 mesh cells)
- Gas holdup: ≈ 0.4%
- Boundary conditions
  – walls at z = 0 and z = 1
  – periodic in x and y
- Liquid & gas initially at rest



Non-dimensional governing equations

$$\mathbf{x}=\frac{\mathbf{x}^*}{L_{ref}^*},\ \mathbf{u}_k=\frac{\mathbf{u}_k^*}{U_{ref}^*},\ t=\frac{t^*U_{ref}^*}{L_{ref}^*},\ \rho_k=\frac{\rho_k^*}{\rho_l^*},\ \mu_k=\frac{\mu_k^*}{\mu_l^*},\ P=\frac{p^*+p_0^*-\rho_l^*\mathbf{g}^*\cdot\mathbf{x}^*}{\rho_l^*U_{ref}^{*\,2}}\quad (k\in l,g)$$

$$\boxed{\frac{\partial}{\partial t}\rho_m\mathbf{u_m}+\nabla\cdot\rho_m\mathbf{u_m}\mathbf{u_m}=-\nabla P+\frac{1}{Re_{ref}}\nabla\cdot\left[\mu_{\mathrm{m}}\left(\nabla\mathbf{u_m}+\nabla\mathbf{u_m}^{\mathrm{T}}\right)\right]-(1-f)\frac{E\ddot{o}_{ref}}{We_{ref}}\frac{\mathbf{g}^*}{g^*}+\frac{a_{int}\kappa\mathbf{n}}{We_{ref}}}$$

$$\boxed{\nabla\cdot\mathbf{u_m}=0}\quad \boxed{\frac{\partial f}{\partial t}+\nabla\cdot f\,\mathbf{u_m}=0}\quad \left(f\equiv\alpha_l,\,0\le f\le 1\right)\quad \mathbf{u_m}\equiv\frac{1}{U_{ref}^*}\frac{f\rho_l^*\mathbf{u}_l^*+(1-f)\rho_g^*\mathbf{u}_g^*}{f\rho_l^*+(1-f)\rho_g^*}$$

$$\rho_m\equiv\frac{f\rho_l^*+(1-f)\rho_g^*}{\rho_l^*}=f+(1-f)\Gamma_\rho,\quad \mu_m\equiv\frac{f\mu_l^*+(1-f)\mu_g^*}{\mu_l^*}=f+(1-f)\Gamma_\mu$$

$$Re_{ref}\equiv\frac{\rho_l^*L_{ref}^*U_{ref}^*}{\mu_l^*},\ E\ddot{o}_{ref}\equiv\frac{\left(\rho_l^*-\rho_g^*\right)g^*L_{ref}^{*\,2}}{\sigma^*},\ We_{ref}\equiv\frac{\rho_l^*L_{ref}^*U_{ref}^{*\,2}}{\sigma^*},\ M=\frac{E\ddot{o}_{ref}We_{ref}^2}{Re_{ref}^4}=\frac{E\ddot{o}_BWe_B^2}{Re_B^4}$$

(dimensional variables are marked by superscript *)

Input parameter for code

1. Fixed values for viscosity ratio: $\Gamma_\mu = 1$
2. Fixed values for $(M, E\ddot{o}_B)$ (two different combinations)
3. Fixed values for reference quantities:

$$L^*_{ref} = 4\,\mathrm{m}, \quad U^*_{ref} = 1\,\mathrm{ms}^{-1}, \quad g^* = 9{,}81\,\mathrm{ms}^{-2}$$

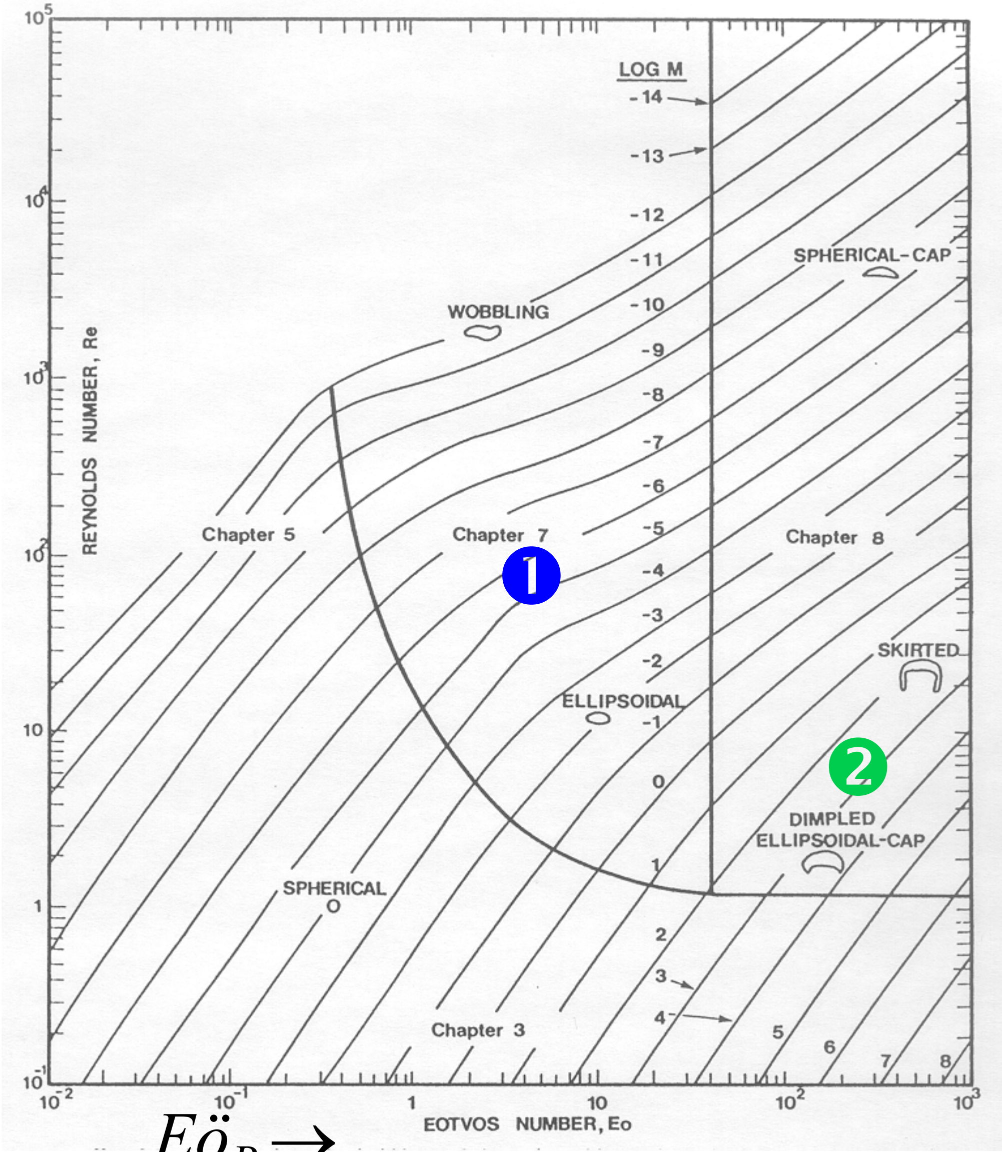
4. Density ratio Γ_ρ to be varied $\Rightarrow$ successively compute

$$E\ddot{o}_{ref} = \left(\frac{L^*_{ref}}{d^*_V}\right)^2 E\ddot{o}_B, \quad We_{ref} = \frac{E\ddot{o}_{ref}}{1-\Gamma_\rho}\frac{U^{*\,2}_{ref}}{g^* L^*_{ref}}, \quad Re_{ref} = \left(\frac{E\ddot{o}_{ref} We^2_{ref}}{M}\right)^{0.25}$$

Note: we do <u>not</u> give explicit values for ρ_l^*, ρ_g^*, μ_l^*, μ_g^*, σ^* !

Combinations (M, $Eö_B$)

$\uparrow$
Re_B



$Eö_B \rightarrow$

1. $M = 3{,}09 \cdot 10^{-6}$, $Eö_B = 3{,}06$
   medium Morton number
   ellipsoidal bubble

2. $M = 266$, $Eö_B = 243$
   high Morton number
   ellipsoidal cap bubble

Simulation parameter case ❶

$$E\ddot{o}_B = 3.06,\ \ M = 3.09\cdot 10^{-6},\ \Gamma_\mu = 1$$

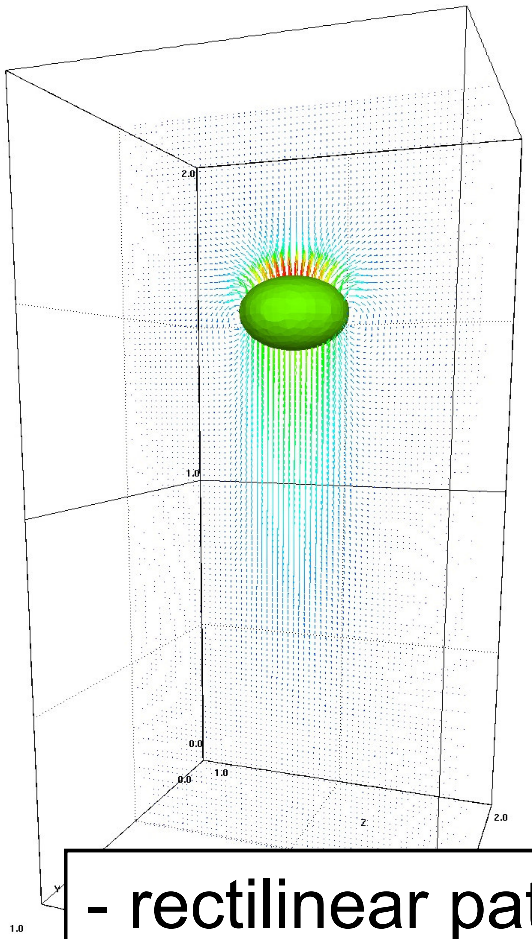
Run	Γ_ρ	**$1 / \Gamma_\rho$**	$E\ddot{o}_{ref}$	We_{ref}	Re_{ref}	Δt	N_t
M2	0.5	**2**	49.05	2.5	100.00	0.0005	1,100
M5	0.2	**5**	49.05	1.563	78.90	0.0003	1,800
M10	0.1	**10**	49.05	1.389	74.39	0.00015	3,200
M50	0.02	**50**	49.05	1.276	71.28	0.00003	13,000
	0	**∞**	49.05	1.25	70.57		

Simulation parameter case ❷

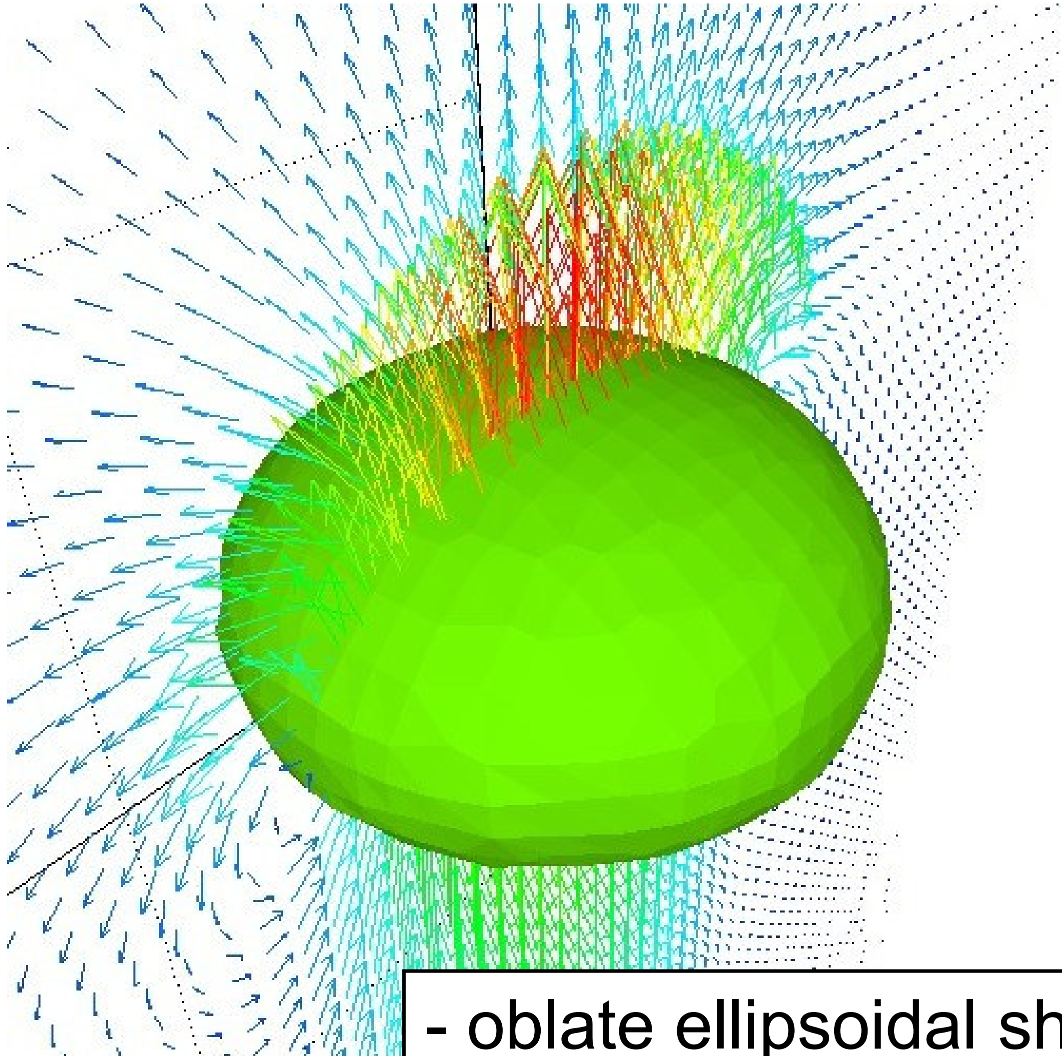
$M = 266,\ E\ddot{o}_B = 243,\ \Gamma_\mu = 1$

Run	Γ_ρ	$1 / \Gamma_\rho$	$E\ddot{o}_{ref}$	We_{ref}	Re_{ref}	Δt	N_t
H2	0.5	**2**	3,888	792.7	55.05	0.0005	5,000
H5	0.2	**5**	3,888	495.4	43.52	0.0001	17,000
H10	0.1	**10**	3,888	440.4	41.03	0.0001	16,000
	0	**∞**	3,888	396.3	38.93		

Flow visualizations case ❶



- rectilinear path
- closed wake

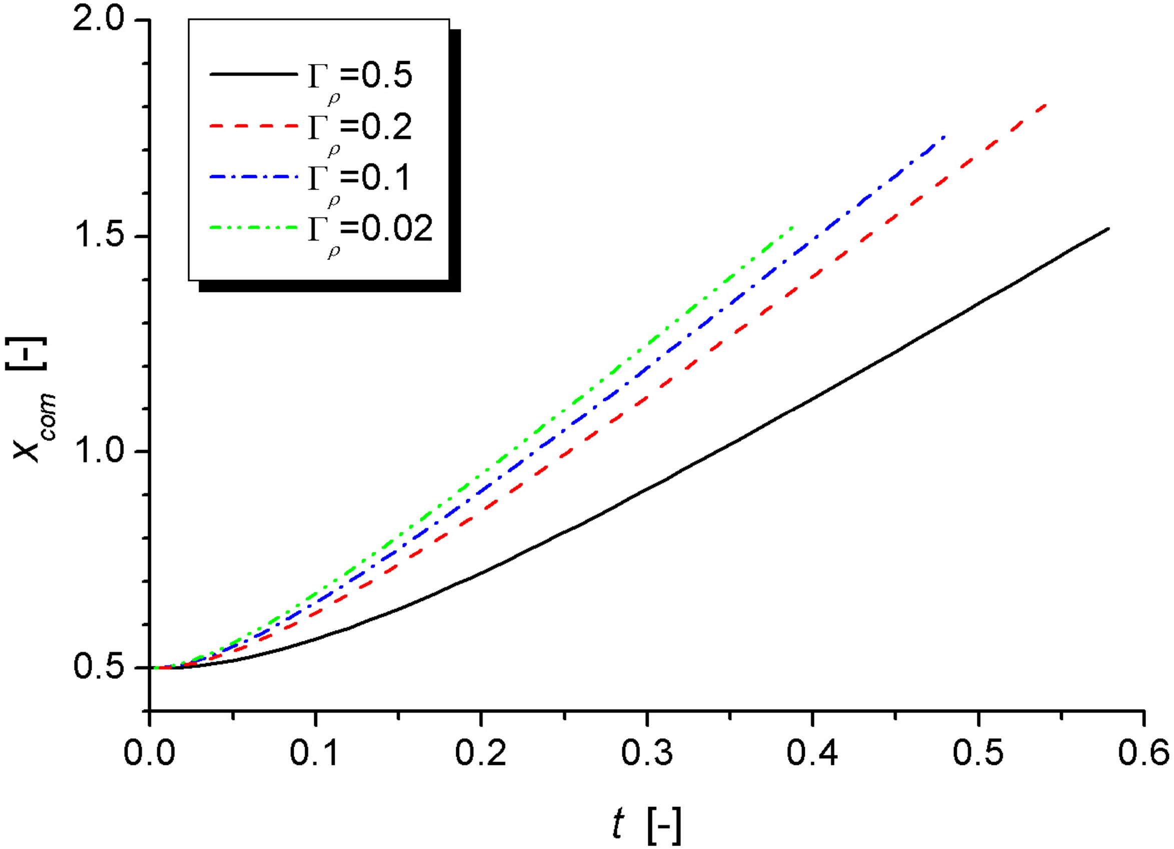


- oblate ellipsoidal shape

Bubble vertical position (case ❶)

Two distinct phases:

- <u>Initial phase:</u>
  bubble accelerates from rest up to terminal velocity
- <u>Subsequent phase:</u>
  bubble rises steadily with terminal velocity



Acceleration of bubble (case ❶)

- Balance between unsteady
  + inertial and buoyancy term

$$\rho_m \frac{\mathrm{D}\mathbf{u}_m}{\mathrm{D}t} = -(1-f)\frac{E\ddot{o}_{ref}}{We_{ref}}\frac{\mathbf{g}^*}{g^*}$$

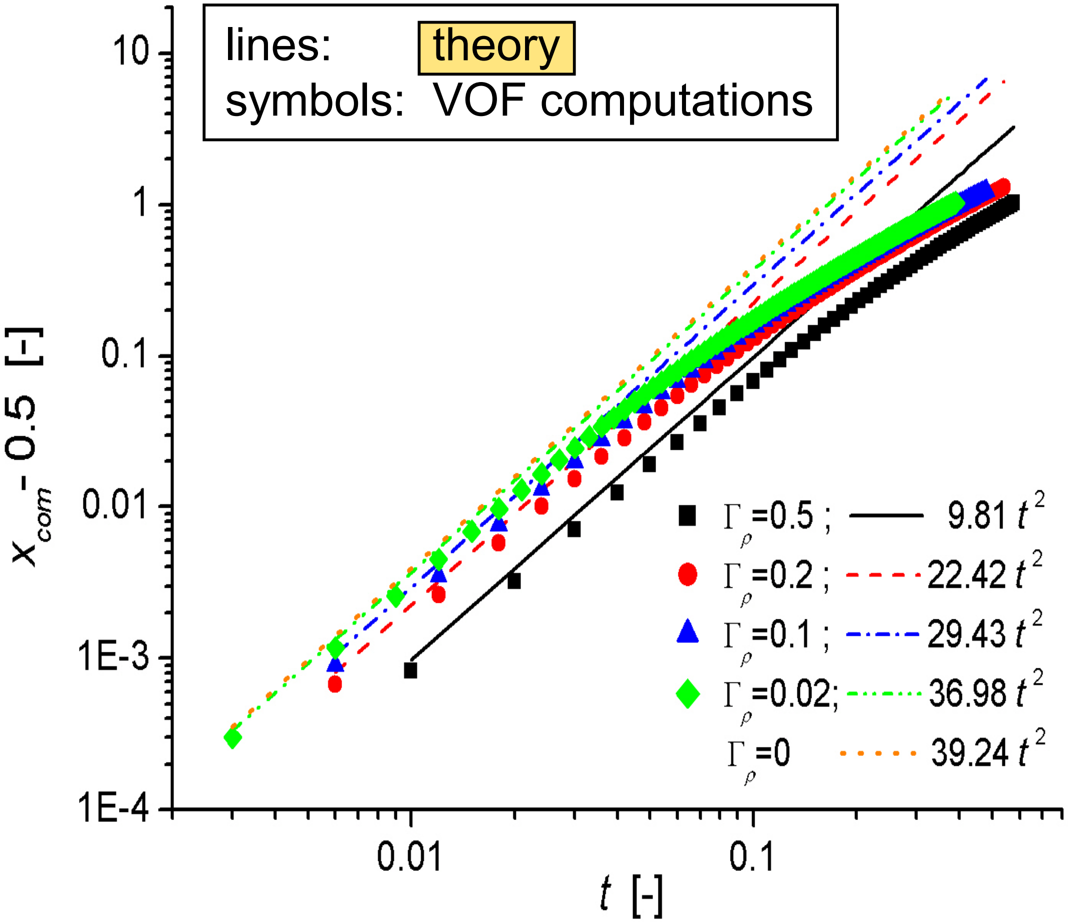
- Approx. for gas phase ($f = 0$):

$$u_{m,x} \approx U_B = \mathrm{d}x_{com}/\mathrm{d}t$$

$$\rho_m \approx \left(\rho_g^* + \boxed{0.5\rho_l^*}\right)/\rho_l^*$$

added mass !

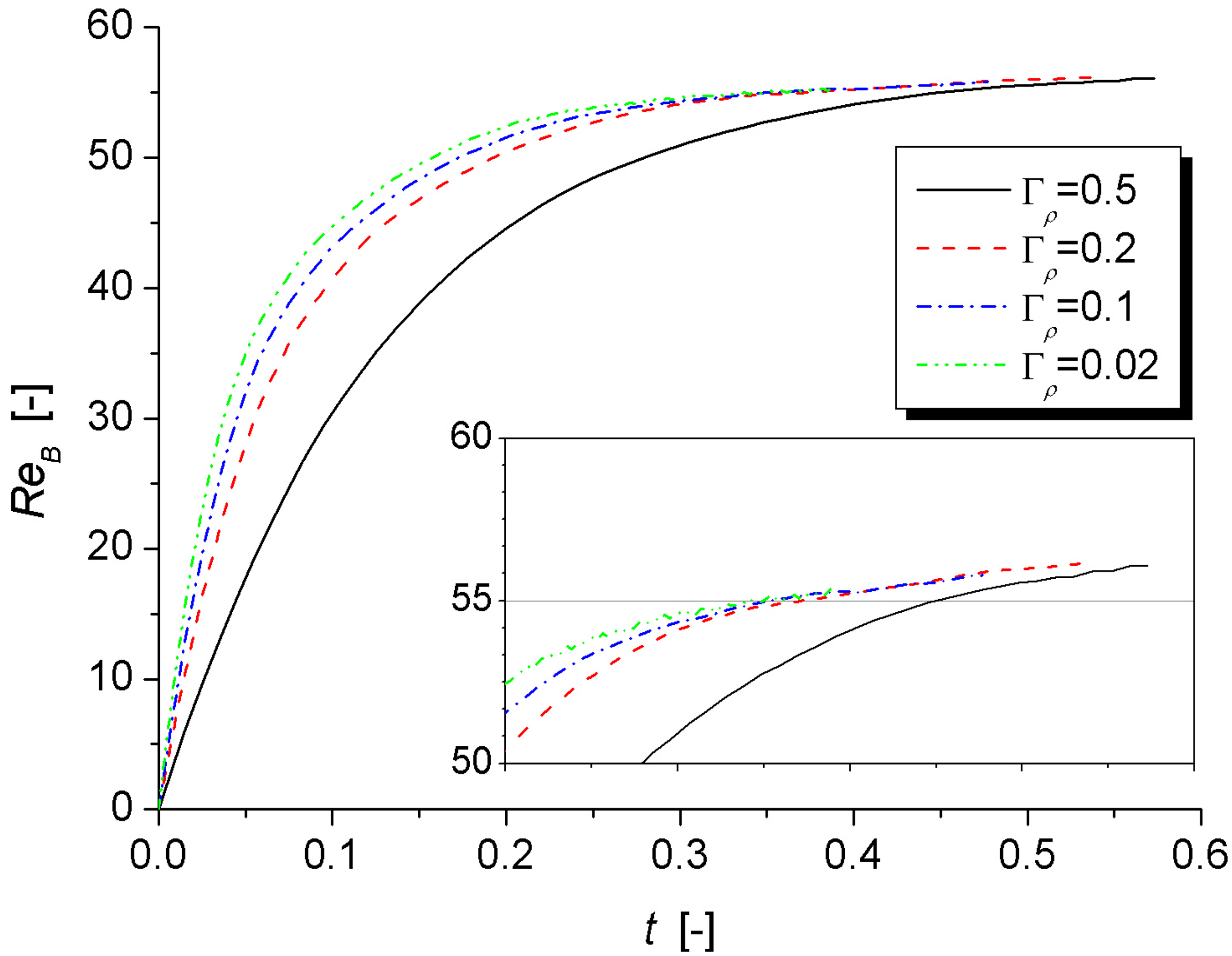
$$\Rightarrow \boxed{x_{com}(t) - 0.5 = \frac{1-\Gamma_\rho}{1+2\Gamma_\rho}\frac{g^* L_{ref}^*}{U_{ref}^{*\,2}} t^2}$$



Bubble Reynolds number (case ❶)

$$Re_B = \frac{d_V^*}{L_{ref}^*}\frac{U_B^*}{U_{ref}^*}Re_{ref} = \frac{1}{4}U_B Re_{ref} = \frac{1}{4}\frac{\mathrm{d}x_{com}(t)}{\mathrm{d}t}Re_{ref}$$

Terminal bubble Reynolds number is about 56 and does <u>not</u> vary with density ratio!



Comparison with „two-phase wave theory“

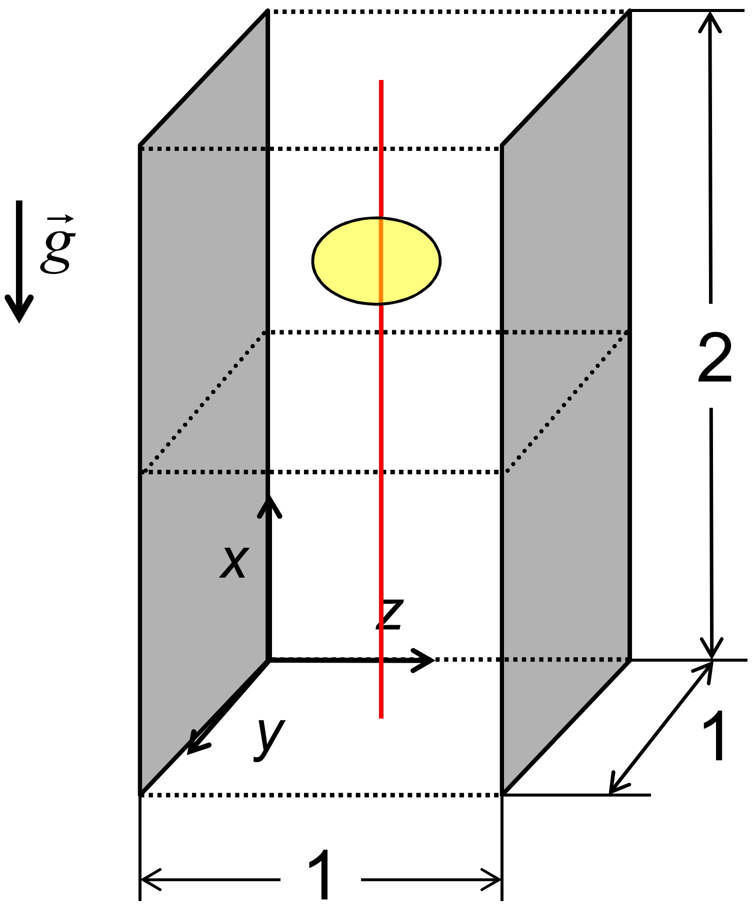
- Mendelson‘s formula recast by Tomiyama et al.:

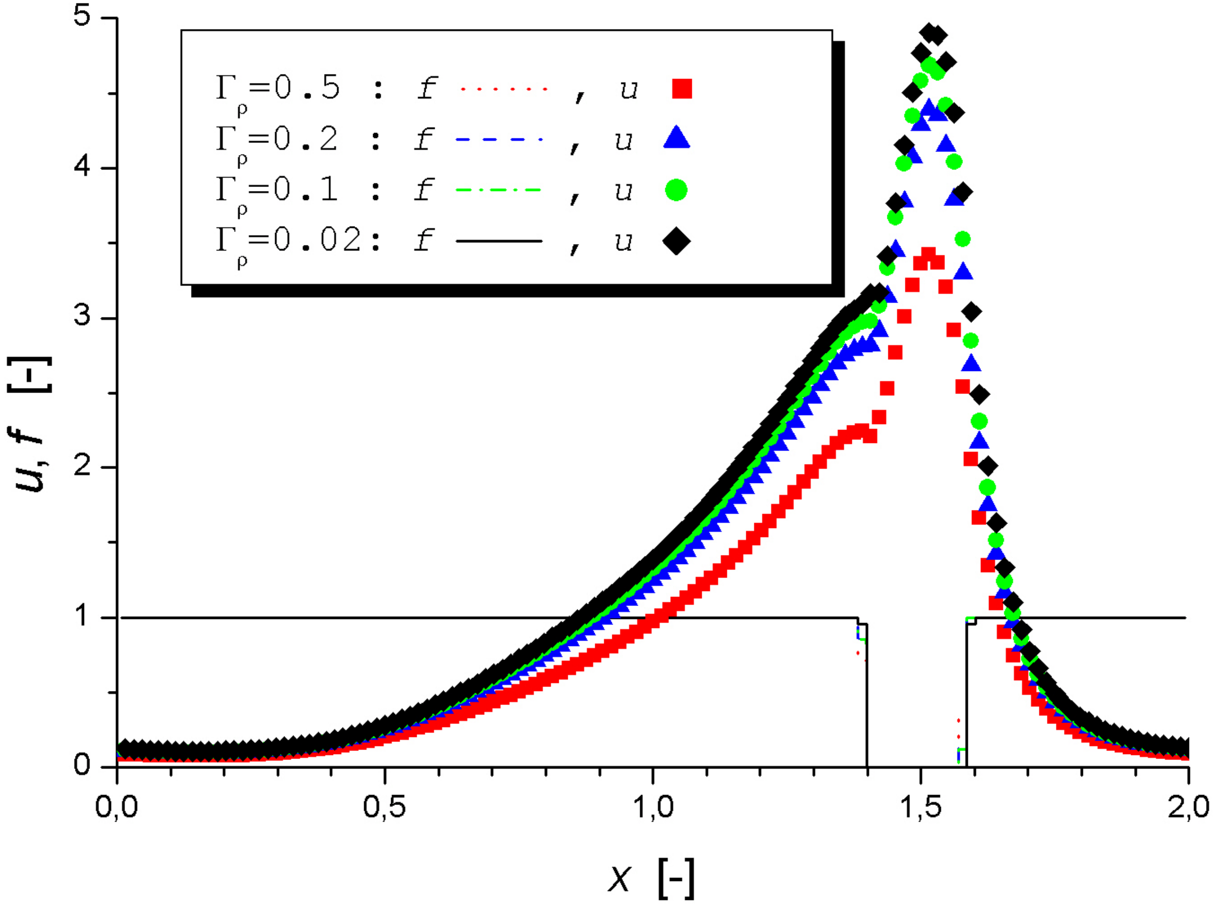
$$U_T^* = \sqrt{\frac{2\sigma^*}{\rho_l^* d_V^*} + \frac{g^* d_V^* (\rho_l^* - \rho_g^*)}{2\rho_l^*}} \Rightarrow \boxed{Re_B = \left(2 + \frac{1}{2} E\ddot{o}_B\right)^{0.5} \left(\frac{E\ddot{o}_B}{M}\right)^{0.25}}$$

		Bubble Reynolds number Re_B	
M	$E\ddot{o}_B$	wave theory	TURBIT-VOF
$3{,}09 \cdot 10^{-6}$	3,06	59,3	56

Local velocity field (case ❶)

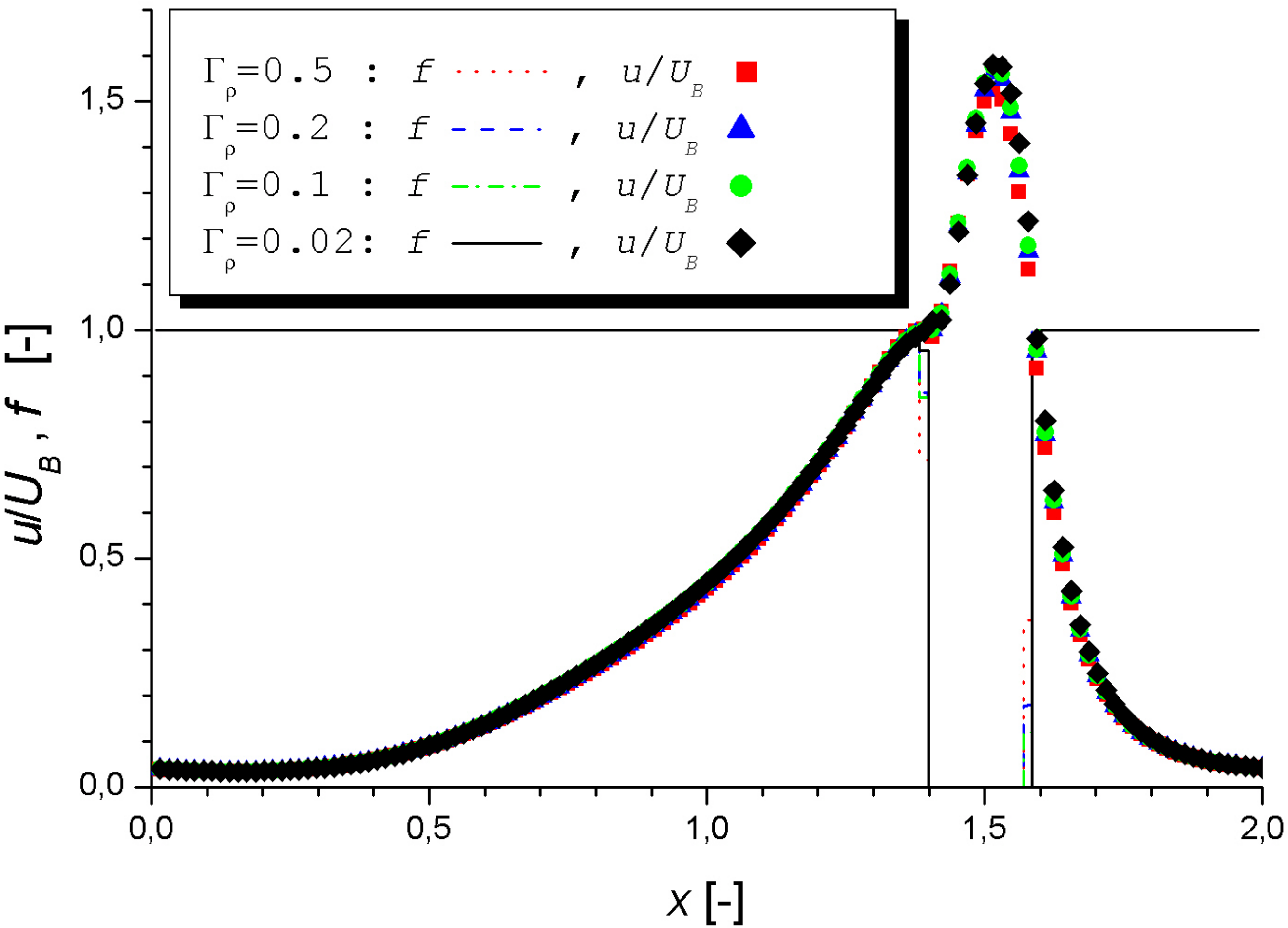
Comparison of profiles for instant t_C where $x_{com}(t_C)=1,5$





Local velocity field (case ❶)

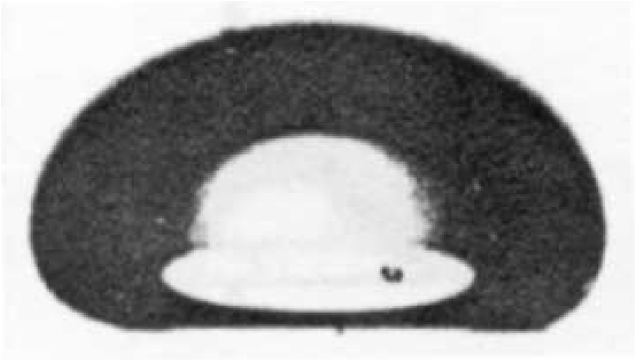
Normalization by respective bubble rise velocity U_B

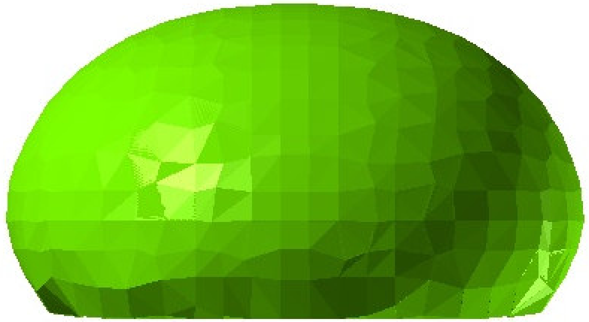


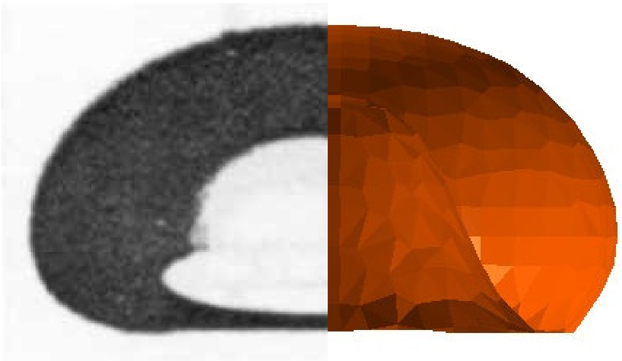
Profiles collapse to a singe curve

Case ❷: Comparison of bubble shape

Experiment Bhaga & Weber* | TURBIT-VOF ($\Gamma_\rho = 0{,}5$)

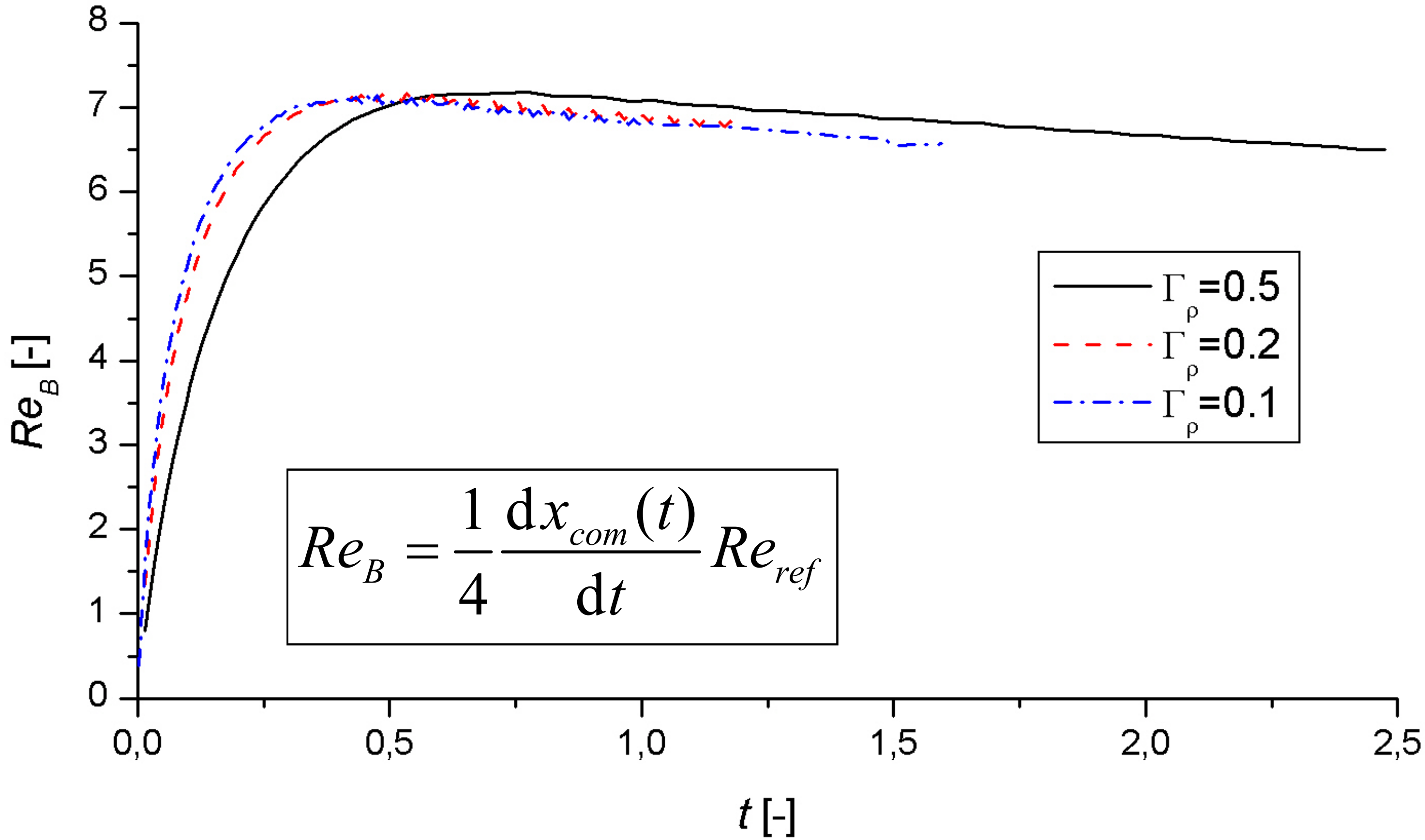






* D. Bhaga, M.E. Weber, J. Fluid Mech. 105:61-85, 1981

Bubble Reynolds number (case ❷)



	Exp. Bhaga & Weber	TURBIT-VOF
Re_B	7,77	6,5

Conclusions

- DNS of ellipsoidal and ellipsoidal cap bubble for fixed values of $E\ddot{o}_B$ and M but different density ratios Γ_ρ
- Bubble acceleration depends on Γ_ρ (added mass force)
- Steady bubble: shape, Reynolds number, and normalized local velocity profile do not depend on Γ_ρ

$\Rightarrow$ DNS results for steady bubbles obtained with $\Gamma_\rho = O(0.1)$ can be transferred to $\Gamma_\rho = O(0.001)$

- *Statistical models for bubble induced turbulence*
  - *formulate models in terms of Eötvös and Morton number*
  - *utilize DNS data for model development and testing*

Outlook

- Experimental investigation of influence of Γ_ρ requires
  - use of a gas-liquid and a liquid-liquid system with same Morton number but different density ratio
  - similarity of Eötvös number can be ensured by appropriate values of equivalent diameter of bubble/drop